

# A Comparative Study of LQR, FLQR, SMC, and STSMC for Self-Balance Control of a Two-Wheel Electric Wheelchair



Sena Teshome Fureta

A Thesis Submitted to the Department of Electrical Power and Control  
Engineering School of Electrical Engineering and Computing

Presented in Partial Fulfillment of the Requirement for the Degree of Master's  
in Electrical Power and Control Engineering (Control)

Office of Graduate Studies  
Adama Science and Technology University

March 2023  
Adama, Ethiopia

# A Comparative Study of LQR, FLQR, SMC, and STSMC for Self-Balance Control of a Two-Wheel Electric Wheelchair

Sena Teshome Fureta

**Advisor:** Dr. Tefera Terefe (Assistant professor)

A Thesis Submitted to the Department of Electrical Power and Control  
Engineering School of Electrical Engineering and Computing

Presented in Partial Fulfillment of the Requirement for the Degree of Master's  
in Electrical Power and Control Engineering (Control)

Office of Graduate Studies  
Adama Science and Technology University

March 2023  
Adama, Ethiopia

## DECLARATION

I hereby declare that this master thesis entitled “A Comparative Study of LQR, FLQR, SMC, and STSMC for Self-Balance Control of a Two-Wheel Electric Wheelchair” is my own work. That is, it has not been submitted for the award of any academic degree, diploma or certificate in any other university. All sources of materials that are used for this thesis have been duly acknowledged through citation.

---

Name of student

---

Signature

---

Date

## **RECOMMENDATION**

I, the advisor of this thesis, hereby certify that I have read the revised version of the thesis entitled “A Comparative Study of LQR, FLQR, SMC, and STSMC for Self-Balance Control of a Two-Wheel Electric Wheelchair” prepared under my guidance by Sena Teshome Fureta submitted in partial fulfillment of the requirements for the degree of Master’s of Science in Electrical Power and Control Engineering, postgraduate in control engineering.

Therefore, I recommend the submission of revised version of the thesis to the department following the applicable procedures.

---

Advisor

---

Signature

---

Date



## APPROVAL PAGE

I the advisors of the thesis entitled “A Comparative Study of LQR, FLQR, SMC, and STSMC for Self-Balance Control of a Two-Wheel Electric Wheelchair” and developed by Sena Teshome Fureta, hereby certify that the recommendation and suggestions made by the board of examiners are appropriately incorporated into the final version of the thesis.

| _____<br>Advisor | _____<br>Signature | _____<br>Date |
|------------------|--------------------|---------------|
|------------------|--------------------|---------------|

We, the undersigned, members of the Board of Examiners of the thesis by Sena Teshome Fureta have read and evaluated the thesis entitled “A Comparative Study of LQR, FLQR, SMC, and STSMC for Self-Balance Control of a Two-Wheel Electric Wheelchair” and examined the candidate during open defence. This is, therefore, to certify that the thesis is accepted for partial fulfillment of the requirement of the degree of Master of Science in control engineering.

|                              |                                                                                     |                   |
|------------------------------|-------------------------------------------------------------------------------------|-------------------|
| _____<br>Chairperson         | _____<br>Signature                                                                  | _____<br>Date     |
| <u>Dr Endalew Ayenew</u>     |  | <u>11/04/2023</u> |
| _____<br>Internal Examiner   | _____<br>Signature                                                                  | _____<br>Date     |
| <u>Dr Beteley Teka (PhD)</u> |  | <u>20/04/2023</u> |
| _____<br>External Examiner   | _____<br>Signature                                                                  | _____<br>Date     |

Finally, approval and acceptance of the thesis is contingent upon submission of its final copy to the Office of Postgraduate Studies (OPGS) through the Department Graduate Council (DGC) and School Graduate Committee (SGC).

|                                               |                    |               |
|-----------------------------------------------|--------------------|---------------|
| _____<br>Department Head                      | _____<br>Signature | _____<br>Date |
| _____<br>School Dean                          | _____<br>Signature | _____<br>Date |
| _____<br>Office of Postgraduate Studies, Dean | _____<br>Signature | _____<br>Date |

## **ACKNOWLEDGEMENTS**

First and foremost, I would like to thank, the Almighty and Merciful, for giving me strength and enabling me to successfully complete my study due to his will.

Second, I would like to thank my advisor, Dr. Tefera Terefe, for his exemplary supervision and advice from the start to the completion of my thesis work.

Third, I would like to express my heartfelt gratitude to my family: my father, mother, sister, and brother, for their prayers and loving support in this difficult journey of the study.

Finally, I would like to thank my friends for their motivation and encouragement.

## ABSTRACT

*In today's world, millions of people are physically disabled and have difficulty moving from one place to another due to disease, accident, and age. A wheelchair is an important device, and it is sometimes the only way for those people to become mobile. A two-wheel wheelchair is less expensive than a three-wheel wheelchair and has better mobility and manoeuvrability than a conventional four-wheel wheelchair. However, this two-wheel wheelchair system is highly nonlinear and unstable, necessitating the use of an active controller. The main aim of the work is to develop an effective controller to balance a two-wheel wheelchair and perform a comparative study between optimal and robust controllers. In this thesis, linear quadratic regulator, fuzzy linear quadratic regulator, sliding mode controller, and super twisting sliding mode controllers were designed, and their performances are compared and evaluated. First, the SIMO nonlinear system dynamics are modelled using Euler-Lagrange equations based on a double inverted pendulum to account for the system's upper body while also expanding the vehicle's workspace and increasing its flexibility. Nonlinearity obviously makes system modelling and control much more complicated. So the Taylor Series expansion was employed to linearize the nonlinear system. Then controllers are designed based on the linearized model to obtain a fast response. To test the effectiveness of the proposed control methods, the controllers were implemented and simulated using MATLAB/Simulink. The response shows that link 1 has a settling time of 1.208sec, 1.485sec, 2.319sec, and 2.895sec with STSMC, SMC, FLQR, and LQR, respectively. Link 2 has a settling time of 1.408sec, 1.631sec, 2.589sec, and 2.947sec with STSMC, SMC, FLQR, and LQR, respectively. And also, the two-wheeled wheelchair with the STSMC controller has attained a 2.72%, 4.8%, and 98.3% fall in distance travelled in comparison with the SMC, FLQR, and LQR controllers, respectively. These transient response performances show that the STSMC outperformed the other proposed controllers.*

**Keywords:** *Two-wheel self-balancing electric wheelchair, Double-link inverted pendulum, Linear quadratic regulator, fuzzy linear quadratic regulator, Sliding mode control (SMC), and super twisting SMC.*

## TABLE OF CONTENTS

| TABLE                                  | PAGE |
|----------------------------------------|------|
| DECLARATION.....                       | i    |
| RECOMMENDATION.....                    | ii   |
| APPROVAL PAGE.....                     | iii  |
| ACKNOWLEDGEMENTS .....                 | iv   |
| ABSTRACT .....                         | v    |
| LIST OF TABLES .....                   | ix   |
| LIST OF FIGURES .....                  | x    |
| LIST OF ABBREVIATIONS .....            | xii  |
| LIST OF SYMBOLS.....                   | xiii |
| CHAPTER 1 .....                        | 1    |
| INTRODUCTION .....                     | 1    |
| 1.1 Background .....                   | 1    |
| 1.2 Statement of the problem.....      | 3    |
| 1.3.1 General objective.....           | 3    |
| 1.3.2 Specific objectives.....         | 3    |
| 1.4 Motivation of the study.....       | 4    |
| 1.5 Significance of the study.....     | 4    |
| 1.6 Scope of the study .....           | 5    |
| 1.7 Limitations of the study .....     | 5    |
| 1.8 Thesis organization .....          | 5    |
| LITERATURE REVIEW .....                | 6    |
| 2.1 Introduction.....                  | 6    |
| 2.2 Self balancing mobile robots ..... | 6    |
| 2.3 Inverted pendulum .....            | 7    |
| 2.3.1 Cart inverted pendulum.....      | 8    |

|                                                              |    |
|--------------------------------------------------------------|----|
| 2.3.2 Rotary inverted pendulum (RIP) .....                   | 8  |
| 2.3.3 Inverted pendulum on wheels .....                      | 9  |
| 2.4 Double inverted pendulum (DIP).....                      | 9  |
| 2.5 Related works .....                                      | 10 |
| CHAPTER 3 .....                                              | 15 |
| METHODOLOGY .....                                            | 15 |
| 3.1 Introduction.....                                        | 15 |
| 3.2 Materials .....                                          | 15 |
| 3.3 Methods .....                                            | 15 |
| 3.4 Self-balancing two-wheel wheelchair system overview..... | 16 |
| 3.5 Block diagram of the system .....                        | 18 |
| 3.6 System modelling .....                                   | 19 |
| 3.7.1 The wheel's energy derivations .....                   | 22 |
| 3.7.2 Link 1 energy derivations.....                         | 23 |
| 3.7.3 Link 2 energy derivations.....                         | 23 |
| 3.8 Linearization of the model .....                         | 27 |
| 3.9 Actuator modelling .....                                 | 28 |
| 3.10 Representation of state space .....                     | 30 |
| 3.11 Controllability and observability .....                 | 33 |
| 3.12 System model analysis.....                              | 34 |
| CHAPTER FOUR .....                                           | 37 |
| CONTROLLER DESIGN .....                                      | 37 |
| 4.1 Introduction.....                                        | 37 |
| 4.2 Sliding mode control.....                                | 37 |
| 4.2.1 Higher order sliding mode controller .....             | 39 |
| 4.3 Design a sliding mode controller for TWEW .....          | 40 |
| 4.3.1 Sliding surface design .....                           | 40 |

|                                                        |    |
|--------------------------------------------------------|----|
| 4.3.2 Control law design .....                         | 44 |
| 4.4 Design STSMC controller .....                      | 45 |
| 4.5 Linear quadratic regulator (LQR) .....             | 46 |
| 4.6 Fuzzy-LQR controller.....                          | 47 |
| 4.6.1 Fusion function.....                             | 48 |
| CHAPTER FIVE .....                                     | 51 |
| RESULTS AND DISCUSSIONS .....                          | 51 |
| 5.1 Introduction.....                                  | 51 |
| 5.2 SMC and STSMC closed loop simulation results ..... | 51 |
| 5.2.1 Test Results for Different Initial Values .....  | 56 |
| 5.2.2 Test results for disturbance .....               | 61 |
| 5.2.3 Test STSMC for weight variation .....            | 63 |
| 5.3 Simulation result using LQR and FLQR .....         | 65 |
| 5.3.1 Results under the influence of disturbance ..... | 66 |
| 5.4 Comparison of controllers .....                    | 67 |
| CHAPTER SIX .....                                      | 72 |
| CONCLUSION AND RECOMMENDATION .....                    | 72 |
| 6.1 Conclusion .....                                   | 72 |
| 6.2 Recommendation .....                               | 73 |
| APPENDIX .....                                         | 80 |
| Appendix A: Simulink block diagrams. ....              | 80 |
| Appendix B: MATLAB script for TWEW. ....               | 82 |

## LIST OF TABLES

|                                                                              |    |
|------------------------------------------------------------------------------|----|
| Table 3.1: Parameters of a DIP (Nurul Fadzlina Jamin, 2016) .....            | 21 |
| Table 3.2: Values of the system parameters (Nurul Fadzlina Jamin, 2016)..... | 32 |
| Table 4.1: Fuzzy rules of membership function.....                           | 49 |
| Table 5.1: Performance of the Controllers for lower pendulum.....            | 69 |
| Table 5.2: Performance of the controllers for upper pendulum.....            | 69 |

## LIST OF FIGURES

|                                                                                                                                                         |    |
|---------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Figure 1.1: Wheelchair system (Aniket Vidyagar, 2022).....                                                                                              | 1  |
| Figure 2.1: Normal non inverted single and double pendulums (George S. Maraslidis, 2022).....                                                           | 7  |
| Figure 2.2: An inverted pendulum on wheels and its inclination with respect to the vertical line (Fadlallah, Khaled M. Goher and Sulaiman O, 2019)..... | 9  |
| Figure 3.1: The work flow of the research .....                                                                                                         | 16 |
| Figure 3.2: Working principle of a self-balancing two-wheel wheelchair (Ngoc Kien Vu and Hong Quang Nguyen, 2021).....                                  | 18 |
| Figure 3.3: Block Diagram of the system.....                                                                                                            | 19 |
| Figure 3.4: Schematic diagram of wheeled DIP system (Nurul Fadzlina Jamin, 2016). ....                                                                  | 20 |
| Figure 3.5: DC motor circuit diagram (Lytigas, 2018).....                                                                                               | 28 |
| Figure 3.6 Lower pendulum angle responses with initial 0.01rad.....                                                                                     | 34 |
| Figure 3.7 Upper pendulum angle responses with initial 0.01rad .....                                                                                    | 34 |
| Figure 3.8 TWEW wheel displacement response.....                                                                                                        | 35 |
| Figure 3.9 Lower pendulum angle responses with initial -0.01rad .....                                                                                   | 35 |
| Figure 3.10 Upper pendulum angle responses with initial -0.01rad.....                                                                                   | 36 |
| Figure 3.11 TWEW wheel displacement response.....                                                                                                       | 36 |
| Figure 4.1: Sliding mode controller graphical representations (Zeb K Uddin W, 2020)....                                                                 | 38 |
| Figure 4.2: Block diagram of sliding mode control (Zhilin Feng, Juntao Fei, 2018).....                                                                  | 39 |
| Figure 4.3: LQR controller block diagram (Rupal Roy, 2021) .....                                                                                        | 47 |
| Figure 4.4: Fuzzy logic controller block diagram (Rupal Roy, 2021) .....                                                                                | 48 |
| Figure 4.5: Block diagram of FLQR (George S. Maraslidis, 2022).....                                                                                     | 49 |
| Figure 4.6: Inputs(e and de) and output membership functions with a triangular shape ....                                                               | 50 |
| Figure 4.7: Surface view of fuzzy logic .....                                                                                                           | 50 |
| Figure 5.1: Wheel linear displacement response using STSMC and SMC with an initial 0m .....                                                             | 52 |
| Figure 5.2: Lower pendulum angular response for SMC and STSMC with an initial 0rad                                                                      | 52 |
| Figure 5.3: Upper pendulum angular response for SMC and STSMC with initial 0rad .....                                                                   | 53 |
| Figure 5.4: Response to angular velocity of link 1 using SMC and STSMC.....                                                                             | 54 |
| Figure 5.6: Control signal using STSMC and SMC.....                                                                                                     | 55 |
| Figure 5.7: Wheel displacement error and angle error of links 1 and 2 using STSMC and SMC.....                                                          | 56 |



|                                                                                                                                                                                                   |    |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----|
| Figure 5.8: System response with initial $\theta_1 = -5\text{rad}$ , $\theta_2 = -5\text{rad}$ , and reference $0\text{rad}$ using SMC and STSMC.....                                             | 57 |
| Figure 5.9: Link 1 angle, Link 2 angle and wheel displacement responses for SMC and STSMC with initial $\theta_1 = 5\text{ rad}$ , $\theta_2 = 5\text{ rad}$ , and reference $0\text{ rad}$ ..... | 58 |
| Figure 5.10: Link 1 angle, Link 2 angle and wheel displacement responses for SMC and STSMC with initial $\theta_1 = -5\text{rad}$ , $\theta_2 = 5\text{rad}$ and reference $0\text{rad}$ .....    | 59 |
| Figure 5.11: Link 1 angle, Link 2 angle and wheel displacement responses for SMC and STSMC with initial $\theta_1 = 5\text{rad}$ , $\theta_2 = -5\text{rad}$ and reference $0\text{rad}$ .....    | 60 |
| Figure 5.12: System perturbation from outside sources .....                                                                                                                                       | 61 |
| Figure 5.13: Responses of the wheel displacement, link 1 and link 2 to an external disturbance with initial $\theta_1 = 2\text{rad}$ and $\theta_2 = 1\text{rad}$ .....                           | 62 |
| Figure 5.14: Wheel displacement response for 60 kg to 90 kg of human weight variation                                                                                                             | 63 |
| Figure 5.15: Link 1 angular response for human weight variation ranging from 60kg to 90kg with initial $\theta_1 = 0.5\text{rad}$ .....                                                           | 63 |
| Figure 5.16: Link 1 angular response for human weight variation ranging from 60kg to 90kg with initial $\theta_2 = 1.5\text{rad}$ .....                                                           | 64 |
| Figure 5.17: A wheel's linear displacement, link 1 and link 2 angular responses for LQR and FLQR .....                                                                                            | 65 |
| Figure 5.18: A wheel's linear displacement and links 1 and 2's angular responses for LQR and FLQR under the influence of disturbance.....                                                         | 66 |
| Figure 5.19: Control signal of FLQR and LQR under external disturbance .....                                                                                                                      | 67 |
| Figure 5.20: wheel displacement responses for all controllers.....                                                                                                                                | 68 |
| Figure 5.21: Link 1 angular responses for all controllerss .....                                                                                                                                  | 68 |
| Figure 5.22: Link 2 angular responses for all controllers.....                                                                                                                                    | 69 |
| Figure 5.23: Response to angular velocity of link 1 for all controllers .....                                                                                                                     | 70 |
| Figure 5.24: Response to angular velocity of link 2 for all controllers .....                                                                                                                     | 71 |
| Figure 5.25: Input signal for all controllers .....                                                                                                                                               | 71 |
| Figure A.1: Simulink block of TWEW model and Simulink of overall system .....                                                                                                                     | 80 |
| Figure A.4: Simulink of Fuzzy LQR .....                                                                                                                                                           | 82 |

## LIST OF ABBREVIATIONS

|        |                                       |
|--------|---------------------------------------|
| BSC    | Back stepping controller              |
| DIP    | Double inverted pendulum              |
| DOF    | Degree of freedom                     |
| FLC    | Fuzzy logic controller                |
| FLQR   | Fuzzy linear quadratic regulator      |
| GA     | Genetic Algorithm                     |
| HOSMC  | Higher Order Sliding Mode Control     |
| IP     | Inverted pendulum                     |
| LQR    | Linear quadratic regulator            |
| LQG    | Linear quadratic Gaussian             |
| MATLAB | Matrix Laboratory                     |
| MIMO   | Multiple input multiple output        |
| PD     | Proportional Derivative               |
| PID    | Proportional Integral and Derivative  |
| PSO    | Particle swarm optimization           |
| SIMO   | Single input multiple output          |
| SISO   | Single input single output            |
| SMC    | Sliding mode controller               |
| SOSMC  | Second order sliding mode controller  |
| STSMC  | Super twisted Sliding mode controller |
| TWEW   | Two-wheel electric wheelchair         |
| VSS    | Variable structure systems            |
| WDIPS  | Wheeled double inverted pendulum      |

## LIST OF SYMBOLS

|            |                                         |
|------------|-----------------------------------------|
| $C_e$      | Back emf constant                       |
| $C_T$      | Torque constant                         |
| $e$        | Error                                   |
| $E$        | Back emf voltage                        |
| $i$        | Current through armature                |
| $g$        | Acceleration due to gravity             |
| $J_w$      | Wheel moment of inertia                 |
| $J_L, J_R$ | Left and right wheel moments of inertia |
| $J_1, J_2$ | Link 1 and link 2 moment of inertia     |
| $KE$       | Kinetic energy                          |
| $KE_T$     | Total kinetic energy                    |
| $KP$       | Potential energy                        |
| $KP_T$     | Total potential energy                  |
| $L$        | Lagrangian function                     |
| $h_1, h_2$ | Length of link 1 and link 2             |
| $m_1, m_2$ | Mass of the link 1 and link 2           |
| $T_i$      | Generalized force                       |
| $M(p)$     | Inertia matrix                          |
| $p_i$      | Generalized coordinate                  |
| $R$        | Nominal terminal resistance             |
| $r$        | Radius of the wheel                     |
| $\tau$     | Motor torque                            |
| $u$        | Control input                           |
| $u_{eq}$   | Equivalent control                      |

|                                 |                            |
|---------------------------------|----------------------------|
| $u_{sw}$                        | Switching control          |
| $V$                             | Applied terminal voltage   |
| $x_w$                           | Wheel distance             |
| $b\dot{\theta}(\dot{\theta}_w)$ | Angular speed of the motor |
| $\theta_1, \theta_2$            | Link 1 and link 2 angle    |
| $\dot{\theta}$                  | Angular velocity           |
| $\ddot{\theta}$                 | Angular acceleration       |

# CHAPTER 1

## INTRODUCTION

### 1.1 Background

In this world, millions of people live with disabilities due to disease, accidents, and ageing (Swostik Sourav Dash, Javeed Shaikh-Mohammed, Vivek Sarda, & S. Sujatha, 2021). Environmental factors such as traffic accidents, natural disasters, and wars all contribute to the rise in disability. Ethiopia had approximately 15 million disabled people in 2003, accounting for 17.6 percent of the total population. Actually, there are several forms of disabilities; mobility, visual, and hearing impairments have been identified as the three most common forms of disability in Ethiopia (Solomon Mekonnen Abebe, 2020). Mobility disabilities are physical impairments that limit or prevent independent movement or the full use of body parts. A wheelchair is a mobility aid for people with walking disabilities. Mobility allows wheelchair users to work, study, participate in social activities, and access services such as healthcare. A wheelchair not only provides mobility, but it also improves quality of life and physical health by reducing common problems like pressure, improving respiration and digestion, and reducing the burden on care givers and family members (Kurczewski P, Gabryelski J, Sydor M, Szperling A, Torzynski D, Zablocki M, 2021) (Swostik Sourav Dash, Javeed Shaikh-Mohammed, Vivek Sarda, & S. Sujatha, 2021).



Figure 1.1: Wheelchair system (Aniket Vidyagar, 2022).

Wheelchairs are mainly classified into two types: manual and electric (Sateesh Reddy Avutu, Sudip Paul B, and Venkateswara Reddy, 2020). Manual wheelchairs are typically less expensive, lighter, and easier to transport. Manual wheelchairs contain fewer parts than powered wheelchairs and thus require less maintenance or care. Even though there are numerous advantages to using a manual wheelchair, it is only possible for people who still have enough upper-body strength to propel their chair by themselves. And also, the repetitive motion of pushing the manual wheelchair hand-rims can quickly fatigue the user and lead to long-term health issues (Mostafa Nikpour, 2020).

The other type of wheelchair, Electric wheelchairs are propelled by electric motors and provide mobility for those who do not have the strength to drive their chairs due to illness, weakness in the upper body, or other factors. When compared to a manual wheelchair, these wheelchairs can significantly reduce shoulder stress, provide a variety of controls, and also allow the user to travel longer distances with less fatigue.

The basic traditional electric wheelchair configuration is built in a traditional four-wheeled style. Although a traditional wheelchair enables mobility, because of its structure, it has several disadvantages. These disadvantages include difficulties such as passing through doorways, avoiding obstacles and sidewalks, and the bulky nature, which complicates things. This issue can be solved by reducing the structure to a two-wheel system, which improves performance and has many advantages, including a zero-turn radius (tight turns), a simpler configuration design, a smaller footprint, cost reduction, and less friction (Mostafa Nikpour, 2020). All these advantages would add to the capabilities of the two-wheeled systems, which could solve numerous challenges and allow easier manoeuvrability in confined and limited travel spaces due to the vehicles' ability to make sharp turns. Such a two-wheel vehicle can be applicable for indoor and outdoor transportation and mobility solutions.

Researchers' interest in two-wheeled robot research has grown in recent years. This is due to the unstable nature of the systems that serve as a benchmark for testing and developing control techniques (Mostafa Nikpour, 2020). The operation of this two-wheel device is based on the IP concept, which is considered to be highly nonlinear and underactuated. Due to the two-wheeled balancing robot's need for an appropriate controller to keep it upright without the aid of an outside force, it is necessary to create a robot that balances on two wheels with a good platform and investigate the effectiveness of different controllers

using an IP model. Different controllers, such as pole placement, a quadratic linear regulator, a fuzzy logic controller, and a PID controller, are currently being used on the two-wheel balancing robots. The robot's wheels can be continuously moved in the direction that it is falling to maintain balance during this sluggish fall.

The goal of this research is to enhance the quality of life for those with disabilities who rely on wheelchairs as their primary mode of transportation and are unable to stand on their own due to permanent injuries to their lower extremities by modelling and simulating a two-wheel self-balancing wheelchair. The design of the proposed two-wheel wheelchair is based on a DIP that can offer more flexibility and manoeuvrability.

## **1.2 Statement of the problem**

A two-wheel wheelchair operates in a similar manner, with a double inverted pendulum. A DIP is an unstable and highly nonlinear system, so controlling the stability requires designing and implementing active control methods for a two-wheeled wheelchair to maintain stability. Many researchers who worked on DIP systems applied different control algorithms. But their control algorithms failed to effectively control the DIP system due to its unstable and highly nonlinear characteristics, and yet many researchers still struggle to choose an effective control approach. In this research, four controllers are designed, and their performances are compared.

## **1.3 Objectives of the study**

### **1.3.1 General objective**

The general objective of this thesis is to study the comparative study of LQR, FLQR, SMC, and STSMC for self-balance control of a two-wheel wheelchair.

### **1.3.2 Specific objectives**

The specific objectives of the thesis are:

- ✓ To derive a mathematical model for a two-wheel wheelchair based on the double inverted pendulum concept.
- ✓ To design LQR, FLQR, SMC, and Super Twisting SMC controllers for the designed model of a two-wheel wheelchair.
- ✓ To simulate and analyse the designed controller using MATLAB/Simulink for verification.
- ✓ To compare and evaluate the robustness and performance of controllers.

## **1.4 Motivation of the study**

A wheelchair is not just a product. It is about enabling physically disabled people to stay healthy, become mobile, work, and live an independent life. We frequently take our ability to move around our homes and communities for granted, but for those who have mobility issues, even the smallest move can prevent them from accessing all aspects of their lives. Being mobile improves a person's ability to learn, participate in the community, interact with others, and contribute to the development of their country. Although a wheelchair can help people become more independent and integrate into their communities, it is not an end in itself. Based on size, comfort, and flexibility, wheelchairs have a significant impact on the level of participation that people with disabilities can achieve. When provided with good quality, they have been able to reduce the time and physical burden for the user and create many opportunities, such as education, work, etc. Therefore, controlling the two wheel wheelchairs can reduce healthcare costs and economic vulnerability while increasing productivity and quality of life.

## **1.5 Significance of the study**

A wheelchair is a critical device that provides mobility assistance for those who have mobility difficulties due to a variety of health conditions and impairments, such as poliomyelitis, muscular dystrophy, spinal cord injury, and stroke, and they are also relevant for older people who have mobility difficulties. The wheelchair not only improves the lives of disabled people but also reduces the burden on caregivers, family members, and the country.

The design and assembly of a wheelchair have a significant effect on the life quality and mobility of an individual. The two-wheeled wheelchair has a smaller footprint than the three- or four-wheeled wheelchair, allowing it to move much faster through corridors and around tight corners. These two-wheel balancing wheelchair capabilities are capable of resolving many societal problems. For this reason, controlling and maintaining the balance of a two-wheeled electric wheelchair means providing a high-quality, inexpensive electric wheelchair to the user.



## **1.6 Scope of the study**

The thesis's scope is driving mathematical modelling of a two-wheel wheelchair based on a DIP and controlling the balance of the system using LQR, FLQR, SMC, and Super Twisting SMC.

## **1.7 Limitations of the study**

Preparing the prototype was extremely difficult due to the limited time I had, the complexity of the system, and the cost of the materials used in the research. As a result, the scope of this thesis work is limited to simulation using the MATLAB/Simulink software.

## **1.8 Thesis organization**

In general, this thesis is organized into six chapters,

Chapter one provides a brief overview of the system, including an introduction, problem statement, thesis objective, motivation of the study, thesis significance, thesis scope, and, finally, thesis organization.

Chapter Two discusses the literature review, which explains previous works on the two-wheeled vehicle and the inverted pendulum. In addition, the concepts used in balancing the inverted pendulums are drawn from previous literature.

Chapter three includes the methodology, which elaborates on the procedure applied in designing the overall system. The derivation of the system's mathematical representation using the Lagrangian method is covered in this chapter.

Chapter four Design the control strategies LQR, FLQR, SMC, and STSMC for the dynamic model of the two-wheel wheelchair.

Chapter five discusses the simulation results of the final system and analyses the results.

Chapter six discusses the conclusions drawn and the recommendation for further work on improving the system.

# CHAPTER TWO

## LITERATURE REVIEW

### 2.1 Introduction

The first powered wheelchair was created in 1665 (Glenn Ruscoe, 2019). In the last two decades, it has seen the most rapid development in wheelchair technologies when researchers began to use technologies developed originally for mobile robots to start creating modern electric-powered wheelchairs or smart wheelchairs (K. Maatoug, M. Njah, and M. Jallouli, 2019).

This chapter conducts a literature review to review the relevant work for the proposed research. Examining the various designs of two-wheeled vehicles, as well as the methodologies employed in their control systems, is essential for understanding the concept more and identifying gaps in the literature. The system being considered has several variations, including a mobile robot, an IP on a cart, a rotational IP, and an IP system on wheels. Furthermore, IP systems with multiple links, such as the DIP system, have been presented.

### 2.2 Self balancing mobile robots

Robots are used in the armed forces, industries, and have been developed for use in a variety of environments such as the home or office, where they serve as security robots, care robots, service robots, vacuum cleaners, and hospitals such as a motorised wheelchair (Hla Myo Tun, Maung Maung Latt, Myat Su Nwe, Zaw Min Naing, Devasis Pradhan, Prasanna Kumar Sahu, 2022). Mobile robots are divided into different categories based on their travel environment (aerial robots, land robots, and underwater robots) and based on mode of locomotion (legged and wheeled) (Mobile Robots, 2022).

**Arial robots:** The term "unmanned aerial vehicle" can also refer to aerial robots (UAVs). They have the ability to fly. For instance, drones, helicopters, etc.

**Land Robots:** These robots can move around on land and inside buildings like homes and offices.

**Underwater robots:** These machines, such as submarines, can navigate themselves and move swiftly through the water.

**Legged robots:** use articulated limbs similar to legs and move by following a mechanism.

**Wheeled robots:** These robots move by rolling on wheels. One-wheeled robots, two-wheeled robots, and various other wheeled robot types are among the many available.

Two-wheeled robots have a significant advantage over other types of robots because they are much faster and can change direction much more quickly while moving. Furthermore, it makes them useful in a variety of real-world applications. The two-wheeled balancing robots, Ninebot and Segway, are becoming increasingly popular in the category of wheeled robots. Since the start of the twentieth century, researchers have been working on self-balancing robots. Different types and forms of balancing robots are used in a variety of applications and environments. A self-balancing robot was created to balance upright and perform forward and rotational motions. The capacity to balance on coaxial wheels and have a zero-radius rotation distinguishes two-wheeled balancing robots from other multi-wheeled robots (Tran Van Thien, Vo Ba Viet Nghia, Nguyen Ngoc Son, & Mai Thang Long, 2022). The inverted pendulum idea is used to build self-balancing robots with two wheels. Two angle-measuring sensors were employed to determine the orientation of the robot, and encoder-equipped motors were used to drive the device. The two-wheeled balancing robots are found to be more unstable versions of the standard IP on a cart.

