

OPTIMAL DESIGN OF INDUCTION INJERA MITAD WITH UNIFORM HEAT
DISTRIBUTION



DUGASA GETACHEW ASFAW

A Thesis Submitted to the Department of Electrical Power and Control Engineering

School of Electrical Engineering and Computing

Presented in Partial Fulfillment of the Requirement for the Degree of Master's in Electrical
power and Control Engineering (Control Engineering)

Office of Graduate Studies

Adama Science and Technology University

Wednesday 12th October, 2022

Adama, Ethiopia

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Advisor: Prof. SEUNGNAM KIM

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DECLARATION

I hereby declare that this Master's thesis, titled "Optimal Design of Induction Injera Mitad With Uniform Heat Distribution," is entirely original to me and has not been submitted to any other university for the purpose of awarding a degree, diploma, or certificate. Each and every source of information used in this thesis has been properly acknowledged through citation.

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We, the advisors of this thesis, hereby certify that we have read the revised version of the thesis entitled “*Optimal Design of Induction Injera Mitad with Uniform Heat Distribution*” was prepared under our guidance by Dugasa Getachew Asfaw submitted in partial fulfillment of the requirements for the degree of Master’s of Science in Electrical power and control Engineering. Therefore, we recommend the submission of a revised version of the thesis to the department following the applicable procedures.

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APPROVAL FOR THE THESIS

We, the advisors of the thesis entitled “*Optimal Design of Induction Injera Mitad with Uniform Heat Distribution*” and developed by Dugasa Getachew Asfaw, hereby certify that the recommendation and suggestions made by the board of examiners are appropriately incorporated into the final version of the thesis.

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APPROVAL OF THE BOARD OF REVIEWERS

We, the undersigned, members of the Board of Examiners of the thesis by Dugasa Getachew Asfaw have read and evaluated the thesis entitled “*Optimal Design of Induction Injera Mitad with Uniform Heat Distribution*” and examined the candidate during the open defense. This is, therefore, to certify that the thesis is accepted for partial fulfillment of the requirement of the degree of Master of Science in Electrical power and Control Engineering

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Final approval and acceptance of the thesis is contingent upon submission of its final copy to the Office of Postgraduate Studies (OPGS) through the Department Graduate Council (DGC) and School Graduate Committee (SGC).

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LIST OF ACRONYMS

AC	Alternate Current
ANSYS	Analysis of Systems
BC	Boundary Condition Equation
DC	Direct Current
EELPA	Ethiopia Electric Light and Power Authority
EMIM	Electro Magnetic Injera Mitad
FEM	Finite Element Method
FB- SRI	Full Bridge Series Resonant Inverter
GA	Genetic Algorithm
HF	High Frequency
HVDC	High Voltage Direct Current
IGBT	Insulated Gate Bipolar Transistor
IH	Induction Heating
IR	InfraRed
MOSFET	Metal Oxide Semiconductor Field Effect Transistor
NTC	Negative Temperature Coefficient
PTC	Positive Temperature Coefficient
PC	Personal Computer
RTD	Resistance Temperature Detector

LIST OF SYMBOLS

Q	Heat loss
λ	Analysis of Systems
σ	Electrical conductivity
δ	Skin Depth
T_A	Ambient Temperature
T_p	Plate Temperature
J	Eddy current Density
h	Convection coefficient
A	Magnetic Vector potential
r_p	Radius of plate
r_0	Radius of insulation
t	Thickness of plate

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ABSTRACT

Injera is one of Ethiopia's most popular foods, and it's recognized for requiring a lot of energy to prepare. In Ethiopia, there are various types of injera baking stoves (mitads) that use biomass energy, including open fire three stone stoves, mirt mitads, and Burayou injera baking stoves. Because Injera baking biomass stoves are inefficient, a lot of firewood is required, resulting in a lot of indoor air pollution, greenhouse gas emissions, and deforestation. Where electricity is available, injera mitad is produced using resistive heaters as a heat source. But, the high energy consumption, non-uniform heat distribution, low efficiency, long cooking time, heavyweight, and lack of safety during cooking are the main drawbacks of traditional Ethiopian electric injera mitad. To address these problems, the optimal design of induction mitad for uniform heat distribution which reduces power consumption; increases efficiency, and improves injera quality and safety during cooking was proposed. Heat uniformity along the radial direction of injera mitad is one of the main factors which affect the efficiency of the system and the quality of injera. So, optimization of the induction mitad magnetic circuit model is necessary to have uniform heat along the radial direction of the mitad. The optimization, design, and model simulations of the induction mitad have been applied using the MATLAB/Chebfun numerical analysis software. To have uniform heat distribution on the working plate, the optimal determination of coil distribution, and optimal parameter selection are done based on numerical computation and system simulation results. Finally, magnetic circuit modeling and simulation of optimized induction mitad with uniform heat distribution have been proposed and tested. The eddy current density distribution in the working plate is optimized by numerical optimization technique and the heat in the plate is distributed uniformly with maximum temperature variation of less than 2°C along the radius of plate. And also, the maximum power consumption of proposed system is 2KW to generate required temperature which is reduced by 50% when compared to traditional electric mitad which consumes 4KW electric power. This first successful attempt to reduce power consumption by half shows a promising result and can lead to an efficient mitad that could be widely used for injera baking.

keywords: Induction Mitad, Matlab Chebfun, Uniform heat, Optimization, Numerical.

CHAPTER ONE

INTRODUCTION

1.1 Back Ground of The Study

1.1.1 Injera

Injera is a sour and savory flatbread with a soft-spongy-like structure and a diameter of around 58 cm. Teff is the main ingredient in injera baking, but other grains including sorghum and barley are sometimes used. Some people nowadays add a few grams of rice flour to their injera to make it whiter. Ethiopians are well-versed in the art of making injera, and it has been passed down from generation to generation for a long time. Figure 1.1, depicts the general structure of well-baked injera. Injera is produced of teff flour, water, and a starter made from leftover batter from a previous baking session. For biomass stoves, a fire will be lit from the clay pan's base, and for electric injera baking stoves, resistors will be powered up. Because of its viscosity, it can be poured into a baking pan rather than rolled out. After that, the baked injera will be taken out of the baking pan. As the temperature required for baking injera depends on the grain type, batter viscosity, and other factors there are no standard temperature ranges and as a result, various temperature values are reported in the literature. Fanta (2020) reported that the required temperature for baking injera is from 150°C to 180°C while Tadesse (2020) presents the temperature ranges for baking injera as 200°C to 250°C . To make the values more reliable, three traditional three-stone open fire mitads and three electric mitads are measured. The temperature of six mitads is measured from five points along the diameter of each mitad using an infrared temperature sensor and the average of six mitads is calculated at each point as shown in Table 1.1. Depending on the literature's value and measurement result, the injera baking temperatures range from 180°C to 250°C on average.

1.1.2 Injera Mitad

The majority of Ethiopians still use a three-stone open fire to bake injera. Since the 1980s, there has been a push to improve biomass injera baking burners and provide electric injera baking stoves for city dwellers (Jones et al., 2017)



Figure 1.1: Both sides of Injera

Electric injera mitad, which uses resistive heaters as a heat source, is used in regions where electrical energy is accessible. It has a high heat loss and, as a result, low- efficiency (Hailu et al., 2017). The traditional Ethiopian electric injera mitad's main flaw is its inefficient energy use. When baking injera with this form of mitad, more energy is consumed, and power losses in the country's power economy will increase in line with the power demand of domestically built electric mitad (Feleke & Oquiño, 2019).

For use in household cooking, high-frequency electrical heating is more energy-efficient than other power frequency electrical heating. In comparison to low-frequency heating, high-frequency electrical heating has a high rate of heat generation, quick response, more even heat distribution, accurate temperature control, full automation, good compactness, and high reliability (Khang & Aung, 2014).

Table 1.1: Measured temperature for different mitads (°C)

No	Type of mitad	left edge	left middle	center	right middle	right edge
1	Three stone	165	176	185	172	161
2	Three stone	198	192	220	202	201
3	Three stone	198	203	216	223	249
	Average	187	190.3	207	199	203.6
1	Electric mitad	201	206	218	229	217
2	Electric mitad	174	193	208	208	188
3	Electric mitad	162	185	216	202	181
	Average	179	194.6	214	213	195.3

Induction heating is the process of heating electrically conductive materials via electromagnetic induction, unlike resistor heating, it is high-frequency electrical heating with various applications due to its advantages: high power ratios and heating created in the work plate, which offers faster heating and increased thermal efficiency (Carretero et al., 2015). It's worth noting that the induction cooker's heating efficiency ranges from 80% to 90%, making it substantially more energy-efficient than a resistive heater (Meng et al., 2011).

These cookers are also easy to clean because the food will not burn if it falls onto the non-heated cooking surface. It does not contaminate the substance being heated because it is a non-contact heating process. The basic theory of induction heating which is a practical application of Faraday's discovery is based on three principles (Lentz, Lorentz, and Joule laws), according to which the magnetic field formed by the inductor produces eddy currents in the working plate, that cause heating.

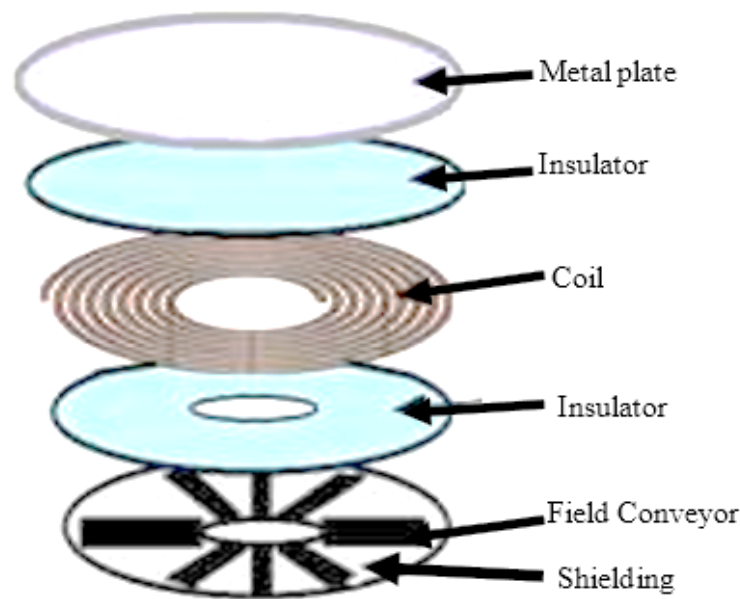


Figure 1.2: The structural arrangement of induction mitad (Feleke & Oquiño, 2019)

The induction mitad's thermal power, which is proportional to the system's working frequency and the magnetic flux density in the working plate, is primarily determined by eddy current density, electrical conductivity, and skin depth Qiu et al. (2014). A great way to increase the thermal power of induction mitad is by adjusting the coil structure to boost the magnetic flux density in the pan. On the other hand, developing induction heating equipment is tough. Inductors are still routinely dimensioned experimentally, based on the manufacturer's experience and the power profile to be injected into the working plate, which is

established through trial and error, and tests are similarly time intensive and expensive (Rezig et al., 2011). To solve this challenge, computer-aided design, which is based on numerical simulation of the magnetic and thermal phenomena studied, provides significant cost, time, and optimization benefits. The ability to predict the magneto-thermal behavior of the complete workspace-inductor enables the electrical and mechanical design of the inductor, as well as the power necessary to treat the workspace.

The basic component of the induction injera mitad is arranged as shown in Figure 1.2. The size of the metal plate was selected based on the required size of the injera. The upper insulation was used as a thermal insulator for the heat generated in the metal plate transferring downward and affecting the coil and electronics part. The coil used to heat the ferrous metal plate was located at the following layer and was driven by high-frequency pulses. At the bottom layer, a magnetic material was placed to complete the flow of electromagnetic flux in the system. The temperature distribution on the working plate is influenced by several factors, these are:

- The structural arrangement of the induction mitad,
- Geometrical structures of the working plate and the inductor.
- The frequency indicates skin thickness (skin depth) and material properties (conductivity and permeability) (Kanssab et al., 2012).

Many electromagnetic processes cause the current distribution within an inductor and workpiece to be uneven. Temperature gradients in the workpiece are caused by this heat source's non-uniformity (Rudnev et al., 2017).

Therefore, an inductor's optimal spacing, the optimal distance between coil and plate, optimal thickness of the plate, and optimal selection of frequency are the main activities of this study to create a homogeneous temperature distribution along the radial direction of the working plate.

1.2 Statement of Problem

The high energy consumption, non-uniform heat distribution, low efficiency, long cooking time, heavyweight, and lack of safety during cooking are the main drawbacks of traditional Ethiopian electric Injera mitad. Each household bake injera two to three times per week. The

locally built electric Injera mitad is an energy-intensive device, consuming 3.5KW to 4KW which is 60%–70% of total family energy requirements due to its high-power consumption and low-efficiency (Feleke & Oquiño, 2019). It is common to observe light bulbs dimming, electricity levels dropping, and a lot of steam and heat being produced while injera is being baked. Once Mitads are running, other electrical appliances like stoves and water heaters won't be turned on. Non-uniform heat distribution on the work plate increases the cooking time and affects the quality of Injera, in addition to energy consumption. The inefficiency and heat distribution non uniformity of the equipment for baking Injera should both be decreased to reduce the machine's energy consumption. Similarly, lowering its power usage and increasing its efficiency will yield remarkable results in the country's power utilization. Therefore, it is better to design optimal mitad to address all of these problems.

1.3 Objectives

1.3.1 General Objective

The general objective of this thesis is to design an induction mitad with uniform heat distribution in the working plate which provides improved power efficiency, uniform heat distribution, and reduced weight.

1.3.2 Specific Objectives

- To model the magneto-thermal model of the system.
- To reduce the power consumption of the system
- To decide the optimal parameter for uniform heat distribution.
- To improve the uniform heat distribution along the radial direction of the induction mitad by optimizing eddy current distribution.
- To determine the critical radius of insulation to prevent heat loss.

1.4 Scope of The Thesis

The thesis focused on the optimal design of induction Injera mitad with uniform heat distribution along the radial direction. It presents the analytical and numerical analysis of induction Injera mitad's magneto-thermal system, critical radius, and skin effect. The solution is simulated with Matlab 2019a/Chebfun to show the eddy current distribution and heat pattern in

the working plate. The heat pattern for different parameter adjustments has been simulated and tested with the prototype to verify the numerical solution. The performance of proposed system is tested with injera baking experiment to validate the obtained simulation result.

1.5 Limitation of The Thesis

The thesis is limited to implementing an experimental setup to analyze the heat distribution along the ray of the working plate. The numerical simulation of the system solution is done and verified with the test. The sticky problem is not solved by this research work as it depends on chemical composition of injera dough and plate treatment which is not the objective of this research. And also, it is limited to temperature distribution analysis, the power conversion is not deeply discussed due to system complexity and time limit.

1.6 Significance of The Thesis

Energy-saving issues have become a global concern in recent years, and improved Induction mitad is one of the answers to the problem of excessive energy use. The need for higher-quality, safer, and energy-efficient products is on the rise. Customers are attracted to heating appliances that are both safe and rapid. Improving the efficiency of electric Injera mitads will lower demand and save energy, as well as spur research and innovation.

Reduced energy usage, which translates into lower electricity costs and a better environment, is the most visible outcome of this greater energy efficiency. In general, the optimal design of uniformly distributed heat induction Injera mitad provides:

- **Rapid heating:** IH technology uses high power densities and little thermal inertia to immediately heat the induction target, reducing wasted heat and drastically shortening heating periods.
- **Efficiency:** Modern efficient coil and power converter designs enable efficiency rates of over 90%, greatly outperforming traditional heating methods (Lucia et al., 2013). Additionally, because just the induction target is heated, minimal heat is lost through the ambient environment, allowing for the achievement of high temperatures.
- **Cleanliness and safety:** Because IH heats the induction target directly, the area immediately around the heating area is less hot, preventing the burning of nearby objects such as spilled food in home IH. Additionally, unlike fossil fuel furnaces, there is no local pollution.

1.7 Thesis Organization

This thesis is organized into five chapters. Chapter one includes the introduction part of the thesis, the problem of the statement, the objective of the project, the significance of the project, the scope of the project, and its limitation.

Chapter two is a literature review of recent works on the evolution of injera mitad, induction stove and mitad heat distribution optimization techniques, power conversion, and Effects of material properties on the performance of induction heating.

Chapter three deals with the methodology used to achieve the predefined objective of this thesis and system modeling which includes a mathematical model of magneto-thermal phenomena, the critical radius of insulation, and induction heating load parameters.

Chapter four presents the discussion on simulation results to assess the effectiveness of the proposed system for distributing heat uniformly along the radial direction of induction mitad.

Finally, chapter five presents the conclusion and recommendation of the thesis based on the results obtained.

CHAPTER TWO

LITERATURE REVIEW

2.1 Theoretical Background

This chapter reviews recent works related to the evolution of injera baking mitad and the problem associated with it in the case of power usage, weight, efficiency, and cooking time. And also, the induction stove and mitad heat distribution optimization techniques along with effects of material characteristics are reviewed. Several studies have been conducted in various nations to address concerns linked to energy sustainability and renewable energy. Efforts to produce efficient and low-carbon equipment, in addition to creating and utilizing clean and renewable energy, are appealing subjects for researchers (Tiandho et al., 2021). Accordingly, many research efforts have previously been focused on improving the Ethiopian traditional injera mitad to improve the efficiency to save energy, reduce indoor air pollution, and CO₂ emission for biomass energy-based mitad. Similarly, electric injera mitad improvement has been conducted by several researchers to reduce electric power consumption. Some research have been done on induction stove heat distribution optimization.

2.2 Review Of Related Work

2.2.1 Quality of Injera

The injeras must be of comparable quality to those baked on conventional clay mitads in order for users to accept a new energy-efficient mitad. The following parameters were derived based on literature and field observations to identify good quality injera (Jones et al., 2017):

- “Eyes” on the top side, Smooth underside, and Thin, crispy edges
- 56-58cm diameter, 3-5 mm thickness, and 330 g weight:

2.3 Existing Injera Baking Mitad

Due to environmental and health concerns, as well as the potential reduction in the amount of energy lost during injera baking, research on increasing the performance of injera baking stoves, has grown in recent years.

2.3.1 Three Stone Open Fire Injera Baking Mitad

In a three-stone open-fire stove, as the name implies, the cooking pan or Mitad (claypan) for baking is supported by three triangular stones. The sorts and sizes of stones used would vary depending on the stones' availability and the pan's diameter. The mitad is usually supported by three (10–15 cm) high stones, as seen in Figure 2.1. To demonstrate how the injera mitad has advanced over time, several inventors have used a three-stone open-fire Injera baking stove as a comparison. The average specific fuel consumption of a three-stone open fire stove is 929 g of wood per kilogram of Injera (Tadesse, 2020).



Figure 2.1: Three stone open fire mitad for baking injera, (Getie & Degefa, 2020)

2.3.2 Burayou Injera Baking Mitad

Mud Injera baking stoves were produced in the early 1980s by the Ministry of Education's Burayou Basic Technology Center. Figure 2.2 depicts a sketch of the Burayou Injera baking stove. This stove was made from a combination of clay and chid (teff straw) and the stove weighs around 100 kg in total. The stove is large and delicate, and the installation requires at least two people. This makes transporting quite difficult as well. The Ambo team adapted the Burayou mud-stove in 1986 and created the Ambo mud-stove, which is used to bake injera (Tadesse, 2020). In the early 1990s, a team of professionals from the Ethiopian Energy Authority and international consultants began a survey to provide a foundation for the development of the Injera baking stove in Ethiopia. The Burayou, Ambo, and Tigrian mud stoves were found to be the most efficient stoves in the country, according to the findings. When tests were conducted on the three Injera baking mud stoves stated above, the difference in performance was linked to the thickness and seasoning ability of the Mitad



Figure 2.2: Burayou mud injera mitad,source: (Tadesse, 2020)

(Shimelis, 1991). Others have invented enclosed Injera baking stoves, which are widely utilized in Ethiopia's northern regions, primarily in Tigray (Tigray Regional State) and Wollo (a province in Amhara Regional State), as well as some sections of Semen Shoa (Oromia Regional State). Unlike three-stone Injera baking stoves, these stoves are installed permanently on the ground or a raised platform made of mud and stones. The stove is built by users based on their best guesses concerning dimensions. The height of the stove varies from 28 to 40 cm in various places, with one or two smoke ports. A typical Tigrian Injera baking stove has two smoke exits and stands around 35 cm tall. A well-constructed enclosed Tigrian Injera baking stove has an efficiency of about 12% when compared to a three-stone open fire Injera baking stove, uses less fuel, is simpler to use, and protects from burning heat. (Bess & Kenna, 1994).

2.3.3 Mirt Injera Mitad

When compared to the other two Injera baking stoves, the Ambo was found to be the most efficient. However, the fuel consumption reduction was minimal. Later, significant changes were made, and a team of professionals developed an efficient mud Injera baking stove, which they termed 'Mirt,' which means 'best' (Bess & Kenna, 1994). As a result, the Mirt mitad structure is transformed into a cement mortar mixture for the construction of the fire chamber enclosure, as shown in Figure 2.3. And the GIZ-Energy Bureau Office in Ethiopia actively promotes this stove, known as the Mirt stove.



Figure 2.3: Mirt injera mitad (Tadesse, 2020)

2.3.4 Solar Injera Baking Stove

The solar thermal Injera baking stove shown in Figure 2.4 has a parabolic dish solar collector on the receiver to concentrate the sun's rays and generate hot steam (Hassen et al., 2011), (Tesfay et al., 2014). On the backside, the steam runs through a pipe to the Injera baking stove, where it is condensed by transmitting heat to the baking pan, or Mitad, which is put on a steel plate in contact with the hot steam. The potential of baking Injera with indirect steam at temperatures ranging from 135 to 160 °C was a key finding of this experiment. The temperature recommended was lower than that found in the literature or in experiments conducted by other researchers or developers using an Injera baking stove (Tesfay et al., 2014), Adem & Ambie (2017). To get a well-formed product, the typical operating temperature was around 250 °C. It requires additional testing and verification, as mentioned in this paper.

2.3.5 Electric Injera Mitad

Traditional cooking stoves use a lot of firewood and are time-consuming and polluting to use. The Ethiopian Electric Light and Power Authority (EELPA) introduced the electric injera baking burner (electric injera mitad) 40 years ago, and demand has grown since then as a result of electrification. Various government and commercial groups created and sold the electric injera mitad to the market to distribute it. The performance of the electric injera baking stove is decided by the company's experience and craftsmanship quality, as the electric injera baking stove is not standardized. A single household electric injera baking stove uses about 3 to 4 kW of electricity on average. A popular electric injera baking stove is shown



Figure 2.4: Concentric solar injera baking(Adem & Ambie, 2017)

in Figure 2.5. The electric injera baking stove is now being manufactured by small manufacturing businesses. The Ethiopian Energy Authority is currently working on an Ethiopian standard for labeling electric injera baking stoves to increase user awareness and allow them to choose their preferred stoves based on performance (Jones et al., 2017).



Figure 2.5: Electric injera mitad (Jones et al., 2017)

The main issue is energy inefficiency, which is caused by their high heat load, building efficiency, and temperature controller implementation. Researchers tested alternative energy sources like solar and thermal cooking, as well as temperature-controlling technology, to improve efficiency (Adem & Ambie, 2017),(Moges & EEA, 2017),(Authority, 2015).

Jones et al. (2017) outlines the major flaws in existing electric mitad designs as follows:

- The excessive resistance, oversized electric wire, and poorly set heating element
- Use of poor construction materials;
- Poor insulation: energy loss during baking is estimated to be between 40% and 50%;
- Lack of temperature control device such as a thermostat, encouraging loss of heat; and
- Overall, the design and workmanship are generally poor and inefficient.