### 2.3 Inverted pendulum

A weight suspended from a pivot, allowing it to freely swing, is referred to as a pendulum. The centre of mass of a normal pendulum is below its pivot point.

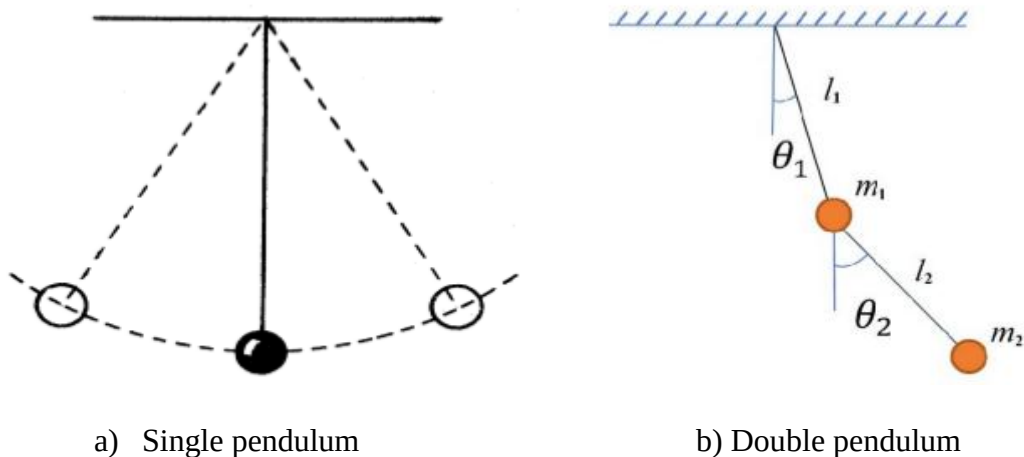


Figure 2.1: Normal non inverted single and double pendulums (George S. Maraslidis, 2022).

An IP is a pendulum with a link that spins above its pivot point (Ibrahim Mustafa Mehedi, Ubaid M. Al-Saggaf, Rachid Mansouri, and Maamar Bettayeb, 2019). It is a highly nonlinear and naturally unstable class of problems that will collapse unless supported. It has to be properly balanced to remain upright; this may be performed by providing torque at the pivot point, shifting the pivot point horizontally as part of a feedback system, and adjusting the rate of rotation of a mass mounted on the pendulum.

The fascination with IP problems stems from their fundamental similarities to a wide range of practical systems, such as balancing a stick on the palm of one's hand, mimicking the human gait, balancing a rocket launcher, and the vertical motion of the human arm (Dang Anh Duy, Le Hoai Phuong, & Vo Duy Cong, 2021). The IP is an underactuated system because the number of degrees of freedom exceeds the number of actuators.

The IP has been developed in several varieties, each representing a different type of real-world system and challenge. IP can be mainly divided into three types: IP on the cart, IP on the rotary, and IP on the wheel (N. A. A. Razali, N. M. Abdul Ghani, N. F. Jamin, and M. F. Masrom, 2018). The majority of the inverted pendulums used are cart-driven. Based on the number of pendulum links, IP systems are also classified as single, double, triple, or quadruple. A single IP is the simplest version of an IP. For greater degrees of freedom (double, triple, and quadruple), inverted pendulums are classified as series IP and parallel IP.

### **2.3.1 Cart inverted pendulum**

The cart IP, also referred to as the pole IP, is the most common configuration presented in dynamic and control systems textbooks. The system is unstable and underactuated. A cart is used to balance the system. Because of the length of the cart, this approach has limits.

### **2.3.2 Rotary inverted pendulum (RIP)**

One of the benchmark problems for many industrial problems in the control system has been the rotary IP system. It consists of a horizontally rotating driven arm and a vertically rotating pendulum linked to the end of the base arm. Compared to cart inverted pendulum modelling and control design, rotary inverted pendulums are slightly more involved and challenging. There are a variety of uses of the RIP system in the practical engineering field, such as rocket stabilisation during take-off, pick-and-place, and so on. Because of its nonlinearity, the RIP is a captivating subject in the control system, making it a suitable

subject for the teaching and practical demonstration of modern control techniques (Kashika Chhabra and Mohd. Rihan, 2017).

### 2.3.3 Inverted pendulum on wheels

As depicted in Figure 2.2 below, a wheeled IP consists of a base platform with a wheel mounted on it. To move the base and balance the system, the wheels are driven forward and backward. When the base or a portion of the base tilts in one direction, the controller accelerates the wheels in that direction to bring the vehicle upright. The wheels are rotated at different velocities to turn the base.

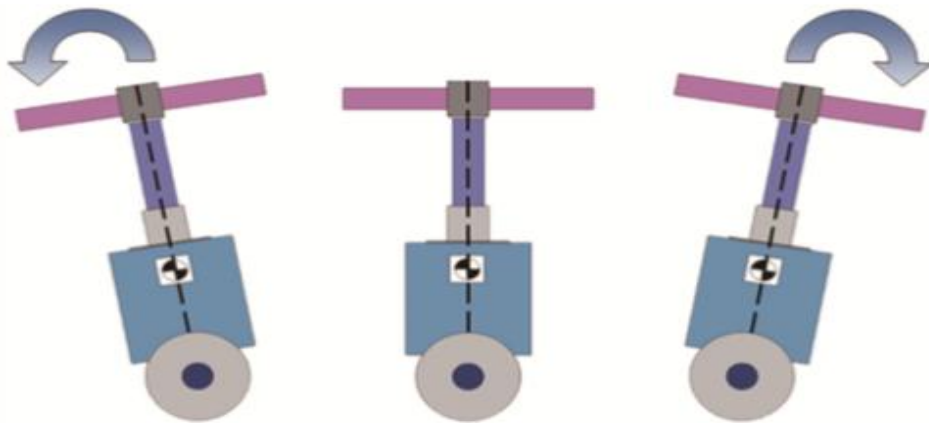


Figure 2.2: An inverted pendulum on wheels and its inclination with respect to the vertical line (Fadlallah, Khaled M. Goher and Sulaiman O, 2019)

## 2.4 Double inverted pendulum (DIP)

The DIP is an enlarged version of a single IP. It adds a pendulum to the single-pendulum system. As a result, the DIP is a highly nonlinear and very unstable system when compared to a single IP. This makes modelling and stabilising the DIP very difficult.

The various characteristics that make the DIP control challenging are as follows (George S. Maraslidis, 2022):

- Highly nonlinear: The dynamic equations of the IP consist of non-linear terms.
- Highly Unstable: The inverted position is the point of unstable equilibrium.
- Under-actuated Mechanical System: The system has fewer actuators than degrees of freedom. The system becomes more cost-effective as a result, but the control problem becomes more difficult.

- Chaotic: tiny changes have a big effect.

The DIP has been widely used as a representative model in stability computation (Ayankoso, M. K. Habib, and S. A., 2020). Different control approaches for the DIP system have been developed by researchers. A DIP can be controlled in two ways: by moving the base and applying torque at the pivot point.

## 2.5 Related works

Various modelling and control mechanisms for balance control of two-wheel robotic manipulators have been presented in various studies. This section reviews studies related to the balance control of two-wheel vehicles using various controllers.

In (Jumi Bharali and Mrinal Buragohain, 2016), three controllers are compared for the stability control of a 3-DOF quarter-car model. The three controllers proposed in this study to stabilise unstable systems are PID, LQR, and fuzzy control theory. According to the findings of this study, LQR is the best controller in terms of cost, but fuzzy logic is the best in terms of settling time.

In (Normaniha Abdul Ghani and Nurul Fadzlina Jamin, 2016), a two-wheeled wheelchair control strategy is proposed. The system is mathematically derived using the Euler-Lagrange method by mimicking the DIP concept. The model was tested using a FLC controller, and the system was found to be stable in an upright position with a good simulation result. The results show that when combined with PSO, the system performs better in terms of settling time, rise time, peak overshoot, and peak undershoot.

In (Mostafa Nikpour, 2020), the researcher investigated a two-wheeled wheelchair's direction control and stability control based on a single-link inverted pendulum and a movable payload mechanism placed under the seat. The model is built using the Euler-Lagrange formulation. The stability and direction of a TWRW are then controlled using a robust second-order sliding mode control. Results from the simulation are shown to validate the proposed method's effectiveness using sliding-mode control under various disturbances, model uncertainties, and rider motion.

(Daniel Gosaye Tesfaye, 2021) designed a two-wheel scooter based on a single inverted pendulum concept to control the system's balance and direction. First, the mathematical model of the scooter is derived using Newton's second law of motion. For balance control,

a super-twisted SMC is used, and for direction control, a PID is used. The system is then simulated in MATLAB/Simulink, and the STSMC and PID parameters are tuned using grey-wolf optimization. The simulation results reveal that the scooter balancing and direction control work flawlessly in the presence of uncertainty and for varying mass variations.

In (Pranav Sevekari, Bhagyashri Tamhane, and Shailaja Kurode, 2020) a higher-order SMC is designed to control the stability of a two-wheeled human transporter (TWHT) system. A reduced-order model is being considered for control development. The controller's effectiveness is compared and examined in the simulation with that of a linear state feedback controller (SFC) to demonstrate its efficacy. The robustness of HOSMC is tested by varying the weight of the rider.

(A. Taifour Ali, 2021) Control a two-wheeled, self-balancing robot using an Arduino PID controller in the presence of external disturbances. The mathematical model for the robot was developed using physics laws and Laplace transforms, and it was then simulated using MATLAB/simulink. By using serial communication to connect the Arduino to MATLAB, a real-time response was plotted. Arduino was used for smoothly controlling the robot. The result is evaluated by manually tuning the PID parameters one by one. However, there was a considerable overshoot and robot oscillation observed when the vehicle moved; this did not ensure the user's comfort.

In (Swati Singh and Akhilesh Swarup, 2021), the performance of a SMC is compared to that of a GA-based LQR for a rotary DIP system. The sliding hyper-plane was created with regular form and linear-quadratic minimization. A settling time comparison is obtained for a performance analysis based on the performances of both controllers. Under nominal conditions, the outcomes obtained by GA-LQR and SMC are nearly identical; however, in the presence of disturbance and parametric uncertainty, SMC outperformed GA-LQR. The biggest limitation of this work is that, despite the use of a sigmoid function to reduce chattering, the chattering effect is not significantly reduced.

(Naoki Tanohata and Hirokazu Seki, 2019) investigated a neural network learning system with human characteristics for driving control of an electric wheelchair. The proposed system could learn human characteristics like unbalanced right and left input power and design the optimal distribution ratio of assisted torque and assistance proportion.

(Vinod Kumar P1 and Dr. Kamala, 2022) also present the design, and balancing control of a two-wheeled robot (TWSBR) using three distinct techniques. PID (proportional-integral-derivative), LQR (linear-quadratic regulator), and ANN (artificial neural network) controllers are used to control the angle and position of the TWSBR. ANN models have shown more impressive results for controlling two-wheeled robots in disturbance cases with different perturbation signals, but controlling with a neural network takes a long time to train, requires a large number of memories, and can occasionally fall into a local optimum.

(Thomas, Keerthi Prakash, and Koshy, 2016) discusses the investigation of different controllers on a TWSBR. The TWSBR works on the inverted pendulum concept. Position and angle control were performed using PID, LQR, and LQG controls. Although the PID controller is very simple to implement, the PID controller design cannot be implemented in a real-world physical environment. A linear quadratic regulator was designed and produces good results. A linear quadratic Gaussian controller was also created and outperforms the other two. The LQG controller is a Kalman filter combined with a regular LQR controller. Sadly, the LQG design lacks some of the desirable LQR problem properties. whereas LQR is easier to implement and less expensive.

(Yusie Rizal, Muhammad Wahyu, Imansyah Noor, Joni Riadi, Feriyadi, and Ronny Mantala, 2021) The purpose of this research is to evaluate how well the adaptive super-twisting controller performs in stabilising the cart-pole IP system compared with other control methods. To address the coupled inputs for control in the system model, the suggested controller was constructed using the decoupling algorithm approach. The proposed stabilising controller was then compared to first-order sliding mode control (FOSMC) and the super-twisting algorithm (STA) in Matlab/Simulink simulations and realistic computer simulations. Three different scenarios were considered in the Matlab/Simulink simulation: a nominal system, a system with uncertainty, and a disturbed system. Under the same conditions, the ASTA and STA controllers performed similarly; they both had less chattering and faster convergence than the FOSMC approach. However, the FOSMC approach delivered the least energy and had the fewest errors.

(Suppachai Howimanporn, 2014) The performances of PID, LQR, and SMC in balancing the X-Y planar IP are compared in this study. The settling times of the three controllers are comparable. The simulation results show that SMC outperforms the others in terms of



robustness. PID, on the other hand, causes a high degree of fluctuation when subjected to disturbance. The disturbance can even cause the system to fail for the LQR controller. The PSO was used to optimise the SMC and LQR parameters, which improved the controller's efficacy and determined the optimal gain. The results of the PSO-based LQR show that it outperforms the conventional LQR. The simulation and experimental results indicate that the SMC with PSO can control the inverted pendulum with better performance in terms of response time, robustness, and fluctuation.

(Tefera T. Yetayew and Daniel G. Tesfaye, 2022) STSMC is used for scooter balancing, and a PID controller is used for direction control (turn left and turn right). The STSMC and PID parameters are tuned using GA, and the controllers' performance is evaluated using MATLAB/Simulink. According to the simulation outcomes, the steady-state error for both GA-tuned and analytically tuned controllers is nearly zero. However, when compared to system response with analytically tuned controllers, GA-tuned parameters provide better convergence to zero and faster dynamic response (settling time).

Taking into account the characteristics and ideas presented in the literature, this study concentrated on a comparable unstable mobile system. This study designed a two-wheel, self-balancing electric wheelchair based on the DIP concept and compared four control strategies to control the stability of a two-wheel wheelchair.

- As presented in the above literature, several investigations have been carried out on inverted pendulums on two wheels for human transportation devices, such as wheelchairs and assistance robots. According to those studies, the researchers have focused on the single-link inverted pendulum on two wheels, which is quite simple but has limited flexibility. Because the entire body is regarded as a single point of mass, a single-link IP does not consider the movement of the upper body. In this study, the proposed two-wheeled wheelchair is based on the DIP concept, which will allow us to consider the upper body movement and will provide greater flexibility and advantages for several functions by increasing the degree of freedom (DOF). The challenge is modelling and controlling the system.
- There are a number of factors that influence wheelchair stability, such as wheel weight, wheel width, seat position, and the weight of the person. Therefore, it necessitates the use of an active controller. Many researchers are still struggling to choose the best controller. In this thesis, a LQR, SMC, FLQR, and STSMC for two self-balancing

wheel chairs are proposed and compared. LQR is one of the most commonly used optimal controllers for such complex systems. As stated in the literature, its control efforts and costs are relatively lower. Because the LQR is an optimal control with certain constraints, the resulting controller usually does not ensure robustness. Optimality does not always guarantee performance. LQR controller performance is based on waiting matrix  $Q$  and  $R$  selection. In this thesis, to adjust the closed-loop controller feedback gains, fuzzy controllers were used in combination with the LQR. On the other hand, a SMC is a robust controller for unstable, highly nonlinear systems. It has characteristics such as quick dynamic response, insensitivity to changes in plant parameters, and resistance to external disturbance. However, this controller has an issue known as chattering. A HOSMC is used to solve the problem of traditional SMC.

## **CHAPTER 3**

### **METHODOLOGY**

#### **3.1 Introduction**

This chapter starts with a discussion of the materials and procedures used to complete the thesis, followed by an overview of the system. Then, using the DIP concept, the mathematical model of the plant was derived.

#### **3.2 Materials**

The materials required for the implementation of this thesis work are appropriate software such as MATLAB/Simulink, MathType 6.0 Equations, and Microsoft Office. MATLAB is an extremely sophisticated programming language used in technical computing that contains a set of toolboxes and Simulink. It combines arithmetic operations, visual analytics, and programming. The thesis documentation is written and edited in Microsoft Word. MathType is used for writing mathematical formulas and equations in Microsoft Office Word.

#### **3.3 Methods**

The study of this research began with the identification of the problem, followed by title selection. To solve the identified problem, the research must necessarily be based on what has already been carried out by other scholars on this selected topic. Therefore, it is crucial to examine the work of these scientists and obtain the necessary and important information on the subject. Therefore, the literature review includes reading books, articles, simulations, and searching the internet for an electric wheelchair. Based on the collected data, the two-wheel electric wheelchair is mathematically modelled. The completed model was developed by applying the DC motor dynamical model to the mathematical model. The nonlinear model derived from the Euler-Lagrange equation of motion is then linearized about its upright position. After that, the model is converted to state space format. Control algorithms for system stabilisation are then designed using the TWDIP system's dynamic model. After designing the system model and controller, the simulation was performed using MATLAB/Simulink software. Finally, all the work is completed with relevant conclusions and recommendations for future work.

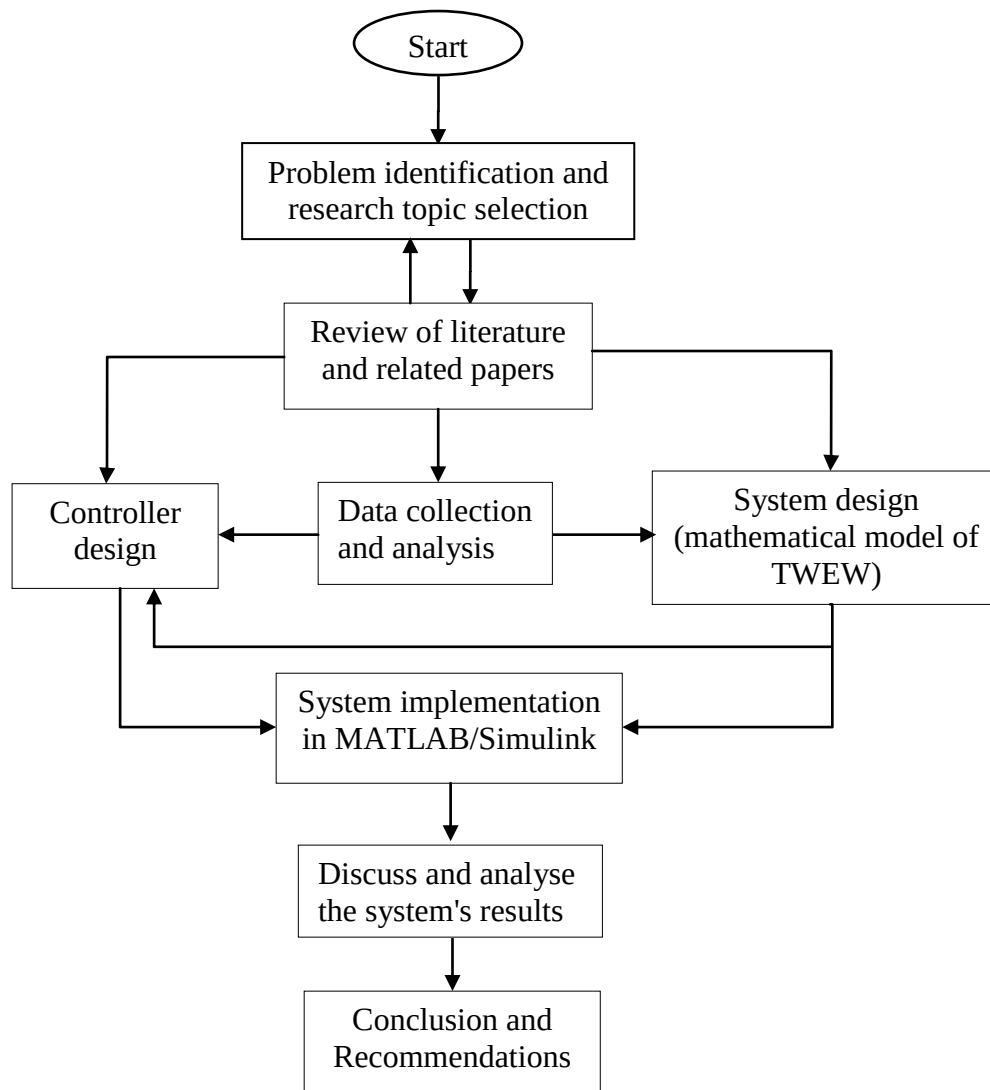


Figure 3.1: The work flow of the research

### 3.4 Self-balancing two-wheel wheelchair system overview

A self-balancing wheelchair can support a person or wheelchair occupant in both sitting and standing positions. The two-wheeled wheelchair proposed in this work consists of two wheels, a chassis, an inverted pendulum, DC motors, and a control unit. A pair of wheels are supported by the chassis and an inverted pendulum. DC motors are used to drive the systems by rotating the systems' wheels, which also maintain the stability of the system using inverted pendulum theory (Ngoc Kien Vu and Hong Quang Nguyen, 2021).

The inverted pendulum issue is similar to trying to keep a broom or a pole upright in our palm. When our eyes observe the pole dropping to one side, they transmit this information

to the brain, which does certain computations and then orders our arm to move to a certain location with a specific velocity to counterbalance the pole's movement. This process is done hundreds of times to keep the pole entirely under our control (KousheekC, 2022). The inverted pendulum operates with a similar approach.

The proposed two-wheel electric wheelchairs work using the DIP concept. The goal is to balance an IP upside-down on a free-moving wheel. In place of eyes, a sensor detects the inclination of the pendulum and provides the signal to a controller, which undertakes specific computations and sends the signal to actuators to rotate the wheel in the direction of fall in such a manner that the pendulum becomes upright again.

The primary purpose of adopting the inverted pendulum theory is to keep the wheels of the wheelchair beneath the centre of mass of the chassis. However, the higher centre of mass creates a larger moment of inertia, which reduces the rate at which the wheelchair falls (Natsuki Sado and Yasuo Kawakami, 2022). This slow fall can be avoided by moving the wheelchair's wheels in the direction of the fall. When the wheelchair tilts forward to maintain stability and balance, the wheels move in the forward direction, and when it tilts backward, the wheels move in the backward direction.

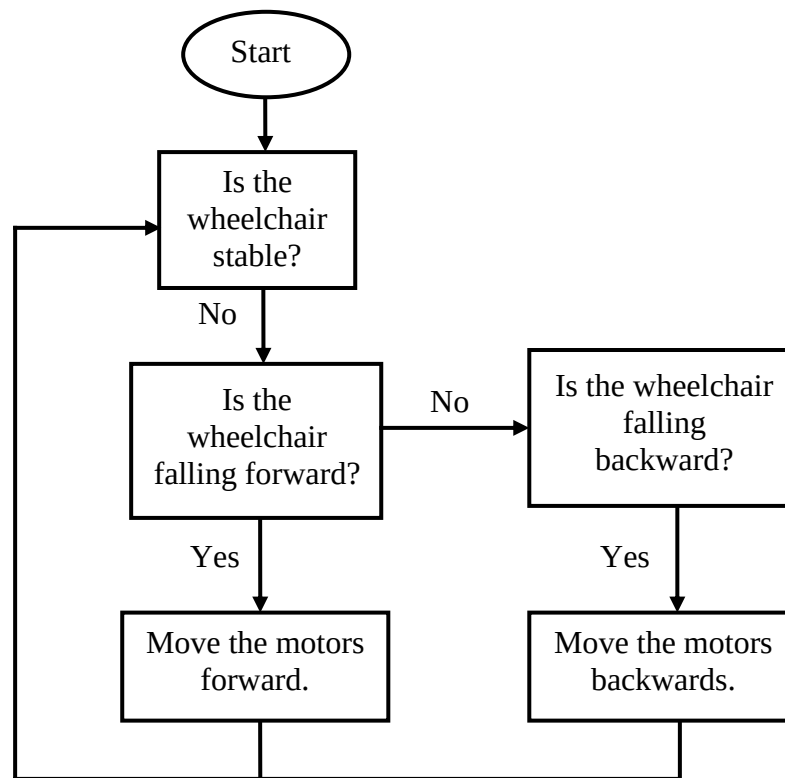


Figure 3.2: Working principle of a self-balancing two-wheel wheelchair (Ngoc Kien Vu and Hong Quang Nguyen, 2021).

### 3.5 Block diagram of the system

A two-wheeled wheelchair is a SIMO nonlinear and unstable system. These wheelchairs are based on a special electromechanical system because they are designed to self-balance on a pair of wheels while the user is standing tall. Unless it is self-balanced, it may roll around its rotational axis. According to the inverted pendulum theory, the wheelchair can remain upright when the motor drives the wheelchair's wheels in the same direction as the fall. Here, the wheelchair's base, on which it stands, is crucial to stabilising it.

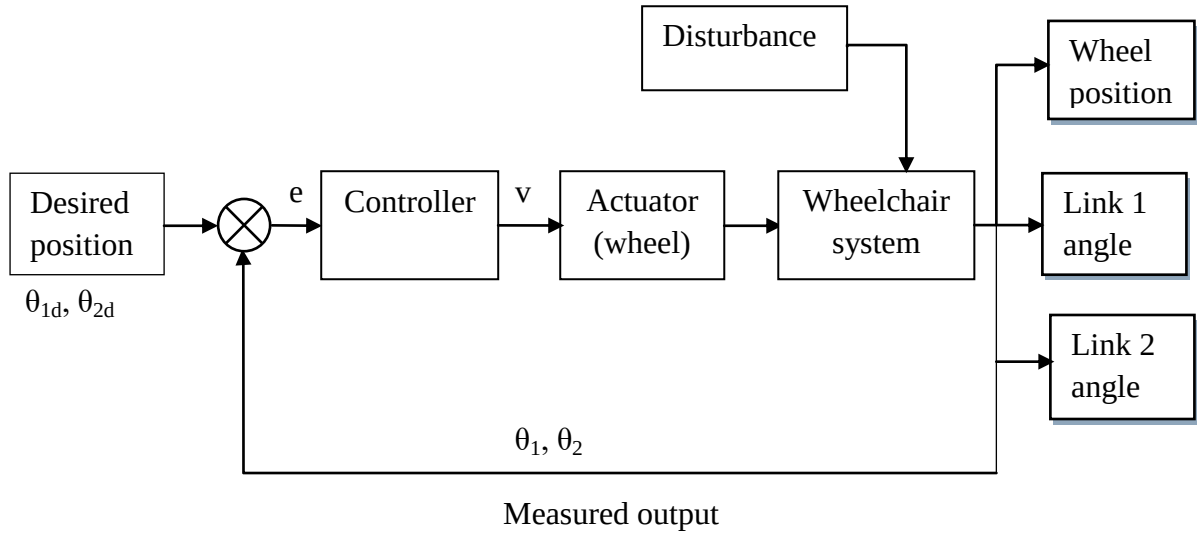


Figure 3.3: Block Diagram of the system.

The block diagram above depicts the overall system. As shown in the above block diagram, the feedback is taken from the angle of the pendulum relative to the vertical axis, and the variable  $e$  represents the error, which is the difference between the desired reference input value and the actual output. This error signal is sent to the controller, and the controller computes this error signal and sends a control signal to the actuator, which is used to move the base of the wheelchair forward and backward by rotating the wheel of the wheelchair to maintain the balance of the wheelchair, and the new output is obtained. The new output is then fed back and compared to the reference to find the new error signal. The controller takes this new error signal and computes again until the wheelchair becomes stable.

### 3.6 System modelling

Understanding the parameters of a wheelchair is critical for developing a control system for the wheelchair. The presentation of those parameters is accomplished through the use of a mathematical model.

A set of equations used in a mathematical model serves to precisely capture the dynamics of the system. For human beings, this is the best tool to describe the physical world precisely and unambiguously. The procedure for determining the mathematical model is "mathematical modeling." A given system can be depicted by different mathematical models as long as the input and initial conditions are the same. It can be represented in either "transfer function form" or "state-space form." SISO-LTI systems use "transfer

function form," whereas MIMO systems use state-space representations with time-domain analyses (Mahmoud Elkafafy & Bart Peeters, 2022).

The Newtonian and Euler-Lagrange approaches are the two most common approaches used to derive the equations of a dynamic system. The primary distinction between the two approaches is based on system constraints. Newton's method models each component of the system independently and determines the forces applied to each component. The Euler-Lagrange technique, on the other hand, uses the potential and kinetic energy of the system components to determine the dynamic model of the system. The Euler-Lagrange method, which is widely used in robotic literature, provides a more systematic and straightforward modelling approach for complex systems (Normaniha Abdul Ghani and Nurul Fadzlina Jamin, 2016).

### 3.7 Mathematical model of the system

As said in the previous section of this document, two-wheel electric wheelchairs use the same concept with a double inverted pendulum. The link 1 includes motors that are connected to the wheels, base frame, battery, and electronics components. The link 1 elements have been represented as a rod with mass  $m_1$  and length  $h_1$ . The second link consists of a seat and human weight. The link 2 elements are represented as a rod with mass  $m_2$  and length  $h_2$ .

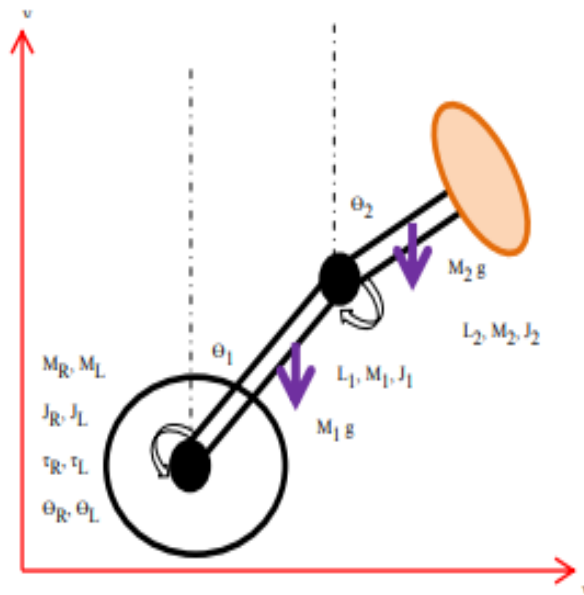


Figure 3.4: Schematic diagram of wheeled DIP system (Nurul Fadzlina Jamin, 2016).



Pendulums are controlled and stabilised by applying external force or torque to the motion generators. In this section, the model of the plant is derived based on a wheeled double-link IP system (WDIPS) using the Euler-Lagrange method.

The system has three degrees of freedom (DOF): the movement of the wheel along the x-axis; the rotational movement of the lower pendulum (link 1), which is connected to the axle between wheels via the revolute joint; and the rotational motion of the upper pendulum (link 2), which is connected to the first link by a revolute joint, as shown in the schematic diagram in Figure 3.4 above.  $p_i = (x_w, \theta_1, \theta_2)$  is the generalised coordinates of the system.  $x_w, \theta_1, \theta_2$  is the linear position, link 1 and link 2 tilt angles, respectively.

The system consists of one input, which is the wheel excitation force, and three outputs: the angular position of the rods and the linear position of the wheel. In this section, the mathematical modelling of the DIP will be performed in order to provide a precise description of the system. This model possesses sets of nonlinear equations of system motions, including the constant parameters of the system. The model simulation is performed using the MATLAB Simulink software. Table 3.1 shows the DIP system constants' notations and units.

Table 3.1: Parameters of a DIP (Nurul Fadzlina Jamin, 2016)

| Parameter            | Unit                 | Description                  |
|----------------------|----------------------|------------------------------|
| $m_w$                | [Kg]                 | Mass of the wheel            |
| $m_1, m_2$           | [Kg]                 | Mass of the pendulums        |
| $h_1, h_2$           | [m]                  | Length of the pendulums      |
| $R$                  | [m]                  | Radius of the wheel          |
| $J_1, J_2$           | [Kg m <sup>2</sup> ] | Pendulums' moment of inertia |
| $J_R, J_L$           | [Kg m <sup>2</sup> ] | Wheels' moment of inertia    |
| $G$                  | m/s <sup>2</sup>     | Gravity constant             |
| $x_w$                | [m]                  | Wheel position               |
| $\theta_1, \theta_2$ | [rad]                | Pendulums' angles            |

The Lagrange's equations used to drive desired equations of motions in this study are written as follow (George S. Maraslidis, 2022):

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{p}_i} \right) - \left( \frac{\partial L}{\partial p_i} \right) = T_i \quad (3.1)$$

where  $p_i (i=1,2,3,...,n)$  is a set of coordinates,  $T_i$  denotes generalised forces or moments acting in the direction of coordinates  $p$ , and  $L$  is the lagrangian function defined as follows:

$$L = KE_T - PE_T \quad (3.2)$$

Where  $KE$  is the kinetic energy and  $PE$  represents the potential energy. For the presented system, total kinetic energy ( $KE_T$ ) and total potential energy ( $PE_T$ ) are obtained by the summation of individual component energies. For the system shown in Figure 3.4, the independent coordinate parameters are defined as:  $x_w$ ,  $\theta_1$ , and  $\theta_2$ . Therefore, by considering the external force  $u$  exerted on the system along the  $x$  coordinate and neglecting the friction of the system, the equation of motion is derived using equation (3.1).

Since the system has three structural elements (a wheel and two pendulum rods), system energies are defined as follows:

$$\begin{cases} KE_T = KE_w + KE_1 + KE_2 \\ PE_T = PE_w + PE_1 + PE_2 \end{cases} \quad (3.3)$$

The indices  $w$ ,  $1$  and  $2$  represent the wheel, first and second pendulum rods, respectively.

The subsystems of the system are divided into wheel displacement, the first link, and the second link, respectively.

### 3.7.1 The wheel's energy derivations

i) The wheel's kinetic energy

The wheels kinetic energy combines the translational and rotational kinetic energy of the driving wheels (Nurul Fadzlina Jamin, 2016).

$$KE_w = \frac{1}{2} m_w v^2 + \frac{1}{2} J_w \dot{\theta}_w^2 \quad (3.4)$$

Where  $\dot{\theta}_w = \frac{\dot{x}_w}{r}$  and  $J_w = J_L + J_R$  are the angle rotated by the wheels and the wheels moment of inertia, respectively, equation (3.4) becomes:

$$\begin{cases} KE_w = \frac{1}{2} m_w \dot{x}_w^2 + \frac{1}{2} (J_L + J_R) \left( \frac{\dot{x}_w}{r} \right)^2 \\ KE_w = \frac{1}{2} \left( m_w + \frac{J_L + J_R}{r^2} \right) \dot{x}_w^2 \end{cases} \quad (3.5)$$

ii) The wheel's potential energy

Since the wheel of the wheelchair touches the ground all the time, the wheel's potential energy is equal to 0.

$$\begin{cases} PE_w = m_w gh, \text{ where } h = 0 \\ PE_w = 0 \end{cases} \quad (3.6)$$

### 3.7.2 Link 1 energy derivations

i) Link 1's kinetic energy

$$\begin{cases} KE_1 = \frac{1}{2} J_1 \dot{\theta}_1^2 + \frac{1}{2} m_1 \left\{ \left[ \frac{d}{dt} (x_w + h_1 \sin \theta_1) \right]^2 + \left[ \frac{d}{dt} (h_1 \cos \theta_1) \right]^2 \right\} \\ KE_1 = \frac{1}{2} m_1 \dot{x}_w^2 + \frac{1}{2} m_1 h_1^2 \dot{\theta}_1^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + m_1 h_1 \dot{x}_w \dot{\theta}_1 \cos \theta_1 \end{cases} \quad (3.7)$$

ii) Link 1's potential energy

$$PE_1 = m_1 gr + m_1 gh_1 \cos \theta_1 \quad (3.8)$$

### 3.7.3 Link 2 energy derivations

i) Link 2's kinetic energy

$$\begin{cases} KE_2 = \frac{1}{2} J_2 \dot{\theta}_2^2 + \frac{1}{2} m_2 \left\{ \left[ \frac{d}{dt} (x_w + h_1 \sin \theta_1 + h_2 \sin \theta_2) \right]^2 + \left[ \frac{d}{dt} (h_1 \cos \theta_1 + h_2 \cos \theta_2) \right]^2 \right\} \\ KE_2 = \frac{1}{2} m_2 \dot{x}_w^2 + \frac{1}{2} m_2 h_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 h_2^2 \dot{\theta}_2^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 + m_2 h_1 \dot{x}_w \dot{\theta}_1 \cos \theta_1 \\ \quad + m_2 h_2 \dot{x}_w \dot{\theta}_2 \cos \theta_2 + m_2 h_1 h_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \end{cases} \quad (3.9)$$

ii) Link 2's potential energy

$$PE_2 = m_2 gr + m_2 gh_1 \cos \theta_1 + m_2 gh_2 \cos \theta_2 \quad (3.10)$$

Substituting the obtained individual kinetic energy in equations 3.5, 3.7, and 3.9 into 3.4, the total kinetic energy is obtained as follows:

$$\begin{aligned}
KE_T = & \frac{1}{2} \left( m_w + \frac{J_L + J_R}{r^2} \right) \dot{x}_w^2 + \frac{1}{2} m_1 \dot{x}_w^2 + \frac{1}{2} m_1 h_1^2 \dot{\theta}_1^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + m_1 h_1 \dot{x}_w \dot{\theta}_1 \cos \theta_1 \\
& + \frac{1}{2} m_2 \dot{x}_w^2 + \frac{1}{2} m_2 h_1^2 \dot{\theta}_1^2 + \frac{1}{2} m_2 h_2^2 \dot{\theta}_2^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 + m_2 h_1 \dot{x}_w \dot{\theta}_1 \cos \theta_1 \\
& + m_2 h_2 \dot{x}_w \dot{\theta}_2 \cos \theta_2 + m_2 h_1 h_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2)
\end{aligned} \tag{3.11}$$

Substituting the obtained individual potential energy in equations 3.6, 3.8, and 3.10 into 3.4, the total potential energy is obtained as follows:

$$PE_T = m_1 gr + m_1 gh_1 \cos \theta_1 + m_2 gr + m_2 gh_1 \cos \theta_1 + m_2 gh_2 \cos \theta_2 \tag{3.12}$$

Using the derived total kinetic and total potential energy in equations 3.11 and 3.12, respectively, the system's Lagrange equation becomes:

$$L = KE_T - PE_T$$

$$\begin{aligned}
L = & \frac{1}{2} m_w \dot{x}_w^2 + \frac{1}{2} \left( \frac{J_L + J_R}{r^2} \right) \dot{x}_w^2 + \frac{1}{2} m_1 \dot{x}_w^2 + \frac{1}{2} m_1 h_1^2 \dot{\theta}_1^2 + \frac{1}{2} J_1 \dot{\theta}_1^2 + m_1 h_1 \dot{x}_w \dot{\theta}_1 \cos \theta_1 + \frac{1}{2} m_2 \dot{x}_w^2 + \frac{1}{2} m_2 h_1^2 \dot{\theta}_1^2 \\
& + \frac{1}{2} m_2 h_2^2 \dot{\theta}_2^2 + \frac{1}{2} J_2 \dot{\theta}_2^2 + m_2 h_1 \dot{x}_w \dot{\theta}_1 \cos \theta_1 + m_2 h_2 \dot{x}_w \dot{\theta}_2 \cos \theta_2 + m_2 h_1 h_2 \dot{\theta}_1 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \\
& - m_1 gr - m_1 gh_1 \cos \theta_1 - m_2 gr - m_2 gh_1 \cos \theta_1 - m_2 gh_2 \cos \theta_2
\end{aligned} \tag{3.13}$$

Using Lagrange's formulation for system dynamics and neglecting the friction of the system, the equation of motion for TWBSW is derived using the following equation:

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{x}_w} \right) - \frac{\partial L}{\partial x_w} = 0 \tag{3.14}$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_1} \right) - \frac{\partial L}{\partial \theta_1} = 0 \tag{3.15}$$

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{\theta}_2} \right) - \frac{\partial L}{\partial \theta_2} = 0 \tag{3.16}$$

where  $u$  is an external force exerted on the system along the  $x$  coordinate to move the wheel of the wheelchair and maintain the stability of the system.

The Lagrangian  $L$  derivative of generalised coordinate  $x_w$  is shown as:

$$\frac{\partial L}{\partial \dot{x}_w} = m_w \dot{x}_w + \left( \frac{J_w}{r^2} \right) \dot{x}_w + m_1 \dot{x}_w + m_1 h_1 \dot{\theta}_1 \cos \theta_1 + m_2 \dot{x}_w + m_2 h_1 \dot{\theta}_1 \cos \theta_1 + m_2 h_2 \dot{\theta}_2 \cos \theta_2 \quad (3.17)$$

$$\begin{aligned} \frac{d}{dt} \frac{\partial L}{\partial \dot{x}_w} = & m_w \ddot{x}_w + \left( \frac{J_w}{r^2} \right) \ddot{x}_w + m_1 \ddot{x}_w + m_1 h_1 \ddot{\theta}_1 \cos \theta_1 - m_1 h_1 \dot{\theta}_1^2 \sin \theta_1 + m_2 \ddot{x}_w + m_2 h_1 \ddot{\theta}_1 \cos \theta_1 \\ & - m_2 h_1 \dot{\theta}_1^2 \sin \theta_1 + m_2 h_2 \ddot{\theta}_2 \cos \theta_2 - m_2 h_2 \dot{\theta}_2^2 \sin \theta_2 \end{aligned} \quad (3.18)$$

$$\frac{\partial L}{\partial x_w} = 0 \quad (3.19)$$

Insert equations 3.18 and 3.19 into equation 3.14.

$$\begin{aligned} u = & m_w \ddot{x}_w + \left( \frac{J_w}{r^2} \right) \ddot{x}_w + m_1 \ddot{x}_w + m_1 h_1 \ddot{\theta}_1 \cos \theta_1 - m_1 h_1 \dot{\theta}_1^2 \sin \theta_1 + m_2 \ddot{x}_w + m_2 h_1 \ddot{\theta}_1 \cos \theta_1 \\ & - m_2 h_1 \dot{\theta}_1^2 \sin \theta_1 + m_2 h_2 \ddot{\theta}_2 \cos \theta_2 - m_2 h_2 \dot{\theta}_2^2 \sin \theta_2 \end{aligned} \quad (3.20)$$

$$\begin{aligned} u = & \left( m_w + m_1 + m_2 + \frac{J_w}{r^2} \right) \ddot{x}_w + (m_1 h_1 + m_2 h_1) \cos \theta_1 \ddot{\theta}_1 + m_2 h_2 \cos \theta_2 \ddot{\theta}_2 \\ & - (m_1 h_1 + m_2 h_1) \sin \theta_1 \dot{\theta}_1^2 - m_2 h_2 \sin \theta_2 \dot{\theta}_2^2 \end{aligned} \quad (3.21)$$

The derivative for Lagrangian  $L$  of generalized coordinate  $\theta_1$  is shown as:

$$\frac{\partial L}{\partial \dot{\theta}_1} = m_1 h_1 \dot{x}_w \cos \theta_1 + m_1 h_1^2 \dot{\theta}_1 + J_1 \dot{\theta}_1 + m_2 h_1^2 \dot{\theta}_1 + m_2 h_1 \dot{x}_w \cos \theta_1 + m_2 h_1 h_2 \dot{\theta}_2 \cos(\theta_1 - \theta_2) \quad (3.22)$$

$$\begin{aligned} \frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_1} = & m_1 h_1 \ddot{x}_w \cos \theta_1 + m_2 h_1 \ddot{x}_w \cos \theta_1 + m_1 h_1^2 \ddot{\theta}_1 + J_1 \ddot{\theta}_1 + m_2 h_1^2 \ddot{\theta}_1 + m_2 h_1 h_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) \\ & - m_1 h_1 \dot{x}_w \dot{\theta}_1 \sin \theta_1 - m_2 h_1 \dot{x}_w \dot{\theta}_1 \sin \theta_1 - m_2 h_1 h_2 \dot{\theta}_2 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2) \end{aligned} \quad (3.23)$$

$$\begin{aligned} \frac{\partial L}{\partial \theta_1} = & -m_1 h_1 \dot{x}_w \dot{\theta}_1 \sin \theta_1 - m_2 h_1 \dot{x}_w \dot{\theta}_1 \sin \theta_1 - m_2 h_1 h_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) \\ & + (m_1 h_1 + m_2 h_1) g \sin \theta_1 \end{aligned} \quad (3.24)$$

Insert equations 3.23 and 3.24 into equation 3.15.

$$\begin{aligned}
0 = & m_1 h_1 \ddot{x}_w \cos \theta_1 + m_2 h_1 \ddot{x}_w \cos \theta_1 + m_1 h_1^2 \ddot{\theta}_1 + J_1 \ddot{\theta}_1 + m_2 h_1^2 \ddot{\theta}_1 + m_2 h_1 h_2 \ddot{\theta}_2 \cos(\theta_1 - \theta_2) \\
& - m_1 h_1 \dot{x}_w \dot{\theta}_1 \sin \theta_1 - m_2 h_1 \dot{x}_w \dot{\theta}_1 \sin \theta_1 - m_2 h_1 h_2 \dot{\theta}_2 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2) \\
& - (-m_1 h_1 \dot{x}_w \dot{\theta}_1 \sin \theta_1 - m_2 h_1 \dot{x}_w \dot{\theta}_1 \sin \theta - m_2 h_1 h_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) \\
& + (m_1 h_1 + m_2 h_1) g \sin \theta_1)
\end{aligned} \tag{3.25}$$

$$\begin{aligned}
0 = & (m_1 h_1 + m_2 h_1) \cos \theta_1 \ddot{x}_w + (m_1 h_1^2 + m_2 h_1^2 + J_1) \ddot{\theta}_1 + m_2 h_1 h_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_2 \\
& + m_2 h_1 h_2 \sin(\theta_1 - \theta_2) \dot{\theta}_2^2 - (m_1 h_1 + m_2 h_1) g \sin \theta_1
\end{aligned} \tag{3.26}$$

The derivative for Lagrangian L of generalized coordinate  $\theta_2$  is shown as:

$$\frac{\partial L}{\partial \dot{\theta}_2} = m_2 h_2^2 \dot{\theta}_2 + J_2 \dot{\theta}_2 + m_2 h_2 \dot{x}_w \cos \theta_2 + m_2 h_1 h_2 \dot{\theta}_1 \cos(\theta_1 - \theta_2) \tag{3.27}$$

$$\begin{aligned}
\frac{d}{dt} \frac{\partial L}{\partial \dot{\theta}_2} = & m_2 h_2^2 \ddot{\theta}_2 + J_2 \ddot{\theta}_2 + m_2 h_2 \ddot{x}_w \cos \theta_2 + m_2 h_1 h_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 h_2 \dot{x}_w \dot{\theta}_2 \sin \theta_2 \\
& - m_2 h_1 h_2 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2)
\end{aligned} \tag{3.28}$$

$$\frac{\partial L}{\partial \theta_2} = -m_2 h_2 \dot{x}_w \dot{\theta}_2 \sin \theta_2 - m_1 h_1 h_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 h_2 g \sin \theta_2 \tag{3.29}$$

Insert equations 3.28 and 3.29 into equation 3.16.

$$\begin{aligned}
0 = & m_2 h_2^2 \ddot{\theta}_2 + J_2 \ddot{\theta}_2 + m_2 h_2 \ddot{x}_w \cos \theta_2 + m_2 h_1 h_2 \ddot{\theta}_1 \cos(\theta_1 - \theta_2) - m_2 h_2 \dot{x}_w \dot{\theta}_2 \sin \theta_2 \\
& - m_2 h_1 h_2 \dot{\theta}_1 (\dot{\theta}_1 - \dot{\theta}_2) \sin(\theta_1 - \theta_2) - (-m_2 h_2 \dot{x}_w \dot{\theta}_2 \sin \theta_2 - m_1 h_1 h_2 \dot{\theta}_1 \dot{\theta}_2 \sin(\theta_1 - \theta_2) + m_2 h_2 g \sin \theta_2)
\end{aligned} \tag{3.30}$$

$$\begin{aligned}
0 = & m_2 h_2 \cos \theta_2 \ddot{x}_w + m_2 h_1 h_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_1 + (m_2 h_2^2 + J_2) \ddot{\theta}_2 - m_2 h_1 h_2 \sin(\theta_1 - \theta_2) \dot{\theta}_1^2 \\
& - m_2 h_2 g \sin \theta_2
\end{aligned} \tag{3.31}$$

The three equations obtained in 3.21, 3.26, and 3.31 can be expressed as follows:

$$\begin{cases} u = \left( m_w + m_1 + m_2 + \frac{J_w}{r^2} \right) \ddot{x}_w + (m_1 h_1 + m_2 h_1) \cos \theta_1 \ddot{\theta}_1 + m_2 h_2 \cos \theta_2 \ddot{\theta}_2 - (m_1 h_1 + m_2 h_1) \sin \theta_1 \dot{\theta}_1^2 \\ \quad - m_2 h_2 \sin \theta_2 \dot{\theta}_2^2 \\ 0 = (m_1 h_1 + m_2 h_1) \cos \theta_1 \ddot{x}_w + (m_1 h_1^2 + m_2 h_1^2 + J_1) \ddot{\theta}_1 + m_2 h_1 h_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_2 + m_2 h_1 h_2 \sin(\theta_1 - \theta_2) \dot{\theta}_2^2 \\ \quad - (m_1 h_1 + m_2 h_1) g \sin \theta_1 \\ 0 = m_2 h_2 \cos \theta_2 \ddot{x}_w + m_2 h_1 h_2 \cos(\theta_1 - \theta_2) \ddot{\theta}_1 + (m_2 h_2^2 + J_2) \ddot{\theta}_2 - m_2 h_1 h_2 \sin(\theta_1 - \theta_2) \dot{\theta}_1^2 - m_2 h_2 g \sin \theta_2 \end{cases} \quad (3.32)$$

### 3.8 Linearization of the model

Linearization is a critical asset for controlling nonlinear systems. The above non-linear equation of the system was linearized about the vertical axis using the Taylor series small angle approximation (at  $\theta_1=0$  and  $\theta_2=0$ ), such that:

$$\begin{aligned} \cos \theta_1 &= \cos \theta_2 = 1, \quad \sin \theta_1 = \theta_1, \quad \sin \theta_2 = \theta_2 \\ \cos(\theta_1 - \theta_2) &= 1, \quad \sin(\theta_1 - \theta_2) = \theta_1 - \theta_2 \\ \dot{\theta}_1 &= \dot{\theta}_2 = 0, \quad \dot{\theta}_1^2 = \dot{\theta}_2^2 = 0, \end{aligned}$$

Then it is possible to represent the system's linearized model in matrix form as follows in equation 3.33.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_w + m_1 + m_2 + \frac{J_w}{r^2} & m_1 h_1 + m_2 h_1 & m_2 h_2 \\ 0 & 0 & 0 & m_1 h_1 + m_2 h_1 & m_1 h_1^2 + m_2 h_1^2 + J_1 & m_2 h_1 h_2 \\ 0 & 0 & 0 & m_2 h_2 & m_2 h_1 h_2 & m_2 h_2^2 + J_2 \end{bmatrix} \begin{bmatrix} \dot{x}_w \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{x}_w \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (m_1 h_1 + m_2 h_1)g & 0 & 0 & 0 & 0 \\ 0 & 0 & (m_2 h_2)g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_w \\ \theta_1 \\ \theta_2 \\ \dot{x}_w \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} [u] \quad (3.33)$$

### 3.9 Actuator modelling

DC motors are the main types of actuators used in mobile robots and robotic applications (B. Supriyo, A. Suharjono, and K. W. Atmaja, 2021). A DC motor was chosen for this thesis as well because it could deliver enough torque and speed at the same time, as well as having a long operating life, the ability to save power, and low electric noise generation. It should also have been necessary that it be powered by a DC power source, which led to the selection of a DC motor over an AC motor. To control the wheelchair's movement, two DC motors are attached to each wheel. The motor mathematical model is derived and incorporated into the dynamic model of the vehicle to present the relationship between the control signal required to stabilise the vehicle and the input voltage. The equivalent motor circuit is presented in the figure below.

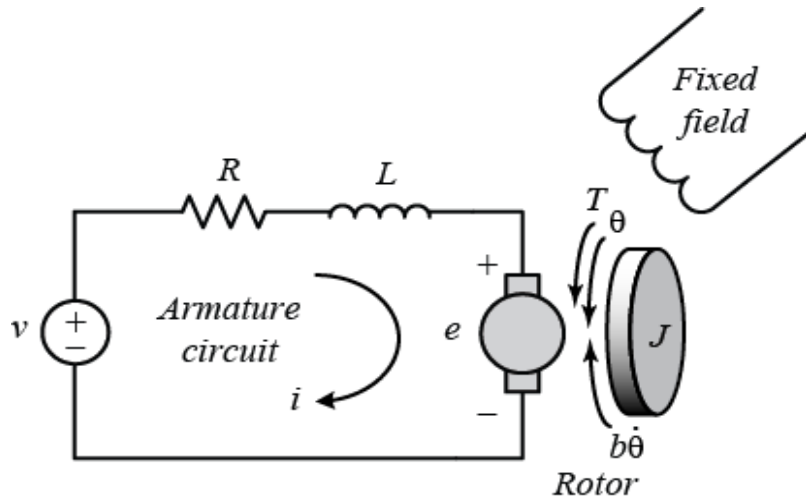


Figure 3.5: DC motor circuit diagram (Lytigas, 2018)

$R$  is armature resistance,  $L$  is armature inductance,  $V$  is input voltage,  $i$  is armature current,  $b\dot{\theta}(=\dot{\theta}_w)$  is angular speed of the motor,  $E$  is back emf,  $T(=\tau)$  is DC motor torque, and  $\theta(=\theta_w)$  is rotated angle. The DC motor model also includes  $C_T$  and  $C_e$  which are torque constant and back emf constant, respectively. Considering the case that the viscosity friction and the armature inductance are negligibly small, equations 3.34 and 3.35 represent the DC motor dynamics (Lytigas, 2018).

$$\tau = C_T i \quad (3.34)$$

$$E = C_e \dot{\theta}_w \quad (3.35)$$

Using Kirchhoff's voltage law, the input voltage of the DC motor circuit becomes:



$$V = iR + C_e \dot{\theta}_w \quad (3.36)$$

Where,  $\dot{\theta}_w = \frac{\dot{x}_w}{r}$ :

$$V = iR + C_e \left( \frac{\dot{x}_w}{r} \right)$$

Solve for current (i):

$$i = \frac{1}{R} \left( V - \frac{C_e \dot{x}_w}{r} \right) \quad (3.37)$$

By inserting Equation 3.37 into 3.34, an expression for motor torque could be derived, as seen in equation 3.38.

$$\tau = \frac{C_T}{R} \left( V - \frac{C_e \dot{x}_w}{r} \right) \quad (3.38)$$

The input u in the system model in Equation 3.35 is in the form of a force. Using  $\tau = ur$ , combining Equation 3.33 and Equation 3.38, the system model can be derived as seen in Equation 3.39.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_w + m_1 + m_2 + \frac{J_w}{r^2} & m_1 h_1 + m_2 h_1 & m_2 h_2 \\ 0 & 0 & 0 & m_1 h_1 + m_2 h_1 & m_1 h_1^2 + m_2 h_1^2 + J_1 & m_2 h_1 h_2 \\ 0 & 0 & 0 & m_2 h_2 & m_2 h_1 h_2 & m_2 h_2^2 + J_2 \end{bmatrix} \begin{bmatrix} \dot{x}_w \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{x}_w \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & (m_1 h_1 + m_2 h_1)g & 0 & 0 & 0 & 0 \\ 0 & 0 & (m_2 h_2)g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_w \\ \theta_1 \\ \theta_2 \\ \dot{x}_w \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \left[ \frac{C_T}{Rr} \left( V - \frac{C_e \dot{x}_w}{r} \right) \right] \quad (3.39)$$

Finally, the model of the overall system, including the model of the DC motor, becomes:

$$\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & m_w + m_1 + m_2 + \frac{J_w}{r^2} & m_1 h_1 + m_2 h_1 & m_2 h_2 \\
0 & 0 & 0 & m_1 h_1 + m_2 h_1 & m_1 h_1^2 + m_2 h_1^2 + J_1 & m_2 h_1 h_2 \\
0 & 0 & 0 & m_2 h_2 & m_2 h_1 h_2 & m_2 h_2^2 + J_2
\end{bmatrix}
\begin{bmatrix}
\dot{x}_w \\
\dot{\theta}_1 \\
\dot{\theta}_2 \\
\ddot{x}_w \\
\ddot{\theta}_1 \\
\ddot{\theta}_2
\end{bmatrix}
\quad (3.40)$$

$$= \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & \frac{-C_T C_e}{R r^2} & 0 & 0 \\
0 & (m_1 h_1 + m_2 h_1)g & 0 & 0 & 0 & 0 \\
0 & 0 & (m_2 h_2)g & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
x_w \\
\theta_1 \\
\theta_2 \\
\dot{x}_w \\
\dot{\theta}_1 \\
\dot{\theta}_2
\end{bmatrix}
+ \begin{bmatrix}
0 \\
0 \\
0 \\
\frac{C_T V}{R r} \\
0 \\
0
\end{bmatrix}$$

### 3.10 Representation of state space

The state-space equation is derived from the linearized model of TWEW. To drive the state-space model of the system, first derive the matrix elements and write the system model in standard form.

$$\begin{aligned}
\dot{x} &= Ax + Bu \\
y &= Cx + Du
\end{aligned}
\quad (3.41)$$

Where  $x \in R^m$ ,  $u \in R^n$ ,  $y \in R^o$ ,  $A \in R^{m \times m}$  and  $B \in R^{m \times n}$ ,  $C \in R^{o \times m}$ ,  $D \in R^{o \times n}$ .

$x$  represents the state,  $u$  represents the input, and  $y$  represents the output.  $A$ ,  $B$ ,  $C$ , and  $D$  are state space matrices.  $A$  and  $B$  are system properties determined by the structure and elements of the system.  $C$  and  $D$  are defined based on the output variables used. For TWEW, the state is:

$$x = [x_w \quad \theta_1 \quad \theta_2 \quad \dot{x}_w \quad \dot{\theta}_1 \quad \dot{\theta}_2]^T \quad (3.42)$$

Define the state variables as follows:

$$x_1 = x_w, \quad x_2 = \theta_1, \quad x_3 = \theta_2, \quad x_4 = \dot{x}_w, \quad x_5 = \dot{\theta}_1, \quad x_6 = \dot{\theta}_2$$

$$\begin{aligned}
\dot{x}_1 &= x_4 = \dot{x}_w \\
\dot{x}_2 &= x_5 = \dot{\theta}_1 \\
\dot{x}_3 &= x_6 = \dot{\theta}_2 \\
\dot{x}_4 &= \ddot{x}_w \\
\dot{x}_5 &= \ddot{\theta}_1 \\
\dot{x}_6 &= \ddot{\theta}_2
\end{aligned} \tag{3.43}$$

Then the state space vector of the system is expressed as:

$$x = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T \tag{3.44}$$

From equation 3.40:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_w + m_1 + m_2 + \frac{J_w}{r^2} & m_1 h_1 + m_2 h_1 & m_2 h_2 \\ 0 & 0 & 0 & m_1 h_1 + m_2 h_1 & m_1 h_1^2 + m_2 h_1^2 + J_1 & m_2 h_1 h_2 \\ 0 & 0 & 0 & m_2 h_2 & m_2 h_1 h_2 & m_2 h_2^2 + J_2 \end{bmatrix} \begin{bmatrix} \dot{x}_w \\ \dot{\theta}_1 \\ \dot{\theta}_2 \\ \ddot{x}_w \\ \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{bmatrix} \tag{3.45}$$

$$= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{-C_T C_e}{R r^2} & 0 & 0 \\ 0 & (m_1 h_1 + m_2 h_1)g & 0 & 0 & 0 & 0 \\ 0 & 0 & (m_2 h_2)g & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_w \\ \theta_1 \\ \theta_2 \\ \dot{x}_w \\ \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{C_T}{R r} \\ 0 \\ 0 \end{bmatrix} [v]$$

Where  $x = [x_1 \quad x_2 \quad x_3 \quad x_4 \quad x_5 \quad x_6]^T = [x_w \quad \theta_1 \quad \theta_2 \quad \dot{x}_w \quad \dot{\theta}_1 \quad \dot{\theta}_2]^T$  and

$$\dot{x} = [\dot{x}_1 \quad \dot{x}_2 \quad \dot{x}_3 \quad \dot{x}_4 \quad \dot{x}_5 \quad \dot{x}_6]^T = [\dot{x}_w \quad \dot{\theta}_1 \quad \dot{\theta}_2 \quad \ddot{x}_w \quad \ddot{\theta}_1 \quad \ddot{\theta}_2]^T$$

By multiplying both sides by  $M^{-1}$  (the coefficient of  $\dot{x}$ ), the state-space matrices A and B are obtained.

$$A = M^{-1}G \quad \text{and} \quad B = M^{-1}H \tag{3.46}$$

Where:

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & m_w + m_1 + m_2 + \frac{J_w}{r^2} & m_1 h_1 + m_2 h_1 & m_2 h_2 \\ 0 & 0 & 0 & m_1 h_1 + m_2 h_1 & m_1 h_1^2 + m_2 h_1^2 + J_1 & m_2 h_1 h_2 \\ 0 & 0 & 0 & m_2 h_2 & m_2 h_1 h_2 & m_2 h_2^2 + J_2 \end{bmatrix},$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \frac{-C_T C_e}{R r^2} & 0 & 0 \\ 0 & (m_1 h_1 + m_2 h_1)g & 0 & 0 & 0 & 0 \\ 0 & 0 & (m_2 h_2)g & 0 & 0 & 0 \end{bmatrix}, \text{ and } H = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{C_T}{R r} \\ 0 \\ 0 \end{bmatrix}$$

Table 3.2: Values of the system parameters (Nurul Fadzlina Jamin, 2016).

| Parameter | Value(unit)             | Description                |
|-----------|-------------------------|----------------------------|
| $m_w$     | 5Kg                     | Mass of the wheel          |
| $m_1$     | 13kg                    | Mass of link1              |
| $m_2$     | 73kg                    | Mass of link 2             |
| $h_1$     | 0.4m                    | Length of link 1           |
| $h_2$     | 0.4m                    | Length of link 2           |
| $R$       | 0.3m                    | Radius of wheel            |
| $J_1$     | 0.6933Kg m <sup>2</sup> | Link 1 moment of inertia   |
| $J_2$     | 3.8933Kg m <sup>2</sup> | Link 2 moment of inertia   |
| $J_w$     | 0.225Kg m <sup>2</sup>  | Moment of inertia of wheel |
| $G$       | 9.81m/s <sup>2</sup>    | Gravitational acceleration |
| $C_T$     | 0.155Nm/A               | Torque constant            |
| $C_e$     | 0.327V/rad/s            | Back emf constant          |
| $R$       | 2.5Ω                    | Resistance                 |

The following state space model is obtained after entering all parameter values into the equations.

$$\begin{aligned}
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \\ \dot{x}_5 \\ \dot{x}_6 \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & -108.5640 & 0 & 0.0300 & 0 & 0 \\ 0 & 338.1502 & -42.2152 & -0.0751 & 0 & 0 \\ 0 & -50.0551 & 50.0551 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0.0276 \\ -0.0689 \\ 0 \end{bmatrix} u \\
y &= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} u \quad (3.47)
\end{aligned}$$

The open loop system's poles are:

$$P = [0 \quad 0.0021 \quad -6.7556 \quad 6.7597 \quad -19.9280 \quad 19.9517]$$

Three of the six state space poles of the linearized model of the system are real positive poles located on the right half of the s-plane, indicating that the system is utterly unstable.