Reviewing the literature to obtain a better understanding of the topic, background, influencing rules, and research. Over the years, the Injera baking Mitad has evolved from an open fire to an electric Injera baking Mitad, to increase energy efficiency. That is, the given energy source's conversion efficiency to heat energy for baking is increased. To accomplish so, progress has been made from the three-stone open fire to Mirt Injera mitad by enclosing the heat-supplying space and using an insulating material. To save the environment, three stone open fire injera baking Mitad has been improved to the biomass type injera baking Mitad, which replaces firewood with more energy-dense biomass energy sources with more utilization efficiency. In addition, the solar and electric-powered Injera baking Mitads were employed, respectively, by capturing solar and electric energy. Non-uniform heat distribution is likely to occur in the case of the traditional electric baking Mitad due to inaccuracies in the manufacturing process and heating elements incorporated in the Mitad. Furthermore, the material characteristic of the Injera baking Mitad, which is clay type, has not improved. In general, it is necessary to improve the material's thermal properties, uniform temperature distribution, and the time required for startup heating.

2.4 Induction Stove and Mitad

In this section the early efforts made by different researchers related to modeling, analysis, and optimization of the coil design for induction cookers to constantly improve the operation performance of induction cookers are discussed.

Feleke & Oquiño (2019), proposed an electromagnetic induction mitad prototype with a uniform coil pattern. This EMIM uses the basic law of electromagnetic induction for its operation. The output power is controlled by controlling the switching frequency of the resonant inverter. Cast iron, ceramic for the main insulation, and copper wire enameled for the working coil were the materials used to build the prototype. Their test results show that

the typical full load power consumption and performance efficiency are 1.5 kW and 79.3 percent, respectively. This work focuses only on the experimental-based prototype without a system model and the difference between the maximum and minimum temperature values is around 39°C which is very non-uniform heat distribution.

Fanta (2020), improved the previous Electromagnetic induction mitad with a non-uniform coil format. Hence, this work is proposed to improve the temperature distribution by using non-uniform spaces between coil turns. And from the final result, the temperature difference between the maximum and minimum value is around 15°C . The gaps in this work are 1). Only coil structure is considered to improve heating performance 2). The magneto-thermal phenomena of the system are not solved either analytically or numerically it is just experimental-based documentation.

Kanssab et al. (2012), presented modeling and optimization of induction cooker using magneto thermal finite element analysis and genetic algorithm. The research presents an inductor construction with four coil-filled throats and optimizes their arrangement. The magneto-thermal phenomenon of the system is modeled by a finite element method (FEM) to determine the temperature distribution on the bottom of the pan. This study considers only the structure of the throat's distribution and the dimension of the inductor. Even if the structure of the inductor is optimized by a genetic algorithm the heat distribution is not optimal as the temperature difference is around 15°C . And also the system magneto-thermal is solved with finite element and genetic algorithm which takes the computer around 14 hours with the best performance computer. The other problem of this work is that only coil span is considered to obtain uniform heat distribution but, there are other parameters to be optimized to have uniform heat distribution.

Heating Performance Improvement and Field Study of the Induction Cooker is proposed by Meng et al. (2009). This research work investigates the structure and associated problems of heating performance for conventional field distribution as it can reflect the heating distribution of induction cookers. Based on the study, a novel coil design is proposed to address conventional cooker heating efficiency problems and drawbacks. And the research gap from this work is that it considers only coil format to improve the heating efficiency but, the system has no model for analyzing, and other parameters affecting the efficiency of the stove are not considered.

The design and modeling of a Fuzzy Logic Controller Based Modified Half-Bridge Resonant

Inverter Fed Induction Heating System was proposed by Pal et al. (2017). The switching of an energy-efficient induction heating system using a fuzzy logic controller without structural consideration is presented in this work. As the structure of induction heating has a vital role in uniform heat distribution this work has to be improved and also temperature distribution is not the concern of the paper.

A design methodology for high-performance domestic induction heating systems under the worktop was proposed by Plumed et al. (2020). They introduce an inductor design method that allows it to operate at a distance of more than 5 mm from a work plate while maintaining the greatest possible performance and efficiency. They tried to design a high-performance induction cooker by optimizing the distance (gap) between coil and work plate and they concluded that if the field is covered with a ferrite shield the distance can be larger than usual. But the difficulty with this paper is that 1). Weak magnetic field linkage between work plate and coil due to the large distance and this increases the power consumption of the system as it needs more time to generate required temperature for baking injera. 2). It considers only distance to obtain high performance but, other factors affecting the heating performance of the induction cooker are there. 3). Due to the large distance between the coil and work plate large size and hence, heavyweight which is not desirable.

Dong et al. (2017), proposed Fuzzy Temperature Control of Induction Cooker. This paper presents an appropriate fuzzy control strategy for controlling the pan temperature of the induction cooker system, without an accurate control model. The temperature uniformity along the radial direction of the cooker is not discussed and the structure of the cooker for optimal temperature distribution is not designed.

Electromagnetic and Thermal Analysis of a Domestic Induction Cooker Coil was proposed by TOPUZ et al. (2019). Induction cookers for residential usage are studied in this research. A 3-dimensional finite element model of a readily available induction cooker is investigated for electromagnetic analysis using an ANSYS electronics workstation. The paper focuses on analyzing the electromagnetic and thermal characteristics of domestic induction cooker coil for existing induction cookers and there is no trial to improve heat uniformity.

The zone control technique was proposed by Miyagi et al. (2006), in which the exciting coil is divided into multiple small coils and each coil is connected to an independent high-frequency power source, with the current and frequency in each coil adjusted to achieve uniform heating. To create a more uniform thermal distribution for household induction

heating, a multiple-coil setup with several independent inverters was proposed. The zone-controlled induction heating system adjusts each current phase angle to be in phase with the others to reduce circulating power among the inverters and expand the current amplitude control range. Multiple inverters or complicated coil configurations are necessarily used in these approaches, increasing the expense and complexity of induction heating systems and also increases the power consumption as all inverters are supplied separately.

Reviewing different articles on heat distribution optimization techniques, the contribution to get uniform heat distribution in the working plate and the gaps regarding heat distribution optimization and power consumption are summarized as follows. Almost all researchers considers only coil span to optimize the heat distribution on the work plate and improvement in power consumption is not the issue in most of the researches. In fact, uniformly distributing heat can increase the efficiency of the system but, heat distribution is not affected only by coil span, other factors that affects heat distribution in the work plate are there. Some research have been done on temperature control of the system without structural consideration of the system. Others tried to generate heat in work plate far from induction coil but this decreases the magnetic field linking the plate due to large distance and to get required temperature for baking injera it needs time hence increasing the power requirement. The mathematical model of the system is not presented

2.5 Power Conversion In Induction Heating

Recently, it has been observed that the use of IH technology in household applications has expanded during the last several years (Lucia et al., 2013). The electromagnetic induction theory serves as the foundation for this induction heating technique. To generate high-frequency AC, induction heating technology often requires a high-frequency (HF) inverter. The induction heating coil receives this high-frequency AC and as a result, high frequency-magnetic flux is generated, which links to the object to be heated. Heat is created in the item due to eddy current and hysteresis losses (Lucia et al., 2013).

The design of an HF inverter has gotten easier since advancements in power semiconductor switches with the possibility of high switching frequency (Koertzen et al., 1995). The FB-SRI architecture is commonly utilized in IH technology because it provides numerous control options (Artigas et al., 2008). The DC link voltage is applied to the FB-SRI in a traditional domestic IH system through a diode bridge or thyristor-based rectifier (Acero et al., 2006).

Figure 2.6, depicts the block diagram of a typical household IH system. Domestic IH systems are typically built in two phases of power conversion. The utility side AC is first converted to uncontrolled DC using a diode bridge rectifier, and then the uncontrolled DC is converted to HF AC using a half-bridge or FB-SRI, which causes high-frequency current to flow in the planar coil (supposed to be primary) and generates a high-frequency magnetic field. The workpiece is connected by a high-frequency magnetic field (supposed to be secondary).

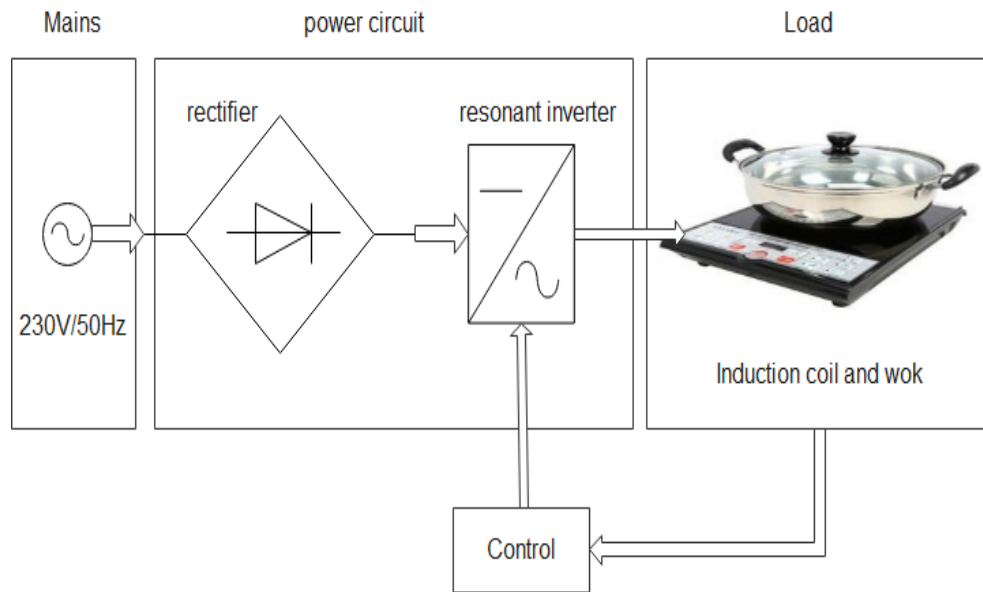


Figure 2.6: General block diagram of induction heating system (Kumar et al., 2019)

The amount of heat dissipated depends on the frequency of the eddy current, magnetic field density, and thickness of the work plate. In addition to eddy current, there is a small contribution of hysteresis effect (Kumar et al., 2019).

2.5.1 Rectifier

Using economical yet reliable rectifier diodes is a simple approach to converting AC to DC. When diode AC/DC converters are used, however, significant harmonics are released to the AC source due to their nonlinearity and uncontrollability. The output DC voltage level is also unregulated with uncontrollable power diodes.

Another method for converting AC to DC power is to employ thyristors rather than power diodes, such as the silicon-controlled rectifier (SCR) or phase-controlled converters. The conducting phase on the AC side can be modified, as its name suggests. In this way, an SCR's output DC voltage can be modified based on load needs. Even though the output DC voltage's controllability is improved, and a considerable number of low-order harmonics

are still created and manifest themselves in the currents pulled from the AC source. The thyristor-based rectification converter, on the other hand, is an active rectifier converter that can function in both rectification and inversion modes (Yang & Zhou, 2018).

Pulse-width modulation (PWM) techniques can be used in rectifiers that use fully adjustable power devices, such as IGBTs and MOSFETs, to fully regulate the output DC voltage, as well as maintain good current quality at the AC power source. The impedance of nonlinear rectifier loads approaches a linear resistance using PWM techniques, making rectification practically harmonic-free and the power supply "clean." As a result, PWM rectifiers are becoming more used in industry as a means of achieving high power factor, low current harmonics, and a steady output DC voltage.

2.5.2 Inverter

Inverters are commonly utilized in systems that operate at medium and high-frequencies. For induction and synchronous motor drives, uninterruptible power supply (UPSs), standby power supplies, induction heating, travel, camping, and HVDC systems, industries employ this device. The optimum input signal for an inverter is a DC signal from a battery or a regulated rectifier output, with an AC output signal that can be a square wave, quasi-square wave, or practically sinusoidal (Hannan et al., 2019). The series resonant inverter is used in this research.

Inverters are categorized into numerous groups based on their configuration and component type. The number of power processing stages (single-stage and multi-stage), transformer and transformerless configurations, the number of levels engaged in the design, and the type of switching employed are all aspects that go into categorizing these designs (Zeb et al., 2018).

2.6 Effects of Material Properties On Induction Heating Performance

2.6.1 Electrical Resistivity and Conductivity

Electrical conductivity describes a material's ability to easily conduct electric current. Electrical resistivity is the reciprocal of conductivity (Neelakanta, 1995), (Lupi & Rudnev, 2014). Metals and alloys are good electrical conductors, with significantly lower electrical resistance to current flow than other materials (e.g., ceramics, plastics, etc.). Electrical resistivity and conductivity have an impact on the depth of heat generation, temperature distribution,

heating efficiency, coil impedance, and other critical aspects of an induction system.

2.6.2 Magnetic Permeability

Magnetic permeability (relative) is the ability of a material to conduct magnetic flux better than a vacuum or air. Relative magnetic permeability has a marked effect on process parameter selection affecting electrical phenomena, including the skin effect, electromagnetic edge, and end effect, as well as proximity and ring effects. This is true in the vast majority of IH and heat-treating applications (Rudnev et al., 2017). In this thesis high magnetic permeability material which is stainless steel is considered.

2.6.3 Skin Effect

When an AC flows through the same conductor, the current distribution is uneven. The largest value of current density will always be found on the conductor's surface with uniform electromagnetic physical properties; from the conductor's surface to its center, the current density will drop. The skin effect is a non-uniform current distribution inside the conductor cross-section that occurs (in variable degrees) wherever AC flow is present. As a result, the skin effect will be noticeable when a workpiece is placed inside an induction coil. This phenomenon is one of the primary causes of eddy current concentration in the workpiece's surface layer ("skin"). The skin effect concentrates around 86% of the power in the conductor's surface layer, which is referred to as the reference (or current penetration) depth δ (Mohan & Gopi, 2020). The frequency, the conductor's material characteristics (ρ and μr), and the geometry of the workpiece all affect how much skin effect there is. When the high frequency is used, if the thickness of the workpiece is big compared to the radius of the workpiece, there will be a noticeable skin effect δ (Rudnev et al., 2017).

2.6.4 Skin Depth

In the theory and practice of induction heating, it is of vital importance as the magnetic field penetrates to this depth from the workpiece's surface, as a result, all heat is generated inside this layer, and this is sometimes referred to as penetration depth. The distribution of the magnetic field, current, and power within the body varies depending on the shape (flat, cylindrical, complicated), size, and material of the body. They are also affected by variations in material properties caused by magnetic saturation, temperature, or composition. Depth is a reference value based only on material qualities and frequency, ignoring body shapes and sizes. The root square of material resistivity is proportional to depth, while the root square

of relative magnetic permeability and current frequency are inversely proportional (Hassan & Mohammad, 2021).

2.6.5 Thermal Conductivity

If the material's ability to transport heat is sufficient, the temperature distribution within a component becomes more even. Thermal conductivity is the capacity to transmit heat throughout a medium in steady-state conditions. And it is defined as the amount of heat (q) transferred through a unit thickness (L) at a unit temperature gradient (T) where the temperature gradient is the only factor affecting the heat transfer under steady-state conditions. The overall magnitude of thermal conductivity is changed not just by changes in thickness and orientation, but also by temperature (Fan et al., 1996). The internal particle velocity and thermal conductivity both increase as the material temperature rises. Heat is transferred at a faster rate with less resistance. It's vital to remember that the relationship between temperature and thermal conductivity is non-linear and difficult to determine without prior research. Table 2.1 shows the thermal conductivity of three different materials. Pure iron has a higher thermal conductivity than cast iron but it is not used for induction cooktops due to its inability of retaining heat and causes temperature fluctuation, cast iron is excellent for induction cooktops due to its unique magnetic and thermal properties, as well as its ability to store heat, which facilitates gradual heating and cooling. As a result, temperature fluctuation is reduced.

Table 2.1: Thermal conductivity of different materials (Fan et al., 1996)

No	Type of materials	Thermal conductivity(W/m.K)
1	Cast iron	55
2	Steel	45
3	Stainless steel	15

When compared to pure steel, stainless steel has a low thermal conductivity of 15 W/(mK), allowing it to store more energy and maintain the surrounding temperature better than other varieties of steel. Stainless steel is commonly utilized in applications such as food processing, ovens, and conveyors that are constantly exposed to high heat surges due to its ability to remain stable when exposed to higher temperatures (Liang & Wu, 2018). The effect of material thermal conductivity on heat distribution will be simulated and discussed later. Selection of material type is crucial for uniform heat distribution as the characteristics of the

material affect the thermal phenomena of the system.

2.6.6 Heat Transfer Coefficient (Convection Coefficient)

The temperature distribution of the pan bottom of the device is influenced by the heat transfer coefficient (convection coefficient). Due to the convection impact of the air nearby, the inductor's axisymmetric structure causes the convection coefficient(h) to be nonlinear (Byun et al., 2000). As a result, in the radial direction of the axisymmetric structure of the analyzed system, it is assumed that the heat transfer coefficient(h) has a constant value.

2.7 Heat Transfer and Insulation

This section describes the heat transfer from the baking surface to the Injera and other sections of the baking Mitad. In general, heat is transferred from the baking plate to the various Mitad components and the product Injera. The following are the principal heat flow paths (Awash, 2011),:

- Heat was generated in the plate by induction and transferred to the injera batter.
- Conduction carried heat from the baking plate to the sides of the baking mitad, which was subsequently lost to the environment by convection and radiation.
- Heat is transferred from the baking mitad through the insulation to the bottom surface and then lost through radiation and convection from the bottom.
- A Small amount of heat flowed from the baking mitad surface to the lid cover, which was then lost to the environment via convection and radiation.

Conduction heat transfer involves the interaction of molecules and atoms in nonmoving liquids and solids. Convectonal heat transfer also involves a solid and a moving fluid with a temperature difference. Heat convection occurs between the top surface of the mitad and Injera, the ambient air, and the outside surface of the mitad. In a system involving heat loss, insulation is usually the most resistant component. The stronger the insulating resistance, the longer the product will last (Teferi & Kolhe, 2021).

Insulation aids in energy conservation, worker protection and comfort, process temperature maintenance, temperature variation and fluctuations reduction, fire protection, and noise and vibration reduction. Although there are many different types of insulation materials on the

market, the majority are constructed of fiberglass, mineral wool, polyethylene, foam, ceramic glass or calcium silicate, clay, and gypsum (Berhanu et al., 2022).

CHAPTER THREE

SYSTEM MODELING AND METHODOLOGY

3.1 Methodology

This chapter deals with the methodology used to achieve the required objective that has been set before. This thesis methodology begins with a thorough assessment of existing injera mitad and heat distribution optimization strategies in the literature as shown in Figure 3.1. Then, a set of procedures, such as solving the maxwell equation for modeling the magnetic

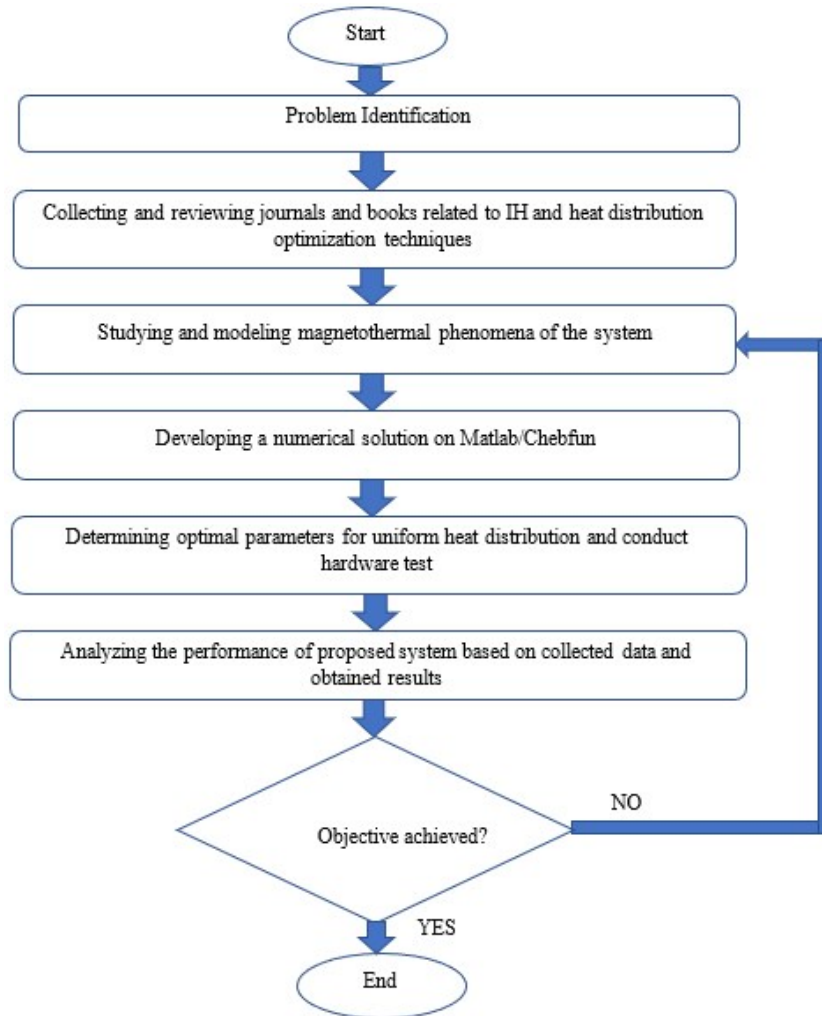


Figure 3.1: Flowchart of system methodology

circuit of the system, solving steady- state heat diffusion equation for thermal analysis, and determining the critical radius of insulation for reducing heat loss to the environment has been followed to achieve the objective of this research. The results and discussions were

formed based on the simulation and experimental results. Finally, conclusions and suggestions are drawn up.

3.2 Modeling Magneto-Thermal System

To construct an optimal induction-heating cooking system, the distribution of temperature on the pan bottom, which is the image of the distribution of induced currents, must be known. By solving the linked Maxwell and thermal equations, physical phenomena in the researched system can be simulated. A 2D solution is attainable due to the system's axial symmetry. The magneto-thermal equation is used to model electromagnetic phenomena using magnetic potential (A). The differential form of Maxwell's equations can be expressed as follows for general time-varying electromagnetic fields:

When electric currents run in the surroundings of an object, a magnetic field is generated and given as:

$$\nabla \times H = J + \frac{\partial D}{\partial t} \quad (3.1)$$

The time rate of change in magnetic flux density B always induces currents in the surrounding area and, in other words, forms a curling electric field in the area where such changes occur.

$$\nabla \times E = -\frac{\partial B}{\partial t} \quad (3.2)$$

The minus sign in equation (3.2) denotes the direction of the induced electric field. The emergence of an AC in the coil circuit is caused by the application of alternating voltage to the induction coil. According to Equation (3.1), an alternating coil current produces an alternating (changing) magnetic field with the same frequency as the source current in its surroundings (coil current). The strength of that magnetic field is determined by the coil current, coil geometry, and coil-to-work-piece electromagnetic coupling. Eddy currents are generated by the changing magnetic field in the workpiece and other objects near the coil.

$$\nabla \cdot B = 0 \quad (3.3)$$

B lines in equation (3.3), have no origin or end points, they always form a continuous loop.

$$\nabla \cdot D = \rho_{charge} \quad (3.4)$$

Where, H-magnetic field intensity, E- electric field intensity, D electric flux density, B- magnetic flux density, J- conduction current density, and ρ_{charge} - electric charge density.

The aforementioned Maxwell's equations are indefinite because there are fewer equations than there are unknowns. When the relationships between the field quantities are stated, and the field quantities are specified for an isotropic medium, these equations become definite.

$$D = \epsilon \epsilon_0 E \quad (3.5)$$

$$B = \mu_r \mu_0 H \quad (3.6)$$

$$J_c = \sigma E \quad (3.7)$$

Where ϵ_r - relative permittivity, ϵ_0 -permittivity of free space, σ -electrical conductivity, μ_r - relative permeability, μ_0 - permeability of free space.