### 3.11 Controllability and observability

Before designing the controller, controllability and observability are checked to ensure that the system states are controllable and observable. A system state space representation is controllable if the controllability matrix,  $C$ , has full rank. And also, the linearized system model is observable if and only if the observability matrix  $O$  has a full rank  $n$ , where  $n$  is the order of the system (Erfan Nozari, 2020).

$$C = [A^0 B \quad A^1 B \quad A^2 B \quad \dots \quad A^{n-1} B] \quad (3.48)$$

$$O = \begin{bmatrix} CA^0 \\ CA^1 \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix} \quad (3.49)$$

The rank of the  $C$  and  $O$  matrix can be found using MATLAB.

In this system, both the controllability matrix  $C$  and the observability matrix  $O$  have rank 6, which corresponds to the system's degree. As a result, the system is controllable and observable.

### 3.12 System model analysis

In this section, simulations have been conducted with the developed model to analyze the unstable and nonlinear behavior of TWEW.

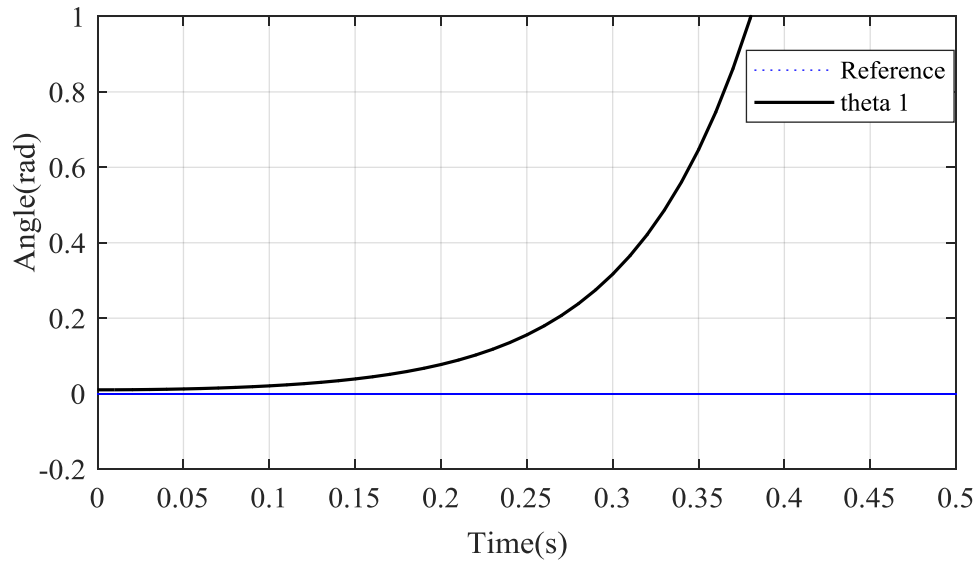


Figure 3.6 Lower pendulum angle responses with initial 0.01rad

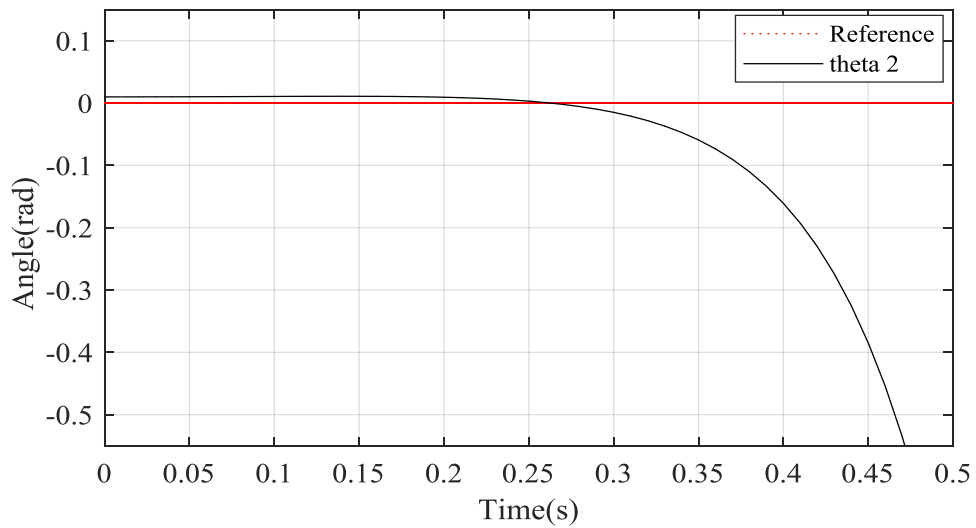


Figure 3.7 Upper pendulum angle responses with initial 0.01rad

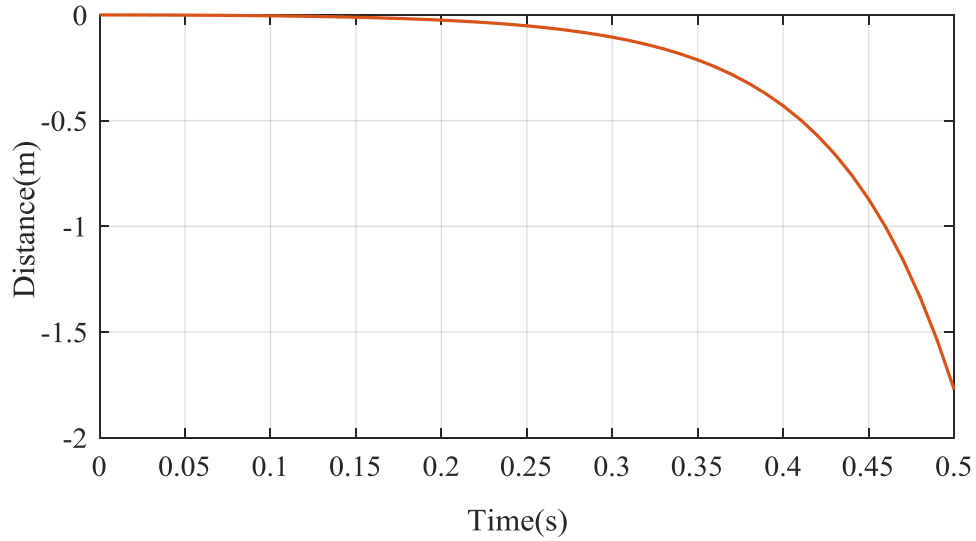


Figure 3.8 TWEW wheel displacement response

First, the initial tilt angle was set to 0.01 radians for both angles. Figures 3.6 and 3.7 show the simulation results for links 1 and 2, respectively. The simulation show that the wheel moves in the negative direction, as shown in Figure 3.8, and link 1 falls in the positive direction and link 2 falls in the negative direction.

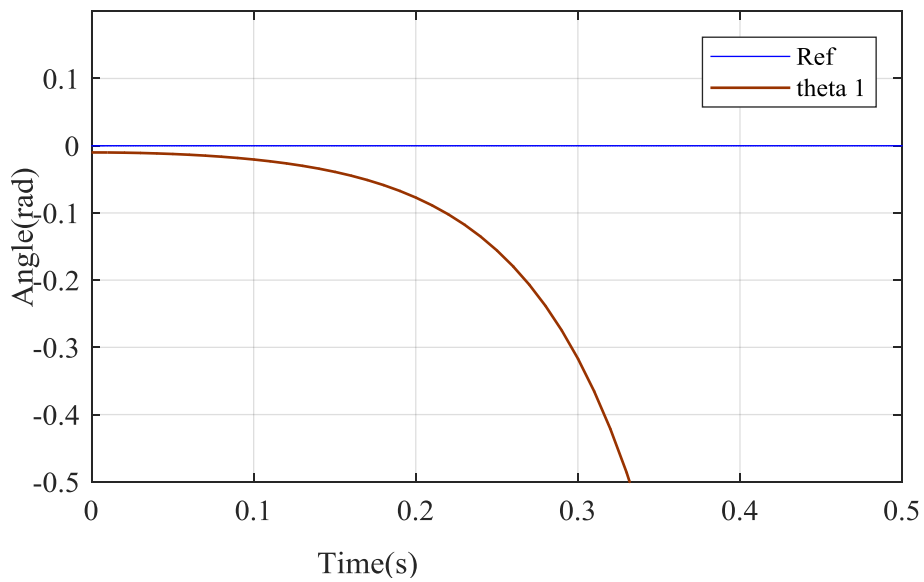


Figure 3.9 Lower pendulum angle responses with initial -0.01rad

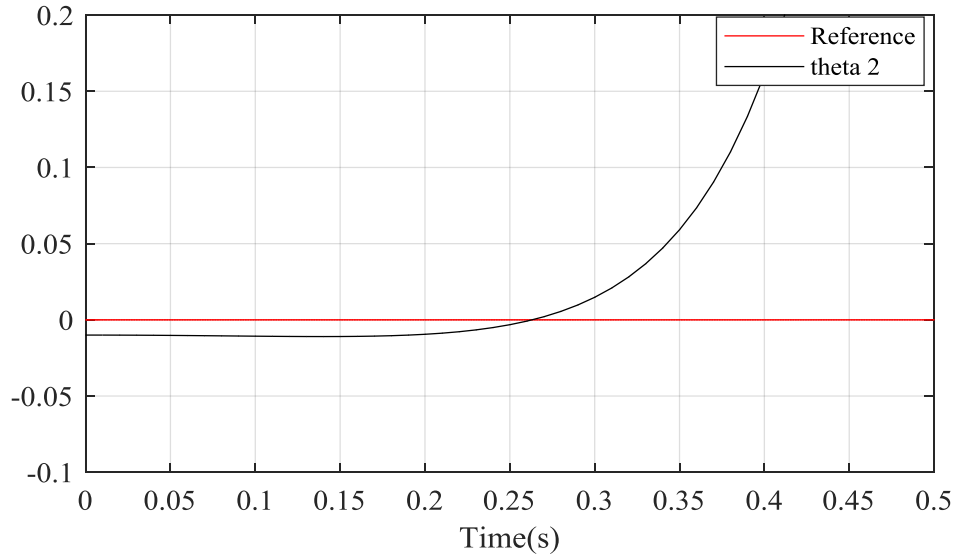


Figure 3.10 Upper pendulum angle responses with initial -0.01rad

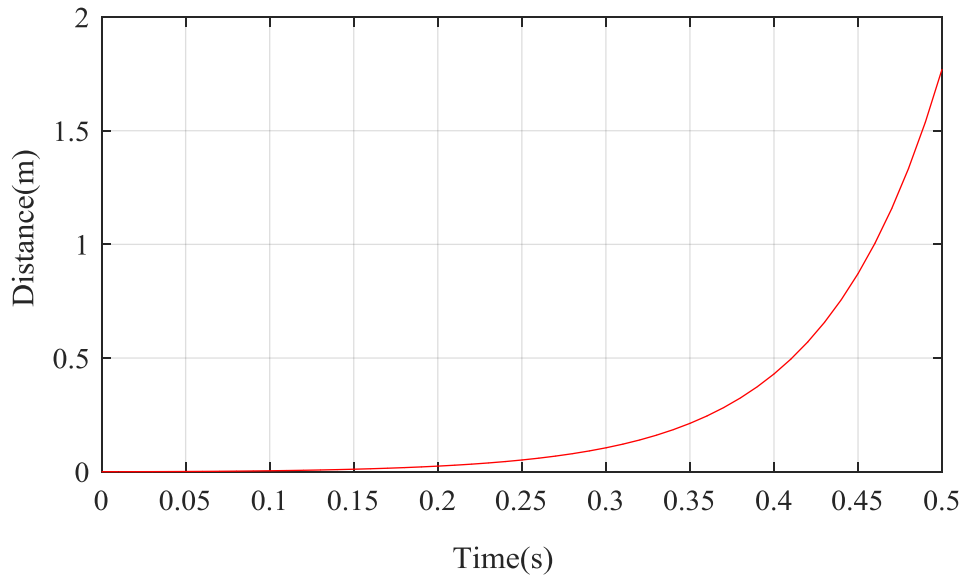


Figure 3.11 TWEW wheel displacement response

Here also, the simulation result indicates that when both angles are initially -0.01, the wheel moves in the positive direction, as shown in Figure 3.11, and link 1 falls in the negative direction, as shown in Figure 3.9, and link 2 falls in the positive direction, as shown in Figure 3.10. This indicates that, as expected, the two-wheel wheelchair is highly unstable and necessitates the use of a feedback control system.



# CHAPTER FOUR

## CONTROLLER DESIGN

### 4.1 Introduction

There will always be a gap between the real plant and its mathematical model. These variations are caused by unknown external disturbances, plant parameters, and unmodeled dynamics (Han Yang, Liu Xiaoma, and Zheng Zhiqiang, 2018). These disturbances and uncertainties have an adverse effect on the design of control laws that provide the desired performance. This has led to an interest in developing active control methods.

There are numerous categories of control algorithms: artificial intelligence-based control theories, model-based control algorithms, and hybrid control procedures. A model-free controller is based on system knowledge that researchers have gained over time. This technique has issues with system dependability. Model-based control techniques should be used to increase stability, robustness, and efficiency. Sliding mode control (SMC), feedback linearization method (FLC), and back-stepping control algorithm (BSC) are widely used model-based controllers (Anoop Chandra & Lal Priya P. S, 2022).

In this chapter, including robust sliding mode controllers, various balance controllers are discussed and designed.

### 4.2 Sliding mode control

Sliding-mode control is a particular form of variable structure control that is recognised as an efficient tool for designing robust controllers for complex high-order linear and nonlinear dynamic plants operating under uncertain conditions. It is designed to force the system states onto a certain surface in state space known as the sliding surface, and when it is reached, sliding mode control maintains the states in the sliding surface's nearby area (Ayad Q. Al-Dujaili, Alaq Falah, Amjad J. Humaidi, Daniel A. Pereira, and Ibraheem K. Ibraheem, 2020).

SMC design is separated into two stages, which are sliding surface design and control law design (Ayad Q. Al-Dujaili, Alaq Falah, Amjad J. Humaidi, Daniel A. Pereira, and Ibraheem K. Ibraheem, 2020). The first stage is determining a sliding surface,  $S$ , also known as a sliding manifold or sliding function. The second stage is designing a control law. The control law of SMC is usually a combination of two control signals: equivalent

control and switching control (Zhilin Feng, Juntao Fei, 2018). The equivalent control completes the controller's main control action, whereas switching control offers additional control action against disturbances and uncertainties.

Equivalent control deals with the model of the system and the sliding surface. The switching control forces the system to the sliding surface,  $S$ , defined by the user and maintains the system's state trajectory on this surface, as shown in Figure 4.1. where  $e$  and  $\dot{e}$  represent the tracking error and its first-time derivative.

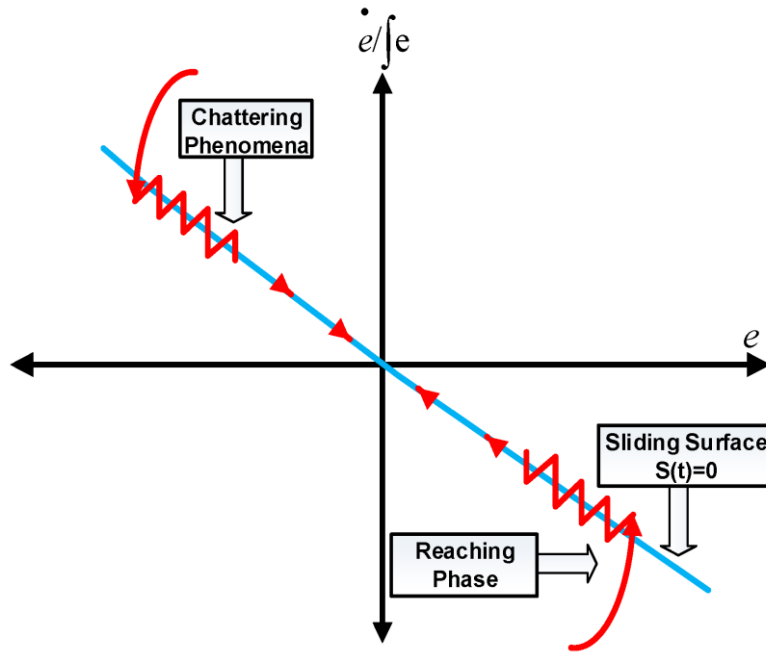


Figure 4.1: Sliding mode controller graphical representations (Zeb K Uddin W, 2020)

Despite its advantages in simplicity and robustness, first-order sliding-mode control is frequently plagued by chattering, which is characterised by a very high-frequency oscillation of the sliding variable around the sliding manifold, as illustrated in the above Figure 4.1. Chattering is extremely undesirable in real-world systems and actuators because it can lead to actuator failure and excessively large control signals (Ho, Sho-Tsung Kao, and Ming-Tzu, 2022). HOSMC is one method for eliminating chattering in control signals (V. Behnamgol, A. R. Vali, and I. Mohammadzaman, 2013).

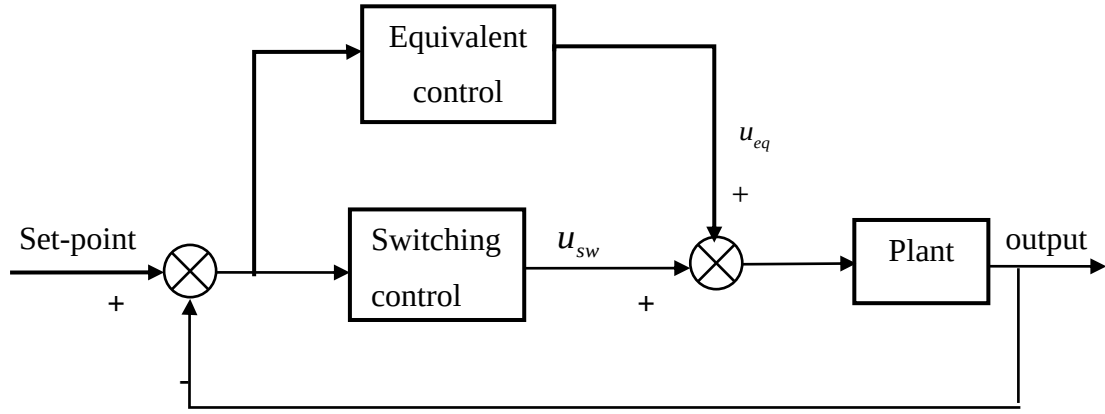


Figure 4.2: Block diagram of sliding mode control (Zhilin Feng, Juntao Fei, 2018)

#### 4.2.1 Higher order sliding mode controller

Higher-order sliding mode is an extension of the standard SMC concept in which the sliding variable and some of its time derivatives are zeroed. In this technique, the higher-order time derivatives of the sliding surface remain zero (Peng, Yang-Rui Li, and Chao-Chung, 2021). The chattering effect is reduced as a result. HOSM has two key features that make it a better choice for designing appropriate controllers. It improves the design's accuracy, which is critical, and provides a continuous control signal. A second-order SMC (SOSMC) is a subset of HOSMC that preserves desirable properties such as invariance and order reduction while improving accuracy and ensuring the finite-time stability of systems (S.D., Parvat Bhagsen, and Ratnaparkhi, 2015). The super-twisting SMC is a highly advanced second-order SMC method with numerous advantages (Ayad Q. Al-Dujaili, Alaq Falah, Amjad J. Humaidi, Daniel A. Pereira, and Ibraheem K. Ibraheem, 2020). It provides a solution that considerably reduces the chattering in control behaviour and makes the states of the system reach the equilibrium point in a finite time.

Notably, the STSMC strategy is primarily intended to maintain system stability through the application of Lyapunov theory. As a result, the super-twisting SMC has the properties of robustness and finite-time convergence for internal and externally uncertain dynamic properties (Kyong-Hyon Kim, Minh-Thien Tran, Hwan-Cheol Park, and Young-Bok Kim, 2020). The super-twisting algorithms were introduced as follows:

$$\begin{cases} \dot{u} = -k_1 |s|^{\frac{1}{2}} \text{sgn}(s) + v \\ \dot{v} = -k_2 \text{sgn}(s) \end{cases} \quad (4.1)$$

### 4.3 Design a sliding mode controller for TWEW

To design SMC for TWEW, the first step is to define the sliding surface.

#### 4.3.1 Sliding surface design

The sliding surface design problem can be interpreted as a state feedback selection problem (Swati Singh and Akhilesh Swarup, 2021). Consider the system state space model,

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (4.2)$$

where  $x \in R^m$ ,  $u \in R^n$ ,  $y \in R^o$ ,  $A \in R^{m \times m}$  and  $B \in R^{m \times n}$   $C \in R^{o \times m}$

The associated sliding surface is defined as follows (Ishan Chawla and Ashish Singla, 2021).

$$s(t) = Sx(t) \quad (4.3)$$

For convenient interpretation of SMC original state space form transformed to regular form using linear transformation ( $\bar{x}(t) = T_r x(t)$ ). The regular form makes the controller design procedure much easier. Orthogonal matrix  $T_r$  derived from QR factorization of B (Manukid Parnichkun and Sondarangallage D.A. Sanjeewa, 2021).

After linear transformation the regular coordinate expressed as,

$$\bar{x}(t) = \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix} \quad (4.4)$$

Where,  $\bar{x}_1(t) \in R^{m-n}$  and  $\bar{x}_2(t) \in R^n$

The regular state-space matrix can be obtained from the original state-space matrix (Ishan Chawla and Ashish Singla, 2021) as follow.

$$A_{new} = T_r A T_r^T = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \quad B_{new} = T_r B = \begin{bmatrix} 0 \\ B_2 \end{bmatrix} \quad (4.5)$$

Then the regular state-space form of the system is expressed as:

$$\begin{bmatrix} \dot{\bar{x}}_1(t) \\ \dot{\bar{x}}_2(t) \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ B_2 \end{bmatrix} u \quad (4.6)$$

Equation 4.2, expressed in regular form as

$$\begin{aligned}\dot{\bar{x}}_1(t) &= A_{11}\bar{x}_1(t) + A_{12}\bar{x}_2(t) \\ \dot{\bar{x}}_2(t) &= A_{21}\bar{x}_1(t) + A_{22}\bar{x}_2(t) + B_2u(t)\end{aligned}\tag{4.7}$$

and the equation in 4.3 becomes

$$s(t) = S_1\bar{x}_1(t) + S_2\bar{x}_2(t)\tag{4.8}$$

During sliding motion,  $s(t) = 0$  (Ishan Chawla and Ashish Singla, 2021).

$$s(t) = S_1\bar{x}_1(t) + S_2\bar{x}_2(t) = 0\tag{4.9}$$

Solve for  $\bar{x}_2(t)$

$$\begin{aligned}\bar{x}_2(t) &= -S_2^{-1}S_1\bar{x}_1(t) \\ \bar{x}_2(t) &= -C\bar{x}_1(t)\end{aligned}\tag{4.10}$$

where  $C = S_2^{-1}S_1$  and  $S_2 \neq 0$

Sliding mode control according to equations 4.7 and 4.10

$$\begin{cases} \dot{\bar{x}}_1(t) = A_{11}\bar{x}_1(t) + A_{12}\bar{x}_2(t) \\ \bar{x}_2(t) = -C\bar{x}_1(t) \end{cases}\tag{4.11}$$

This is a (m-n)th-order system in which  $\bar{x}_2$  has the role of a linear full-state feedback control signal.

From equation 4.11:

$$\dot{\bar{x}}_1(t) = (A_{11} - A_{12}C)\bar{x}_1(t)\tag{4.12}$$

This equation represents the mathematical equation of the system when all six states are on the sliding surface.

During sliding, the minimum design prerequisite is that dynamics must be asymptotically stable so that  $\bar{x}_1 \rightarrow 0$  as  $t \rightarrow \infty$  (Swati Singh and Akhilesh Swarup, 2021). Now we must choose  $C = S_2^{-1}S_1$  to place negative poles in the system closed loop equation of 4.12. This can be obtained by using any modified standard design procedure that prescribes a linear full-state feedback controller for a linear dynamical system (Swati Singh and Akhilesh

Swarup, 2021); ( Manukid Parnichkun and Sondarangallage D.A. Sanjeewa, 2021). The linear quadratic minimization method was used in this case.

$$S_2 C = S_1 \quad (4.13)$$

In the following work, the hyperplane matrix  $S$  is determined from  $C$  by letting  $S_2 = I_m$ , which gives

$$ST_r^T = [C \quad I_m] \quad (4.14)$$

This approach minimises the calculation in proceeding from  $C$  to  $S$  and so reduces the possibility of numerical errors.

Consider now how to design the switching hyperplane by minimising a cost function where the integrand is quadratic. This procedure will allow certain elements to be given desirable weights.

$$J = \frac{1}{2} \int_{ts}^{\infty} x^T Q x dt \quad (4.15)$$

$Q$  is both symmetric and positive-definite, and  $ts$  is the time at which sliding begins. The goal is to minimise the equation  $J$ . It is assumed that the state of the system at time  $ts$ ,  $x(ts)$  is a known initial condition and is such that  $x(t) \rightarrow 0$  as  $t \rightarrow \infty$ . The matrix  $Q$  is transformed and partitioned compatibly with  $\bar{x}_1(t)$ :

$$T_r Q T_r^T = \begin{bmatrix} Q_{11} & Q_{12} \\ Q_{21} & Q_{22} \end{bmatrix} \quad (4.16)$$

where  $Q_{21} = Q_{12}^T$ ,  $J$  can be expressed in terms of the  $\bar{x}_1(t)$  coordinate system as:

$$J = \frac{1}{2} \int_{ts}^{\infty} \bar{x}_1^T Q_{11} \bar{x}_1 + 2 \bar{x}_1^T Q_{12} \bar{x}_2 + \bar{x}_2^T Q_{22} \bar{x}_2 dt \quad (4.17)$$

$$J = \frac{1}{2} \int_{ts}^{\infty} \bar{x}_1^T (Q_{11} - Q_{12} Q_{22}^{-1} Q_{21}) \bar{x}_1 + (\bar{x}_2 + Q_{22}^{-1} Q_{21} \bar{x}_1)^T Q_{22} (\bar{x}_2 + Q_{22}^{-1} Q_{21} \bar{x}_1) dt \quad (4.18)$$

Let's define,

$$\begin{aligned}\bar{Q} &= Q_{11} - Q_{12}Q_{22}^{-1}Q_{21} \\ u &= \bar{x}_2 + Q_{22}^{-1}Q_{21}\bar{x}_1\end{aligned}\tag{4.19}$$

$J$  can also be written as

$$J = \frac{1}{2} \int_{t_0}^{\infty} \bar{x}_1^T \bar{Q} \bar{x}_1 + u^T Q_{22} u dt \tag{4.20}$$

From the original equation

$$\dot{\bar{x}}_1(t) = A_{11}\bar{x}_1(t) + A_{12}\bar{x}_2(t) \tag{4.21}$$

After eliminating the  $\bar{x}_2$  contribution using Equation 4.19, the modified equation becomes

$$\dot{\bar{x}}_1(t) = \bar{A}\bar{x}_1(t) + A_{12}u(t) \tag{4.22}$$

where

$$\bar{A} = A_{11} - A_{12}Q_{22}^{-1}Q_{21} \tag{4.23}$$

The problem then becomes that of minimising the functional 4.20 that is subject to system 4.22. A unique positive and definite solution  $P_1$  is guaranteed for the ARE, which is associated with the problem defined by equations 4.20 and 4.22.

$$\bar{A}^T P_1 + P_1 \bar{A} - P_1 A_{12} Q_{22}^{-1} A_{12}^T P_1 + \bar{Q} = 0 \tag{4.24}$$

The optimal u-minimization equation is given by

$$u = -Q_{22}^{-1} A_{12}^T P_1 \bar{x}_1 \tag{4.25}$$

This equation is used in equation (4.19) to give

$$\bar{x}_2 = -Q_{22}^{-1} (A_{12}^T P_1 + Q_{21}) \bar{x}_1 \tag{4.26}$$

The comparison with (4.11) gives the expression  $C = Q_{22}^{-1} (A_{12}^T P_1 + Q_{21})$

The value of  $Q$ , in our case, is

$$Q = \text{diag}[5 \times 10^6 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1] \tag{4.27}$$

Then the hyperplane matrix S is

$$S = [42.7043 \quad 3.1623 \quad 76.8433 \quad -67.3893 \quad -43.5951 \quad 1.0000] \quad (4.28)$$

#### 4.3.2 Control law design

To design the control law, Lyapunov stability criteria were used. Here, in our case, the Lyapunov energy function was assumed to be

$$V = \frac{1}{2} s^2 \quad (4.29)$$

Where s is the sliding manifold, in order to meet the Lyapunov stability criteria, the following equation must be satisfied:

$$\dot{V} = s\dot{s} < 0, \text{ (Reaching law)} \quad (4.30)$$

If condition 4.30 is fulfilled, the state trajectory is driven towards the sliding surface in finite time. Therefore, a control variable must be chosen to satisfy the above condition. Consider the equation 4.2,

$$\dot{x}(t) = Ax(t) + Bu(t)$$

The sliding surface is given as  $s(t) = Sx(t)$ . By differentiating the sliding surface, we get

$$\dot{s}(t) = S\dot{x}(t) \quad (4.31)$$

Substitute equation 4.2 into 4.31.

$$\dot{s}(t) = S(Ax(t) + Bu(t)) \quad (4.32)$$

The constant rate reaching law is,  $\dot{s}(t) = -b \text{sign}(s)$

The control signal can be obtained as

$$u = -(SB)^{-1}[SAx + b \text{sign}(s)] \quad (4.33)$$

where b is some positive number, which is taken as 0.01 for our system. The control law is made up of two components:

$$u_c = u_{eq} + u_{sw} \quad (4.34)$$



To derive the equivalent control, equate the sliding surface's derivative to zero.

$$\begin{cases} u_{eq} = -(SB)^{-1}[SAx] \\ u_{sw} = -(SB)^{-1}(bsign) \end{cases} \quad (4.35)$$

#### 4.4 Design STSMC controller

The discontinuous sliding surface derivative and the continuous function of the sliding variable make up the two components of the super-twisting SMC. The continuous function is useful for dealing with the chattering effect caused by the discontinuous function (Ismail, Vina Wahyuni Eka Putranti, and Zool Hilmi, 2017). The super-twisting SMC is depicted as

$$u_{st} = v_1 + v_2 \quad (4.36)$$

Where  $v_1$  is a continuous function and  $v_2$  is the discontinuous time derivative of the sliding variable defined as follows:

$$v_1 = -k_1 |s|^{\frac{1}{2}} \text{sgn}(s) \quad (4.37)$$

$$v_2 = \int -k_2 \text{sgn}(s) \quad (4.38)$$

$$u_{st} = -k_1 |s|^{\frac{1}{2}} \text{sgn}(s) + \int -k_2 \text{sgn}(s) \quad (4.39)$$

Where  $k_1$  and  $k_2$  are small positive quantities and 0.2 and 0.01 respectively. Like the SMC, the STSMC control law is made up of two components:

$$u_c = u_{eq} + u_{sw} \quad (4.40)$$

Equivalent control is derived by equating the sliding surface's derivative to zero. From equation 3.42, the sliding surface's derivative is  $\dot{s}(t) = S(Ax(t) + Bu(t))$  then

$$u_{eq} = -(SB)^{-1}[SAx] \quad (4.41)$$

The control law for the STSMC is as follows:

$$u = -(SB)^{-1}[SAx + -k_1 |s|^{\frac{1}{2}} \text{sgn}(s) - \int k_2 \text{sgn}(s)] \quad (4.42)$$

## 4.5 Linear quadratic regulator (LQR)

A LQR is a subset of optimal control techniques that takes into account both the states of the dynamical system and the control input (Lal Bahadur Prasad, Barjeev Tyagi, and Hari Om Gupta, 2015). Optimal refers to a class of methods for developing a control policy that produces the best possible behaviour that leads to the maximisation of performance in relation to the specified criterion.

The linear state-space equation is obtained after the linearization of the system's nonlinear equations about the upright equilibrium position.

$$\dot{x} = Ax + Bu \quad (4.43)$$

When working with LQR, we assume that all  $n$  states are available for feedback and that the states are completely controllable, and that there is a feedback gain matrix  $K$  (Peng, Yang-Rui Li, and Chao-Chung, 2021). Such that the state feedback control input is given by

$$u = -Kx \quad (4.44)$$

The control  $u = -Kx$  leads to

$$\dot{x} = (A - BK)x \quad (4.45)$$

$K$  must be calculated in order to minimise a given performance criterion or cost function when designing an optimal control law. The following is a quadratic cost function that must be minimized (Dang Anh Duy, Le Hoai Phuong, & Vo Duy Cong, 2021):

$$J = \int (x^T Q x + u^T R u) dt \quad (4.46)$$

Cost function  $J$  can be interpreted as an energy function that optimises the states,  $x$ , and the control signal  $u$  of the system, where  $x$  is the system's state variable weighted by  $Q$  and  $u$  is the system's control input weighted by  $R$ . This cost function is minimised in LQR to find the gain matrix  $K$  for the state feedback controller law. Minimizing the cost function  $J$  results in moving  $x$  to zero with little control energy, ensuring that the systems are stable. The two weighting matrices  $Q > 0$  and  $R > 0$ , which are symmetric and have a positive definite gain, should be chosen. The gain of LQR, vector  $K$ , is given by (Dang Anh Duy, Le Hoai Phuong, & Vo Duy Cong, 2021):

$$K = R^{-1} B^T P = [K_1 \quad K_2 \quad K_3 \quad \dots \quad K_n] \quad (4.47)$$

Where  $n$  is the number of state variables and  $P$  is a positive definite symmetric constant matrix derived from the matrix Algebraic Riccati Equation solution (ARE):

$$A^T P + PA - PBR^{-1}B^T P + Q = 0 \quad (4.48)$$

Since calculating  $K$  manually is difficult, the MATLAB `lqr` instruction is used to solve the gain vector  $K$  for full-state feedback of the system. The  $A$ ,  $B$ ,  $Q$ , and  $R$  matrices are the four arguments for the LQR command.

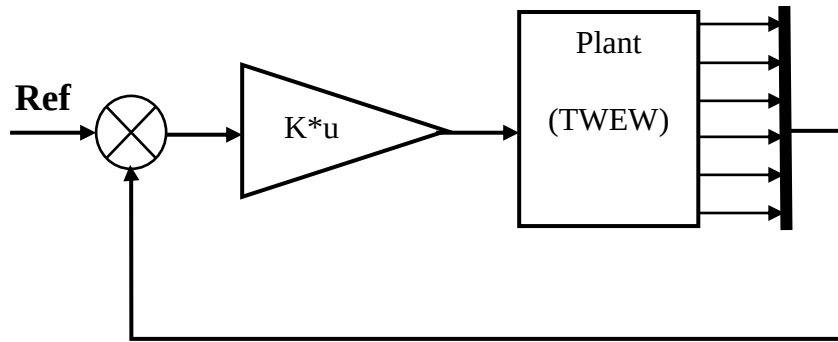


Figure 4.3: LQR controller block diagram (Rupal Roy, 2021)

#### 4.6 Fuzzy-LQR controller

This controller is a hybrid of fuzzy and LQR algorithms. Fuzzy logic control (FLC) is a control technology based on artificial intelligence. It is a Boolean extension that transforms an expert's knowledge into a control system algorithm (instead of 1 or 0, fuzzy logic is based on the degree of truth). FLC consists of three stages: input, processing, and output (George S. Maraslidis, 2022). Before producing the output, the FLC's inputs must pass through the three fundamental processes (stages) of fuzzification, decision-making, and defuzzification (Ahmed Faisal Ghaleb, 2022).

**Fuzzification:** In this stage, the input variable is converted into a linguistic variable using predefined membership functions (MFs).

**Decision-making:** Generate fuzzified output according to the rule set.

**Defuzzification:** converting the fuzzified output into the crisp output required to control the system.

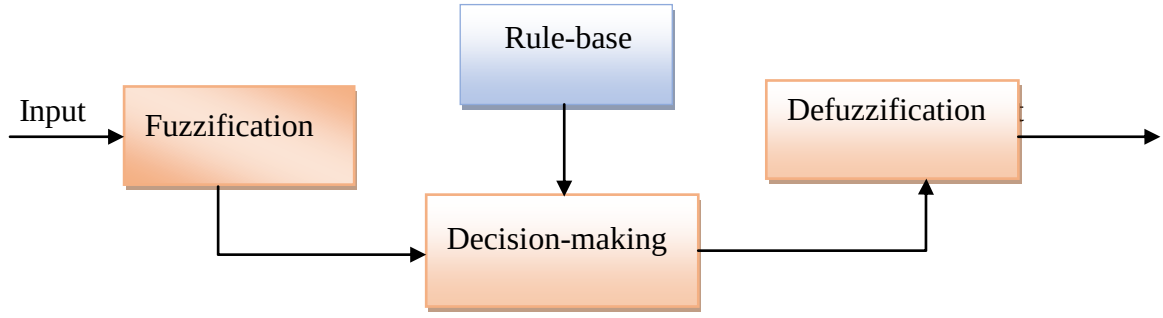


Figure 4.4: Fuzzy logic controller block diagram (Rupal Roy, 2021)

FLC operates by executing rules that relate controller inputs to desired outputs. Our system has six variables. Since the rule is determined by the number of input variables:  $rule = m^n$ , where  $m$  is the number of membership functions and  $n$  is the number of input variables, it is difficult to apply a fuzzy controller to this system. To implement FLC, we use the fusion function. Using the fusion function, six state variables can be merged into two groups to produce two output variables, greatly simplifying the FLC controller (Zied Ben Hazem, Mohammad Javad Fotuh,i and Zafer Bingü, 2020).

#### 4.6.1 Fusion function

As shown in the figure below, the Fusion function transforms multiple variables into comprehensive errors ( $e$ ) and error changes ( $de$ ). There are several approaches to developing the fusion function (Zied Ben Hazem, Mohammad Javad Fotuh,i and Zafer Bingü, 2020). The linear fusion function (LFF) for the TWEW is as follows:

$$F(x) = \begin{bmatrix} k_{x_w} & k_{\theta_1} & k_{\theta_2} & 0 & 0 & 0 \\ 0 & 0 & 0 & k_{\dot{x}_w} & k_{\dot{\theta}_1} & k_{\dot{\theta}_2} \end{bmatrix} \quad (4.49)$$

Then,  $e$  and  $de$  can be calculated as follows:

$$\begin{bmatrix} e \\ de \end{bmatrix} = F(x)x^T \quad (4.50)$$

Where state vector,  $x^T = [x_w \quad \theta_1 \quad \theta_2 \quad \dot{x}_w \quad \dot{\theta}_1 \quad \dot{\theta}_2]$ ,

$$\begin{cases} e = k_{x_1} x_w + k_{x_2} \theta_1 + k_{x_3} \theta_2 \\ de = k_{x_4} \dot{x}_w + k_{x_5} \dot{\theta}_1 + k_{x_6} \dot{\theta}_2 \end{cases} \quad (4.51)$$

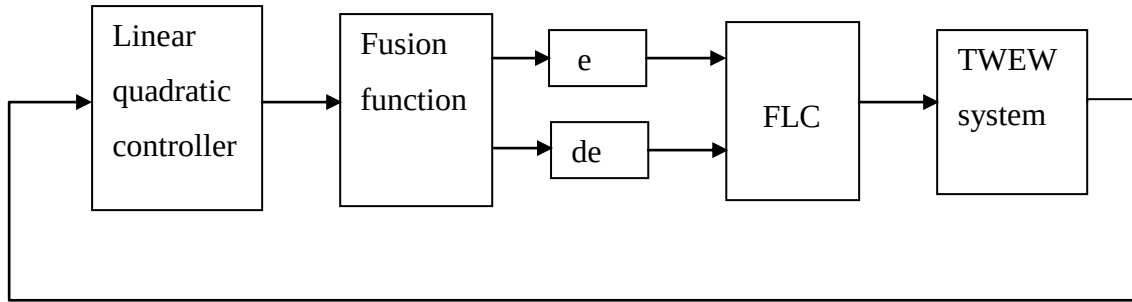


Figure 4.5: Block diagram of FLQR (George S. Maraslidis, 2022)

Figure 4.5 depicts a FLC structure based on the LQR controller. The LQR output is fed to the fusion function, which generates two variables as inputs to the FLC. A Mamdani-type fuzzy membership function is used in this system. The system membership function is shown in Figure 4.6 below. The membership has a triangular shape. Seven fuzzy variables are used in this system. As shown in Table 4.1 below, the 7x7 membership function generates the rules of the system. The fuzzy variable is the same for two inputs, which are error and change of error, and output. And the FLC gain value was estimated and chosen using trial and error as  $k_e = 0.3$ ,  $k_{de} = 0.3$ , and  $k_u = 20$ .

Table 4.1 Fuzzy rules of membership function

| $\begin{matrix} de \\ e \end{matrix}$ | NB | NM | NS | Z  | PS | PM | PB |
|---------------------------------------|----|----|----|----|----|----|----|
| NB                                    | NB | NB | NB | NM | NM | NS | Z  |
| NM                                    | NB | NB | NM | NM | NS | Z  | PS |
| NS                                    | NB | NM | NM | NS | Z  | PS | PM |
| Z                                     | NM | NM | NS | Z  | PS | PM | PM |
| PS                                    | NM | NS | Z  | PS | PM | PM | PB |
| PM                                    | NS | Z  | PS | PM | PM | PB | PB |
| PB                                    | Z  | PS | PM | PM | PB | PB | PB |

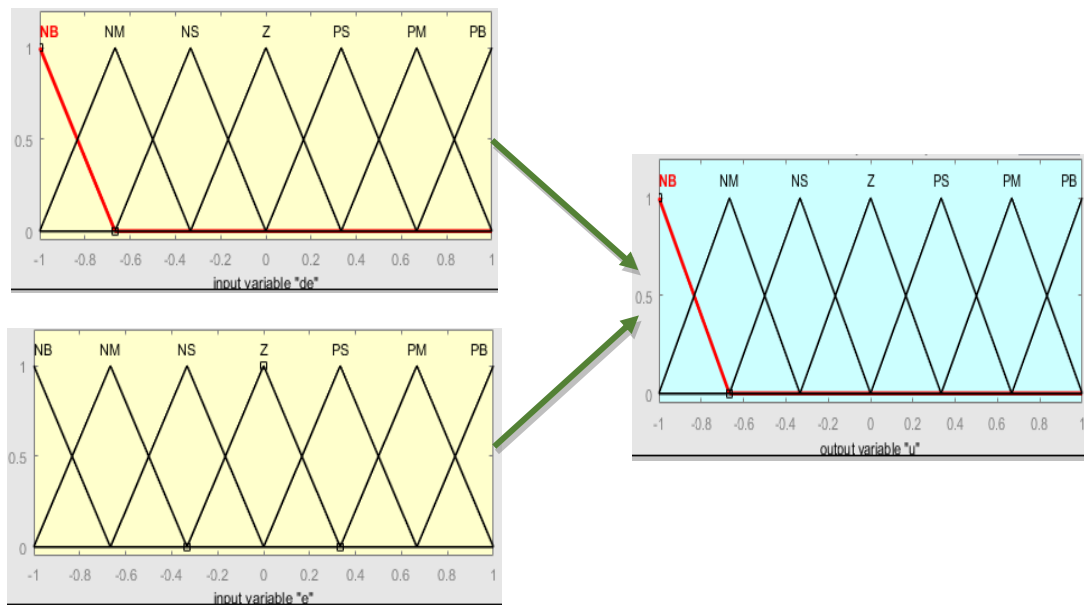


Figure 4.6: Inputs(e and de) and output membership functions with a triangular shape

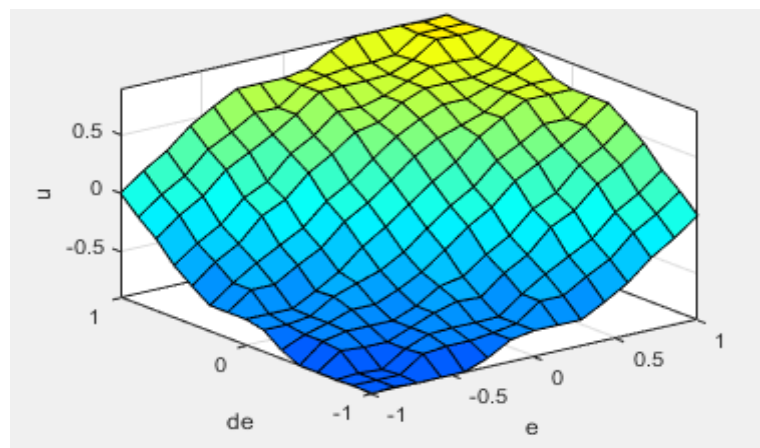


Figure 4.7: Surface view of fuzzy logic

# CHAPTER FIVE

## RESULTS AND DISCUSSIONS

### 5.1 Introduction

Simulating the behaviour of a process or system in the real world over time is known as "simulation." Before something can be simulated, a model must first be created, and this model must capture the essential traits, behaviours, or functions of the chosen physical or conceptual system or procedure. While the model depicts the system as a whole, the simulation depicts how the system behaves over time.

This section presents four controller simulation results to stabilise an electric wheelchair on two wheels and compares the performance of controllers under the influence of disturbance. Sections 5.2 and 5.3 explain the responses of the SMC and STSMC controllers and the LQR and FLQR, respectively. Lastly, Section 5.4 presents the comparison of all four controllers.

Simulations have been conducted in MATLAB/Simulink version R2017a using the system's dynamic model to validate the performance of the proposed controller. The system mainly needs to control three of six state variables to a certain desired value, which are the wheel linear displacement, link 1, and link 2 angles. In Simulink, the signal response of all state variables can be shown, but it must modify the matrix  $C$  to get all state variables as the outputs, i.e.,  $C$  equals a  $3 \times 3$  identity matrix for three state variable output and a  $6 \times 6$  identity matrix for six state variable output.

### 5.2 SMC and STSMC closed loop simulation results

The simulation results of two controllers, SMC and STSMC, to balance a two-wheeled wheelchair upside down on a freely rotating wheel are discussed in this section. The two-wheeled self-balancing wheelchair's system, as described in Chapter 3, is based on the idea of an inverted pendulum. Stabilizing the wheelchair depends on the base on which it stands. The sensor determines the position of the pendulum and sends a signal to a controller, which makes some computations and sends the signal to actuators to move the wheel horizontally in order to keep the system stable.



Figure 5.1: Wheel linear displacement response using STSMC and SMC with an initial 0m

When both angle links 1 and 2 are in the upright position at 0 degrees and the wheel movement is close to zero, the system is said to be stable. The results of the closed-loop simulation for the displacement of the wheel are shown in Figure 5.1. From the result, it can be seen that the STSMC for linear motion of the wheel outperforms the SMC controller because the wheel does not deviate largely to balance the system. Small positional wheel deviations and chattering were seen in systems with SMC.

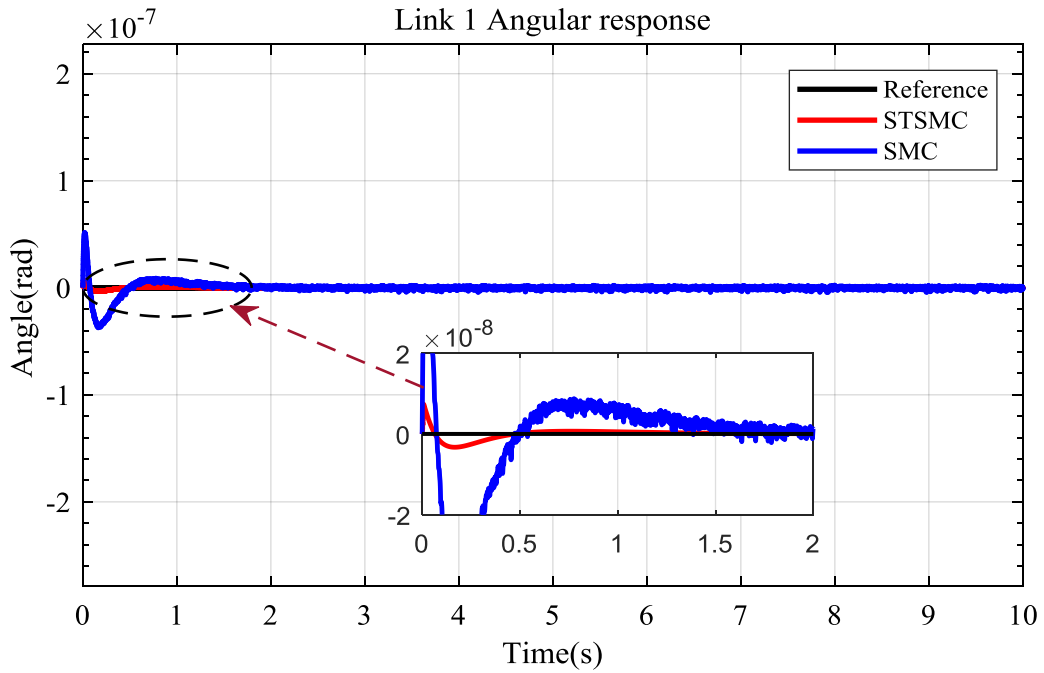


Figure 5.2: Lower pendulum angular response for SMC and STSMC with an initial 0rad



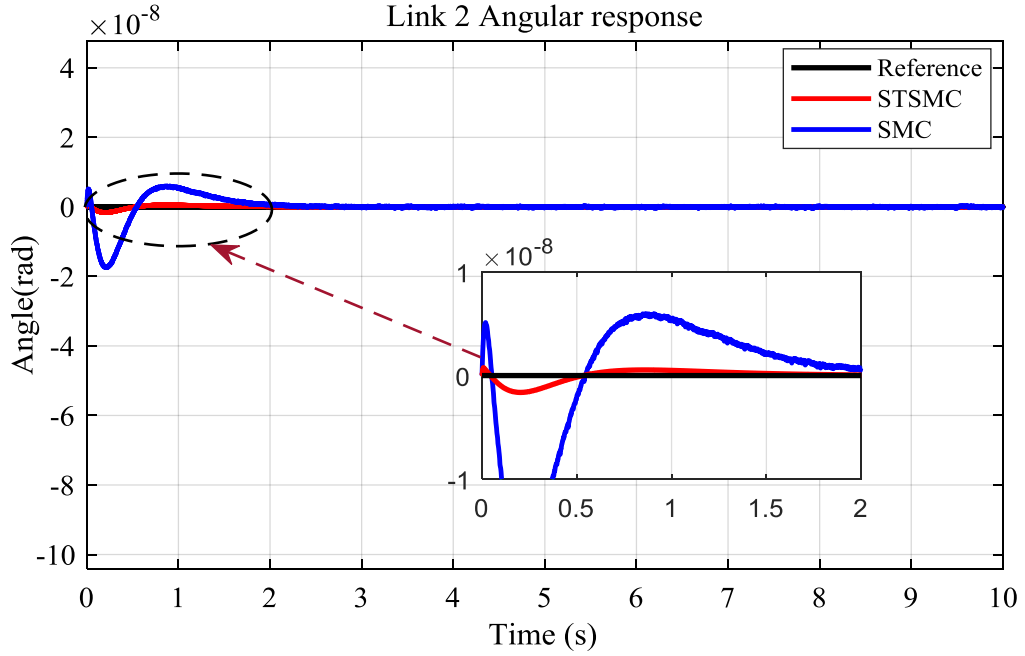


Figure 5.3: Upper pendulum angular response for SMC and STSMC with initial 0rad

Figures 5.2 and 5.3 above show the lower pendulum angle (link 1) and the upper pendulum angle (link 2) responses of a two-wheeled wheelchair for an initial condition of  $x = [0 \ 0 \ 0 \ 0 \ 0 \ 0]^T$ .

According to the simulation results, the system can be stabilised by both a sliding mode controller and a super twisted sliding mode controller. As shown in the figures, the angular displacements of the wheelchair angles converge to zero degrees for both controllers, but chattering is visible at SMC in the zoomed views of Figures 5.3 and 5.3. However, in the case of the STSMC controller, it is significantly suppressed, and the convergence pendulum angle is faster. And also, the pendulum's angle with the STSMC controller has a lower overshoot and settling time than the pendulum with the SMC.

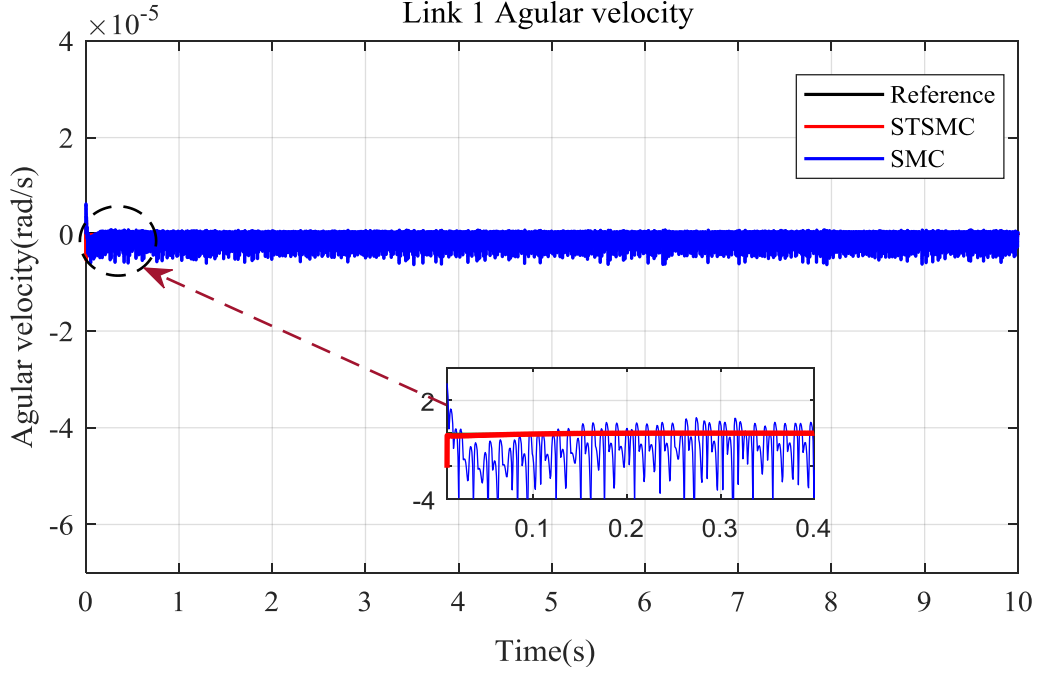


Figure 5.4: Response to angular velocity of link 1 using SMC and STSMC

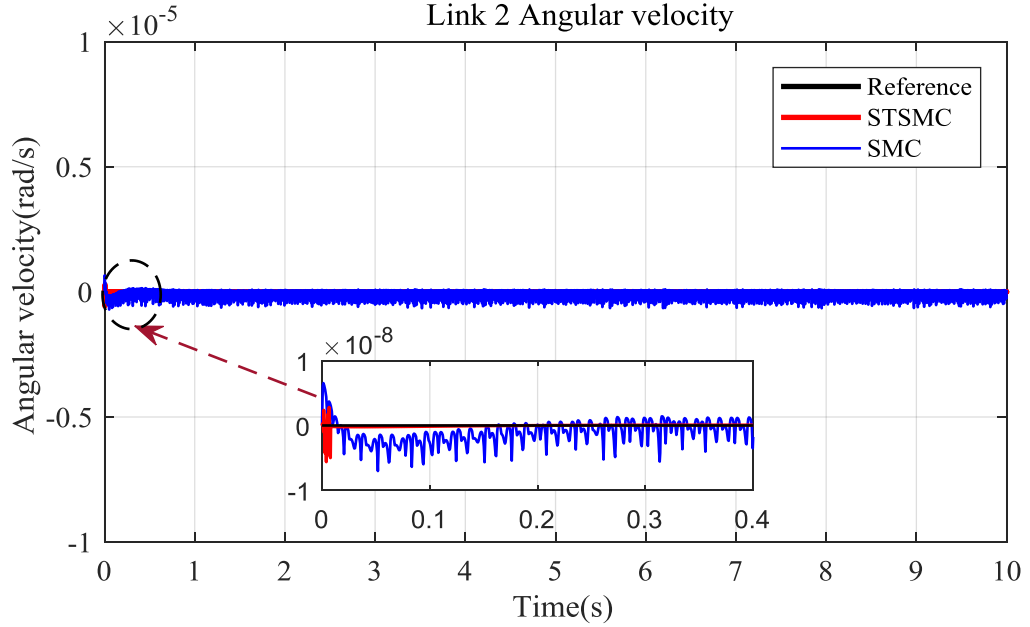


Figure 5.5: Response to angular velocity of link 2 using STSMC and SMC

As observed in Figures 5.4 and 5.5, for STSMC, the angular velocities of the upper and lower pendulum start to increase and return to zero with a small overshoot, a small settling time, and fast convergence, but the responses with the SMC have a longer settling time compared to STSMC, and as observed in zoomed view, the response with an SMC has too much chattering. Thus, the STSM controller is more efficient than the SMC controller.

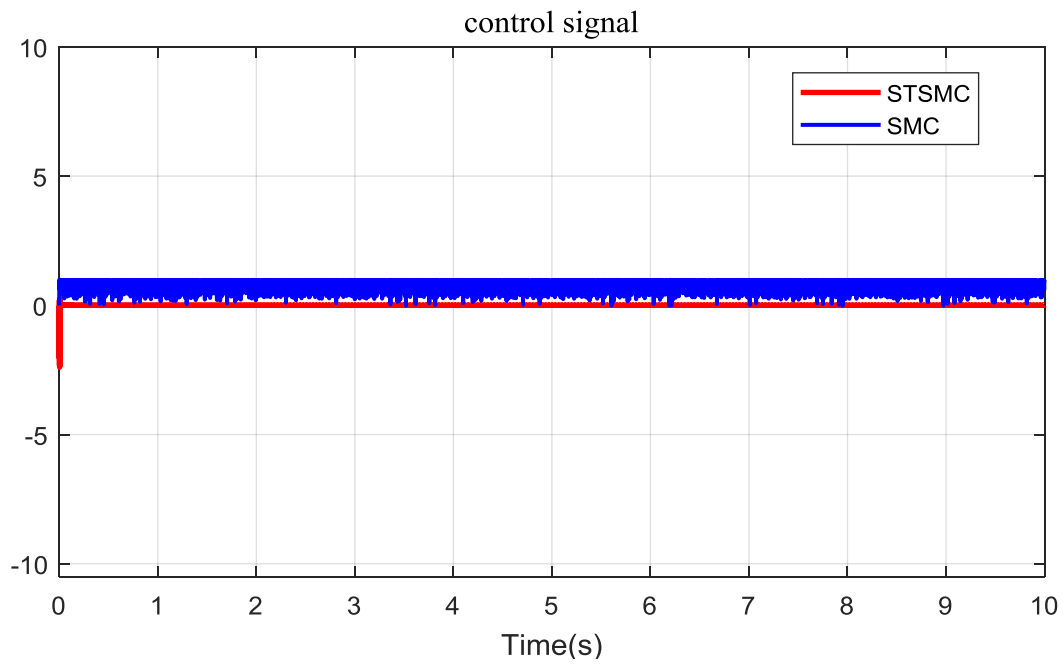
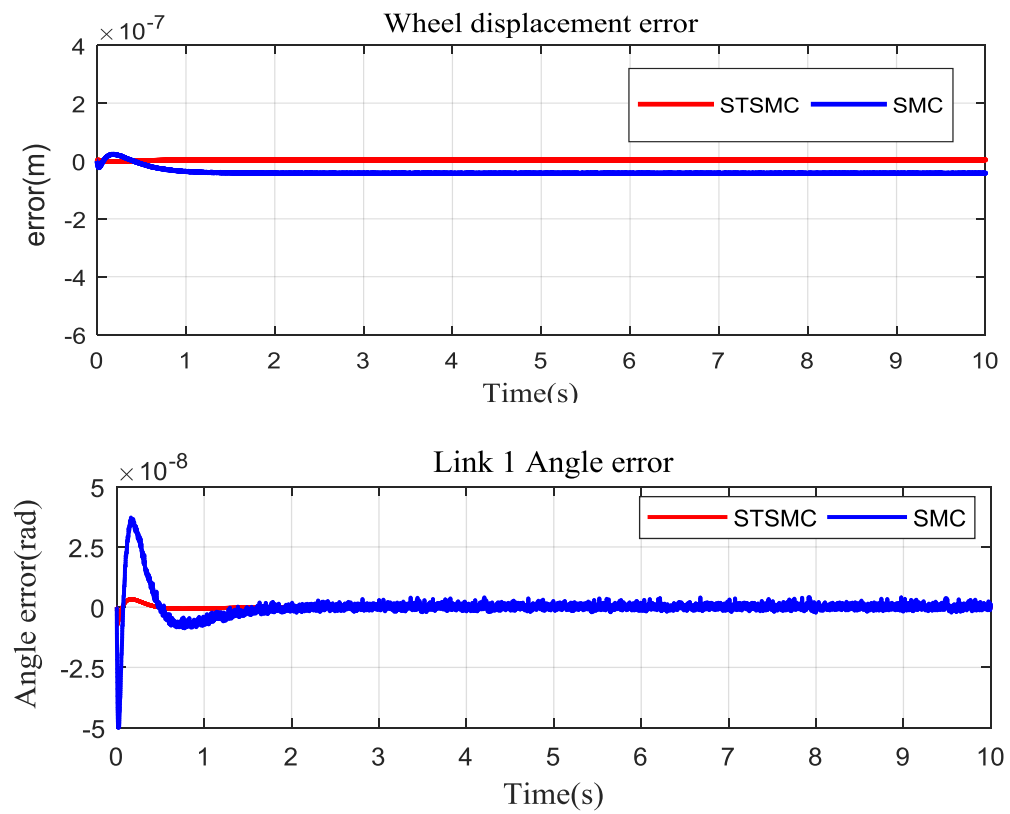


Figure 5.6: Control signal using STSMC and SMC

From the obtained result in Figure 5.6, chattering effects are observed on the control input signal of the system using SMC.