As magnetic flux density satisfies the zero divergence condition, magnetic vector potential (A) can be introduced as

$$B = \nabla \times A \quad (3.8)$$

From equation (3.2) and (3.8) above the curl of electric field can be expressed as:

$$\nabla \times E = -\nabla \times \frac{\partial A}{\partial t} \quad (3.9)$$

By integrating equation (3.9) electric field is given as:

$$E = -\frac{\partial A}{\partial t} - \nabla \phi \quad (3.10)$$

Where, ϕ is electric scalar potential

Substitute equation (3.5) and (3.7) in to (3.1) the curl of magnetic field intensity is obtained and expressed as:

$$\nabla \times H = \sigma E + \frac{\partial(\epsilon \epsilon_0 E)}{\partial t} \quad (3.11)$$

The operation frequency of most induction heating systems is typically less than 10 MHz, and the induced conduction current density is substantially greater than the displacement current density. As a result, the last term in the above equation can be eliminated. In other words for sinusoidal waves, $\frac{\partial \epsilon E}{\partial t} = \frac{\epsilon \partial E}{\partial t} = j \epsilon \omega E$ the term σE is much greater than $\epsilon \omega E$ or

$\sigma \gg \varepsilon \omega$, $\sigma \approx 10^7$ mhos/meter for metals, $\varepsilon \approx 10^{-11}$. For frequencies on the order of 10^7 Hz, $\omega \approx 10^8$. Therefore, $10^7 \gg 10^8 \times 10^{-11}$.

$$\nabla_x H = \sigma E \quad (3.12)$$

Applying vector algebra on equation (3.1), (3.2), (3.6) and (3.12)

$$\begin{aligned} \nabla_x \left(\frac{1}{\mu_r} \nabla_x E \right) &= \nabla_x \left(\frac{1}{\mu_r} \left(-\frac{\partial B}{\partial t} \right) \right) \\ &= \nabla_x \left(-\frac{1}{\mu_r} \frac{\partial B}{\partial t} \right), \end{aligned} \quad (3.13)$$

Substituting equation (3.6),

$$\begin{aligned} \nabla_x \left(-\frac{1}{\mu_r} \mu_r \mu_0 \frac{\partial H}{\partial t} \right) &= -\nabla_x \left(\mu_0 \frac{\partial H}{\partial t} \right) = -\mu_0 \left(\nabla_x \frac{\partial H}{\partial t} \right) \\ &= -\mu_0 \frac{\partial}{\partial t} (\nabla_x H) = -\mu_0 \frac{\partial}{\partial t} (\sigma E) = -\mu_0 \sigma \frac{\partial E}{\partial t} \end{aligned} \quad (3.14)$$

Therefore,

$$\nabla_x \left(\frac{1}{\mu_r} \nabla_x E \right) = -\sigma \mu_0 \frac{\partial E}{\partial t} \quad (3.15)$$

From equation (3.9)

$$-\nabla_x \left(\frac{1}{\mu_r} \frac{\partial (\nabla_x A)}{\partial t} \right) = -\sigma \mu_0 \frac{\partial E}{\partial t} \quad (3.16)$$

From equation (3.7)

$$\frac{1}{\mu_r \mu_0} (\nabla_x \nabla_x A) = J_s - \sigma \frac{\partial A}{\partial t} \quad (3.17)$$

$$(\nabla_x (\nabla_x A)) = \nabla (\nabla \cdot A) - \nabla^2 A \quad (3.18)$$

Since the gradient of divergent of a vector field is zero the first term of equation (3.18) becomes zero and equation (3.17) becomes

$$\frac{1}{\mu_r \mu_0} (-\nabla^2 A) = J_s - \sigma \frac{\partial A}{\partial t} \quad (3.19)$$

By rearranging the terms,

$$\frac{1}{\mu_r \mu_0} \nabla^2 A = -J_s + \sigma \frac{\partial A}{\partial t} \quad (3.20)$$

Where $J_s = -\sigma \nabla \phi$ is the source (excitation) current density in the induction coil.

It can be demonstrated that for the vast majority of IH applications, the mathematical model

can be further simplified by assuming that the currents have a steady-state nature. As a result of this assumption, the electromagnetic field quantities in Maxwell's equations can be deduced to be harmonically oscillating functions with a single frequency. It is therefore possible to introduce a time-harmonic electromagnetic field as:

$$\begin{aligned}\frac{\partial A}{\partial t} &= \frac{\partial}{\partial t} \text{Re}[Ae^{j\omega t}] \\ \frac{\partial A}{\partial t} &= \text{Re}[j\omega Ae^{j\omega t}] = j\omega A\end{aligned}\quad (3.21)$$

Therefore, equation (3.20) can be written as

$$\frac{1}{\mu_r \mu_0} \nabla^2 A = -J_s + j\omega \sigma A \quad (3.22)$$

Where ∇^2 is the Laplacian, which has different forms in Cartesian and cylindrical coordinates. In Cartesian coordinates,

$$\nabla^2 A = \frac{\partial^2 A}{\partial X^2} + \frac{\partial^2 A}{\partial Y^2} + \frac{\partial^2 A}{\partial Z^2} \quad (3.23)$$

The induction cooking system was analyzed using a cylindrical coordinate due to the circular shapes of the primary and secondary coils. Since there is axis symmetry in the mitad along the radial direction, the Laplacian in cylindrical coordinate with axis symmetry can be written as:

$$\nabla^2 A = \frac{\partial^2 A}{\partial R^2} + \frac{1}{R} \frac{\partial A}{\partial R} - \frac{A}{R^2} + \frac{\partial^2 A}{\partial Z^2} \quad (3.24)$$

Therefore, equation (3.22) can be rewritten as

$$\frac{1}{\mu_r \mu_0} \left(\frac{\partial^2 A}{\partial R^2} + \frac{1}{R} \frac{\partial A}{\partial R} + \frac{\partial^2 A}{\partial Z^2} - \frac{A}{R^2} \right) = -J_s + j\omega \sigma A \quad (3.25)$$

In cylindrical coordinates system, if one single turn delta function coil at (r_0, z_0) is employed

$$\frac{\partial^2 A}{\partial r^2} + \left(\frac{1}{r} \right) \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{A}{r^2} + \mu I \delta(r - r_0) \delta(z - z_0) = j\omega \mu \sigma A \quad (3.26)$$

In Air, layer 1,3,4 the differential equation become

$$\frac{\partial^2 A}{\partial r^2} + \frac{\partial^2 A}{\partial z^2} + \frac{1}{r} \frac{\partial A}{\partial r} - \frac{A}{r^2} = 0 \quad (3.27)$$

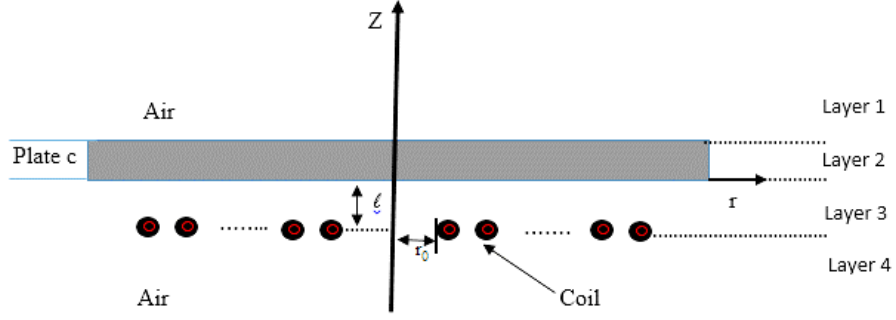


Figure 3.2: Side view of induction heating with single spiral coil (Meng et al., 2012)

The differential equation in induction plate is given by:

$$\frac{\partial^2 A}{\partial r^2} + \left(\frac{1}{r}\right) \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial z^2} - \frac{A}{r^2} = j\omega\mu\sigma A \quad (3.28)$$

By separation of variable the solution of $A(r,z)$ can be obtained. Given that $A(r,z) = R(r)Z(z)$ and dividing by $R(r)Z(z)$

$$\frac{R(r)''Z(z)}{R(r)Z(z)} + \frac{R(r)Z(z)''}{R(r)Z(z)} + \frac{1}{r} \frac{R(r)'}{R(r)Z(z)} - \frac{R(r)Z(z)}{r^2 R(r)Z(z)} = \frac{j\omega\mu\sigma R(r)Z(z)}{R(r)Z(z)} \quad (3.29)$$

$$\frac{R(r)''}{R(r)} + \frac{1}{r} \frac{R(r)'}{R(r)} + \frac{Z(z)''}{Z(z)} - \frac{1}{r^2} = j\omega\mu\sigma \quad (3.30)$$

$$\frac{1}{R(r)} \left[\frac{\partial^2 R(r)}{\partial r^2} \right] + \frac{1}{rR(r)} \left[\frac{\partial R(r)}{\partial r} \right] + \frac{1}{Z(z)} \frac{\partial^2 Z(z)}{\partial z^2} - \frac{1}{r^2} - j\omega\mu\sigma = 0 \quad (3.31)$$

By introducing variable alpha (α), the Z dependency can be written as:

$$\frac{1}{Z(z)} \frac{\partial^2 Z(z)}{\partial z^2} = \alpha^2 + j\omega\mu\sigma \quad (3.32)$$

$$\frac{1}{R(r)} \left[\frac{\partial^2 R(r)}{\partial r^2} \right] + \frac{1}{rR(r)} \left[\frac{\partial R(r)}{\partial r} \right] + \alpha^2 + j\omega\mu\sigma - \frac{1}{r^2} - j\omega\mu\sigma = 0 \quad (3.33)$$

$$\frac{1}{R(r)} \left[\frac{\partial^2 R(r)}{\partial r^2} \right] + \frac{1}{rR(r)} \left[\frac{\partial R(r)}{\partial r} \right] + \alpha^2 - \frac{1}{r^2} = 0 \quad (3.34)$$

The solution of $Z(z)$ can be obtained from equation (3.32) as follows Let $Z(z)=e^{mz}$

$$\frac{\partial^2 Z(z)}{\partial z^2} = (\alpha^2 + j\omega\mu\sigma)Z(z)$$

$$\frac{\partial^2 Z(z)}{\partial z^2} - (\alpha^2 + j\omega\mu\sigma)Z(z) = 0$$

$$m^2 e^{mz} - (\alpha^2 + j\omega\mu\sigma)e^{mz} = 0$$

$$m^2 - (\alpha^2 + j\omega\mu\sigma) = 0$$

$$m = \pm (\alpha^2 + j\omega\mu\sigma)^{\frac{1}{2}}$$

The general solution is given as

$$Z(z) = Ee^{m_1 z} + Be^{m_2 z}$$

$$Z(z) = Ee^{(\alpha^2 + j\omega\mu\sigma)^{1/2} z} + Be^{-(\alpha^2 + j\omega\mu\sigma)^{1/2} z} \quad (3.35)$$

Defining

$$\alpha_i = (\alpha^2 + j\omega\mu\sigma)^{\frac{1}{2}} \quad (3.36)$$

Since z goes to infinity, $A(r, z)$ should be finite, hence negative is not allowed to the value of α .

$$\frac{1}{R(r)} \left[\frac{\partial^2 R(r)}{\partial r^2} \right] + \frac{1}{rR(r)} \left[\frac{\partial R(r)}{\partial r} \right] - \frac{1}{r^2} + \alpha^2 = 0 \quad (3.37)$$

The solution of equation (3.37) is that of first order Bessel's equation (see Appendix A) and its solution has the form

$$R(r) = CJ_1(\alpha r) + DY_1(\alpha r) \quad (3.38)$$

Hence the solution of $A(r, z)$ is given as:

$$A(r, z) = [Ee^{\alpha_i z} + Be^{-\alpha_i z}][CJ_1(\alpha r) + DY_1(\alpha r)] \quad (3.39)$$

The constants E , B , C and D are function of α and are different depending on the value of α . The solution of $A(r, z)$ is an integral of continues variable α over the whole range.

$$A(r, z) = \int_0^\infty [Ee^{\alpha_i z} + Be^{-\alpha_i z}][CJ_1(\alpha r) + DY_1(\alpha r)] d\alpha \quad (3.40)$$

In layer 1, as Z goes to plus infinity $E(\alpha)$ is regarded to be zero. Because of divergence of Y_1 at origin $D(\alpha) = 0$ in all layers. In layer 4, as Z goes to minus infinity $B(\alpha)$ is regarded to be zero. The vector potential in four regions ($z \geq c$, $c \geq z \geq 0$, $0 \geq z \geq -\ell$, $-\ell \geq z$) are

given as follow:

$$A^{(1)}(r, z) = \int_0^\infty B_1 e^{-\alpha z} J_1(\alpha r) d\alpha, \quad z \geq c \quad (3.41)$$

$$A^{(2)}(r, z) = \int_0^\infty [C_2 e^{+\alpha_1 z} + B_2 e^{-\alpha_1 z}] J_1(\alpha r) d\alpha. \quad c \geq z \geq 0 \quad (3.42)$$

$$A^{(3)}(r, z) = \int_0^\infty [C_3 e^{-\alpha z} + B_3 e^{+\alpha z}] J_1(\alpha r) d\alpha \quad 0 \geq z \geq -\ell \quad (3.43)$$

$$A^{(4)}(r, z) = \int_0^\infty B_4 e^{\alpha z} J_1(\alpha r) d\alpha \quad -\ell \geq z \quad (3.44)$$

Where, $J_1(\alpha r)$ is an 1st order Bessel function of the first kind and $B_1 = B_2 = C_3 = B(\alpha)xC(\alpha)$,
 $C_2 = B_3 = B_4 = E(\alpha)xC(\alpha)$

Applying boundary condition between the four layers

$$A^{(1)}(r, c) = A^{(2)}(r, c) \quad (3.45)$$

$$\frac{\partial}{\partial z} A^{(1)}(r, z)|_{z=c} = \frac{\partial}{\partial z} A^{(2)}(r, c)|_{z=c} \quad (3.46)$$

$$\frac{\partial}{\partial z} A^{(2)}(r, z)|_{z=0} = \frac{\partial}{\partial z} A^{(3)}(r, c)|_{z=0} \quad (3.47)$$

$$A^{(3)}(r, -\ell) = A^{(4)}(r, -\ell) \quad (3.48)$$

$$\frac{\partial}{\partial z} A^{(3)}(r, z)|_{z=-\ell} = \frac{\partial}{\partial z} A^{(4)}(r, z)|_{z=-\ell} - \mu I \delta(r - r_0) \quad (3.49)$$

Applying boundary conditions to the equations from (3.41) to (3.44) and using Fourier Bessel equation to solve for constants. Equating equation (3.41) and (3.42) and substituting boundary value,

$$\begin{aligned} \frac{\partial}{\partial z} (\int_0^\infty B_1 e^{-\alpha z} J_1(\alpha r) d\alpha) &= \frac{\partial}{\partial z} (\int_0^\infty [C_2 e^{\alpha_1 z} + B_2 e^{-\alpha_1 z}] J_1(\alpha r) d\alpha) \\ B_1 e^{-\alpha z}|_{z=c} &= C_2 e^{\alpha_1 z} + B_2 e^{-\alpha z}|_{z=c} \\ B_1 e^{-\alpha c} &= C_2 e^{\alpha_1 c} - B_2 e^{-\alpha c} \\ B_1 e^{-\alpha c} - C_2 e^{\alpha_1 c} - B_2 e^{-\alpha c} &= 0 \end{aligned} \quad (3.50)$$

Derivate with respect to z

$$-\alpha B_1 e^{-\alpha z} - \alpha_1 C_2 e^{\alpha_1 z} + \alpha_1 B_2 e^{-\alpha_1 z} = 0$$

$$-B_1 e^{-\alpha c} - \frac{\alpha_1}{\alpha} C_2 e^{\alpha_1 c} + \frac{\alpha_1}{\alpha} B_2 e^{-\alpha_1 c} = 0 \quad (3.51)$$

Equating equation (3.42) and (3.43)

$$\frac{\partial}{\partial z} (\int_0^\infty [C_2 e^{\alpha_1 z} + B_2 e^{-\alpha_1 z}] J_1(\alpha r) d\alpha) = \frac{\partial}{\partial z} (\int_0^\infty [C_3 e^{-\alpha z} + B_3 e^{\alpha z}] J_1(\alpha r) d\alpha)$$

$$C_2 e^{\alpha_1 z} + B_2 e^{-\alpha_1 z} \Big|_{z=0} = C_2 e^{-\alpha z} + B_3 e^{\alpha z} \Big|_{z=0}$$

$$C_2 + B_2 = C_3 + B_3$$

$$C_2 + B_2 - C_3 - B_3 = 0 \quad (3.52)$$

Derivating with respect to z

$$\frac{\partial}{\partial z} (C_2 e^{\alpha_1 z} + B_2 e^{-\alpha_1 z}) = \frac{\partial}{\partial z} (C_3 e^{-\alpha z} + B_3 e^{\alpha z})$$

$$\alpha_1 C_2 e^{\alpha_1 z} - \alpha_1 B_2 e^{-\alpha_1 z} = \alpha C_3 e^{-\alpha z} + \alpha B_3 e^{\alpha z}$$

$$\frac{\alpha_1}{\alpha} C_2 - \frac{\alpha_1}{\alpha} B_2 - B_3 + C_3 = 0 \quad (3.53)$$

Equating equation (3.42) and (3.43)

$$\frac{\partial}{\partial z} (\int_0^\infty [C_3 e^{-\alpha z} + B_3 e^{\alpha z}] J_1(\alpha r) d\alpha) = \frac{\partial}{\partial z} (\int_0^\infty B_4 e^{\alpha z} J_1(\alpha r) d\alpha)$$

$$B_3 e^{-\alpha \ell} + C_3 e^{\alpha \ell} - B_4 e^{-\alpha \ell} = 0 \quad (3.54)$$

$$\frac{\partial}{\partial z} (\int_0^\infty [C_3 e^{-\alpha z} + B_3 e^{\alpha z}] J_1(\alpha r) d\alpha) = \frac{\partial}{\partial z} (\int_0^\infty B_4 e^{\alpha z} J_1(\alpha r) d\alpha) - \mu I \delta(r - r_0)$$

$$C_3 e^{-\alpha z} + B_3 e^{\alpha z} - B_4 e^{\alpha z} \Big|_{z=-\ell} + \mu I \delta(r - r_0) = 0$$

$$B_3 e^{-\alpha \ell} + C_3 e^{\alpha \ell} - B_4 e^{-\alpha \ell} + \mu I \delta(r - r_0) = 0$$

Derivating in terms of z

$$-\alpha C_3 e^{-\alpha z} + \alpha B_3 e^{\alpha z} - \alpha B_4 e^{\alpha z} \Big|_{z=-\ell} + \mu I \delta(r - r_0) = 0$$

Divide both side by alpha

$$-C_3 e^{-\alpha z} + B_3 e^{\alpha z} - B_4 e^{\alpha z} + \frac{\mu I \delta}{\alpha} (r - r_0) = 0$$

$$B_3e^{-\alpha\ell} - C_3e^{\alpha\ell} - B_4e^{-\alpha\ell} + \mu I r_0 J_1(\alpha r_0) = 0 \quad (3.55)$$

Equation (3.50) to (3.55) can be expressed in the form of matrix as:

$$\begin{bmatrix} e^{-\alpha c} & -e^{-\alpha_1 c} & -e^{\alpha_1 c} & 0 & 0 & 0 \\ -e^{-\alpha c} & \frac{\alpha_1}{\alpha} e^{-\alpha_1 c} & -\frac{\alpha_1}{\alpha} e^{\alpha_1 c} & 0 & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 & 0 \\ 0 & \frac{-\alpha_1}{\alpha} & \frac{\alpha_1}{\alpha} & -1 & 1 & 0 \\ 0 & 0 & 0 & e^{-\alpha\ell} & e^{\alpha\ell} & -e^{-\alpha\ell} \\ 0 & 0 & 0 & e^{-\alpha\ell} & -e^{\alpha\ell} & -e^{-\alpha\ell} \end{bmatrix} \begin{bmatrix} B_1 \\ B_2 \\ C_2 \\ B_3 \\ C_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\mu I r_0 J_1(\alpha r_0) \end{bmatrix} \quad (3.56)$$

By adding equation (3.50) and (3.51)

$$\begin{aligned} B_1 e^{-\alpha c} C_2 e^{\alpha_1 c} - B_2 e^{-\alpha c} - B_1 e^{-\alpha z} - \frac{\alpha_1}{\alpha} C_2 e^{\alpha_1 z} + \frac{\alpha_1}{\alpha} B_2 e^{-\alpha_1 z} &= 0 \\ \left(\frac{\alpha_1}{\alpha} B_2 - B_2\right) e^{-\alpha_1 c} - \left(\frac{\alpha_1}{\alpha} C_2 + C_2\right) e^{\alpha_1 c} &= 0 \\ B_2 \left(\frac{\alpha_1}{\alpha} - 1\right) e^{-\alpha_1 c} - C_2 \left(\frac{\alpha_1}{\alpha} + 1\right) e^{\alpha_1 c} &= 0 \\ B_2 \left(\frac{\alpha_1 - \alpha}{\alpha}\right) e^{-\alpha_1 c} - C_2 \left(\frac{\alpha_1 + \alpha}{\alpha}\right) e^{\alpha_1 c} &= 0 \\ B_2 &= \frac{e^{\alpha_1 c} (\alpha_1 + \alpha)}{e^{-\alpha_1 c} (\alpha_1 - \alpha)} C_2 \end{aligned} \quad (3.57)$$

From summation of equation (3.54) and (3.55)

$$\begin{aligned} B_3 e^{-\alpha\ell} + C_3 e^{\alpha\ell} - B_4 e^{-\alpha\ell} + B_3 e^{-\alpha\ell} - C_3 e^{\alpha\ell} - B_4 e^{-\alpha\ell} + \mu I r_0 J_1(\alpha r_0) &= 0 \\ 2B_3 e^{-\alpha\ell} &= 2B_4 e^{-\alpha\ell} - \mu I r_0 J_1(\alpha r_0) \\ B_4 &= B_3 + \frac{\mu I r_0 J_1(\alpha r_0)}{2e^{-\alpha\ell}} \end{aligned} \quad (3.58)$$

Substituting equation (3.58) in to equation (3.54)

$$\begin{aligned} B_3 e^{-\alpha\ell} + C_3 e^{\alpha\ell} - \left(B_3 + \frac{\mu I r_0 J_1(\alpha r_0)}{2e^{-\alpha\ell}}\right) e^{-\alpha\ell} &= 0 \\ C_3 e^{\alpha\ell} - \frac{\mu I r_0 J_1(\alpha r_0)}{2} &= 0 \\ C_3 &= \frac{\mu I r_0 J_1(\alpha r_0)}{2e^{\alpha\ell}} = \frac{\mu I r_0 J_1(\alpha r_0)}{2} e^{-\alpha\ell} \end{aligned} \quad (3.59)$$

Subtracting equation (3.52) from equation (3.53)

$$\begin{aligned}
\frac{\alpha_1}{\alpha}C_2 - \frac{\alpha_1}{\alpha}B_2 + C_3 - B_3 - (C_2 + B_2 - C_3 - B_3) &= 0 \\
\left(\frac{\alpha_1}{\alpha} - 1\right)C_2 - \left(\frac{\alpha_1}{\alpha} + 1\right)B_2 + 2C_3 &= 0 \\
\left(\frac{\alpha_1}{\alpha} - 1\right)C_2 - \left(\frac{\alpha_1}{\alpha} + 1\right)\frac{e^{\alpha_1 c}(\alpha_1 + \alpha)}{e^{-\alpha_1 c}(\alpha_1 - \alpha)}C_2 + 2\frac{\mu I r_0 J_1(\alpha r_0)}{2}e^{-\alpha \ell} &= 0 \\
\frac{e^{-\alpha_1 c}\alpha(\alpha_1 - \alpha)(\alpha_1 + \alpha) - e^{\alpha_1 c}(\alpha_1 + \alpha)^2}{e^{-\alpha_1 c}\alpha(\alpha_1 - \alpha)}C_2 &= -2\frac{\mu I r_0 J_1(\alpha r_0)}{2}e^{-\alpha \ell} \\
C_2 = \frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{2e^{-\alpha_1 c}\alpha(\alpha_1 - \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] & \quad (3.60)
\end{aligned}$$

Substituting equation (3.60) in to equation (3.57)

$$\begin{aligned}
B_2 &= \left[\frac{e^{\alpha_1 c}(\alpha_1 + \alpha)}{e^{-\alpha_1 c}(\alpha_1 - \alpha)} \right] \frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{2e^{-\alpha_1 c}\alpha(\alpha_1 - \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] \\
B_2 &= \frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{2\alpha e^{\alpha_1 c}(\alpha_1 - \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] \quad (3.61)
\end{aligned}$$

Substitute equation (3.60),(3.61) in to (3.52)

$$\begin{aligned}
B_1 e^{-\alpha c} - e^{\alpha_1 c} \left[\frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{2e^{-\alpha_1 c}\alpha(\alpha_1 - \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] \right] - \\
e^{-\alpha c} \left[\frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{2\alpha e^{\alpha_1 c}(\alpha_1 - \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] \right] &= 0 \\
B_1 e^{-\alpha c} &= \frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{2\alpha e^{\alpha_1 c}(\alpha_1 + \alpha)e^{-\alpha_1 c} + 2e^{-\alpha_1 c}\alpha(\alpha_1 - \alpha)e^{\alpha_1 c}}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] \\
B_1 &= \frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{4\alpha\alpha_1 e^{\alpha c}}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] \quad (3.62)
\end{aligned}$$

Substituting equation (3.59), (3.60) and (3.61) in to equation (3.52)

$$\begin{aligned}
B_3 &= \frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{2\alpha e^{\alpha_1 c}(\alpha_1 - \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] + \\
\frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{2e^{-\alpha_1 c}\alpha(\alpha_1 - \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] - \frac{\mu I r_0 J_1(\alpha r_0)}{2}e^{-\alpha \ell} & \\
B_3 &= \frac{\mu I r_0}{2}J_1(\alpha r_0)e^{-\alpha \ell} \left[\frac{e^{\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha) + e^{-\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] \quad (3.63)
\end{aligned}$$

Substituting equation (3.63) in to equation (3.58)

$$B_4 = B_3 + \frac{\mu I r_0 J_1(\alpha r_0)}{2e^{-\alpha \ell}} \quad (3.64)$$

Substitute B_3

$$B_4 = \frac{\mu I r_0}{2} J_1(\alpha r_0) e^{-\alpha \ell} \left[\frac{e^{\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha) + e^{-\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] + \frac{\mu I r_0 J_1(\alpha r_0)}{2e^{-\alpha \ell}}$$

$$B_4 = \frac{\mu I r_0}{2} J_1(\alpha r_0) e^{-\alpha \ell} \left[\frac{e^{\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha) + e^{-\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} + e^{2\alpha \ell} \right] \quad (3.65)$$

So far all of six unknowns are derived and the magnetic vector potential in each layer can be obtained as follow: From equation (3.41)

$$A^{(1)}(r, z) = \int_0^\infty B_1 e^{-\alpha z} J_1(\alpha r) d\alpha$$

Substitute B_1

$$A^{(1)}(r, z) = \frac{\mu I r_0}{2} \int_0^\infty J_1(\alpha r_0) J_1(\alpha r) e^{-\alpha \ell} \cdot \left[\frac{2\alpha \alpha_1 e^{\alpha c}}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] d\alpha \quad (3.66)$$

From equation (3.42)

$$A^{(2)}(r, z) = \int_0^\infty [C_2 e^{+\alpha_1 z} + B_2 e^{-\alpha_1 z}] J_1(\alpha r) d\alpha$$

Substitute B_2 and C_2

$$A^{(2)}(r, z) = \mu I r_0 \int_0^\infty J_1(\alpha r_0) J_1(\alpha r) e^{-\alpha \ell} \alpha \cdot \left[\frac{e^{-\alpha_1(c-z)}(\alpha_1 - \alpha) + e^{\alpha_1(c-z)}(\alpha_1 + \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] d\alpha \quad (3.67)$$

From equation (3.43)

$$A^{(3)}(r, z) = \int_0^\infty [C_3 e^{-\alpha z} + B_3 e^{+\alpha z}] J_1(\alpha r) d\alpha$$

Substitute C_3 and B_3

$$A^{(3)}(r, z) = \frac{\mu I r_0}{2} \int_0^\infty J_1(\alpha r_0) J_1(\alpha r) e^{-\alpha \ell} \cdot \left\{ e^{-\alpha z} + \left[\frac{e^{\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha) + e^{-\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] e^{-\alpha z} \right\} d\alpha \quad (3.68)$$

From equation (3.44)

$$A^{(4)}(r, z) = \int_0^\infty B_4 e^{\alpha z} J_1(\alpha r) d\alpha$$

Substitute B_4

$$A^{(4)}(r, z) = \frac{\mu I r_0}{2} \int_0^\infty J_1(\alpha r_0) J_1(\alpha r) e^{\alpha(z-\ell)} \cdot \left\{ \frac{e^{\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha) + e^{-\alpha_1 c}(\alpha_1 - \alpha)(\alpha_1 + \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} + e^{2\alpha \ell} \right\} d\alpha \quad (3.69)$$

As a result, using the superposition principle, the magnetic vector potential on the induction plate with a single planar n-turns spiral coil can be derived as follows.

$$A^{(2)}(r, z) = \sum_{i=1}^n \mu I r_i \int_0^\infty J_1(\alpha r_i) J_1(\alpha r) e^{-\alpha \ell} \alpha \cdot \left[\frac{e^{-\alpha_1(c-z)}(\alpha_1 - \alpha) + e^{\alpha_1(c-z)}(\alpha_1 + \alpha)}{e^{\alpha_1 c}(\alpha_1 + \alpha)^2 - e^{-\alpha_1 c}(\alpha_1 - \alpha)^2} \right] d\alpha \quad (3.70)$$

The eddy current density in the induction plate can therefore be calculated as follows:

$$J_e = -j\omega\sigma A \quad (3.71)$$

And the eddy current obtained enables the calculation of the generated power losses and it is given as:

$$q_G = \frac{|J_e|^2}{\sigma} \quad (3.72)$$

3.3 Governing Heat Equation

In the cylindrical coordinates with a general transient three-dimensional heat conduction problem for $T = T(r, \theta, z, t)$, consider an elementary volume $dv = r dr d\theta dz$ By Fourier law

$$\begin{aligned} dQ_r &= -k(r d\theta dz) \frac{\partial T}{\partial r} \\ dQ_{r+dr} &= dQ_r + \frac{\partial}{\partial r} (dQ_r) dr \\ dQ_r - dQ_{r+dr} &= -\frac{\partial}{\partial r} \left(-k r d\theta dz \frac{\partial T}{\partial r} \right) dr \\ dQ_r - dQ_{r+dr} &= -\frac{\partial}{\partial r} \left(-k r d\theta dz \frac{\partial T}{\partial r} \right) dr \\ dQ_r - dQ_{r+dr} &= k r d\theta dz dr \frac{\partial^2 T}{\partial r^2} + k d\theta dz dr \frac{\partial T}{\partial r} \end{aligned} \quad (3.73)$$

Similarly, $d\theta_\theta = -k dz dr \frac{\partial T}{\partial \theta}$

$$\begin{aligned} dQ_{\theta+d\theta} &= dQ_\theta + \frac{\partial}{\partial \theta} (dQ_\theta) r d\theta \\ dQ_\theta - dQ_{\theta+d\theta} &= -\frac{\partial}{\partial \theta} \left(-k dz dr \frac{\partial T}{\partial \theta} \right) r d\theta \end{aligned}$$

$$\begin{aligned}
& kdzdrd\theta \frac{1}{r} \frac{\partial^2 T}{\partial \theta^2} \\
& dQ_z = -kdrd\theta \frac{\partial T}{\partial z} \\
& dQ_{z+dz} = dQ_z + \frac{\partial}{\partial z}(dQ_z)dz = kdrd\theta \frac{\partial^2 T}{\partial z^2} dz
\end{aligned} \tag{3.74}$$

The rate of heat generation from internal heat source is given as:

$$q_G drrd\theta dz \tag{3.75}$$

The rate of energy accumulated within the cylindrical volume is given by

$$\rho(drrd\theta dz)c \frac{\partial T}{\partial t} \tag{3.76}$$

By energy balance

$$\rho drrd\theta dz c \frac{\partial T}{\partial t} = krd\theta dz dr \frac{\partial^2 T}{\partial r^2} + kd\theta dz dr \frac{\partial T}{\partial r} + kdzdrd\theta \frac{1}{r} \frac{\partial^2 T}{\partial \theta^2} + kdrd\theta \frac{\partial^2 T}{\partial z^2} + q_G drrd\theta dz \tag{3.77}$$

Dividing both sides by $drrd\theta dz$

$$\begin{aligned}
\rho c \frac{\partial T}{\partial t} &= k \frac{\partial^2 T}{\partial r^2} + k \frac{1}{r} \frac{\partial T}{\partial r} + k \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + k \frac{\partial^2 T}{\partial z^2} + q_G \\
\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_G}{k} &= \frac{1}{\alpha} \frac{\partial T}{\partial t}
\end{aligned} \tag{3.78}$$

where $\alpha = \frac{k}{\rho c}$ is the material's thermal diffusivity, which measures how speedily heat moves through it. Equation (3.78) is the general heat conduction in three-dimensional cylindrical coordinates. Using Laplacian operator $\nabla^2 T = \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}$ it can be written as:

$$k\nabla^2 T + q_G = \rho c_p \frac{\partial T}{\partial t} \tag{3.79}$$

For steady state system (no time dependency)

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q_G}{k} = 0 \tag{3.80}$$

In case of axis symmetry (no angle dependency)

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} + \frac{q_G}{k} = 0 \quad (3.81)$$

So, for mitad which is symmetrical along radial direction equation (3.81) holds. To solve the partial differential equation, boundary conditions are required. Many different types of boundary conditions can be applied to the system's domain boundary. A faulty boundary condition imposition might cause the solution to diverge or converge to the incorrect answer, which is critical for addressing the computational problem. Therefore, in this thesis two types of boundary conditions are applied, Neumann boundary condition and mixed boundary condition (Robin).

3.4 Neumann Boundary Conditions

The Neumann boundary condition can be defined along the axis of symmetry if the body being heated is geometrically symmetrical.

$$\frac{\partial T}{\partial n} = 0 \quad (3.82)$$

From equation (3.82) the temperature gradient in a direction normal to the axis of symmetry is zero, according to the Neumann boundary condition. In other words, there is no heat transfer along the symmetry axis. In the case of a perfectly insulated workpiece, this boundary condition can also be used. Heat transfer is reduced by insulation, but it is not eliminated unless the thickness is infinite. However, heat transfer via an appropriately insulated surface can be considered zero since adequate insulation decreases heat transfer through a surface to insignificant levels. Therefore, a well-insulated surface can be modeled as a surface with zero heat flux (Welty et al., 2020). So, in this study, the lower boundary of mitad is assumed to be perfectly insulated and the left boundary which is along the axis of symmetry is the boundary where the temperature gradient is zero hence Neumann boundary condition is applied.

3.5 Mixed Boundary Condition

Since most heat transfer surfaces are exposed to an environment at a specific temperature, a mixed boundary condition (convection) is probably the boundary condition that is encoun-

tered most frequently in practice. Accordingly, the right surface and the top of the mitad are exposed to the environment and the mixed boundary condition is applied to solve the heat conduction given in the equation (3.81). As a result, the mixed boundary condition can be written as follows:

$$\frac{\partial T}{\partial n} + \frac{h}{\lambda} (T_{\infty} - T_a) = 0 \quad (3.83)$$

Where h is convection coefficient, λ is thermal conductivity T_{∞} is steady state temperature and T_a is ambient temperature. For analyzing the heat transfer at the boundary of the plate the above-mentioned two boundary conditions are applied. The thermal conductivity and convection coefficient depends on the type of material used in the system.

3.6 Determining Critical Radius of Insulation

Consider a mitad with a constant temperature at a circular edge that transfers heat into the surrounding air by convection and an insulating layer to be applied to the circular edge. The variation in heat loss from the mitad surface when the insulation thickness is varied has to be investigated.

The outside exposed surface temperature will decrease as insulation is added to the mitad due to increased conduction resistance, but the surface area available for convective heat dissipation will increase resulting in additional heat loss. The combination of these two opposing effects results in the optimal thickness of insulation (Karwa, 2020).

Let the plate of radius r_p and thickness t , at temperature T_p be covered with insulation as shown in Figure 3.3 At the outside radius of the insulation r_0 , a surface convection coefficient h_a is assumed for heat transfer to the atmosphere at temperature T_a . As r_0 increases, insulation thickness ($r_0 - r_p$) increases and heat loss (Q) decreases. Again, as r_0 increases, surface area (A_0) increases, and Q increases. There is, thus, an optimum r_0 at which Q is the maximum. The temperature difference between the plate and the surrounding environment is calculated as follows:

$$T_p - T_a = Q \left[\frac{\ln(r_0/r_p)}{2\pi k_i t} + \frac{1}{2\pi r_0 h_a} \right] \quad (3.84)$$

From equation (3.84) the heat dissipated to the environment from the circular edge surface is given as:

$$Q = \frac{2\pi t (T_p - T_a)}{\frac{1}{k_i} \ln \left[\frac{r_0}{r_p} \right] + \frac{1}{h_a r_0}} \quad (3.85)$$

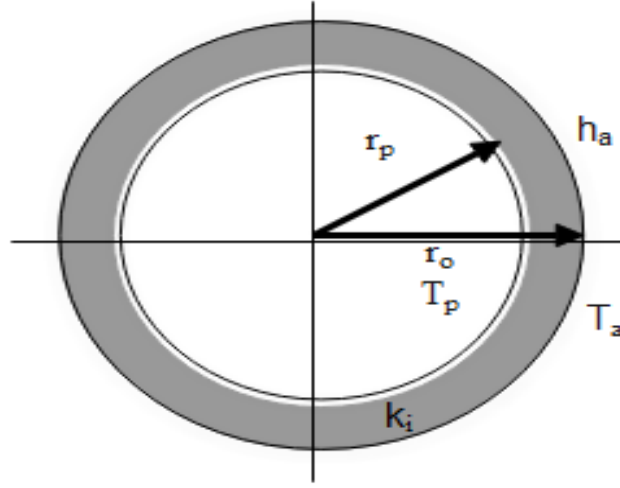


Figure 3.3: Critical radius of insulation (Cengel & Heat, 2003)

From equation (3.85) T_p, T_a, r_p, k_i and h_a are all constants, then the heat loss Q depends only on r_o . As r_o increases $\frac{1}{k_i} \ln \left[\frac{r_o}{r_p} \right]$ increase but $\frac{1}{h_a r_o}$ decrease. Differentiating equation (3.85) with respect to r_o and Equating to zero will give critical radius of insulation.

$$\frac{dQ}{dr_o} = -2\pi t(T_p - T_a) \left[\frac{1}{k_i} \frac{r_p}{r_o} - \frac{1}{h_a r_o^2} \right] = 0 \quad (3.86)$$

From equation (3.86) the critical radius of insulation can be solved as follow.

$$(r_o)_{critical} = \frac{k_i}{h_a} \quad (3.87)$$

The thermal conductivity of clay is 0.4W/mK and the critical radius of insulation for the

Table 3.1: Parameter used to calculate heat loss and critical radius of insulation Liu et al. (2021)

No	Parameters	Value
1	Thickness of plate	5mm
2	Radius of plate	29cm
3	Thermal conductivity of clay	0.25 W/mK
4	Convection coefficient	12.5
5	Plate temperature	220 ⁰ _c
6	Ambient temperature	25 ⁰ _c

proposed mitad is calculated by equation (3.87) and it is 3.2cm for a proposed system based on the parameters given in Table 3.1. So, until this critical radius is reached the heat loss increases and at this critical value the heat loss will be maximum but beyond this value, the

heat loss decreases for an increase in radius.

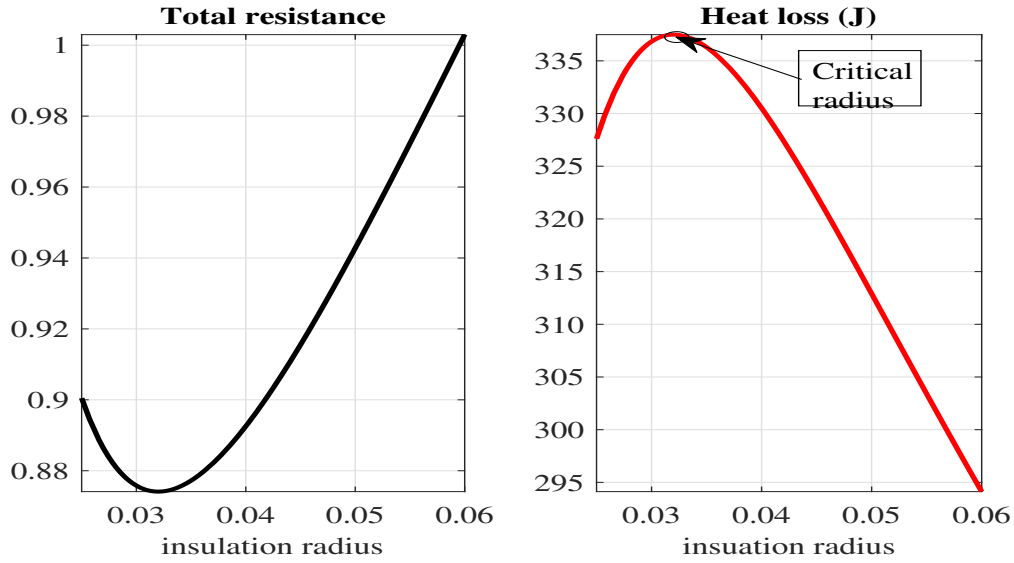


Figure 3.4: Relationship between insulation thickness and heat loss

Figure 3.4 shows the relationship between the insulation heat resistance and heat loss at the side of the mitad. It is visible that when the radius of insulation increases the heat loss to the environment decreases. But, this is due to the increase in conductive resistance which is the first term of the denominator of the equation (3.85). On the other hand, the convective resistance increases as the radius increases and the design should consider these two opposing phenomena. Insulation's objective is to reduce overall undesired heat dissipation. As a result, the optimum insulation thickness should be larger than the critical insulation thickness. However, due to the high cost of supplying more insulation thickness, is not economically viable. Since heat loss to the side is not desired in mitad the radius of insulation should increase beyond the critical radius with consideration of cost and size.

3.7 Simulink Model of Power Converter

The system power converter has a full bridge rectifier and High- frequency inverter as the main components as shown in Figure2.6. The rectifier rectifies the AC source to pulsating DC and the LC filter removes the harmonics and changes to smooth DC. Then the DC link voltage from the filter is applied to the inverter and converted to high-frequency AC voltage. Finally, this high-frequency current is given to the working coil to generate a high-frequency magnetic field, which induces high-frequency eddy current in the working plate. And this high-frequency eddy current generates heat in the working plate by the joule effect. The induction load (equivalent resistance and inductance) is derived as shown in Appendix B and

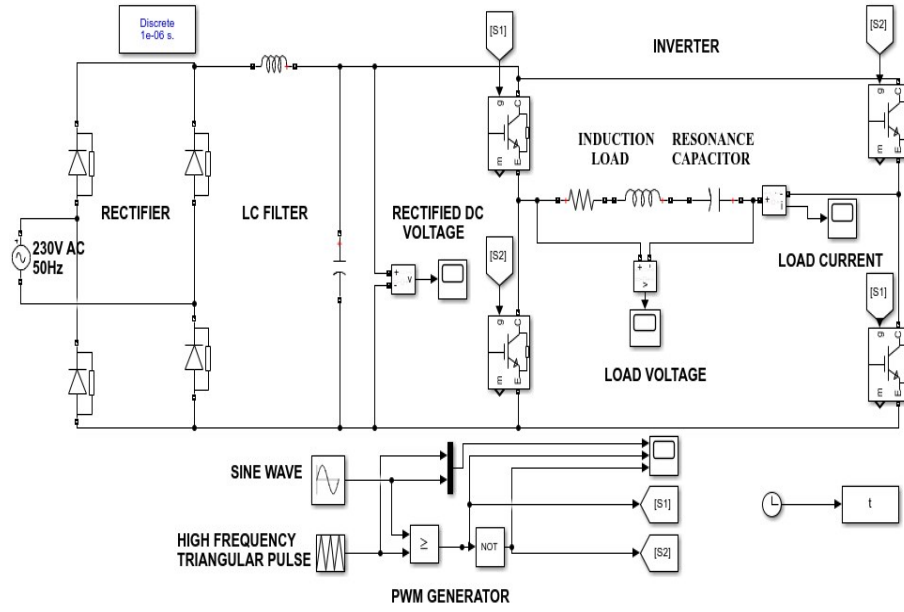


Figure 3.5: Simulink model of the system

the capacitor is added to form a resonant condition. The simulation parameter values are listed in appendix C. Here, the switching device used is IGBT as it has better performance for high frequency and low power systems. Additionally, the low power consumption, low operating voltage, and high speed of IGBTs have made them one of the essential and primary components of the electronics design industry (Tayade & Lanudkar, 2020).

3.8 Materials Used For Simulation and Experimental Setup

In this research work, the following software and hardware components are employed: MATLAB 2019a/Chebfun for the simulation; infrared temperature sensor, thermistor, ferric stainless steel plate, cast iron, enameled copper wire, digital multimeter, breadboard, and wood are used to conduct the test.

3.8.1 Matlab Chebfun

In 2002, Lloyd N. Trefethen and his student Zachary Battles launched the Chebfun project at the University of Oxford's Mathematical Institute. Chebfun is a numerical computing software system that is free and open-source. Chebyshev technology is used to do piecewise polynomial interpolation, which is the mathematical basis of Chebfun. Chebfun has continuous counterparts of linear algebra concepts like QR and singular value decomposition, as well as a wide range of linear and nonlinear differential and integral operator capabilities. Chebfun2 is a two-variable function extension that works with x-y plane rectangles (Driscoll et al., 2014). Chebyshev polynomials and piecewise polynomial interpolants are the math-

emational basis of Chebfun, hence the name "Cheb." The program intends to bring symbolic computing tools like Maple and Mathematica together with the speed of floating-point numeric calculations. In this thesis, Matlab Chebfun has been applied to analyze the magneto thermal phenomena of the system and it is a new approach to use Chebfun to solve the induction mitad partial differential equation. When compared to the finite element method Chebfun is fast and accurate as it has continuous counterparts of linear algebra and nonlinear differential and integral operators and it has a high speed for calculating floating points. As mentioned in the literature, the finite element method needs around 14 hours of CPU processing time to solve magnetic vector potential while Chebfun needs around 4-5 hours for the same problem.