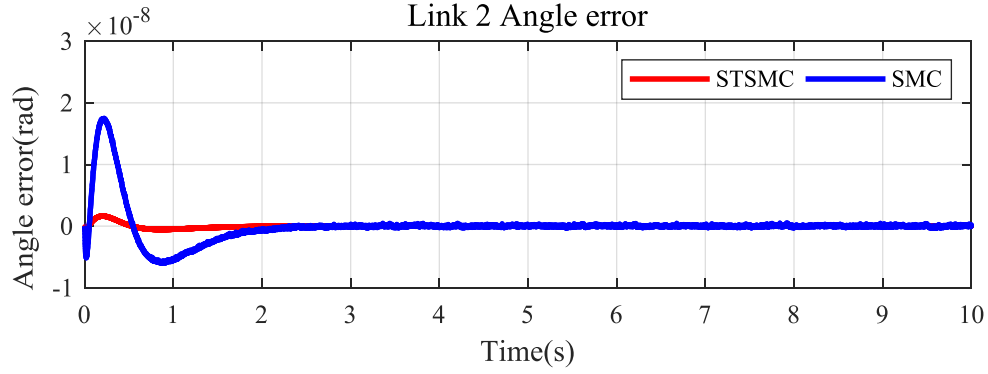


Figure 5.7: Wheel displacement error and angle error of links 1 and 2 using STSMC and SMC

According to Figure 5.7, the wheelchair's wheel displacement linear position error and links 1 and 2 angular position error converged quickly into zero with a small overshoot using both SMC and STSMC, except the effect of chattering is seen in SMC.

### 5.2.1 Test Results for Different Initial Values

For all computations up to this point, we have used an initial value of zero, meaning that the angles of links 1 and 2 were at their initial positions and both at zero degrees. An analysis of the four primary scenarios for initial system values and the measured outputs for controllers is presented below. The four common scenarios are as follows:

- Both pendulums start at a negative angle.
- Both pendulums start at a positive angle.
- The first pendulum starts at a negative angle, while the second pendulum starts at a positive angle.
- The first pendulum starts at a positive angle, while the second pendulum starts at a negative angle.

**Scenario 1:** Both link 1 and link 2 start at a negative angle (-5 rad).

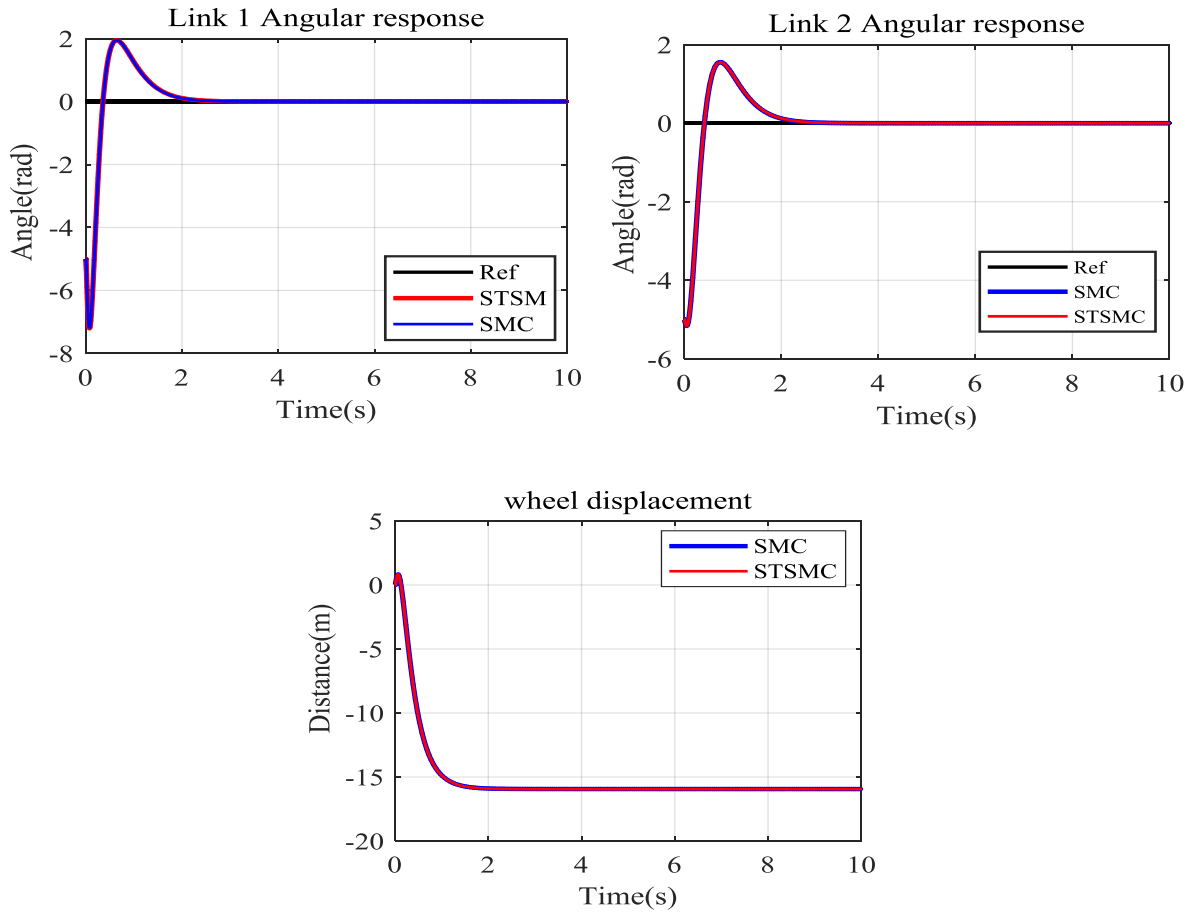


Figure 5.8: System response with initial  $\theta_1 = -5\text{rad}$ ,  $\theta_2 = -5\text{rad}$ , and reference  $0\text{rad}$  using SMC and STSMC

Positive angles move clockwise, while negative angles move anticlockwise. Figure 5.8 shows the response of the system when both Link 1 and Link 2 begin at a negative angle (-5 rad). This means that both angles are inclined backward. As explained in the previous chapter, to maintain the stability of the system, the wheel of the system accelerates in the direction of inclination. Here, according to the results, the wheel of the system moves backward by 16 metres in less than 2 seconds to balance the system.

**Scenario 2:** Both link 1 and link 2 begin at a positive angle (5rad).

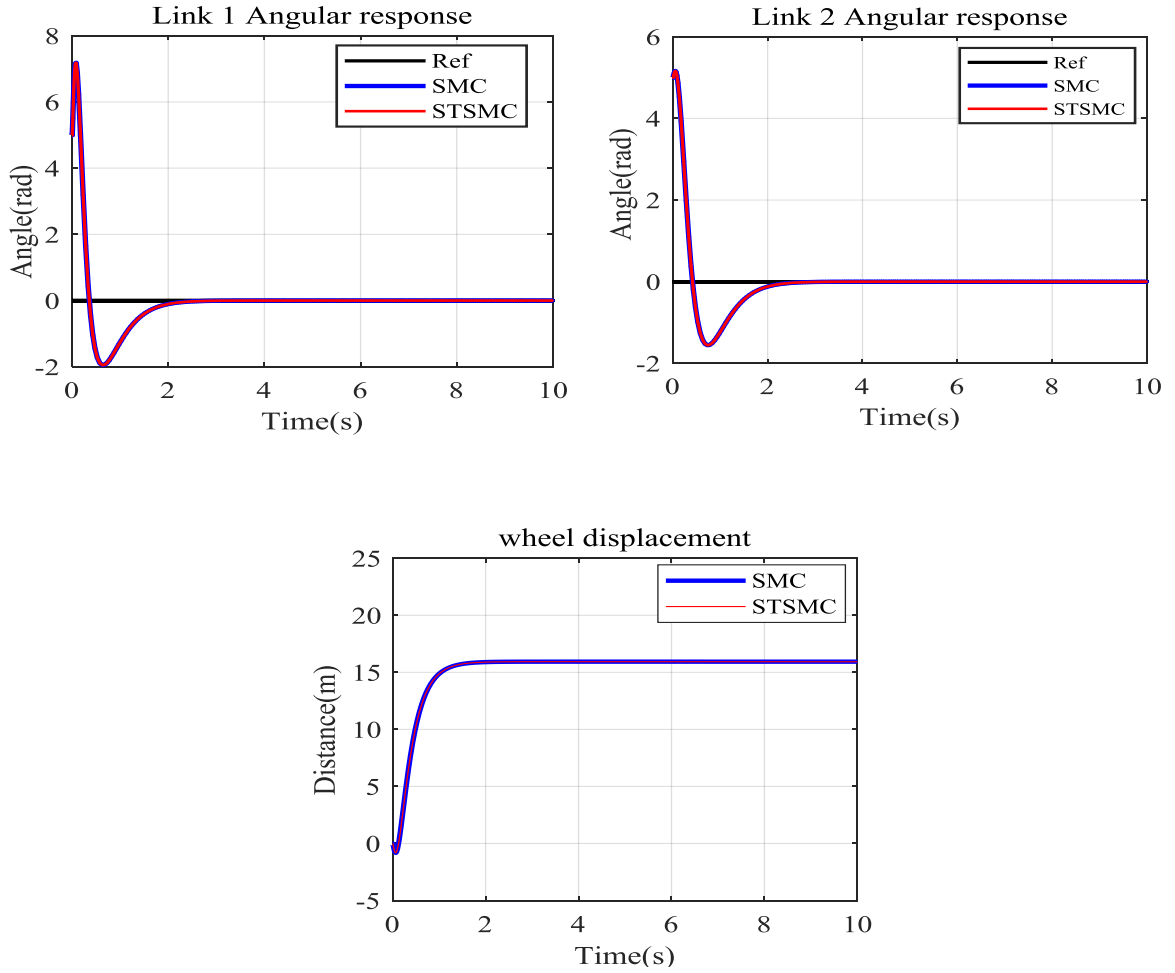


Figure 5.9: Link 1 angle, Link 2 angle and wheel displacement responses for SMC and STSMC with initial  $\theta_1 = 5$  rad,  $\theta_2 = 5$  rad, and reference 0 rad

Figure 5.9 shows the result of the system when both links 1 and 2 begin at a positive angle (5 rad). Because both pendulums were tilted forward, the system's wheel moved forward 16 meters to maintain stability.

**Scenario 3:** The first pendulum begins at a negative angle, while the second pendulum begins at a positive angle ( $\theta_1 = -5\text{rad}$ ,  $\theta_2 = 5\text{rad}$ ).

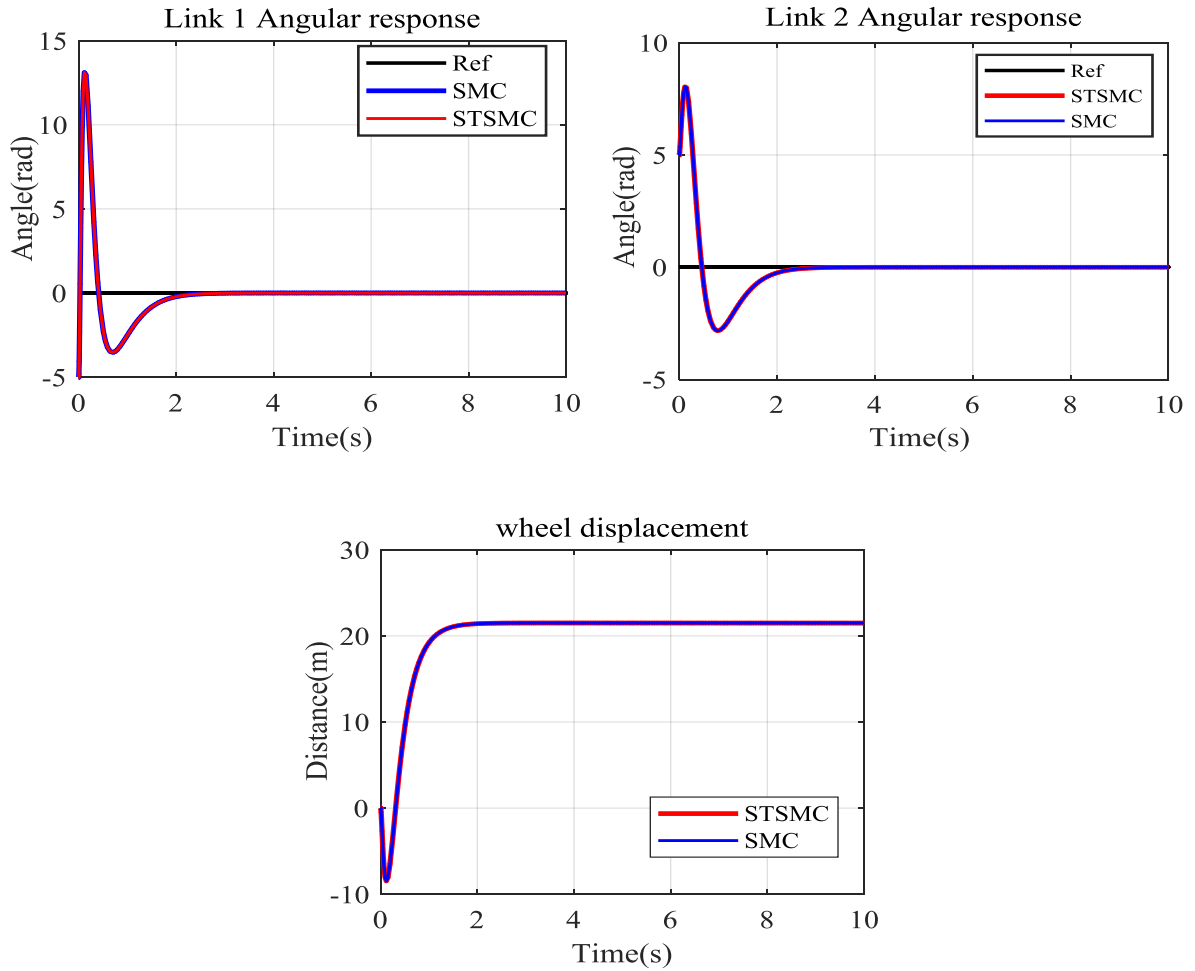


Figure 5.10: Link 1 angle, Link 2 angle and wheel displacement responses for SMC and STSMC with initial  $\theta_1 = -5\text{rad}$ ,  $\theta_2 = 5\text{rad}$  and reference 0rad

Figure 5.10 shows the result of the system when Link 1 starts at a negative angle (-5 rad) and Link 2 at a positive angle (5 rad). The lower pendulum is tilted backwards, and the upper pendulum is tilted forwards. From the result, we observed that the wheel of the wheelchair was accelerated forward by 21 metres in the direction of the upper pendulum inclination. This indicates that the direction of the upper body is more critical to the stability of the system than that of the lower body.

**Scenario 4:** The first pendulum begins at a positive angle, while the second pendulum begins at a negative angle ( $\theta_1 = 5\text{rad}$ ,  $\theta_2 = -5\text{rad}$ ).

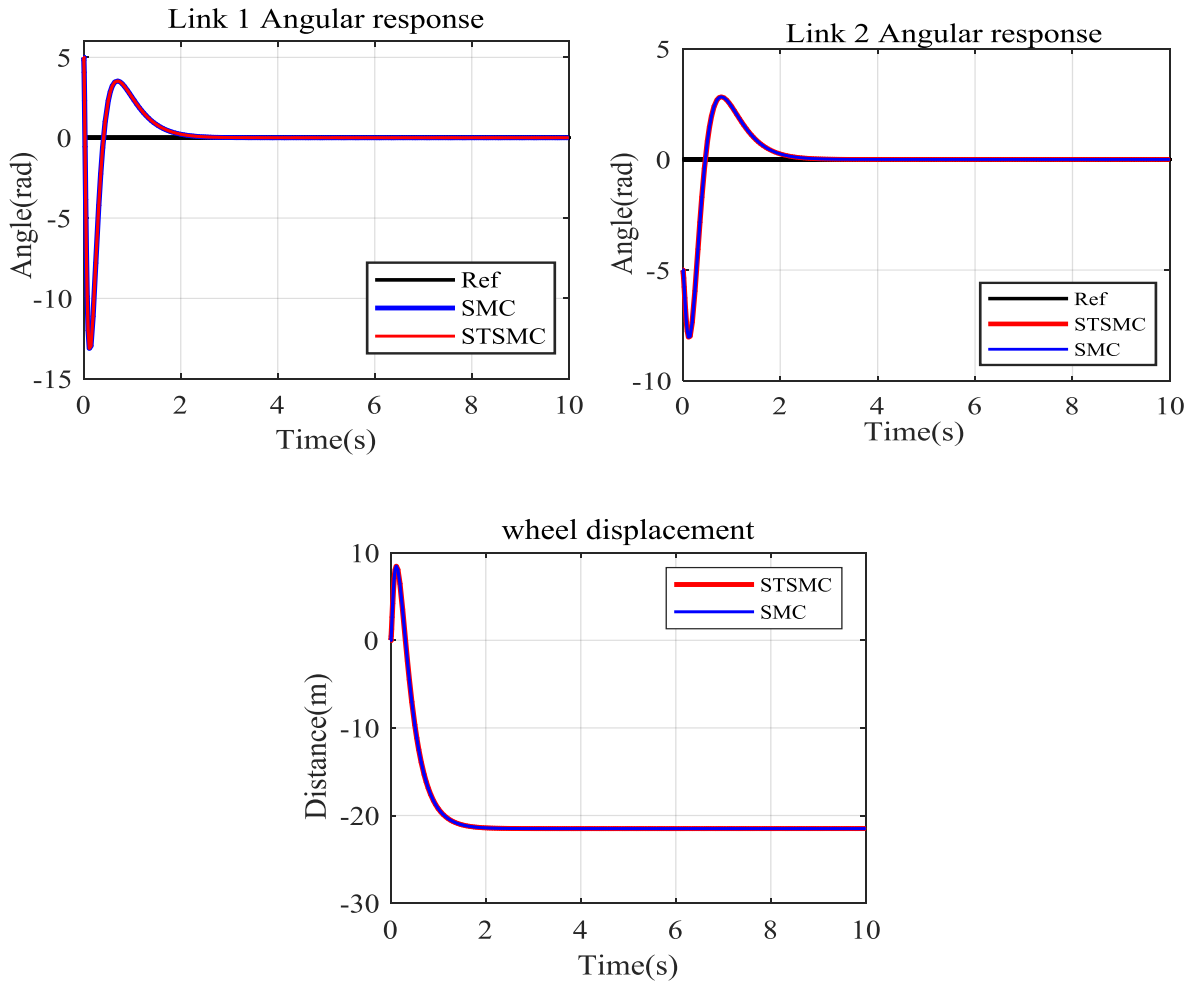


Figure 5.11: Link 1 angle, Link 2 angle and wheel displacement responses for SMC and STSMC with initial  $\theta_1 = 5\text{rad}$ ,  $\theta_2 = -5\text{rad}$  and reference  $0\text{rad}$

Figure 5.11 shows the result of the system when link 1 starts at a positive angle (5 rad) and link 2 starts at a negative angle (-5 rad). Here, the lower pendulum is tilted forward and the upper pendulum is tilted backward. From the result, we observed that the wheel of the wheelchair accelerated backwards by 21 meters.

Scenarios 3 and 4 are the most difficult to control compared to scenarios 1 and 2, where both angles were launched from the same direction. As can be seen in the figures above, the wheel of the system moves a long way to stabilise the wheelchair, so it takes a lot of energy.



Generally

- ✓ It can be seen that both the SMC and STSM controllers are capable of achieving robust two-wheel wheelchair stabilisation with a low percentage of overshoot for variations in the initial tilt angle.
- ✓ Both SMC and STSMC controllers are able to stabilise the system in less than 2.5 seconds.
- ✓ It can be seen here that displacement reaches its final value in less than 3 seconds and that the system is more stable.

### 5.2.2 Test results for disturbance

Wheelchair stability is influenced by a number of variables, including wheel weight, rod or surface condition, seat position, and user weight. In this section, the robustness and the ability of the controller to balance an unstable two-wheel wheelchair in the presence of external disturbances are evaluated and analysed using MATLAB/Simulink implementation results. A square perturbation signal is added to the system as a rod surface disturbance (Stone).

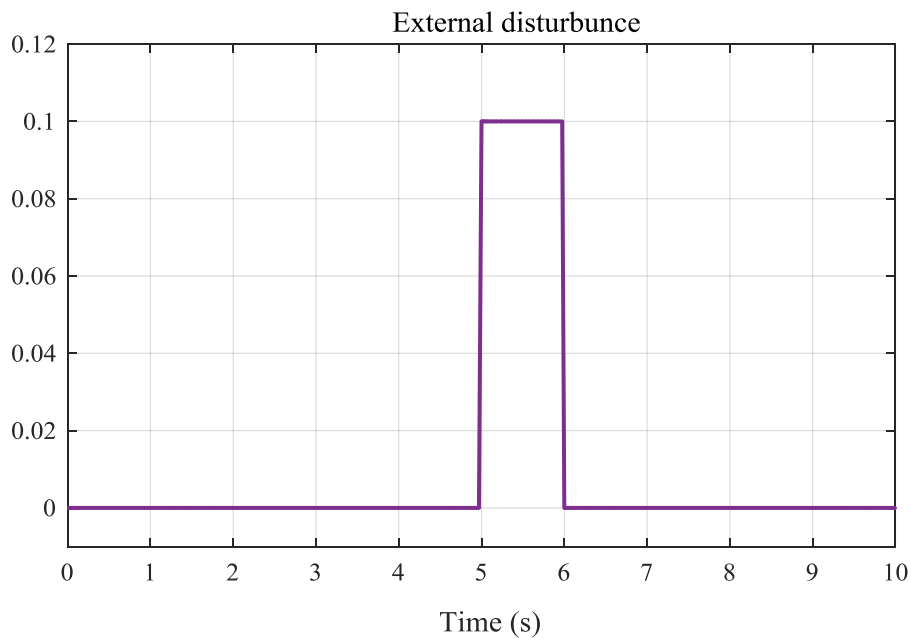


Figure 5.12: System perturbation from outside sources

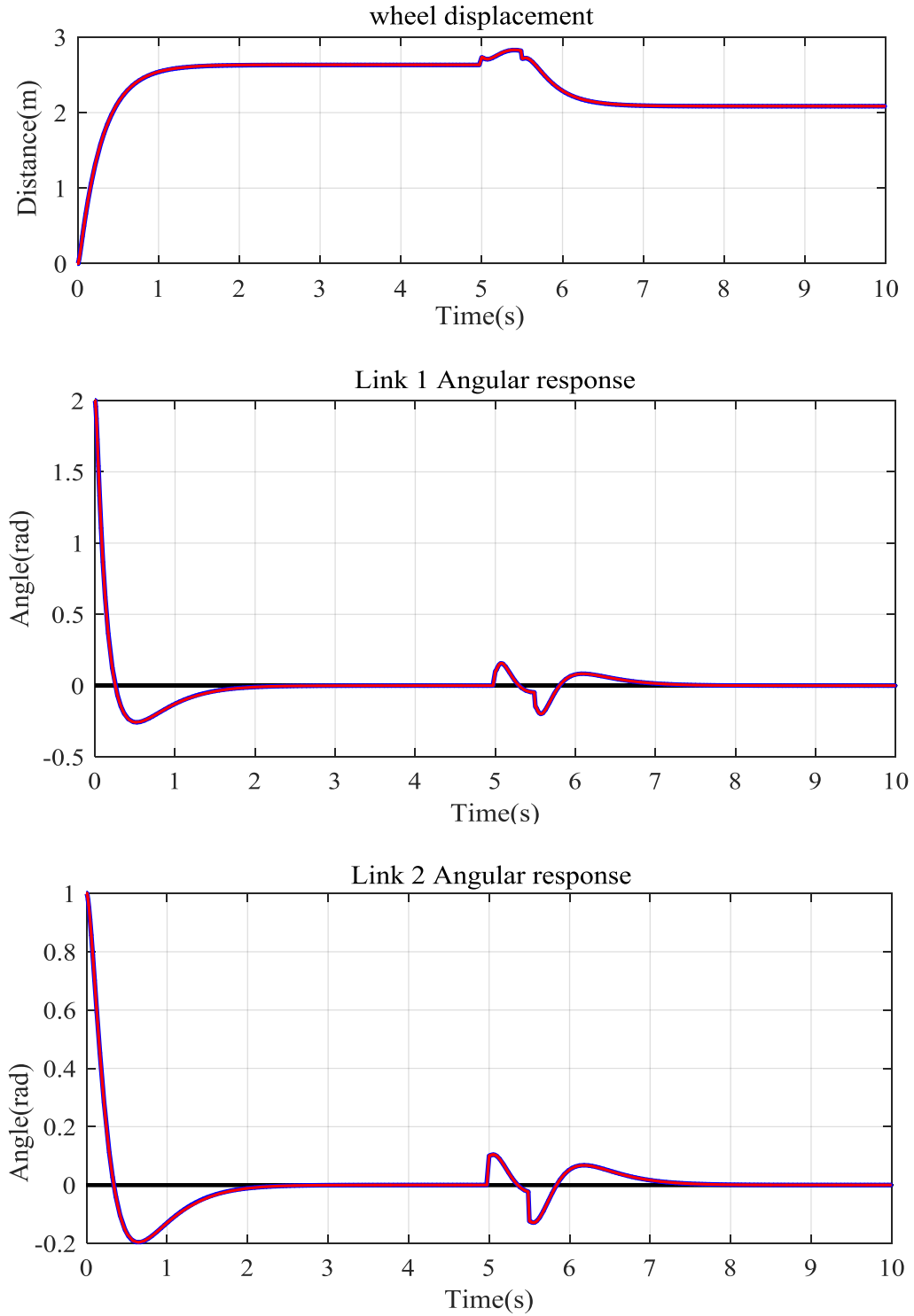


Figure 5.13: Responses of the wheel displacement, link 1 and link 2 to an external disturbance with initial  $\theta_1 = 2rad$  and  $\theta_2 = 1rad$

The simulation result of the system with disturbance using STSMC and SMC is shown in Figure 5.13. Here, after 5 seconds, the disturbance is applied to the system, and the

system's robustness has been taken into account using SMC and STSMC. The figures clearly show that both controllers can maintain system balance despite the disturbance.