3.8.2 Induction Coil Preparation

The coil arrangement for the proposed system is done based on the simulation result obtained. Figure 3.6a shows the induction coil formed from insulated copper wire and tested whether it is suitable for induction heating or not. From the test, it is possible to generate heat with insulated copper wire but the heat from the bottom of the plate causes the insulation to melt hence, it is not recommended to use it. Figure 3.6b shows Litz wire is wounded to

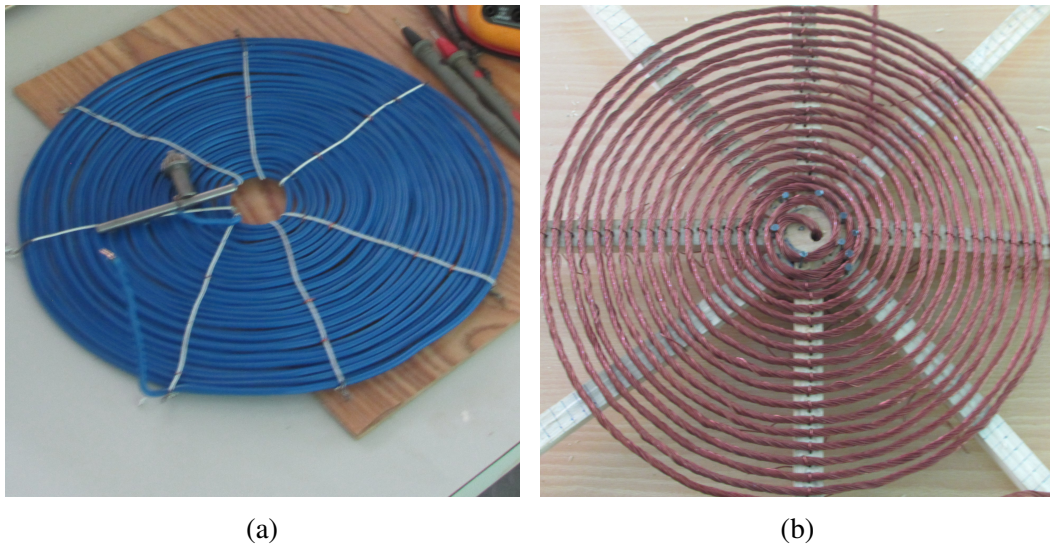


Figure 3.6: Spiral coil made from a) Insulated copper b) Enameled copper wire

generate heat in the working plate. The Litz wire is formed from 24 single enameled copper wires of 0.7mm diameter each and the spiral coil of 8mm uniform spacing between each turn is wounded. The Litz wire has enough capability to withstand the heat encountered from the bottom of the plate.

3.8.3 Wood Frame

Figure 3.7 shows the top view of the developed system. The frame which has 60cm x 60 cm x 10 cm dimensions is formed from wood to shield the system and protect the coil and power circuit from external damage. The coil and power converter circuit are placed inside this frame and a 6mm wood plate is placed over this frame to insulate the power circuit and coil from heat generated in the plate. The input and output interface terminal is attached to this wood frame to provide the interaction of the user with the system. The lid cover made from aluminum is used to cover the mitad to resist heat transfer to the environment from the upper boundary during baking injera and also it is important to get well-baked injera.

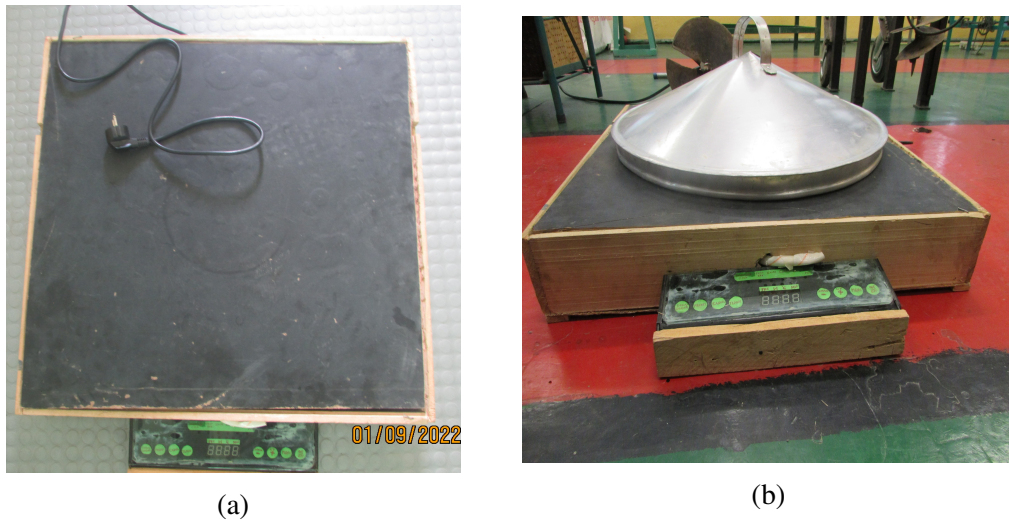


Figure 3.7: Proposed system top view with and without cover

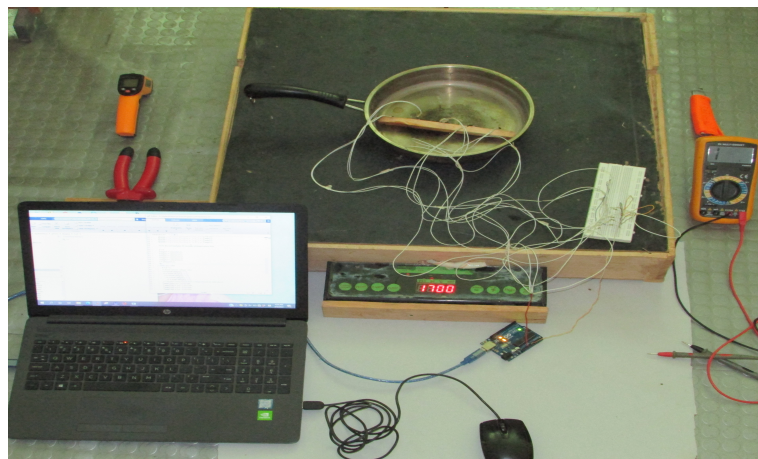


Figure 3.8: Temperature distribution measurement setup

Figure 3.8 shows the hardware arrangement for measuring and plotting the temperature distribution on the plate. Five thermistors are connected to Arduino Uno to read the analog

temperature from five points on the plate and the Arduino is connected to Matlab through hardware package support to save, process, and plot the live data from the sensors.

From the measurement setup, it can be seen that the hardware in the loop is measured to show temperature distribution along the diameter of the stainless-steel pan. Arduino Uno is used for data acquisition, PC is used to write Matlab program for Arduino to read analog temperature directly from thermistors, five 100K resistors are used to resist the flow of sensor signal to ground, and breadboard is used to construct the circuit on it. And also, other devices like a digital multimeter, Infrared temperature sensor, cutter, pliers, screwdrivers, and driller are used to achieve the objective of this research prototype.

The user interface part of the system has different buttons for different commands. It has eight pressure switches to give commands such as power ON/OFF, MENU, UP, DOWN, TIMER, CANCEL, PRESET, and INJERA/WOT to the system for operation and the seven segment display is used to show the values of temperature, power and time depending on the input command. The LEDs glows for the different command to make the system more understandable and user-friendly.

When the system is turned ON the buzzer sounds beep sound until the metal plate is placed over the system and if there is no metal the system automatically turns OFF the system after 10 seconds to save the power.

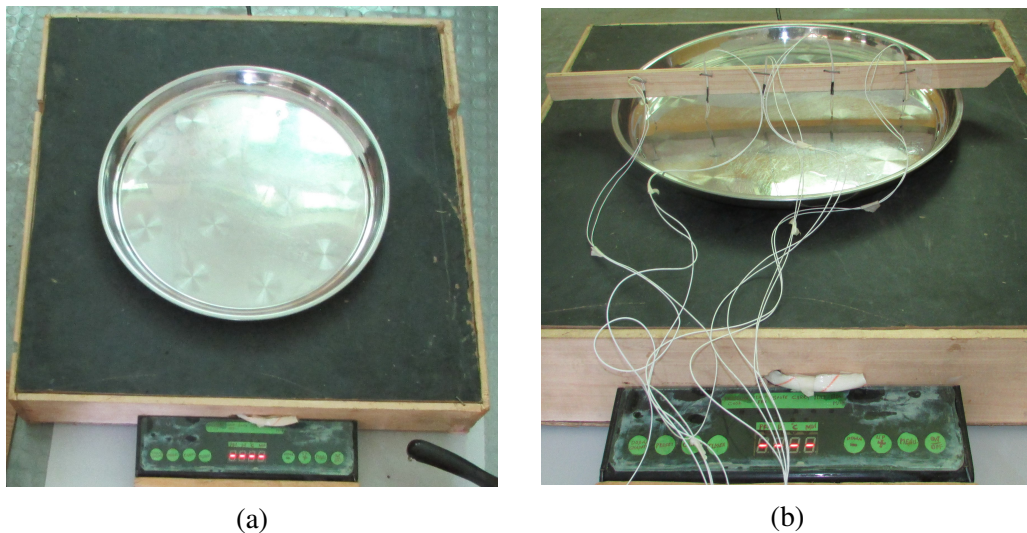


Figure 3.9: Stainless steel plate of 35 cm diameter

Figure 3.9a shows the 35 cm diameter stainless steel plate being heated and the temperature sensors placed along the diameter of the plate to measure temperature at each point shown in Figure 3.9b. The sensors are placed 7cm apart from each other along the diameter of the

plate and the temperature data at each point is plotted on Matlab to analyze the heat pattern on the plate. Figure 3.10a shows the 45 cm diameter cast iron plate being heated and the

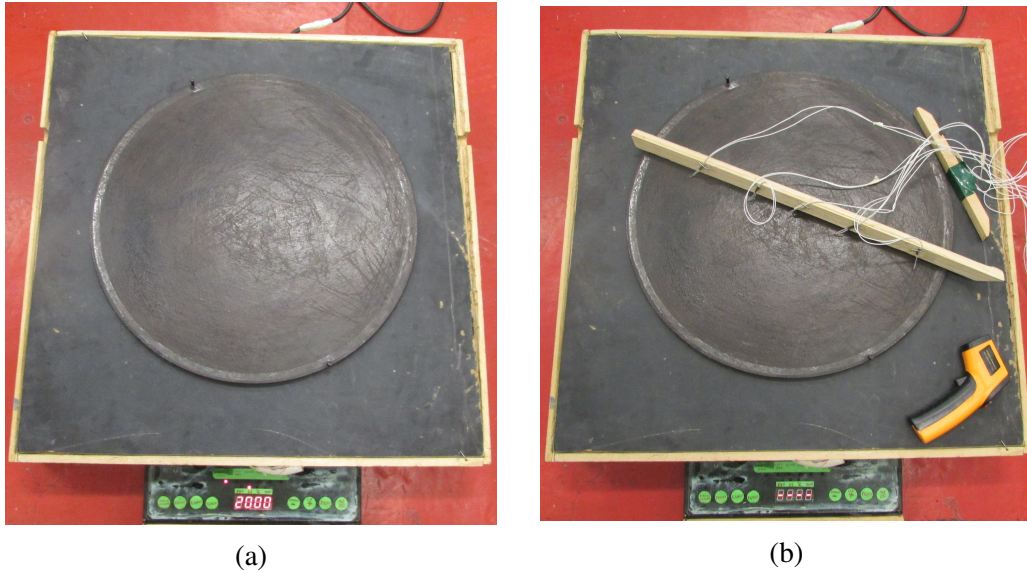


Figure 3.10: a) Cast iron plate of 45 cm diameter b) Temperature measurement

temperature sensors placed along the diameter of the plate to measure temperature at each point as shown in Figure 3.10b. The sensors are placed 9cm apart from each other along the diameter of the plate and the temperature data at each point is plotted on Matlab to analyze the heat pattern on the plate.

CHAPTER FOUR

RESULTS AND DISCUSSIONS

This chapter presents the simulation and hardware results to assess the effectiveness of the proposed system for distributing heat uniformly along the radial direction of induction multitad. In this section, a graphical representation presents the result of calculated quantitative performance measures from analytic and numerical simulation.

4.1 Power Conversion Result

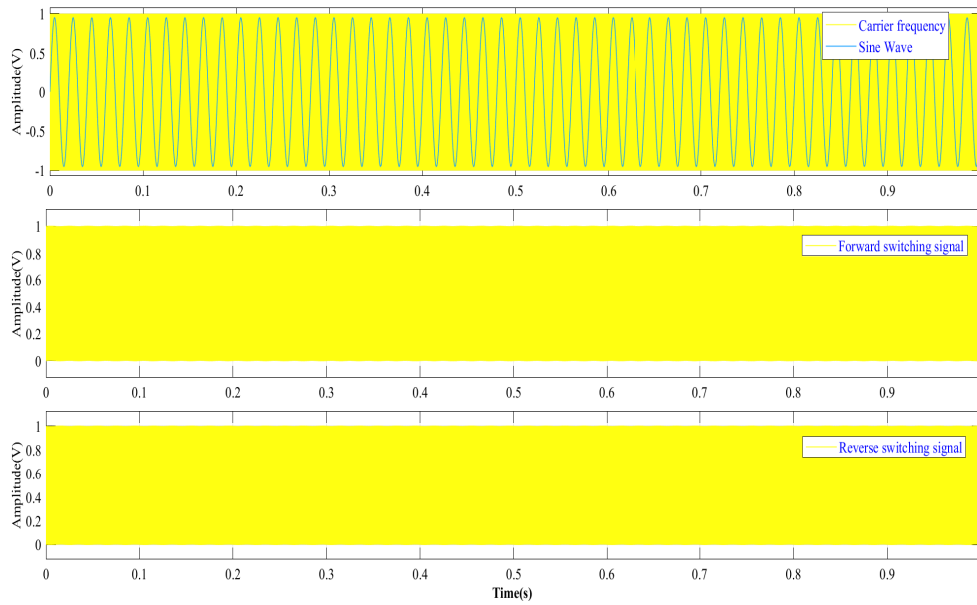


Figure 4.1: Switching signal

Figure 4.1, shows the switching pulse applied to the switching device (IGBT) generated by the PWM generator. The switching pulse is generated by a pulse width modulation mechanism in which a low-frequency sinusoidal signal is compared to a high-frequency saw-tooth signal to generate a switching frequency of 24KHz, to generate high-frequency voltage and current applied to the induction coil.

The LC filter shown in Figure 3.5, filters the rectified output voltage of the full bridge rectifier and generates smooth DC voltage shown in Figure 4.2. The filtered DC voltage from the full bridge rectifier has been sent to the FB-SRI to be converted to high-frequency AC. Finally, the FB-SRI produces 230V and 8.73 A RMS high-frequency voltage (V_{out}) and current (I_{out}). The waveforms of V_{out} and I_{out} are shown in Figure 4.3. The FB-SRI switching

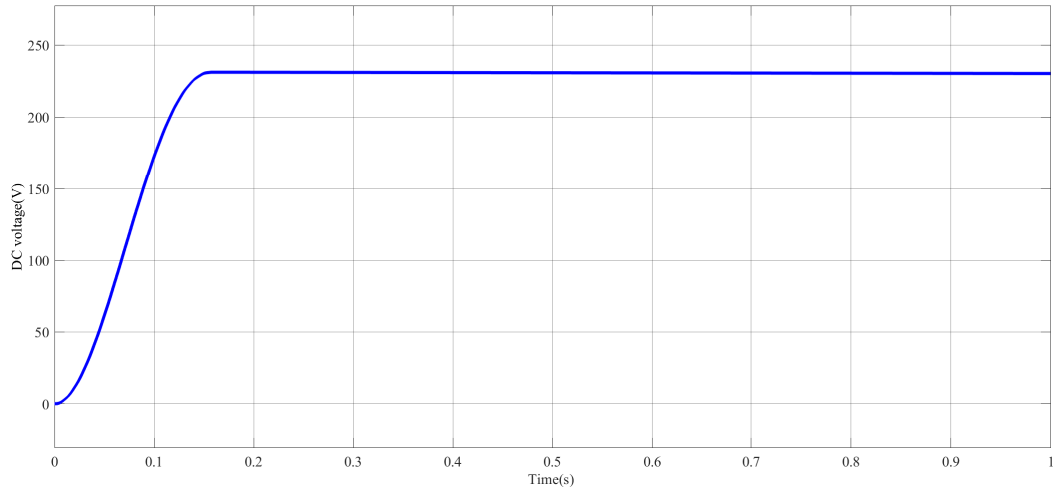
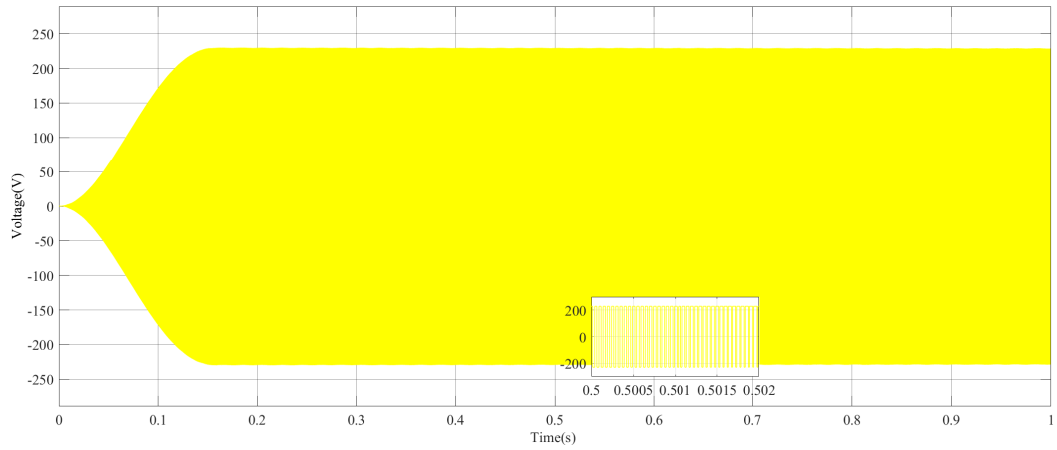
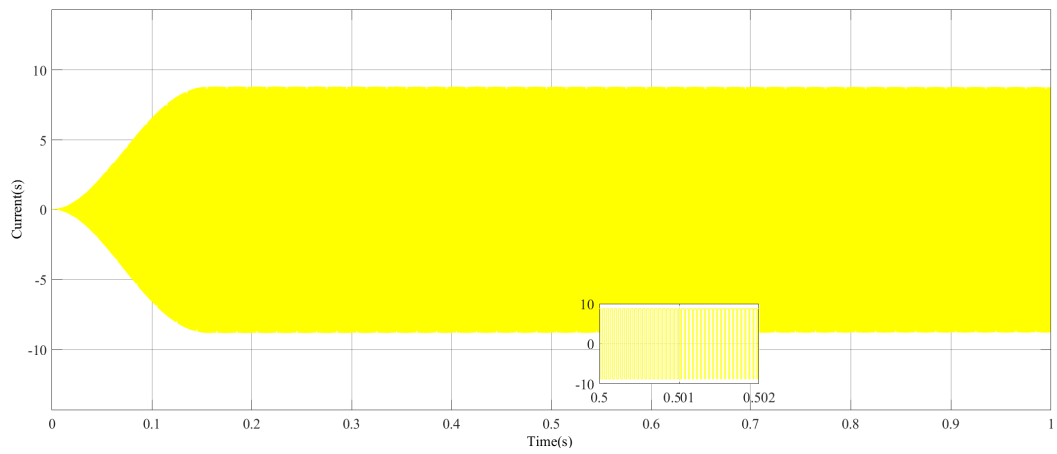


Figure 4.2: Filtered DC voltage

frequency has been determined as 24 kHz using the relationship $f_r = \frac{1}{2\pi\sqrt{LC}}$.



(a) Output Voltage



(b) Output Current

Figure 4.3: Wave form of output Voltage and Current

4.2 Heat Distribution Result

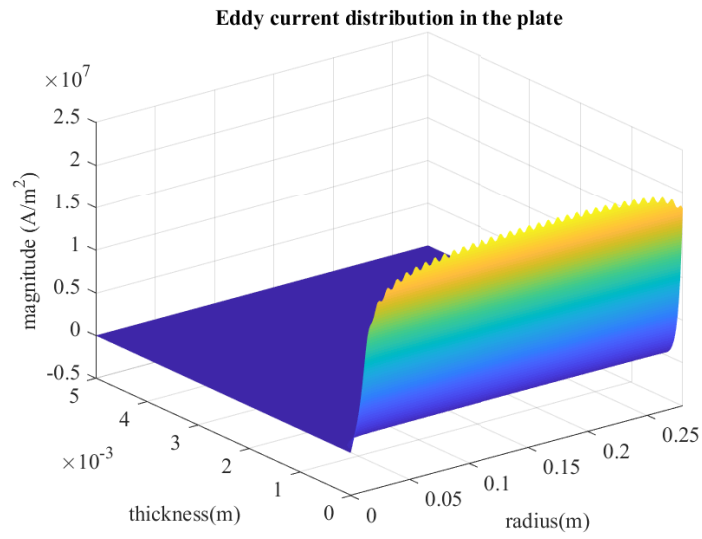


Figure 4.4: Eddy current distribution of 35 turns coil

Figure 4.4 shows the eddy current distribution along a radial direction which generates heat in the plate by the joule effect. This eddy current is generated by applying high-frequency AC to 35 turns coil winded uniformly with 8mm spacing between each turn as shown in Figure 4.6. The thickness of the stainless steel plate and the gap between the coil and plate is 5mm and 6mm respectively with a 24KHz frequency. From the figure, it can be observed that the eddy current distribution along the radial direction is uniformly distributed with a magnitude of around 2×10^7 A.

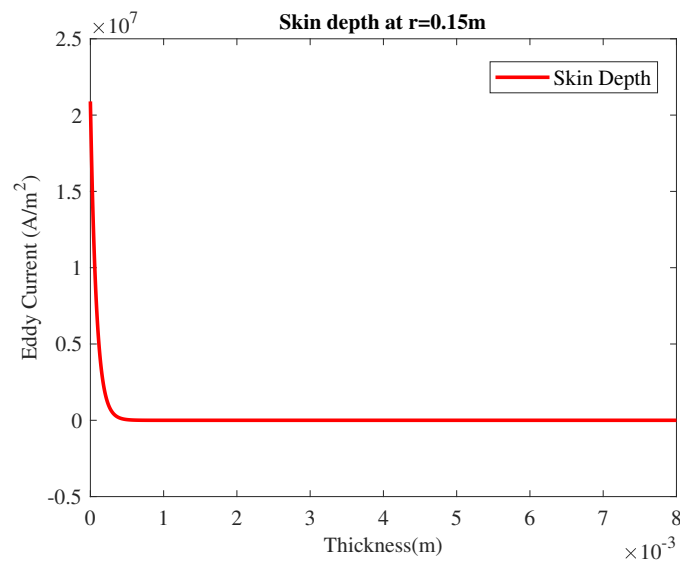


Figure 4.5: Eddy current penetration depth

Figure 4.5 shows the skin depth at $r=0.15\text{m}$ of eddy current shown in Figure 4.4. The penetration depth (skin depth) is similar along the radius of the plate and this improves the eddy current distribution and makes the heat distribution uniform. Since the skin depth of the eddy current is similar along the radius of the plate, the current penetrates the plate uniformly and the heat generated in the plate also become uniform. As discussed in chapter two, the skin effect which is determined by the skin depth depends on the magnetic field frequency, conductivity, and permeability of the working plate.

In this study, the frequency of the current supplied to the coil is 24kHz and if a stainless-steel plate which has 10^6mho conductivity and $1000\text{-}1800$ relative permeability is used, by taking the average value of relative permeability the skin depth is calculated as:

$$\delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{1}{\pi f\mu\sigma}}$$

$$\delta = \sqrt{\frac{1}{\pi f\mu\sigma}} = \sqrt{\frac{1}{3.14 \times 24 \times 10^3 \times 1400 \times 4 \times 3.14 \times 10^{-7} \times 10^6}}$$

$$\delta = 8.687 \times 10^{-5} \text{mm} \approx 0.1 \text{mm}$$

Also, from the numerical result of the eddy current shown in Figure 4.12a, the eddy current skin depth is around 0.1mm as shown in Figure 4.5. Therefore, the analytical and numerical solutions are similar and verified.

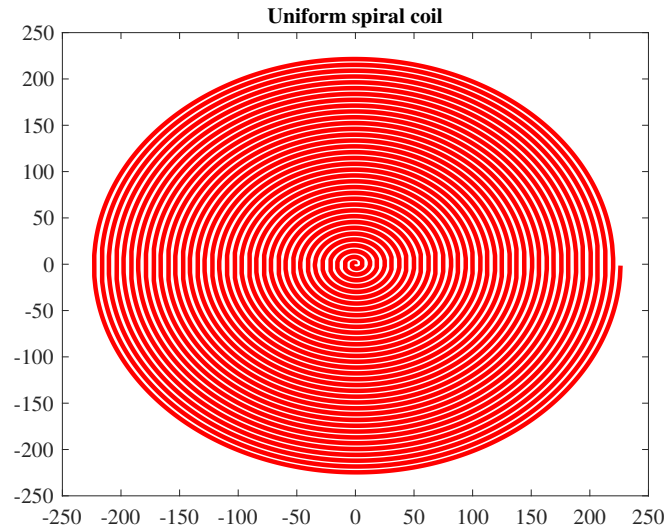


Figure 4.6: Uniformly spaced induction coil

Figure 4.6 shows the uniform spiral coil placed under the plate to generate a high-frequency eddy current. The coil arrangement is the base for eddy current density and heat distribution. To have uniform heat distribution, the coil span has to be arranged optimally.

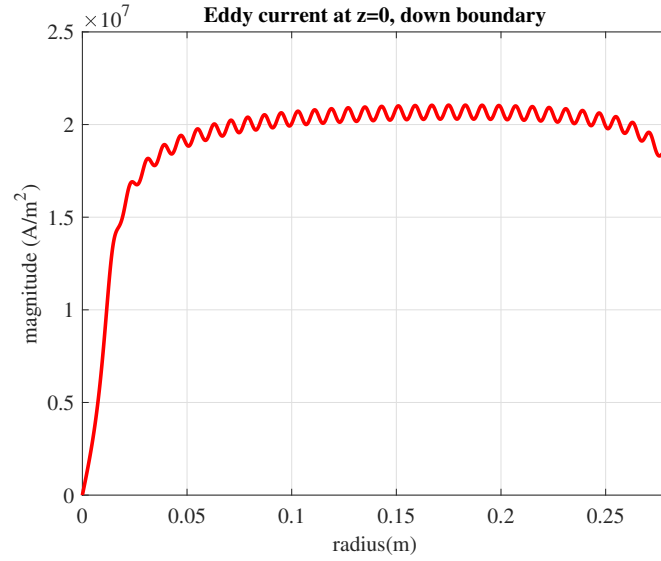


Figure 4.7: Eddy current distribution along radial direction at lower boundary

Figure 4.7 shows the eddy current at the lower boundary which penetrates the plate and generates heat in the plate. The fluctuation at the top of the graph is due to the space between each coil. As the coil span increases the eddy current becomes more fluctuating and similarly, when the coil is very dense the eddy current will be smooth.

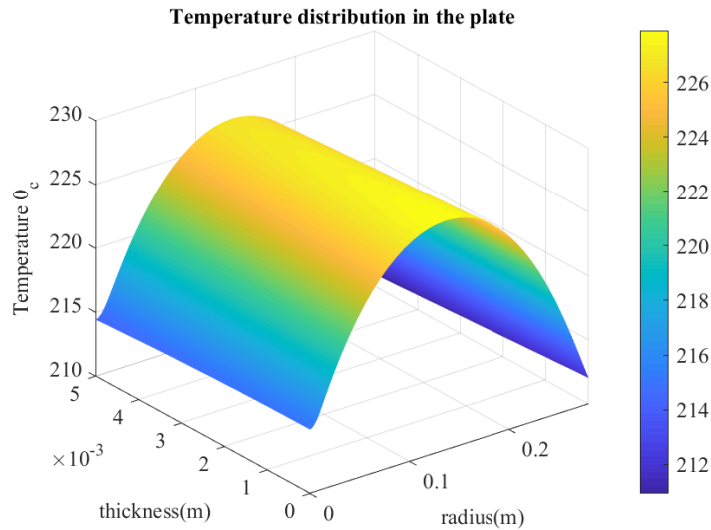


Figure 4.8: Temperature distribution in the whole plate

Figure 4.8, shows the simulation results for temperature distribution in the radial direction of a plate and along the thickness of the plate. As it can be seen from the figure, the temperature difference between the maximum and minimum value is less than 15°C which is very small as compared to other literature values. From the figure, the yellow color shows the high-temperature area while the blue color shows the area with low-temperature.

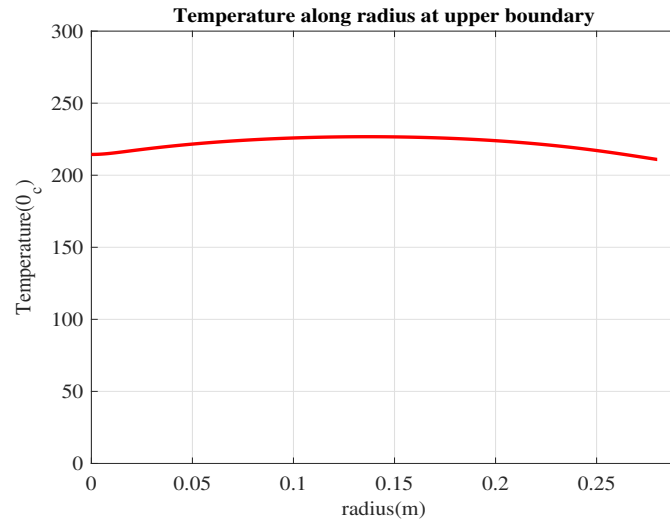


Figure 4.9: Temperature distribution on the top of the mitad plate.

Figure 4.9 shows the temperature distribution at the upper boundary (at $z=0.005$) which is the temperature needed to bake injera along the radial direction. From this figure, it can be seen that the temperature of the plate varies by less than 15°C from center to edge.

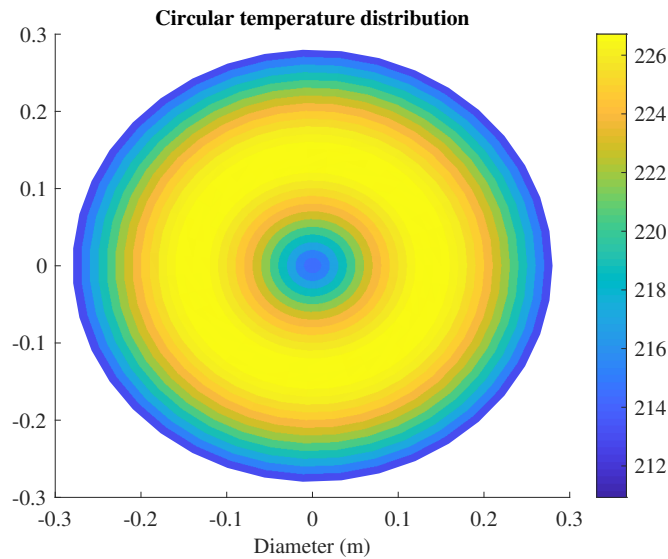


Figure 4.10: Circular temperature distribution

Figure 4.10 depicts the circular temperature distribution of the induction plate. Rather than concentrating in a tiny ring, the temperature dispersion covers a larger region of the induction plate, considerably improving the induction mitad's heating efficiency.

Figure 4.11 shows the thermal flow at the boundary of the system and it depicts there is very small heat transfer to the environment from the upper and right boundaries. The thermal flux is zero at lower and left boundaries due to insulation and axis of symmetry respectively.

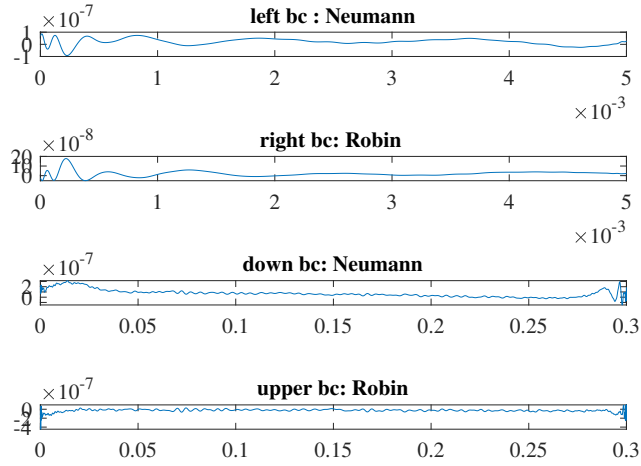


Figure 4.11: Heat transfer at boundaries

Therefore, at these boundaries, the heat transfer is very small (order of 10^{-7}) as can be seen from the figure.

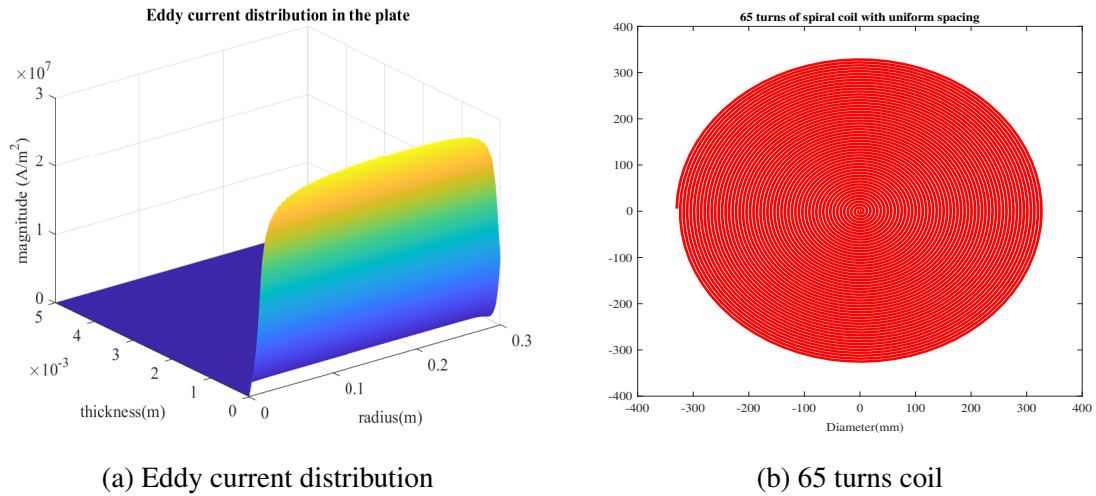


Figure 4.12: 65 turns coil and corresponding eddy current distribution

Figure 4.12a shows the eddy current distribution along a radial direction which generates heat in the plate by the joule effect. This eddy current is generated by applying high-frequency AC to 65 turns coil wound uniformly with 4.4mm spacing between each turn as shown in Figure 4.12b. For the same thickness of the plate, the gap between the coil and plate and frequency, as in the previous problem the eddy current magnitude is increased due to the densely wound coil. The top of the eddy current is smooth as compared to the previous one. This is due to magnetic field concentration resulting from many numbers of turns. From

Figure, it can be concluded that the coil span affects the eddy current distribution. When the coil is wound with many numbers of turns, the eddy current becomes smooth and there is no fluctuation on the top of the eddy current along the radial direction.

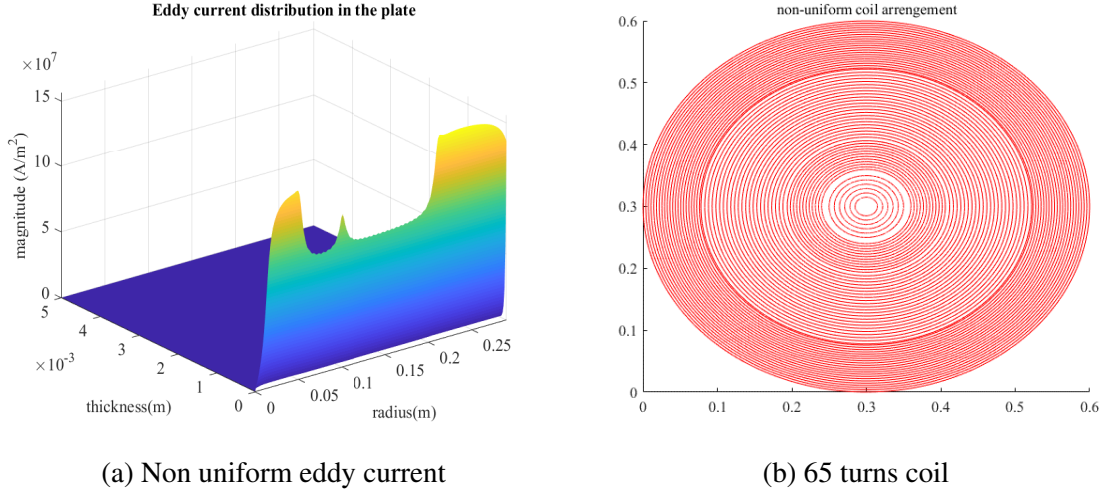


Figure 4.13: Non uniform coil spacing and corresponding eddy current distribution

Figure 4.13a shows the non-uniform eddy current distribution. It is generated by dividing the radius of the plate in to four regions as shown in Figure 4.13b. The coil is spaced in such a way that the eddy current increases at the edge of the plate to improve the temperature at that point. The four regions are:

$$1^{st} \text{ region} = 0.015 + 8\text{turns} \times 0.00325 = 0.041\text{m}$$

$$2^{nd} \text{ region} = 10\text{turns} \times 0.0053 = 0.053\text{m}$$

$$3^{rd} \text{ region} = 18\text{turns} \times 0.0064 = 0.1152\text{m}$$

$$4^{th} \text{ region} = 25\text{turns} \times 0.0036 = 0.09\text{m}$$

$$\text{Total radius} = 0.299\text{m}$$

The thickness of the plate used for this result is 5mm with 24KHz frequency. Since 25 turns of the coil is placed at the edge, the eddy current generated at this point is high which in turn produces high heat at the 4th region as shown in Figure 4.15a.

Figure 4.14 shows the temperature distribution at the upper boundary (top of the plate) generated by the eddy current shown in Figure 4.13a. The temperature is high at the edge due to the high eddy current distribution at this point. From this, it can be concluded that the temperature distribution is an image of eddy current and is highly affected by coil position.

Figure 4.15 depicts the circular temperature distribution of the heat generated by nonuniform coil spacing. Figure 4.15a shows the heat distribution of the coil with more turns at the edge

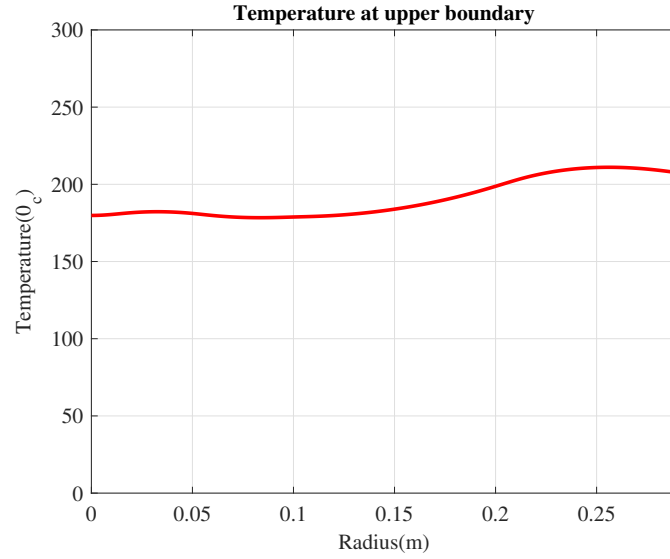


Figure 4.14: Non uniform temperature distribution at top of mitad

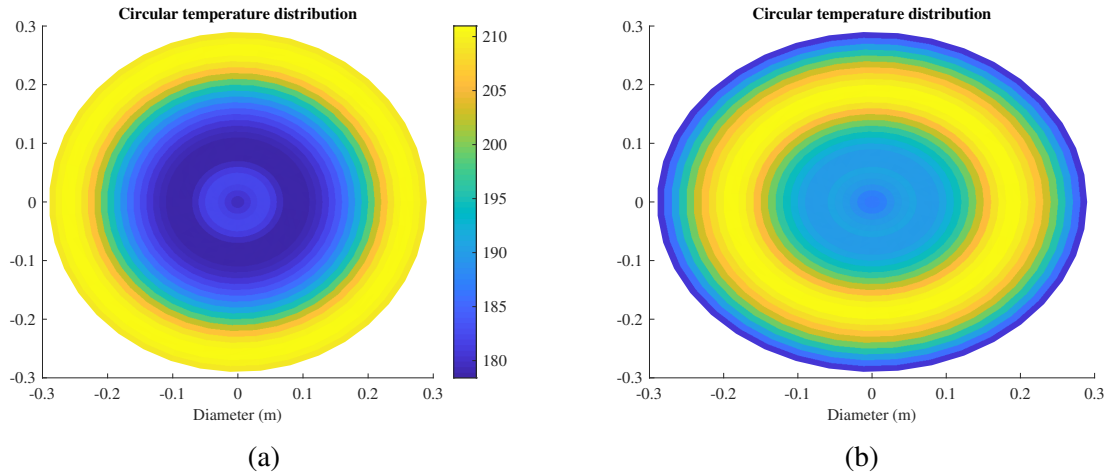


Figure 4.15: Heat distribution for different coil structure

and as a result high heat at the edge of the plate. Figure 4.15b shows another arrangement of the coil with non-uniform spacing and the heat distribution is like a ring in the middle. This is due to the greater number of turns of the induction coil and hence eddy current distribution at that point. The temperature distribution on the plate surface depends on the distribution of eddy current under the plate.

Figure 4.16a shows the eddy current generated by 65 turns, 5mm spacing, 4mm gap between coil and plate, and 24KHz frequency. The improvement here is that the diameter of the coil is greater than the diameter of the plate. This improves the problem of low eddy current and heat at the edge by increasing the magnetic field linking the plate. The magnetic field chooses the path with less magnetic reluctance and since the magnetic reluctance of air is

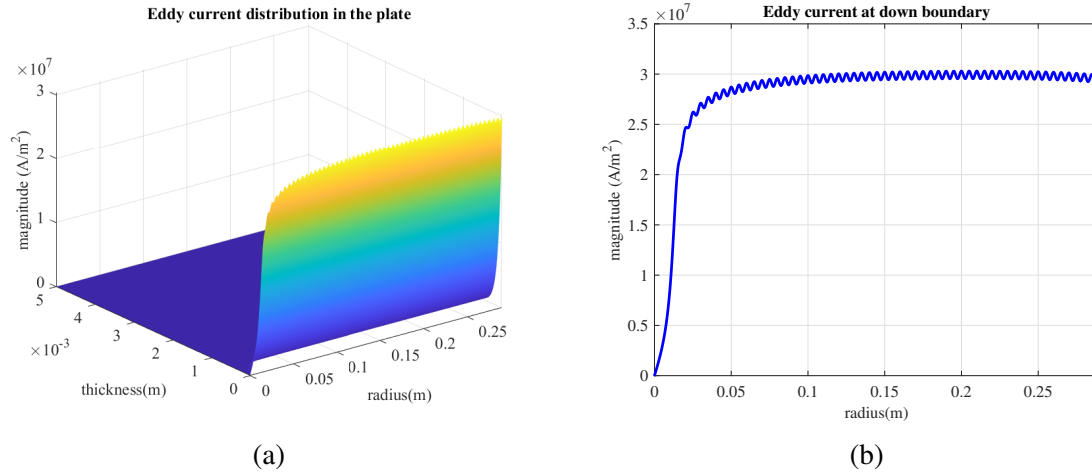


Figure 4.16: Eddy current distribution in whole plate and at lower boundary

greater than plate reluctance, the field flows to the plate from the side as shown in Figure 4.17 and this increases the field alternating in the plate and hence increase the eddy current generated at edge point.

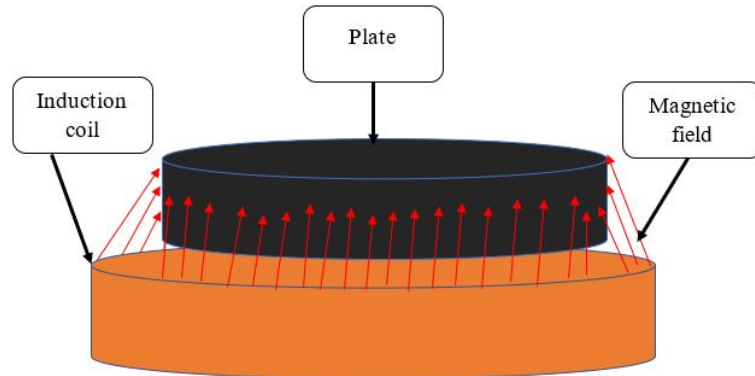


Figure 4.17: Magnetic field flowing toward plate from side

Figure 4.16b shows the eddy current at the lower boundary which penetrates the plate and generates heat in the plate. It rises fast and becomes uniform under the plate. Its magnitude is around 3×10^7 A/m² and almost uniform along the radius of the plate except for the center of the plate. The fluctuation at the top of the eddy current is due to the space between each coil. As the coil span increases the eddy current becomes more fluctuating and similarly when the coil is very dense the eddy current becomes smooth. Figure 4.18, shows the simulation results for temperature distribution in the radial direction of a plate generated by eddy current shown in Figure 4.16a. As it can be seen from the figure, the temperature difference between the maximum and minimum value is around 2 °C which is very small as compared to other literature values.

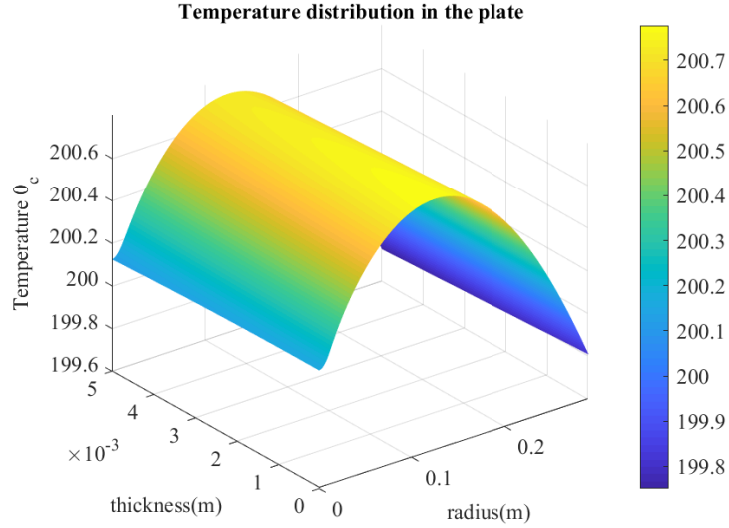


Figure 4.18: Temperature distribution along radius

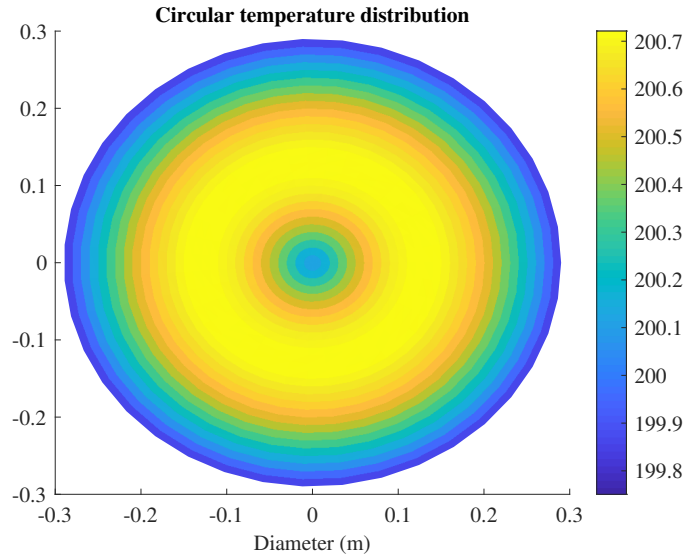


Figure 4.19: Circular temperature distribution

Figure 4.20 shows heat distribution on the top of the plate for different materials. It shows heat generated by 35 turns of the coil, 5mm spacing between each turns, and 4mm gap between coil and plate with 24 kHz frequency of coil current for different material to analyze the effect of thermal conductivity on heat distribution. From the figure, cast iron with thermal conductivity of 58W/mK exhibits a more uniform heat distribution compared to others. The difference between the maximum and minimum value is around 2.139°C . Stainless steel provides uniform heat distribution than pure steel due to its ability to retain heat. The minimum temperature difference obtained with stainless steel is around 8.21°C and this is less uniform compared to cast iron with other parameters constant. The last type of metal considered is

steel with thermal conductivity of 54W/mK as given in Table 2.1. The minimum temperature difference attained with steel plate is around 21.291°C which is very nonuniform heat distribution.