### 5.2.3 Test STSMC for weight variation

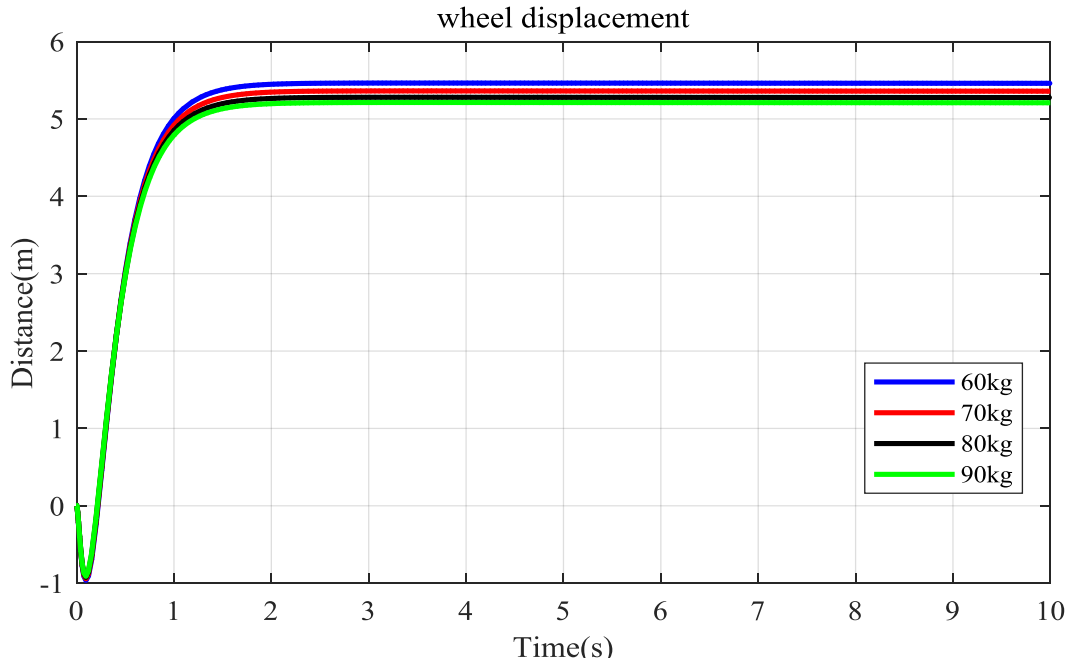


Figure 5.14: Wheel displacement response for 60 kg to 90 kg of human weight variation

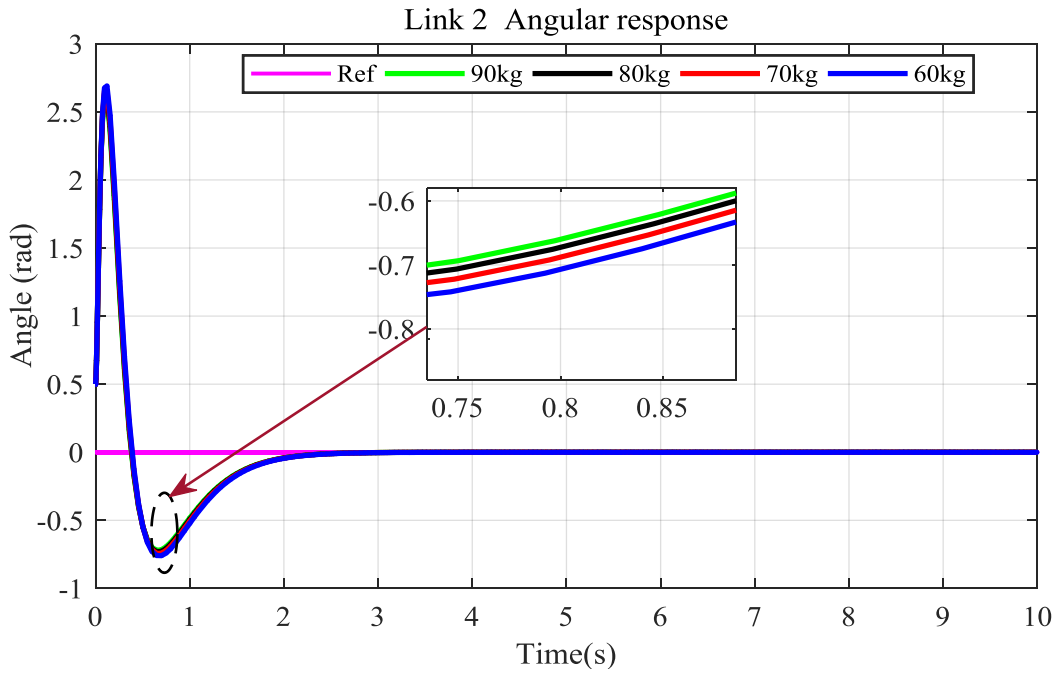


Figure 5.15: Link 1 angular response for human weight variation ranging from 60kg to 90kg with initial  $\theta_1 = 0.5rad$

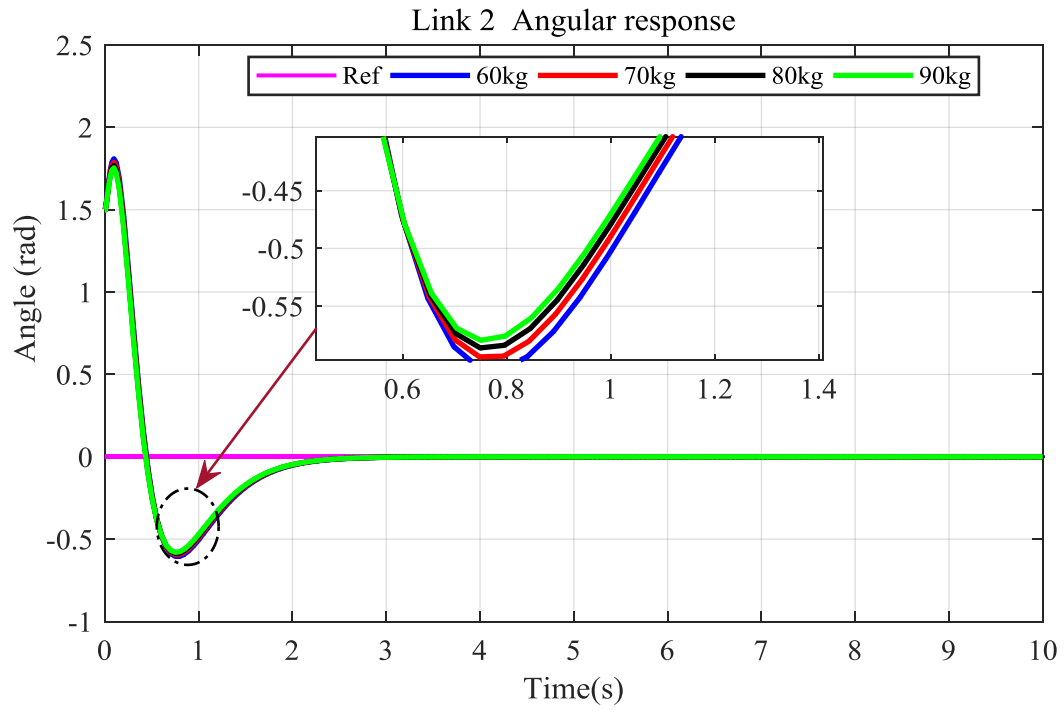


Figure 5.16: Link 1 angular response for human weight variation ranging from 60kg to 90kg with initial  $\theta_2 = 1.5rad$

In this work, a wheelchair can carry a human load of up to 100 kg. Here in this section, the performance of the controller is tested for human weight variation. The results in Figures 5.14, 5.15, and 5.16 show that the STSMC is robust against mass variation from 60 kg to 90 kg.

### 5.3 Simulation result using LQR and FLQR

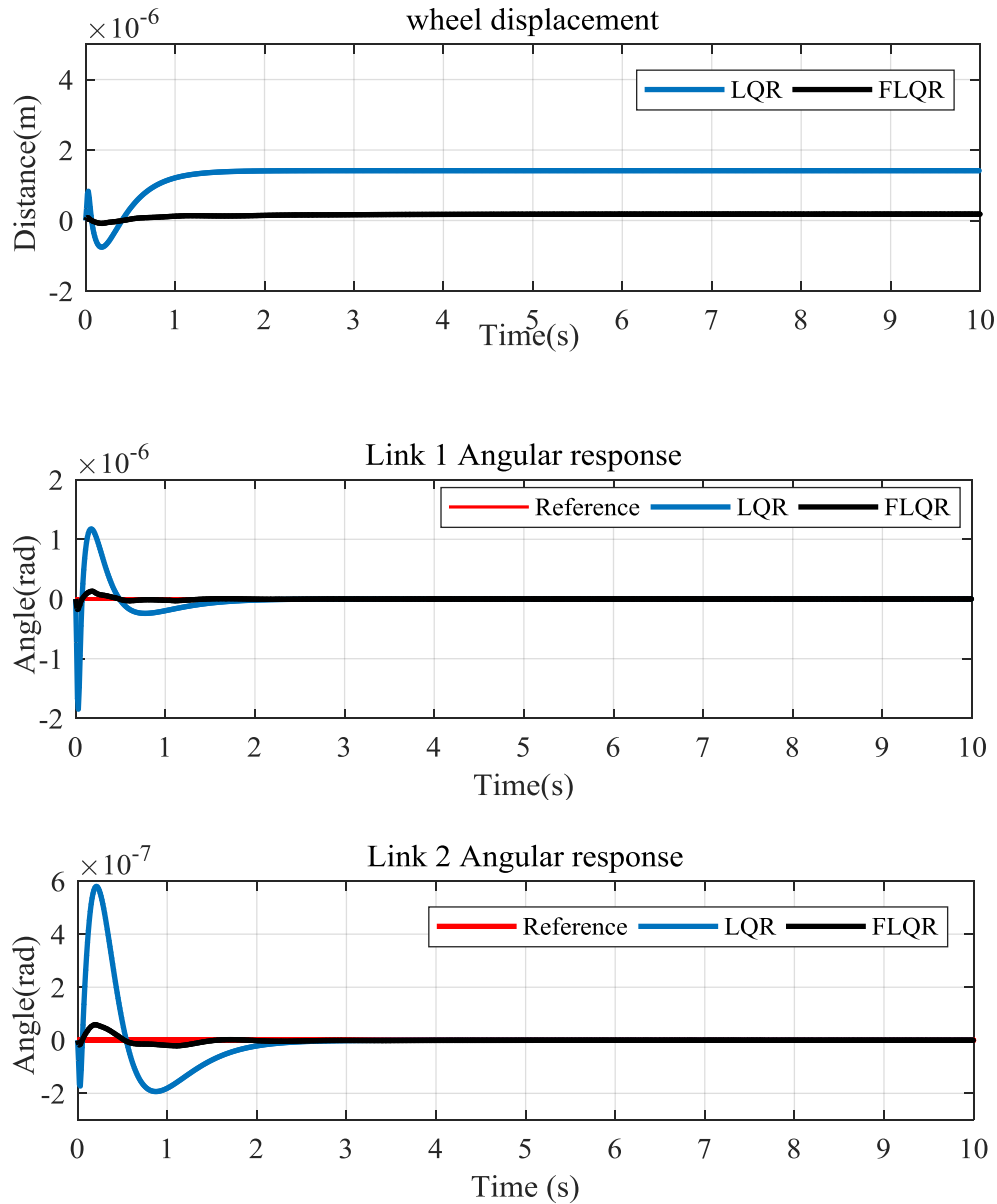


Figure 5.17: A wheel's linear displacement, link 1 and link 2 angular responses for LQR and FLQR

The above Figure 5.17 shows the closed loop simulation results for wheel displacement, lower pendulum, and upper pendulum angular responses of a two-wheeled wheelchair. According to the simulation results, the system can be stabilised by the LQR controller and fuzzy LQR. Both link 1 and link 2 of the two-wheeled wheelchair angles begin to increase and return to zero degrees for the two controllers, as shown in the figures, but the pendulum with the LQR controller has a higher overshoot and a longer settling time than the pendulum with the FLQR. According to the simulation results, the system with the

fuzzy LQR controller performs better in terms of less overshoot, quicker pendulum angle convergence, and less wheel position deviation.

### 5.3.1 Results under the influence of disturbance

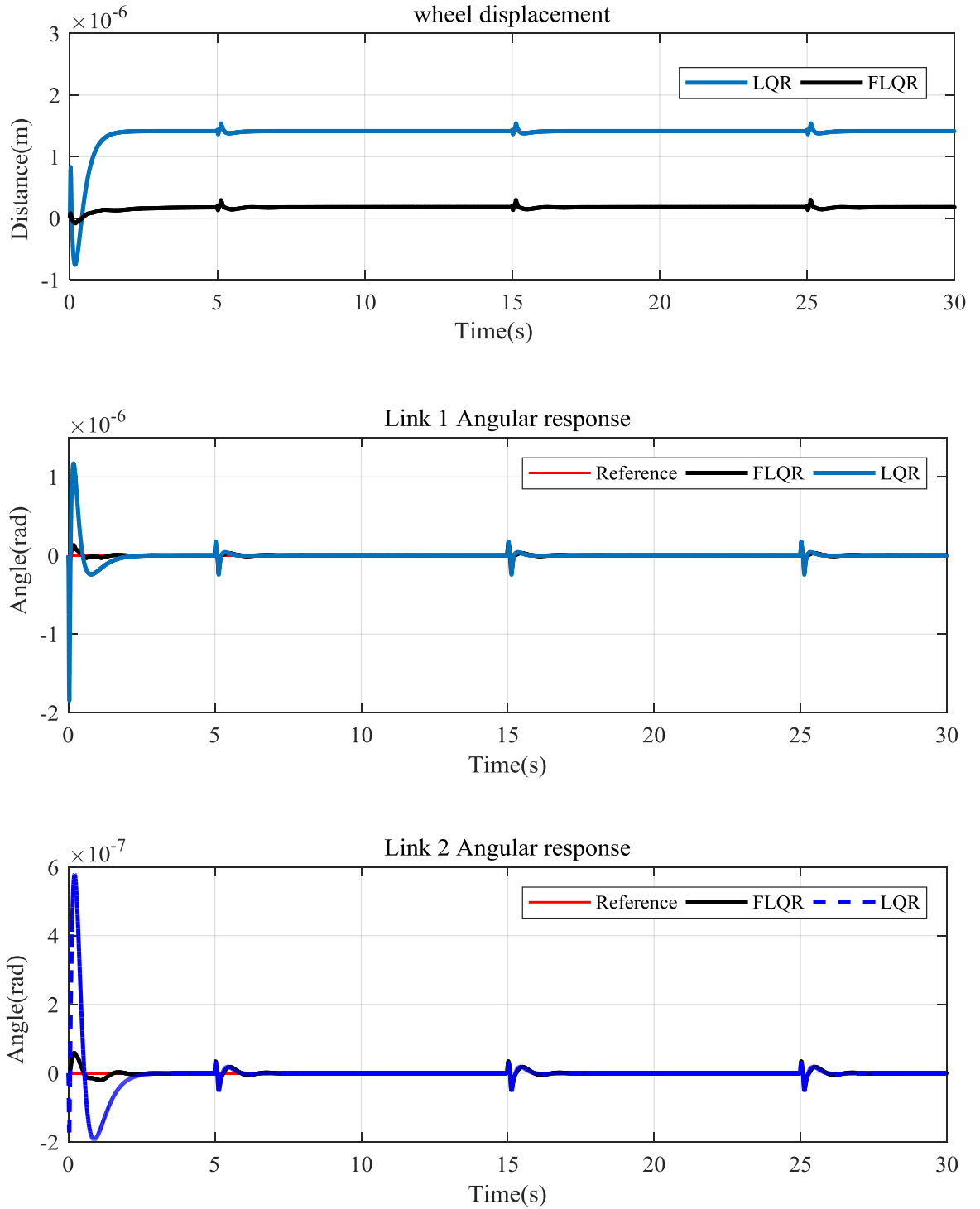


Figure 5.18: A wheel's linear displacement and links 1 and 2's angular responses for LQR and FLQR under the influence of disturbance

Figure 5.18 depicts the simulation result for the system subjected to external disturbances after 5 seconds in a period of 10s. According to the obtained results, the stability of the two-wheel electric wheelchair was successfully maintained under the influence of disturbances using both LQR and FLQR. The FLQR controller outperforms the LQR controller.

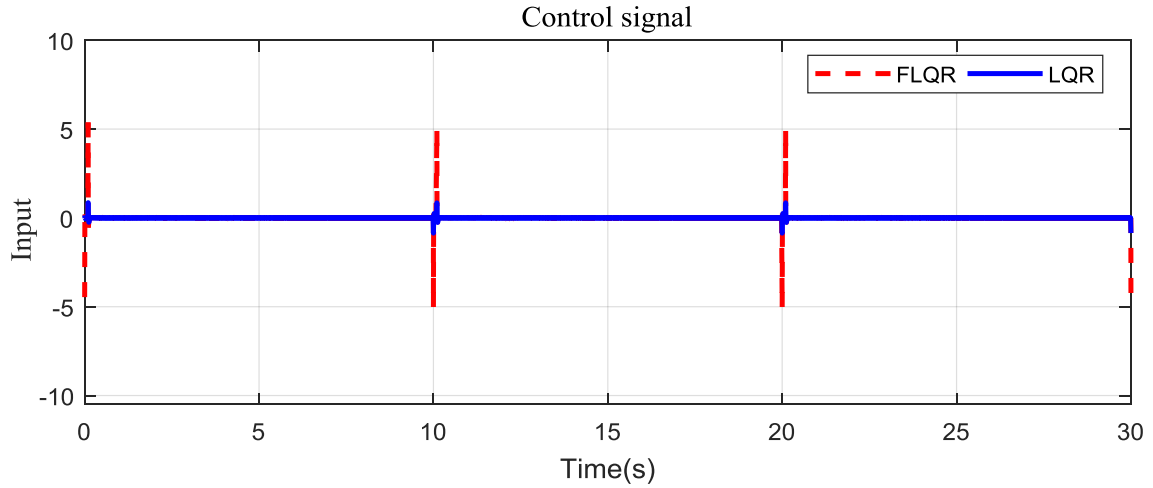


Figure 5.19: Control signal of FLQR and LQR under external disturbance

Figure 5.19 shows the control signal of FLQR and LQR controllers under external disturbances. According to the results, the LQR-controlled system requires less effort or energy to operate.

From the above figures, we conclude that the fuzzy logic controller is the best in terms of settling time but that LQR is the best in terms of cost.

#### 5.4 Comparison of controllers

In this section, the simulation results of the modelled linear and non-linear controllers (SMC, STSMC, LQR, and FLQR) for stabilisation of the two wheel wheelchair are discussed. According to the simulation results, a comparative study of all controllers is given.

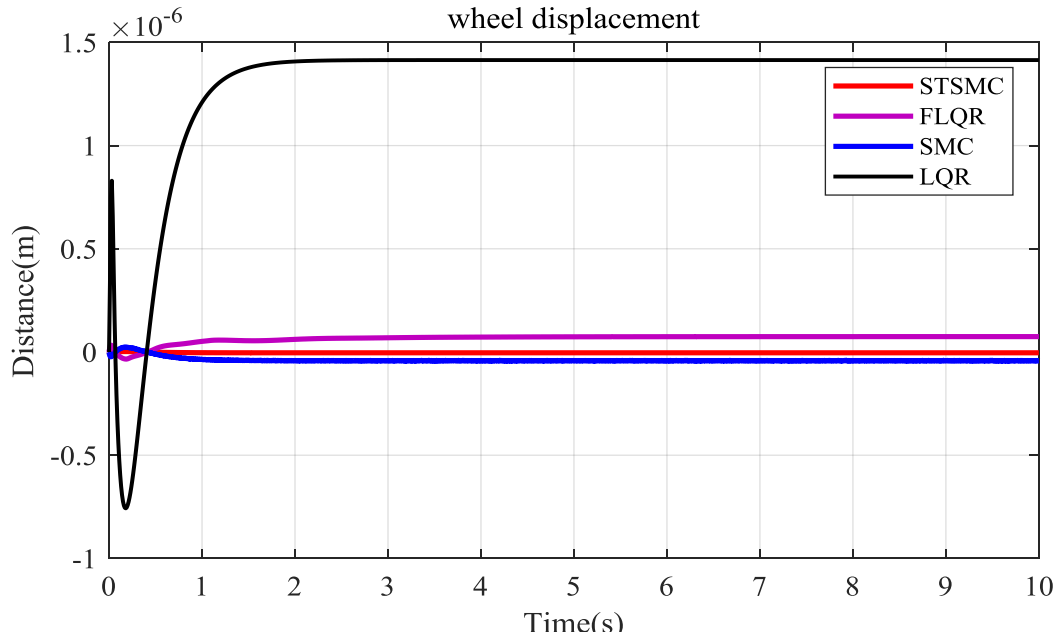


Figure 5.20: wheel displacement responses for all controllers

Figure 5.20 shows that the linear motion of wheel displacement for STSMC produces better performance than the other. It is because the system does not travel far from 0 meters when the upper and lower pendulum angles do not tilt forward and backward. The graph also shows that LQR is more deviated than the other.

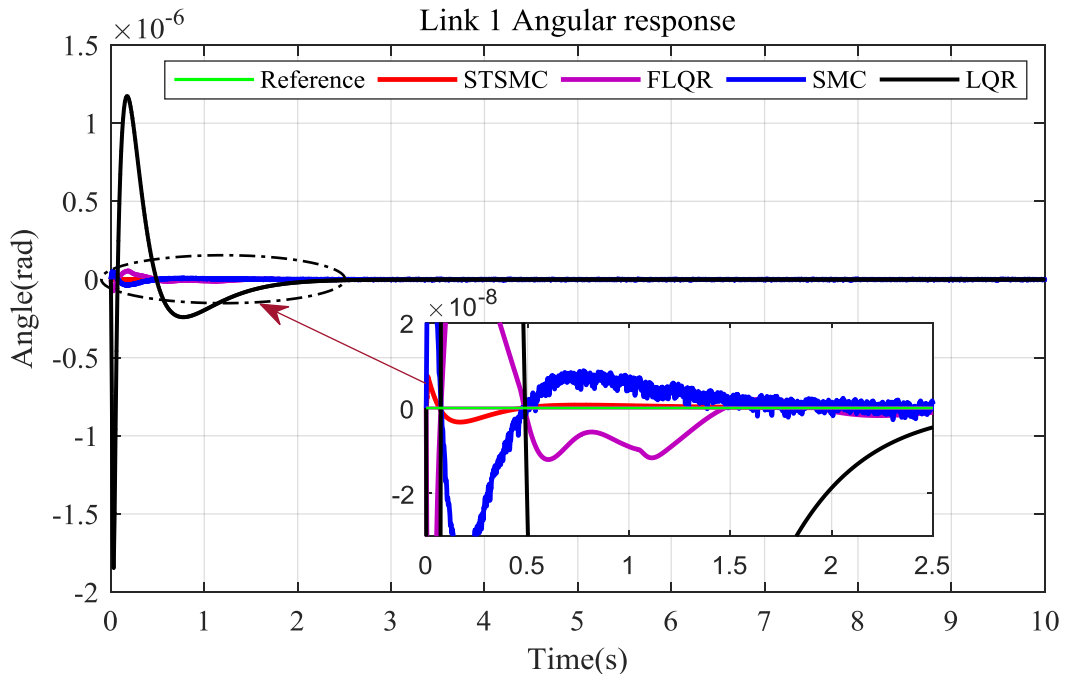


Figure 5.21: Link 1 angular responses for all controllers



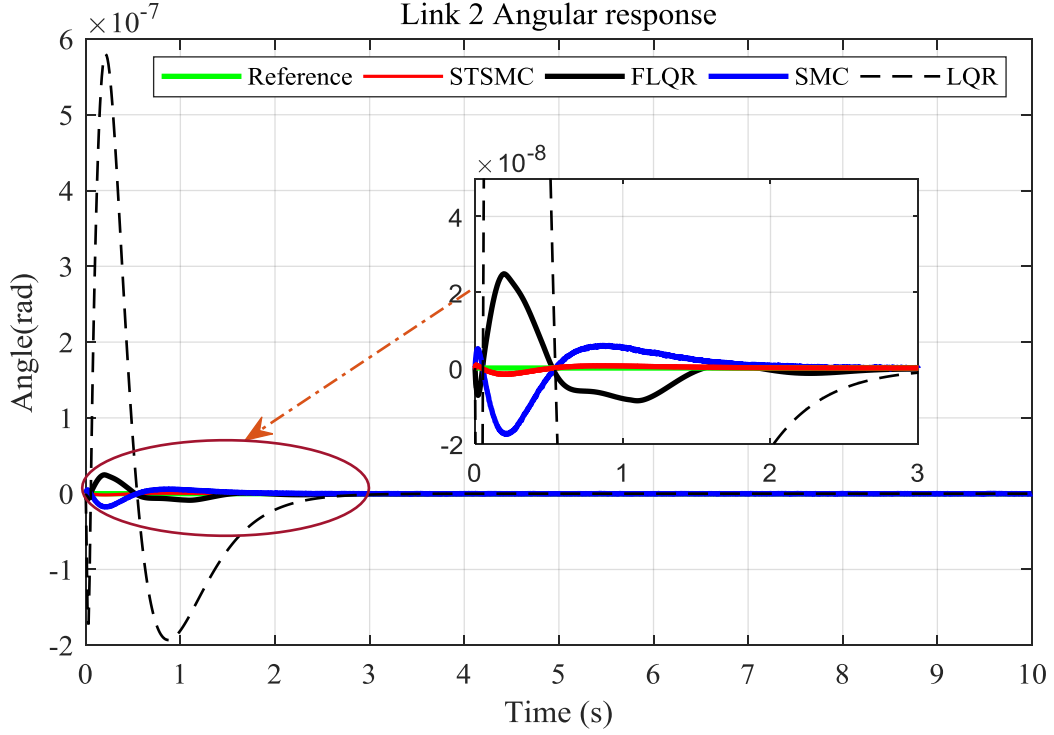


Figure 5.22: Link 2 angular responses for all controllers

Figures 5.20, 5.21, and 5.22 show wheel displacement, the lower pendulum angle, and the upper pendulum angle response, respectively. According to the simulation results, a two-wheeled wheelchair can be stabilised by the LQR controller, fuzzy LQR, SMC, and STSMC. The two-wheeled wheelchair angles converge to zero degrees for all controllers, as shown in the figures, but the pendulum's angle with the LQR controller has a higher overshoot and a longer settling time than the pendulum with the FLQR, SMC, and STSMC. And as shown in the zoomed upper and lower pendulum responses, the system with the FLQR and LQR controllers settles for a longer period of time in the link 1 and link 2 angular displacements. SMC and STSMC are robust and have a low settling time, but chattering can be seen at SMC in the zoomed views. However, in the case of the STSMC controller, it is significantly suppressed, and pendulum angle convergence is quicker. Thus, it is a better option to design STSMC in order to retain the system's robustness while attenuating the harmful chattering effect. Table 5.1 and 5.2 summarises the performance of the controllers in terms performance measurement parameters.

Table 5.1: Performance of the Controllers for lower pendulum

| Performance parameters | Controller              |                         |                         |                         |
|------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
|                        | SMC                     | STSMC                   | LQR                     | FLQR                    |
| Rising time            | 0.019                   | 0.0101                  | 0.171                   | 0.170                   |
| Settling time          | 1.485                   | 1.208                   | 2.895                   | 2.319                   |
| Peak overshoot         | $5.141 \times 10^{-8}$  | $0.7066 \times 10^{-8}$ | $11.73 \times 10^{-7}$  | $5.413 \times 10^{-8}$  |
| Peak undershoot        | $-3.525 \times 10^{-8}$ | $-0.329 \times 10^{-8}$ | $-24.08 \times 10^{-8}$ | $-7.336 \times 10^{-8}$ |

Table 5.2: Performance of the Controllers for upper pendulum

| Performance parameters | Controller             |                         |                         |                         |
|------------------------|------------------------|-------------------------|-------------------------|-------------------------|
|                        | SMC                    | STSMC                   | LQR                     | FLQR                    |
| Rising time            | 0.1428                 | 0.1409                  | 0.1800                  | 0.1717                  |
| Settling time          | 1.631                  | 1.408                   | 2.947                   | 2.589                   |
| Peak overshoot         | $5.969 \times 10^{-9}$ | $0.5413 \times 10^{-9}$ | $57.38 \times 10^{-8}$  | $24.86 \times 10^{-9}$  |
| Peak undershoot        | $17.47 \times 10^{-9}$ | $-1.627 \times 10^{-9}$ | $-19.30 \times 10^{-8}$ | $-8.588 \times 10^{-9}$ |

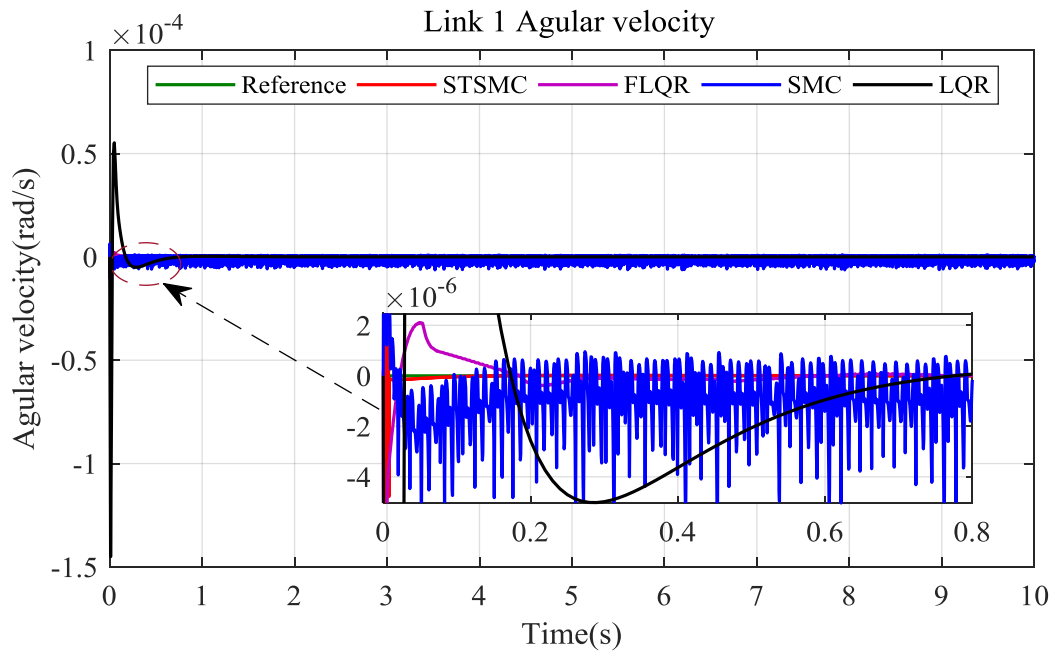


Figure 5.23: Response to angular velocity of link 1 for all controllers

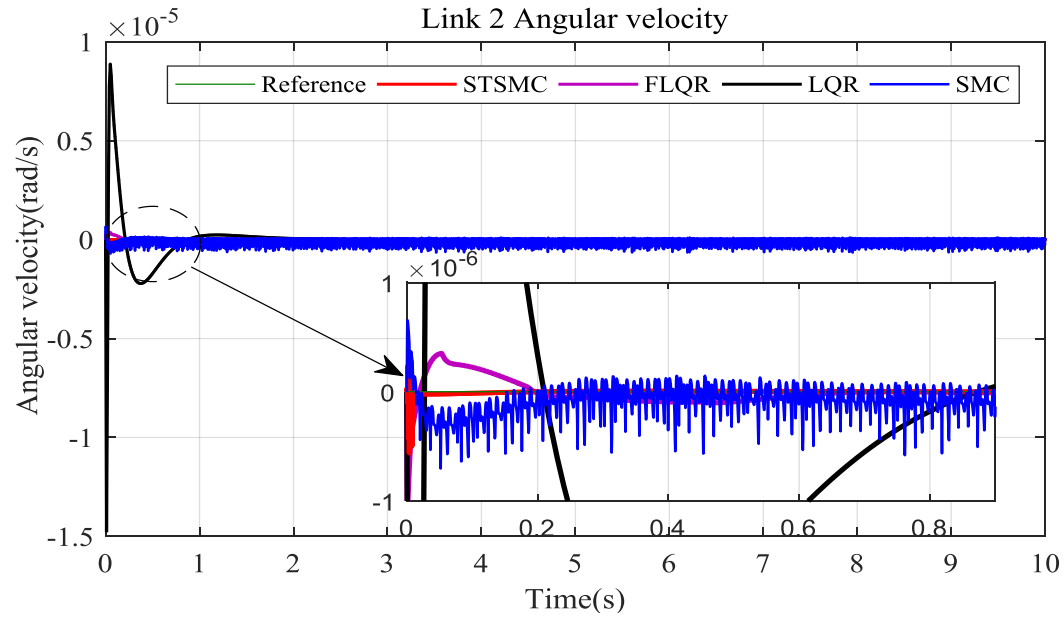


Figure 5.24: Response to angular velocity of link 2 for all controllers

The two-wheeled wheelchair angular velocity also converges to zero degrees for all controllers, as shown in the figures, but the pendulum's velocity with the LQR controller has a higher overshoot and a longer settling time than the pendulum with the FLQR, SMC, and STSMC.