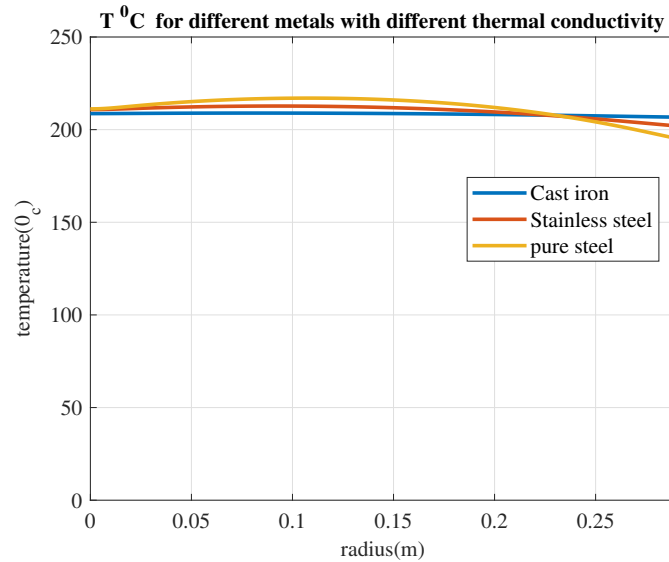


Figure 4.20: Heat distribution for different thermal conductivity

Figure 4.21a shows the eddy current distribution of double layer winding generated by 66 turns of the coil. The eddy current strength is very high compared to the single-layer winding discussed above. From the figure, the eddy current magnitude is around $13 \times 10^7 \text{ A/m}^2$ which is much greater than the eddy current generated by single layer winding. This high magnitude of eddy current improves high heat generation in the mitad plate which in turn improves the heating time. However, due to the high heat generated in the plate and the small thickness of the plate, it may cause bending of the plate or might be beyond the curie temperature value of the plate and may result in the destruction of the plate. To eliminate over heating of the plate efficient temperature control system is required. And also, due to the many numbers of turns winded, the cost of production increases and it is not economical to use it.

Figure 4.21b shows the eddy current at the lower boundary (bottom of the plate) generated by 66-turn double layer winding. As it can be seen from the figure the magnitude of eddy current at the bottom of the plate is high as compared to single-layer winding. This enhances the system to attain high temperatures within a short period.

Figure 4.22 shows the upper boundary temperature of 66 turns double layer winding with 5mm thickness of plate, 4mm gap between coil and plate, and 24KHz frequency. From the figure, it can be seen that the temperature value is increased to around 725°C which is more

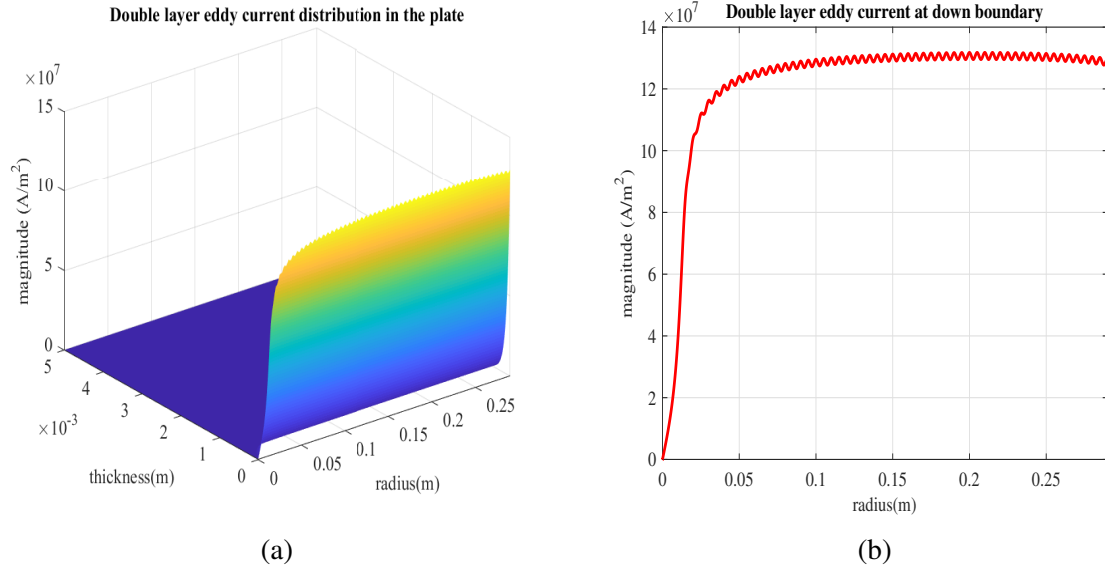


Figure 4.21: Double layer eddy current in whole plate and at lower boundary

than three times the single layer winding. It is better for fast heating but it is beyond the required temperature value for baking injera and this may cause the destruction of the plate depending on the type of material due to very high temperature beyond curie temperature value.

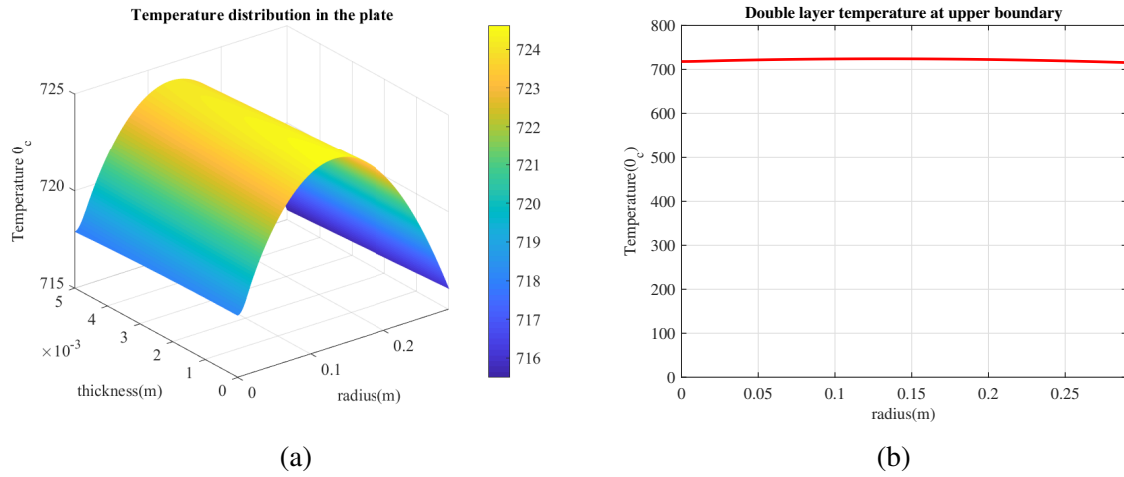


Figure 4.22: Double layer winding heat distribution along radial direction

Figure 4.23 depicts the double layer winding circular temperature distribution in the induction plate. Rather than concentrating in a tiny ring, the temperature dispersion covers a larger region of the induction plate, considerably improving the induction mitad's heating efficiency.

Figure 4.24 shows the temperature distribution at the upper boundary for the 3mm distance between the induction coil and plate and the 3mm thickness of the plate. From the figure,

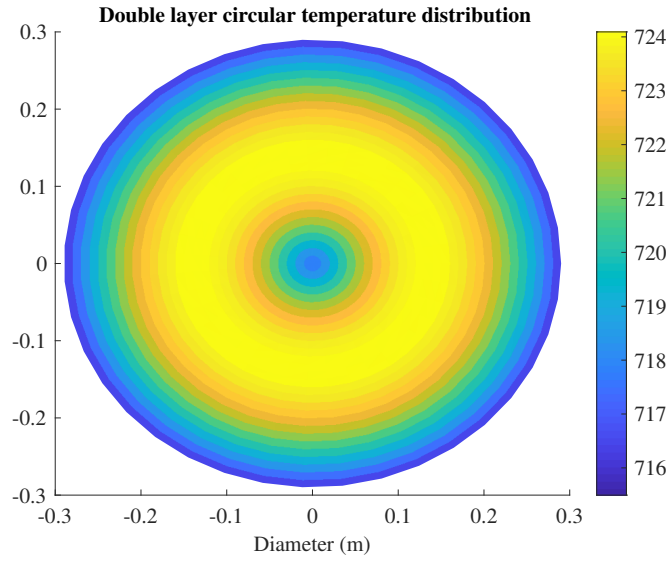


Figure 4.23: Double layer circular temperature distribution

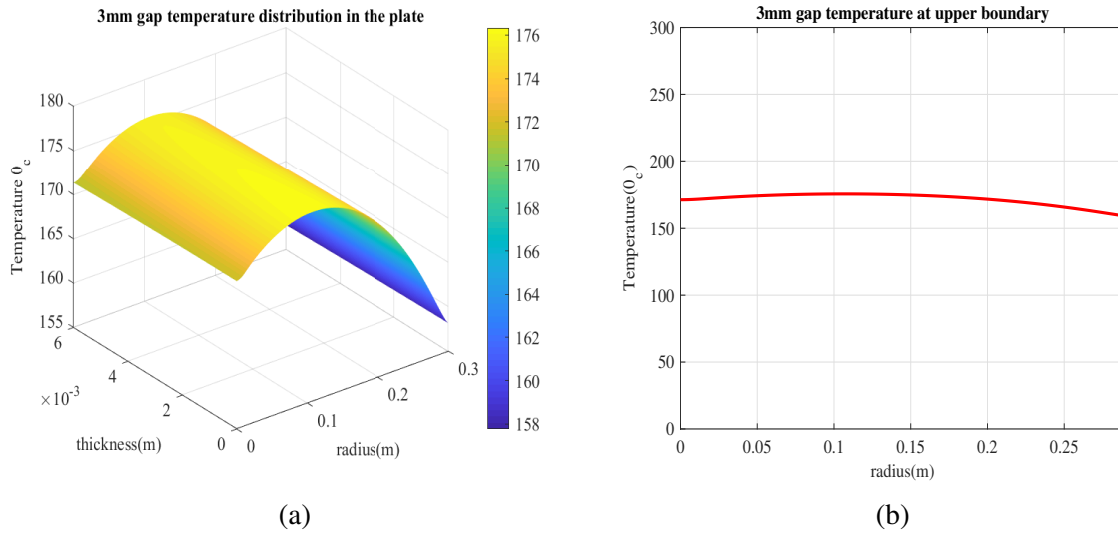


Figure 4.24: Upper boundary temperature for 3mm distance between coil and plate

the temperature distribution is less uniform compared to the 6mm gap with a 5mm thickness of plate discussed before. As shown in the figure the difference between the maximum and minimum temperature value is 18°C and also the temperature value is decreased below the required value. This is due to magnetic field distortion caused when the coil and plate are very close to each other and magnetic field opposition caused by eddy current to the original field. Figure 4.25 presents the temperature distribution for a large distance between coil and plate. When the distance between the induction coil and plate is 8mm the temperature value decreases to 129°C due to the weak magnetic field linking the plate and it is not enough to bake injera with this temperature. From this, it can be concluded that the eddy current and

temperature distribution are affected by the distance between the coil and the workpiece.

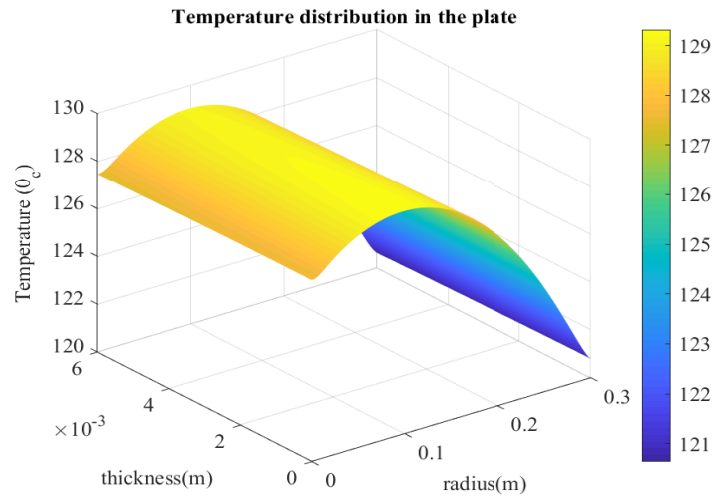


Figure 4.25: Temperature distribution for 8mm distance between coil and plate

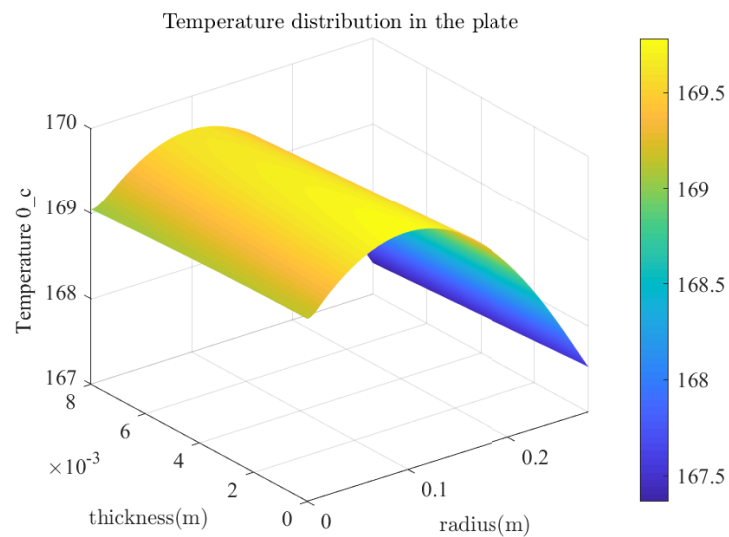


Figure 4.26: Temperature distribution for 8mm thickness of plate

The thickness of the plate has a great impact on the temperature and eddy current distribution. Figure 4.26 shows the effect of the thickness of the plate on temperature distribution. The thickness of the plate simulated here is 8mm and when the thickness increases the temperature distribution becomes more uniform as the plate heats up slowly. However, the value of temperature decreases as the eddy current penetration is small when compared to plate thickness and this increases the time to heat up and the weight of the system which is not desired.

4.3 Experimental Results

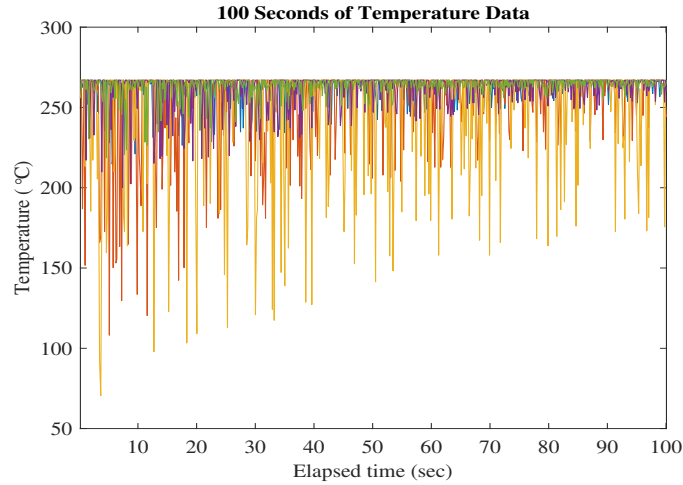


Figure 4.27: Temperature measurement from five sensors for 100 sec

Figure 4.27 shows the temperature data measured for 100 seconds from five sensors placed along the diameter of the stainless steel pan. The system is turned on simultaneously to plot live data on Matlab and from the figure, the temperature rises with fluctuation initially and becomes uniform as the time increases.

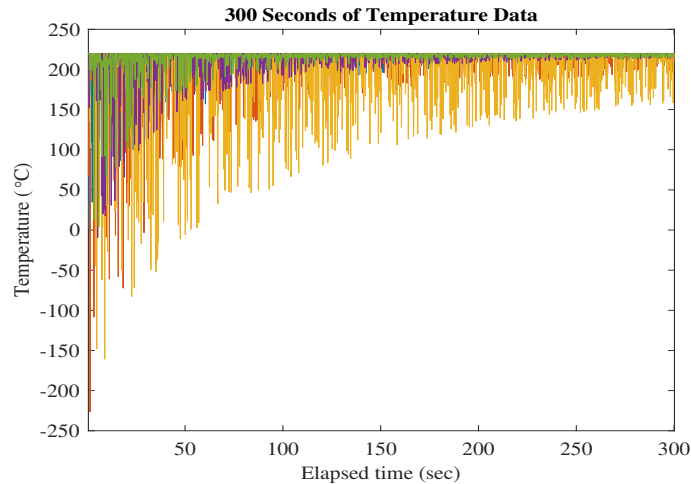


Figure 4.28: Temperature measurement from five sensors for 300 sec

Figure 4.28 shows the temperature data measured for 300 seconds(5 minutes) from five sensors placed along the diameter of the stainless steel pan. The system hardware and Matlab code are turned on simultaneously to read and plot live data on Matlab. And from the figure, the temperature rises with fluctuation initially and becomes uniform as time increases. When compared to 10 seconds and 100 seconds the heat distribution uniformity is better for 300 seconds.

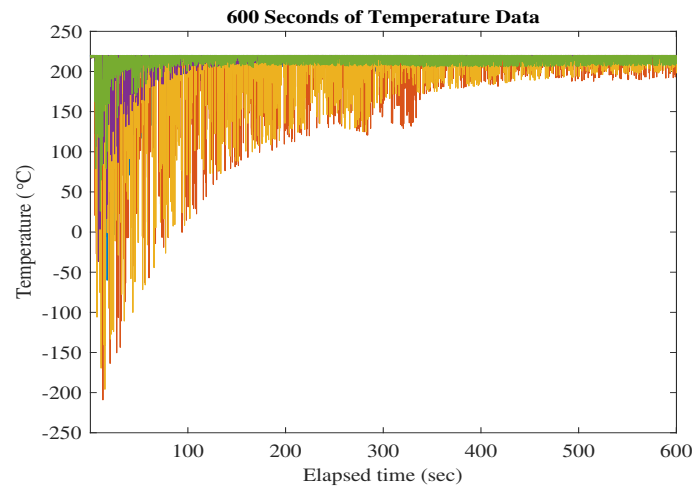


Figure 4.29: Temperature measurement from five sensors for 600 sec

Figure 4.29 shows the temperature data measured for 600 seconds (10 minutes) from five points placed along the diameter of the pan. As shown in the figure the temperature at five points rises with fluctuation at an initial time and becomes uniform as the time increases.

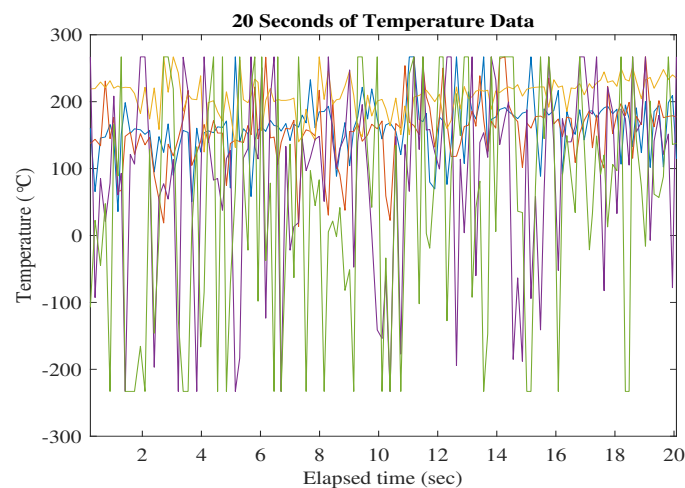


Figure 4.30: Temperature distribution along the radius of 35 cm diameter plate

When compared to traditional electric injera mitad, within 10 minutes it is impossible to get injera baking temperature. Figure 4.31 shows the heat distribution along the diameter of 36 cm stainless steel plate recorded by five thermistors and plotted on Matlab. From the figure, it can be seen that the temperature value at each point increases and fluctuates around 200 °C and this is due to vibration during a dry test of the plate. When compared to a small diameter pan the temperature value is decreased as in the case of a small diameter plate the magnetic field from all directions is concentrated on a small area. But when the size increases the field is distributed and there is a small reduction in temperature value. Figure 4.32 shows the heat

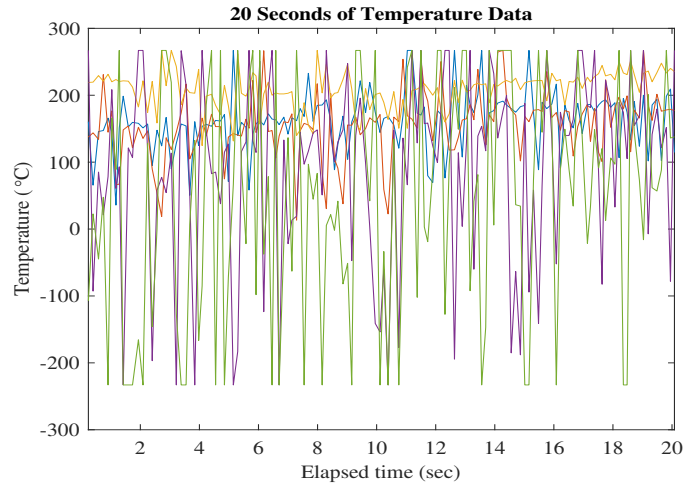


Figure 4.31: Temperature distribution along the radius of 35 cm diameter plate

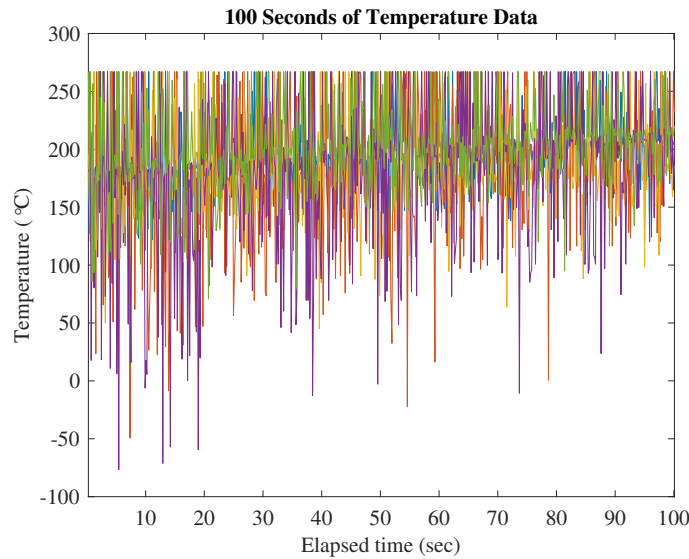


Figure 4.32: Temperature distribution in the cast iron plate for 100 seconds

distribution along the diameter of a 45 cm cast iron plate recorded for 100 seconds. Here, the temperature is around 200°C with sensor inaccuracy. From this, it can be seen that the system is fast enough to generate the heat required to bake injera within 100 seconds but it needs more time to be uniform. Figure 4.33 shows the heat distribution along the diameter of a 45 cm cast iron plate recorded for 300 seconds. As can be seen from the figure, the heat distribution uniformity increases with time. When time increases the temperature reading from five sensors starts overlapping and reduces fluctuation. Generally, the simulation and hardware results of the system are presented and analyzed. The system performance for different parameters and materials is tested and a promising result is obtained. There are some compromises between simulation results and hardware results due to measurement

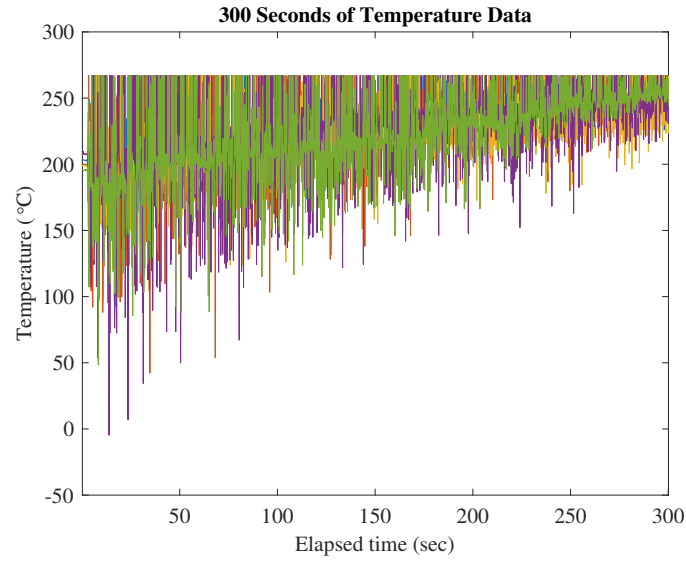


Figure 4.33: Temperature distribution in the cast iron plate for 300 seconds

inaccuracy, lack of exact specifications of materials, and sensor noise. To make the system



Figure 4.34: Injera baking test a) while baking b) baked injera on the plate

more efficient, it needs special production of materials like plate, Litz wire, and frame. Even though there is some gap between simulation and hardware results, there are a lot of improvements such as low power consumption uniform heat distribution of heat, fast start-up heating time, and lightweight.

Figure 4.34 shows the injera baking test on proposed system. From the test, it can be concluded that the designed induction mitad can provide required temperature and uniform heat

distribution and as a result, Injera with its beautiful texture and flavor can be baked by optimal induction injera mitad using cast iron work piece provided that the cast iron surface must be well treated to reduce sticky problem.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.1 CONCLUSION

Injera will continue to be the staple food of Ethiopians for years to come. Nowadays, biomass is the main energy source used for baking injera followed by electricity. As seen in different literature, the energy consumption of existing traditional electric mitad is around 60% - 70% of the total household demand which initiates research and development work for improved injera baking system. Electricity-based mitads are a sound alternative for the energy-inefficient traditional three-stone open fire mitad. However, also their energy efficiency has to be improved drastically to prevent an overload on the electricity grid. To improve this efficiency optimal design of induction injera mitad with uniform heat distribution has been studied. In this thesis, various aspects related to induction injera mitad temperature distribution have been comprehensively investigated and studied, including magnetic field, thermal field, power conversion, and critical radius of insulation. The research provides meaningful theoretical findings as well as feasible application solutions for induction heating appliances for baking injera. To develop a uniform heat distribution a numerical optimization has been applied and the optimal parameter is determined. A good distribution of eddy current and hence heat along a radial direction of mitad is obtained. The analysis is performed for several coil arrangements, the gap between coil and plate, and different thicknesses of the plate, and shows that heat distribution is affected by various parameters. Matlab/Chebfun simulation is applied to observe the eddy current distribution, heat pattern along the radial direction, and circular field of the cooking plate. From the simulation result, the heat distribution is more uniform when the coil diameter is larger than the plate diameter. This is due to the electromagnetic field linking the plate from the edge in addition to the underside of the plate and this increases the heat at the edge which is always heated lower than the center due to diffusion. This thesis also presents the mathematical approach to determine the critical radius of insulation to reduce heat loss to the environment from the side of the mitad. It is concluded that the optimal heat distribution is obtained with 65 turns, 5mm spacing between each turn, 6mm gap between coil and plate, and 24KHz frequency of the current flowing in

the induction coil with coil diameter greater than plate diameter by three or four turns. And with these optimal parameters, the temperature difference between the maximum and minimum value is reduced to less than 2°C as well as the power consumption of the system is reduced by half when compared to traditional electric injera mitad. From hardware result, the maximum power consumption of proposed system 2KW which is half of the commercial injera mitad. So, the proposed system power consumption is reduced to 50% when compared to traditional electric injera mitad and improves the country's power efficiency. Every household uses electric injera mitad where there is electricity and if optimal induction injera mitad is replaced a remarkable reduction in power consumption and hence very low electricity cost will be attained. And this improves the life standards of the society and enhances the growth of country by providing saved energy for other production sectors. Also, the temperature difference between the maximum and minimum value is less than 10°C . The difference between hardware result and simulation result is due to inaccuracy of material used for hardware part and also calibration materials. Additionally, the start-up heating time is reduced to 5-7 minutes to generate the temperature required for baking injera which is fast and makes the system efficient.

5.2 Recommendation

Injera baking is power consuming activity that needs efficient cooking devices and complex activity by its nature as it is affected by various parameters. Therefore, it is difficult to improve all things simultaneously. The thesis focuses more on thermal distribution along the ray of mitad and most of the analysis performed in this study is on heat distribution. The power conversion system is not discussed deeply in this study so it is recommended to further study the power conversion part of the system along with heat distribution. Even though the uniform heat distribution is achieved with a very minimum temperature difference, the sticky problem has not been considered in this study. Therefore, future researchers can further consider the sticky problem to make the system more reliable and preferable. The critical radius of insulation is determined for the side of the mitad but it is better if determined for the bottom of the mitad as there is small heat transfer from the bottom.

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References

- Acero, J., Alonso, R., Burdío, J. M., Barragán, L. A., & Puyal, D. (2006). Frequency-dependent resistance in litz-wire planar windings for domestic induction heating appliances. *IEEE Transactions on Power Electronics*, 21(4), 856–866.
- Adem, K. D., & Ambie, D. A. (2017). A review of injera baking technologies in ethiopia: Challenges and gaps. *Energy for Sustainable Development*, 41, 69–80.
- Artigas, J. I., Urriza, I., Acero, J., Barragan, L. A., Navarro, D., & Burdío, J. M. (2008). Power measurement by output-current integration in series resonant inverters. *IEEE Transactions on Industrial Electronics*, 56(2), 559–567.
- Authority, E. E. (2015). Energy efficiency standards and labeling.
- Awash, T. (2011). *Experimental investigation on performance characteristics and efficiency of electric injera baking pans (mitad)* (Unpublished doctoral dissertation). MSc Thesis, School of Postgraduate Studies, Addis Ababa University, Ethiopia.
- Berhanu, H., Bekele, A., Venkatachalam, C., & Sivalingam, S. (2022). Performance improvement of an electric injera baking pan (mitad) using copper powder as additive material. *Energy for Sustainable Development*, 68, 242–257.
- Bess, M., & Kenna, J. (1994). Cooking efficiency improvement and new fuels marketing project. *Project final report*, 95, 1995–96.
- Byun, J.-K., Choi, K., Roh, H.-S., & Hahn, S.-y. (2000). Optimal design procedure for a practical induction heating cooker. *IEEE Transactions on Magnetics*, 36(4), 1390–1393.
- Carretero, C., Alonso, R., Acero, J., & Burdio, J. M. (2015). Optimized 4-coil inductor system arrangement for induction heating appliances. In *Iecon 2015-41st annual conference of the ieee industrial electronics society* (pp. 004948–004952).
- Cengel, Y., & Heat, T. M. (2003). A practical approach. *Second edi.*
- Dong, Z., Li, Y., Zhang, S., & Shang, F. (2017). Fuzzy temperature control of induction cooker. In *Iecon 2017-43rd annual conference of the ieee industrial electronics society* (pp. 3051–3056).

- Driscoll, T. A., Hale, N., & Trefethen, L. N. (2014). *Chebfun guide*. Pafnuty Publications, Oxford.
- Fan, H., et al. (1996). The influence of temperature on the thermal conductivity of cast irons. *Materials Review*, 3, 23–25.
- Fanta, F. (2020). Improving performance of electromagnetic induction injera mitad. <https://www.irjet.net/archives/V7/i4/IRJET-V7I4349.pdf>.
- Feleke, D., & Oquino, V. H. (2019). Design and development of electromagnetic induction injera mitad.
- Getie, M. Z., & Degefa, M. (2020). Assessment of trend and determinant factors for household energy utilization choice in urban areas of ethiopia: Case of eastern amhara. *Journal of Energy Management and Technology*, 4(1), 67–74.
- Hailu, M. H., Kahsay, M. B., Tesfay, A. H., & Dawud, O. I. (2017). Energy consumption performance analysis of electrical mitad at mekelle city. *Momona Ethiopian Journal of Science*, 9(1), 43–65.
- Hannan, M. A., Ghani, Z. A., Hoque, M. M., Ker, P. J., Hussain, A., & Mohamed, A. (2019). Fuzzy logic inverter controller in photovoltaic applications: Issues and recommendations. *IEEE Access*, 7, 24934–24955.
- Hassan, S. G., & Mohammad, E. J. (2021). Study of the skin depth in different metallic atoms. *Materials Today: Proceedings*, 42, 2749–2751.
- Hassen, A. A., Amibe, D. A., & Nydal, O. J. (2011). Performance investigation of solar powered injera baking oven for indoor cooking. In *Ises solar world congress proceedings, kassel, germany* (pp. 186–196).
- Jones, R., Diehl, J. C., Simons, L., & Verwaal, M. (2017). The development of an energy efficient electric mitad for baking injeras in ethiopia. In *2017 international conference on the domestic use of energy (due)* (pp. 75–82).
- Kanssab, A., Zaoui, A., & Feliachi, M. (2012). Modeling and optimization of induction cooking by the use of magneto-thermal finite element analysis and genetic algorithms. *Frontiers of Electrical and Electronic Engineering*, 7(3), 312–317.

- Karwa, R. (2020). *Heat and mass transfer*. Springer Nature.
- Khang, M., & Aung, S. (2014). Design and comparison of conductor size for inductino cooker coil. *International Journal of Scientific Engineering and Technology Research*, 3, 1240–1244.
- Koertzen, H. W., Van Wyk, J. D., & Ferreira, J. A. (1995). Design of the half-bridge, series resonant converter for induction cooking. In *Proceedings of pesc'95-power electronics specialist conference* (Vol. 2, pp. 729–735).
- Kumar, A., Sarkar, D., & Sadhu, P. K. (2019). Boost power factor correction converter fed domestic induction heating system. In *2019 20th international conference on intelligent system application to power systems (isap)* (pp. 1–6).
- Liang, X., & Wu, C. (2018). Investigation on thermal conductivity of steel fiber reinforced concrete using mesoscale modeling. *International Journal of Thermophysics*, 39(12), 1–19.
- Liu, L., He, H., Dyck, M., & Lv, J. (2021). Modeling thermal conductivity of clays: A review and evaluation of 28 predictive models. *Engineering Geology*, 288, 106107.
- Lucia, O., Acero, J., Carretero, C., & Burdio, J. M. (2013). Induction heating appliances: Toward more flexible cooking surfaces. *IEEE Industrial Electronics Magazine*, 7(3), 35–47.
- Lupi, S., & Rudnev, V. (2014). Electromagnetic and thermal properties of materials.
- Meng, L., Cheng, K. W. E., & Chan, K. (2009). Heating performance improvement and field study of the induction cooker. In *2009 3rd international conference on power electronics systems and applications (pesa)* (pp. 1–5).
- Meng, L., Cheng, K. W. E., & Chan, K. W. (2011). Systematic approach to high-power and energy-efficient industrial induction cooker system: circuit design, control strategy, and prototype evaluation. *IEEE transactions on power electronics*, 26(12), 3754–3765.
- Meng, L., et al. (2012). Field analysis and high frequency conversion for multi-coils induction heating appliances.

- Miyagi, D., Saitou, A., Takahashi, N., Uchida, N., & Ozaki, K. (2006). Improvement of zone control induction heating equipment for high-speed processing of semiconductor devices. *IEEE transactions on magnetics*, 42(2), 292–294.
- Moges, G., & EEA, D. G. (2017). Electric injera mitad energy efficiency standards & labeling, challenges and prospects. In *Vienna energy forum* (p. 35).
- Mohan, D. G., & Gopi, S. (2020). Induction assisted friction stir welding: a review. *Australian Journal of Mechanical Engineering*, 18(1), 119–123.
- Neelakanta, P. S. (1995). *Handbook of electromagnetic materials: monolithic and composite versions and their applications*. CRC press.
- Pal, P., Sadhu, P. K., & Pal, N. (2017). Design and simulation of fuzzy logic controller based modified half bridge resonant inverter fed induction heating system.
- Plumed, E., Acero, J., Lope, I., & Burdío, J. M. (2020). Design methodology of high performance domestic induction heating systems under worktop. *IET Power Electron.*, 13(2), 300–306.
- Qiu, L., Wen, X., Li, H., & Li, T. (2014). Study on effect of material property on thermal power in induction cooker system with finite element method. *International Journal of Applied Electromagnetics and Mechanics*, 46(1), 35–42.
- Rezig, M., Srairi, K., & Aloui, L. (2011). Modeling and optimization of a geometrique induction cooking for improving the temperature. *International Journal of Materials Engineering*, 1(1), 17–20.
- Rudnev, V., Loveless, D., & Cook, R. L. (2017). *Handbook of induction heating*. CRC press.
- Shimelis, T. (1991). Attempts in renewable energy technologies and major stove activities in ethiopia. *country paper presented at the Fourth Regional Biomass Energy Course organised by RWEPA/KENGO*.
- Tadesse, M. (2020). The developmental patterns of injera baking stoves: review on the efficiency, and energy consumption in ethiopia. *International Journal of Mechanical Engineering*, 7(1), 7–16.

- Tayade, V., & Lanudkar, S. (2020). A review of emerging devices beyond mosfet for high performance computing. In *2020 international conference on emerging smart computing and informatics (esci)* (pp. 34–38).
- Teferi, F. T., & Kolhe, K. P. (2021). Design and analytical study for influence of thermal insulation on ethiopian electric injera baking pan (mitad). *Journal of Engineering Science and Technology*, 16(6), 5071–5086.
- Tesfay, A. H., Kahsay, M. B., & Nydal, O. J. (2014). Design and development of solar thermal injera baking: steam based direct baking. *Energy Procedia*, 57, 2946–2955.
- Tiandho, Y., Indriawati, A., Putri, A. K., & Afriani, F. (2021). Induction stoves: An option for clean and efficient cooking in indonesia. In *Iop conference series: Materials science and engineering* (Vol. 1034, p. 012068).
- TOPUZ, N. E., DAWOOD, K., Ünsal, K., ODABAŞ, G., & KÖMÜRGÖZ, G. (2019). Electromagnetic and thermal analysis of a domestic induction cooker coil. In *2019 4th international conference on power electronics and their applications (icpea)* (pp. 1–5).
- Welty, J., Rorrer, G. L., & Foster, D. G. (2020). *Fundamentals of momentum, heat, and mass transfer*. John Wiley & Sons.
- Yang, Y., & Zhou, K. (2018). Modeling and control of single-phase ac/dc converters. In *Control of power electronic converters and systems* (pp. 93–115). Elsevier.
- Zeb, K., Uddin, W., Khan, M. A., Ali, Z., Ali, M. U., Christofides, N., & Kim, H. (2018). A comprehensive review on inverter topologies and control strategies for grid connected photovoltaic system. *Renewable and Sustainable Energy Reviews*, 94, 1120–1141.

APPENDICES

Appendix A: Bessel Functions

Bessel function appears in a wide variety of physical problems. Bessel functions and closely related functions form a rich area of mathematical analysis with many representations, interesting and useful properties, and many interrelations. In standard form, Bessel's equation is either written as

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (x^2 - v^2) y = 0,$$

or given as

$$\frac{d^2 y}{dx^2} + \frac{1}{x} \frac{dy}{dx} + \left(1 - \frac{v^2}{x^2}\right) y = 0.$$

Where the real parameter v determines the nature of the two linearly independent solutions of the equation. A more general form of Bessel's equation that arises in many applications is

$$x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + (\lambda^2 x^2 - v^2) y = 0,$$

Or equivalently

$$\frac{d^2 y}{dx^2} + x \frac{dy}{dx} + \left(\lambda^2 - \frac{v^2}{x^2}\right) y = 0.$$

Their general solution is

$$y = C J_v(\lambda x) + D Y_v(\lambda x)$$

In which $J_v(x)$ is named Bessel functions of the first kind of order v , $Y_v(x)$ is the Bessel function of the second kind of order v . When the order is an integer ($v=n$), the following relationship is true.

$$J_{-n}(x) = (-1)^n J_n(x).$$

The Bessel function of the second kind $Y_v(x)$ is defined as

$$Y_v(x) = \frac{J_v(x) \cos(vn) - J_{-v}(x)}{\sin(vn)}$$

When v is an integer n or zero

$$Y_n(x) = \lim_{v \rightarrow n} Y_v(x)$$

The series expansions for $J_n(x)$ and $Y_n(x)$ are

$$J_n(x) = x^n \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{2^{2k+n} k! (n+k)!},$$

$$Y_n(x) = \frac{\left(\frac{1}{2}x\right)^{-n}}{n} \sum_{k=0}^{n-1} \frac{(n-k-1)!}{k!} \left(-\frac{1}{4}x^2\right)^k + \frac{2}{\pi} \ln \frac{x}{2} J_n(x)$$

$$- \frac{\left(\frac{1}{2}x\right)^n}{\pi} \sum_{k=0}^{\infty} \left\{ \psi(k+1) + \psi(n+k+1) \right\} \frac{\left(-\frac{1}{4}x^2\right)^k}{k! (n+k)!}$$

Some special values for $J_n(x)$ and $Y_n(x)$ are

$$J_n(0) = 0,$$

$$\lim_{x \rightarrow \infty} J_n(x) = 0, [n = 0, 1, 2, \dots],$$

$$\lim_{x \rightarrow 0} Y_n(x) = -\infty, [n = 0, 1, 2, \dots],$$

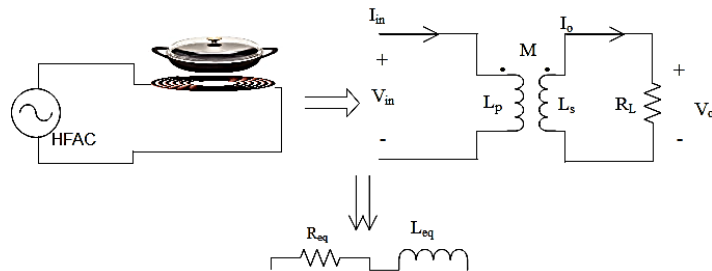
$$\lim_{x \rightarrow \infty} Y_n(x) = 0, [n = 0, 1, 2, \dots],$$

The Fourier Bessel equation can be expresses as

$$F(\alpha') = \int_0^{\infty} F(\alpha) \int_0^{\infty} J_n(\alpha r) J_n(\alpha' r) \alpha r dr d\alpha$$

Appendix B: Induction load calculation

The mathematical modeling of the domestic IH coil and its load has been done with the help of a transformer equivalent circuit in which the IH coil is treated as a primary winding and the object/pan which is to be heated is treated as a secondary winding with single turn consideration. By applying KVL at primary and secondary side



$$V_{in} = j\omega L_p I_{in} - j\omega M I_o - V_o = -j\omega M I_{in} + j\omega L_s I_o$$

Evaluating I_o in terms of I_{in}

$$I_o = \frac{j\omega M}{R_L + j\omega L_s} I_{in} \text{ and } R_L = V_o / I_{in}$$

Substituting the value of I_o from equation

$$V_{in} = \left(j\omega L_p + \frac{\omega^2 M^2}{R_L + j\omega L_s} \right) I_{in}$$

Equation can be written in terms of impedance (Z_{in})

$$Z_{in} = \left(j\omega L_p + \frac{\omega^2 M^2}{R_L + j\omega L_s} \right)$$

Since the primary side inductor is non coupled single inductor, expanding second term of equation we get

$$Z(j\omega) = \frac{\omega^2 M^2}{R_L + j\omega L_s} = \frac{\omega^2 M^2}{R_L + j\omega L_s} \times \frac{R_L - j\omega L_s}{R_L - j\omega L_s} \quad Z(j\omega) = \frac{R_L \omega^2 M^2}{R_L^2 + j\omega^2 L_s^2} - j\omega \left(\frac{\omega^2 L_s M^2}{R_L^2 + j\omega^2 L_s^2} \right)$$

By equating real part to real part and imaginary part to imaginary part,

$$R_{eq} = \frac{R_L \omega^2 M^2}{R_L^2 + j\omega^2 L_s^2} \quad L_{eq} = \frac{\omega^2 L_s M^2}{R_L^2 + j\omega^2 L_s^2}$$

Equation and gives the value of equivalent resistance R_{eq} and inductance L_{eq} . A resonant capacitor C_{eq} is connected in series to create resonance condition.

Appendix C: Simulation parameters

No	Parameters	Value
1	Filter inductor	1mH
2	Filter Capacitor	3.8 μ F
3	Equivalent load resistance	24 Ω
4	Equivalent load inductance	20 μ H
5	Equivalent capacitance	0.8 μ F
6	Switching frequency	24KHz
7	Output power	2.6KW

Appendix D: Sample Matlab code

```
%%
clear all; clc
set(0, 'defaulttextinterpreter', 'Latex');

tic,
th = 0.005;           % the plate thickness , m
radius = 0.295;       % the plate radius
interRadius = 0.015;
interDis = 0.008;
dis = 0.006;          % the distance (coil, plate), length
fr = 24*10^3;         % hz
omega = 2*pi*fr;
permVac = 4*pi* 10^(-7) ; % the vacuum permeability
permRel = 1500;
perm = permVac*permRel;
cond = 10.1^6;
Current = 10; % Coil Currnet
alpha = 10000;
domain = [0 0.28 0 th];
r1= interRadius: interDis:radius;
%%
alpha1 = @(x,r,z) sqrt(x.^2 + 1j*omega*perm*cond) ;
num = @(x,r,z) (exp(-alpha1(x,r,z).*(th-z)) .*(alpha1(x,r,z),
    z)-x) + exp( alpha1(x,r,z).*(th-z)) .*(alpha1(x,r,z) + x
    ));
den = @(x,r,z) exp(alpha1(x,r,z).*th).*(alpha1(x,r,z) + x)
    .^2 - exp(-alpha1(x,r,z).*th) .*(alpha1(x,r,z)-x).^2;
H = 0;
for i = 1:35
```

```

tic
i
f = @(x,r,z) besselj(1,x*r1(i)).*besselj(1,x.*r).*exp
    (-x*dis).*x;
Temp = @(r,z) sum(chebfun(@(x) f(x,r,z) .*num(x,r,z) ./
    den(x,r,z), [ 0 alpha])));
Flux = chebfun2(@(r,z) perm*Current*r1(i)*Temp(r,z),
    domain,'vectorize','on');
H = H + Flux;
toc

end

coilgretr_5mmthc_6mmgp_48mmspc_24khz= 1j*omega*cond*H;
plot(abs(coilgretr_5mmthc_6mmgp_8mmspc_24khz));
save coilgretr_5mmthc_6mmgp_8mmspc_24khz

```