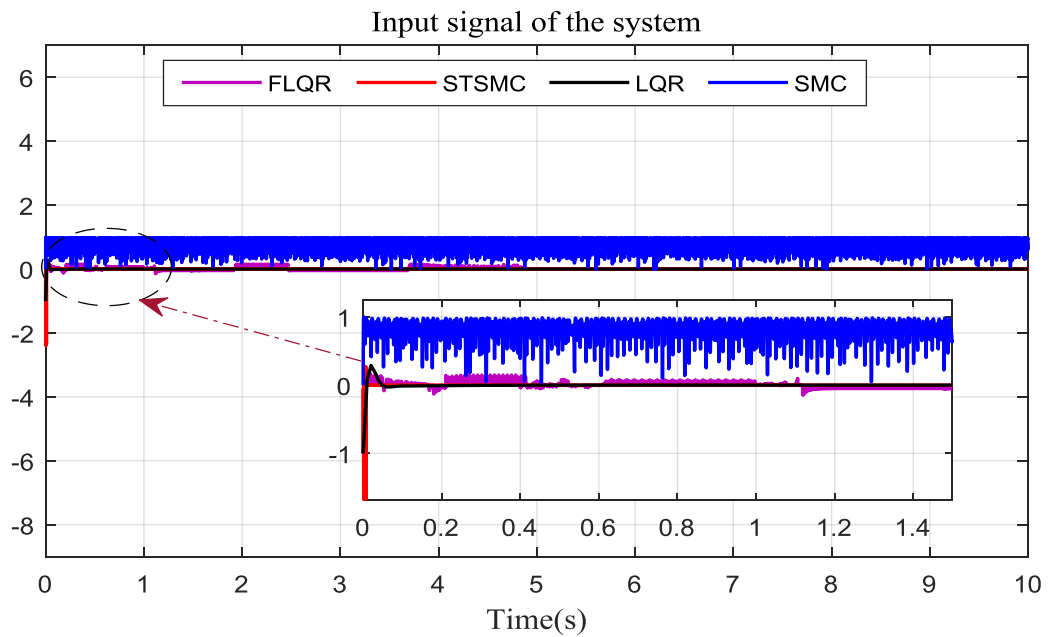


Figure 5.25: Input signal for all controllers

Figure 5.25 shows the control input signal of all controllers. According to the results, the LQR-controlled system requires less effort or energy to operate.

## CHAPTER SIX

### CONCLUSION AND RECOMMENDATION

#### 6.1 Conclusion

The goal of this study was to model and develop control techniques for a two-wheeled electric wheelchair based on a double inverted pendulum. However, because a two-wheel wheelchair is a system that is unstable, highly nonlinear, multivariable, and underactuated, designing an active controller is necessary. In this thesis, LQR and SMC are designed for two-wheel self-balancing electric wheelchairs. First, the mathematical equations have been derived based on a DIP structure using the Euler-Lagrange approach. Then, it is represented as a state-space model of a two-wheeled wheelchair. The model has been tested using an LQR, FLQR, SMC, and STSMC controller, and it shows the system is able to be stable in an upright position with all controllers. LQR is a straightforward controller for complex, nonlinear, multivariable systems, but it has limited robustness and high overshoot. According to the simulation results, in the case of link 1, STSMC reduces peak overshoot by 98.4%, SMC by 95.6%, and FLQR by 95.4% relative to LQR, while in the case of link 2, STSMC reduces peak overshoot by 99.1%, SMC by 98.6%, and FLQR by 95.6%. As a result, SMC and STSMC are the best choices for such a system because they are robust, have a small overshoot, and are insensitive to parameter variations. However, one of the most significant disadvantages of using SMC is that it produces chattering, which can be hazardous to mechanical systems. Super-twisted sliding mode can be used to mitigate the harmful effects of chattering. The system with STSMC maintains its robustness, and the chattering effect is reduced to some extent. The robustness of the two-wheel wheelchair control system was also tested in the presence of disturbances and with a variation in the initial tilt angle. The results obtained show that the STSMC controller was capable of tracking the reference input with a good response and a reasonable overshoot.

## **6.2 Recommendation**

For people who are unable to walk due to a lower body injury, a wheelchair is their only mode of transportation. In this thesis, the balance of a two-wheel electric wheelchair is controlled using four controllers. However, this does not address all of the issues faced by the physically disabled. Improving the quality of the wheelchair by adding some features such as direction control and battery management is important. In addition to this, I recommend the implementation of the designed system hardware.

**Acknowledgments:** This research thesis was funded by Adama Science and Technology University under the grant number of ASTU/SM-R/538/22, Adama, Ethiopia.

## REFERENCES

- Manukid Parnichkun and Sondarangallage D.A. Sanjeewa. (2021). Control of rotary double inverted pendulum system using LQR sliding surface based sliding mode controller. *Journal of Control and Decision*, 1-14.
- A. Taifour Ali, A. M.-A.-A. (2021). Design and Implementation of Two-Wheeled Self-Balancing Robot Using PID Controller. *IEEE*, PP.1-5. doi:109/ICCCEEE49695.2021.9429579
- Ahmed Faisal Ghaleb, A. A.-O. (2022). Optimum of fractional order fuzzy logic controller with several evolutionary optimization algorithms for inverted pendulum. *International Review of Applied Sciences and Engineering*, 1-12. doi:10.1556/1848.2021.00375
- Aniket Vidyagar, P. G. (2022). Design and Development of Lever Operated Wheelchair. *International Research Journal of Engineering and Technology (IRJET)*, 671-677.
- Anoop Chandra & Lal Priya P. S. (2022). Higher Order Sliding Mode Controller for a Quadrotor UAV with a Suspended Load. *ELSEVIER*, 610-615.
- Ayad Q. Al-Dujaili, Alaq Falah, Amjad J. Humaidi, Daniel A. Pereira, and Ibraheem K. Ibraheem. (2020). Optimal super-twisting sliding mode control design of robot manipulator: Design and comparison study. *International Journal of Advanced Robotic Systems*, 1-17.
- Ayankoso, M. K. Habib, and S. A. (2020). Modeling and Control of a Double Inverted Pendulum using LQR with Parameter Optimization through GA and PSO. *21st International Conference on Research and Education in Mechatronics (REM)*, 1-6.
- B. Supriyo, A. Suharjono, and K. W. Atmaja. (2021). Modelling and Control of DC Motor Speed and Position for Wheel Mobile Robot Application. 80-84.
- Dang Anh Duy, Le Hoai Phuong, & Vo Duy Cong. (2021). Stability Analysis and Assessment of Double Inverted Pendulum with LQR and LQG Controllers. *Journal of Mechanical Engineering Research and Developments*, 408-417.
- Daniel Gosaye Tesfaye. (2021). *Grey Wolf Optimizer Based Super Twisted Sliding Mode Controller (STSMC) for Self-balancing Control of a Two-wheel Electric Scooter*.

Msc Thesis, ASTU, The Department of Electrical Power and Control Engineering  
School of Electrical Engineering and Computing , Adama.

Erfan Nozari. (2020). *Stability, Controllability & Observability*.

Fadlallah, Khaled M. Goher and Sulaiman O. (2019). Control of a Two-wheeled Machine with Two-directions Handling Mechanism Using PID and PD-FLC Algorithms. *Machine intelligence research*, 511-533. doi:10.1007/s11633-019-1172-0

George S. Maraslidis, M. G. (2022). Design of a Fuzzy Logic Controller for the Double Pendulum Inverted on a Cart. *informations*, 1-24.

Glenn Ruscoe. (2019, 01 24). *history.physio*. Retrieved from History of the Wheelchair: <https://history.physio/history-of-the-wheelchair/>

Han Yang, Liu Xiaoma, and Zheng Zhiqiang. (2018). A Novel Vector Backstepping and Sliding Mode. *IEEE*, 479-483.

Hla Myo Tun, Maung Maung Latt, Myat Su Nwe, Zaw Min Naing, Devasis Pradhan, Prasanna Kumar Sahu. (2022). Research on Self-balancing Two Wheels Mobile Robot Control System Analysis. *Electrical Science & Engineering*, 7-20. doi:<https://doi.org/10.30564/ese.v4i1.4398>

Ho, Sho-Tsung Kao, and Ming-Tzu. (2022). Balance Control of a Configurable Inverted Pendulum on an Omni-Directional Wheeled Mobile Robot. *Appl. Sci*, 1-26.

Ibrahim Mustafa Mehedi, Ubaid M. Al-Saggaf, Rachid Mansouri, and Maamar Bettayeb. (2019). Stabilization of a double inverted rotary pendulum through fractional order integral control scheme. *International Journal of Advanced Robotic Systems*, 1-9.

Ishan Chawla and Ashish Singla. (2021). Real-Time Stabilization Control of a Rotary Inverted Pendulum Using LQR-Based Sliding Mode Controller. *springer*, 2589-2596.

Ismail, Vina Wahyuni Eka Putranti, and Zool Hilmi. (2017). Super Twisting Sliding Mode Control with Region Boundary Scheme for an Autonomous Underwater Vehicle. *Recent Developments in Sliding Mode Control Theory and Applications*, 67-88. doi:10.5772/67579

- Jumi Bharali and Mrinal Buragohain. (2016). A Comparative Analysis of PID, LQR and Fuzzy. *1st IEEE International Conference on Power Electronics, Intelligent Control and Energy Systems (ICPEICES-2016)* , 1-4.
- K. Maatoug, M. Njah, and M. Jallouli. (2019). Electric Wheelchair Trajectory Tracking Control based on Fuzzy Logic Controller. *19th International Conference on Sciences and Techniques of Automatic Control and Computer Engineering (STA)*, 191-195. doi:10.1109/STA.2019.8717190
- Kashika Chhabra and Mohd. Rihan. (2017). Design of Linear Quadratic Regulator for Rotary Inverted Pendulum Using LabVIEW. *IEEE*.
- KousheekC. (2022, 8 19). *article*. Retrieved from ubate.
- Kurczewski P, Gabryelski J, Sydor M, Szperling A, Torzynski D, Zablocki M. (2021). Development of Transport for Disabled People on the Example of Wheelchair Propulsion with Cam-Thread Drive. *Energies*. Retrieved from <https://doi.org/10.3390/en14238137>
- Kyong-Hyon Kim, Minh-Thien Tran, Hwan-Cheol Park, and Young-Bok Kim. (2020). A Study on an Adaptive Super-Twisting Sliding Mode Control Design with Perturbation Estimation. 53-63.
- Lal Bahadur Prasad, Barjeev Tyagi, and Hari Om Gupta. (2015). Optimal Control of Nonlinear Inverted Pendulum System. *International Journal of Automation and Computing*, 661-670. doi:10.1007/s11633-014-0818-1
- Lytigas. (2018). Modelling a Braking DC Motor Speed Controller. *Electrical Engineering Stack Exchange*. Retrieved from <https://electronics.stackexchange.com/q/408870>
- M. Nikpour, L. Huang, and A. M. Al-Jumaily. (2020). Stability and Direction Control of a Two-Wheeled Robotic Wheelchair Through a Movable Mechanism. *in IEEE Access*, pp. 45221-45230.
- Mahmoud Elkafafy & Bart Peeters. (2022). Robust Identification of Stable MIMO Modal State Space Models. *springer*, pp81-95. doi:[https://doi.org/10.1007/978-3-031-05445-7\\_10](https://doi.org/10.1007/978-3-031-05445-7_10)



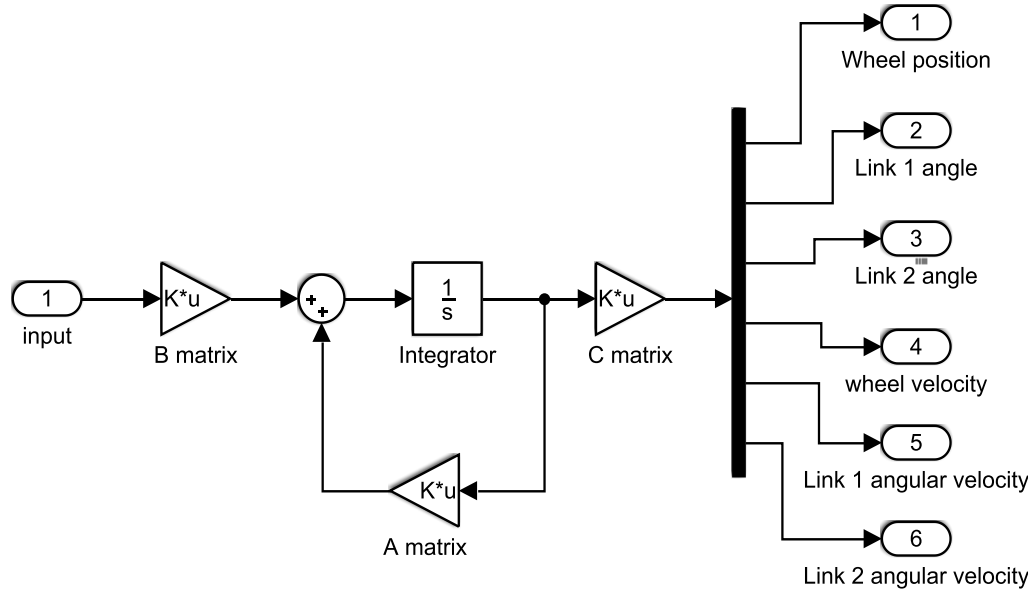
- Minh-Thien Tran, Kyong-Hyon Kim, Hwan-Cheol Park and Young-Bok Kim. (2020). A Study on an Adaptive Super-Twisting Sliding Mode Control Design with Perturbation Estimation. 53-63.
- Mobile Robots. (2022, Nov 17). Retrieved from <https://www.geeksforgeeks.org/mobile-robots/>
- Mostafa Nikpour, L. H. (2020). Stability and Direction Control of a Two-Wheeled Robotic Wheelchair Through a Movable Mechanism. *in IEEE Access*, pp. 45221-45230.
- N. A. A. Razali, N. M. Abdul Ghani, N. F. Jamin, and M. F. Masrom. (2018). Stability Control of Wheelchair System Using Interval Type-2 Fuzzy Logic Control (IT2FLC). *IEEE*, 162-167.
- Naoki Tanohata and Hirokazu Seki. (2019). Driving Control of Electric Power Assisted. *IEEE*, 4185-4190.
- Natsuki Sado and Yasuo Kawakami. (2022). Lower limb in sprinters with larger relative mass but not larger normalized moment of inertia. 1-22. doi:10.1101/2022.02.25.482057
- Ngoc Kien Vu and Hong Quang Nguyen. (2021). Design Low-Order Robust Controller for Self-Balancing Two-Wheel Vehicle. *Research Article*, 22.
- Normaniha Abdul Ghani and Nurul Fadzlina Jamin. (2016). Two-Wheeled Wheelchair Stabilization Control. *IEEE International Conference on Automatic Control and Intelligent Systems (I2CACIS)*, 180-185.
- Nurul Fadzlina Jamin, G. a. (2016). Two-Wheeled Wheelchair Stabilization Control Using Fuzzy Logic Controller Based Particle Swarm Optimization. *IEEE*, 78-83.
- Peng, Yang-Rui Li, and Chao-Chung. (2021). Super-Twisting Sliding Mode Control Law Design for Attitude Tracking Task of a Spacecraft via Reaction Wheels. *Hindawi*, 1-13.
- Pranav Sevekari, Bhagyashri Tamhane, and Shailaja Kurode. (2020). Robust Control for Stable and Safe Performance of a Two Wheeled Human Transporter. *ELSEVIER*, 616-621.

- Rupal Roy, M. I. (2021). A Review on Comparative Remarks, Performance Evaluation and Improvement Strategies of Quadrotor Controllers. *Technologies*, 1-22.
- S.D., Parvat Bhagsen, and Ratnaparkhi. (2015). A second Order Sliding Mode Controller Application in Industrial Process. *International Journal of Engineering Trends and Technology*, 217-222.
- Sateesh Reddy Avutu, Sudip Paul B, and Venkateswara Reddy. (2020). Modeling and Comparative Analysis of Direct Drive Wheel for Manual Wheelchair. *IEEE*.
- Solomon Mekonnen Abebe, A. N. (2020). Disability and its prevalence and cause in. 1-16.
- Suppachai Howimanporn. (2014). *DEVELOPMENT AND BALANCING CONTROL OF AN X-Y PLANAR INVERTED PENDULUM*. Thailand: Asian Institute of Technology.
- Swati Singh and Akhilesh Swarup. (2021). Control of Rotary Double Inverted Pendulum using Sliding Mode Controller. *IEEE Xplore*, 1-6. doi:10.1109/CONIT51480.2021.9498525
- Swostik Sourav Dash, Javeed Shaikh-Mohammed, Vivek Sarda, & S. Sujatha. (2021). Design journey of an affordable manual standing. *Disability and Rehabilitation: Assistive Technology*, 1-12. doi: 10.1080/17483107.2021.1892839
- Tefera T. Yetayew and Daniel G. Tesfaye. (2022). Genetic Algorithm Tuned Super Twisted Sliding Mode Controller (STSMC) for Selfbalancing Control of a Two-Wheel Electric Scooter. *ResearchGate*, 270-287.
- Thomas, Keerthi Prakash, and Koshy. (2016). Study of Controllers for a Two Wheeled Selfbalancing Robot. *International Conference on Next Generation Intelligent Systems*.
- Tran Van Thien, Vo Ba Viet Nghia, Nguyen Ngoc Son, & Mai Thang Long. (2022). Adaptive neural sliding mode control for two wheel self balancing robot. *International Journal of Dynamics and Control*, 771-784.
- V. Behnamgol, A. R. Vali, and I. Mohammadzaman. (2013). Second Order Sliding Mode Control With Finite Time. *Amirkabir International Journal of Science & Research*, 41-52.

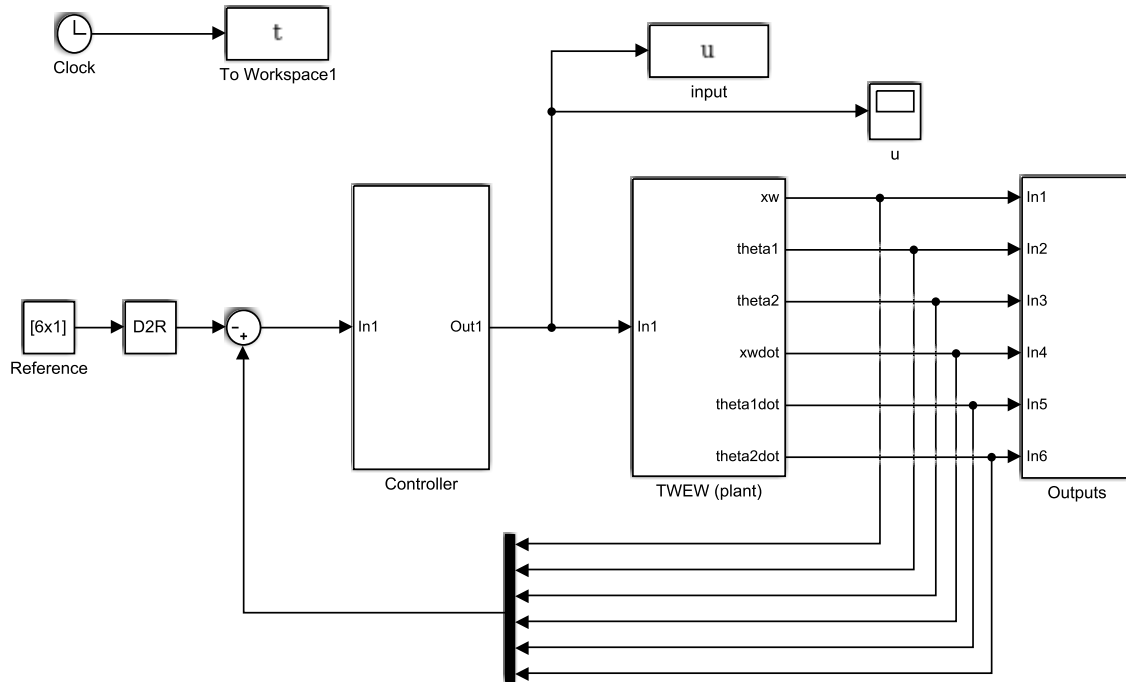
- Vinod Kumar P1 and Dr. Kamala. (2022). Design, Dynamic Modelling, and Control of a Two-wheeled Self-Balancing Robot (TWSBR). *International Journal of Mechanical Engineering*, 650-668.
- Yusie Rizal, Muhammad Wahyu, Imansyah Noor, Joni Riadi, Feriyadi, and Ronny Mantala. (2021). Design of an Adaptive Super-Twisting Control for the Cart-Pole Inverted Pendulum System. *Jurnal Ilmiah Teknik Elektro Komputer dan Informatika*, 161-174.
- Zeb K Uddin W, B. T.-J. (2020). Design of Super Twisting Sliding Mode Controller for a Three-Phase Grid-connected Photovoltaic System under Normal and Abnormal Conditions. *energies*, 1-20. Retrieved from <https://doi.org/10.3390/en13153773>
- Zhilin Feng, Juntao Fei. (2018). Design and analysis of adaptive SuperTwisting sliding mode control for a. *PLoS ONE 13(1)*, 1-18. doi:10.1371/journal.pone.0189457
- Zied Ben Hazem, Mohammad Javad Fotuhi and Zafer Bingü. (2020). Development of a Fuzzy-LQR and Fuzzy-LQG stability control for a double link rotary inverted pendulum. *ELSEVER*, 10529-10556.

# APPENDIX

## Appendix A: Simulink block diagrams.

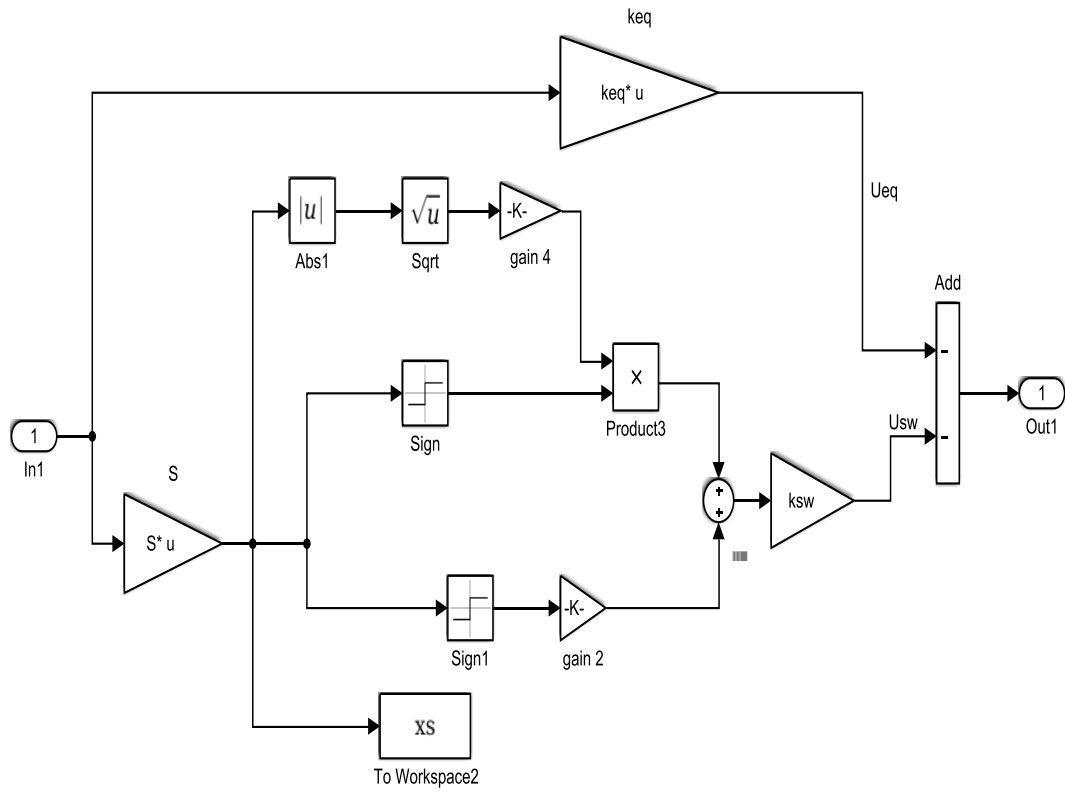


a) Simulink block of TWEW model

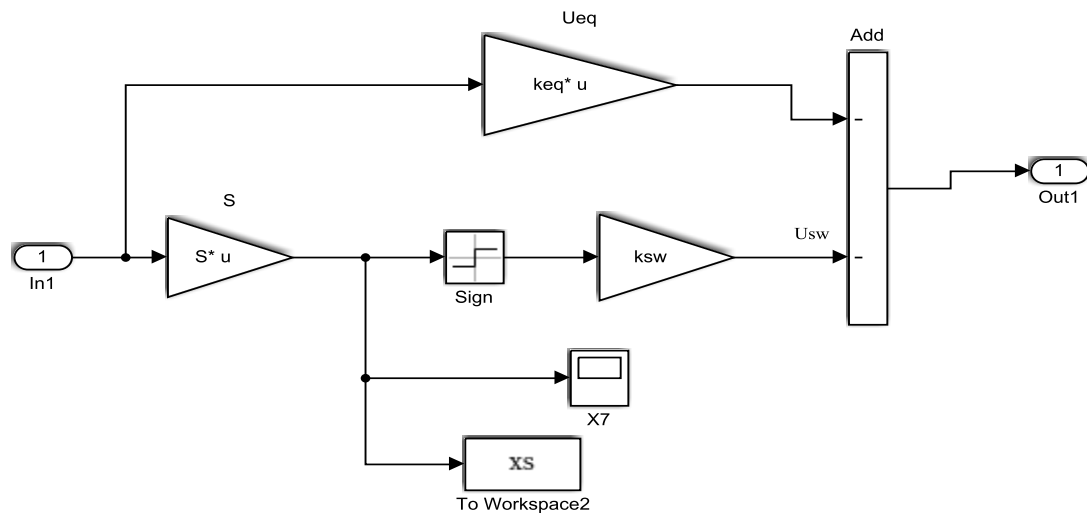


b) Simulink of overall system

Figure A.1: Simulink block of TWEW model and Simulink of overall system



a) STSMC simulink



b) SMC simulink

Figure A.2: STSMC and SMC simulink

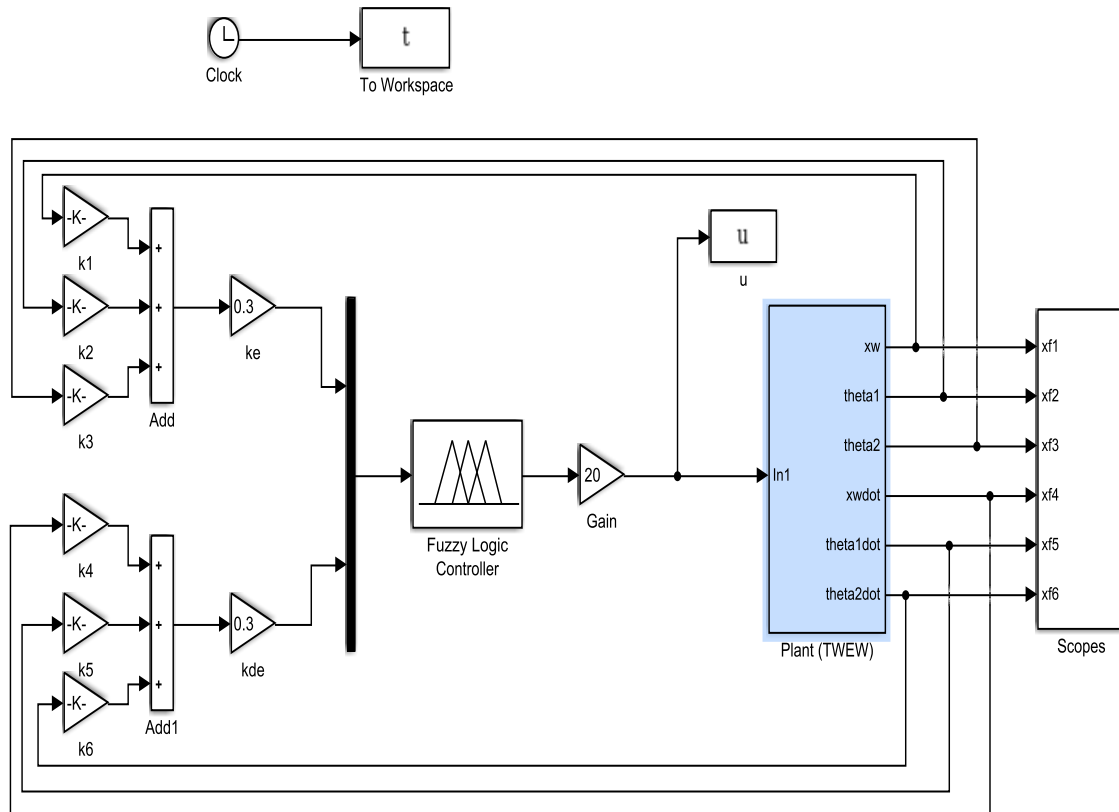


Figure A.4: Simulink of Fuzzy LQR

## Appendix B: MATLAB script for TWEW.

% Define parameters for TWEW (Two wheel electric wheelchair)

$m_w = 2;$

$r = 0.3;$

$m_1 = 13;$

$m_2 = 90;$

$h_1 = 0.4;$

$h_2 = 0.4;$

$J_w = (m_w * r * r) / 2;$

$J_1 = (m_1 * h_1 * h_1) / 3;$

$J_2 = (m_2 * h_2 * h_2) / 3;$

$CT = 0.155;$

$C_e = 0.327;$

$R = 2.5;$

$b_1 = m_w + m_1 + m_2 + J_w / (r * r);$

$b_2 = (m_1 + m_2) * L_1;$

```

b3 = (m2*h2);
b4 = (m1+m2)*(h1*h1);
b5 = (m2*h1*h2);
b6 = (m2*h2*h2)+J2;
b7 = (m1 + m2)*h1*9.81;
b8 = m2*h2*9.81;
f3 = (CT*Ce)/(R*r*r);
f4 =CT/(R*r);
% define for the M,G,and H matrix
F = [b1 b2 b3;b2 b4 b5; b3 b5 b6];
L = [0 0 0; 0 b7 0 ; 0 0 b8];
N = [f3 0 0;0 0 0; 0 0 0];
M = [eye(3) zeros(3);zeros(3) F];
G = [zeros(3) eye(3);L N ];
H = [ 0; 0; 0; f4; 0; 0];
% state space matrices for TWEW
A = M\G;
B = M\H;
C = [eye(6)];
D = (zeros(3,3));
% find the poles of A
Poles = eigs(A);
% find controllability matrix
C = ctrb(A,B);
% find observability matrix
O = obsv(A,C);

```