QUANTUM CORRELATIONS OF TWO-MODE LIGHT PRODUCED BY NON-DEGENERATE OPTICAL PARAMETRIC OSCILLATOR

By

Zelalem Tarekegn

A Thesis Submitted to the Department of Applied Physics

School of Applied Natural Science

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Declaration

I hereby declare that this Msc thesis is my own original work and has not been presented in any other universities, and that all sources of material used for the thesis have been duly acknowledged.

Name: Zelalem Tarekegn

Signature ———

This M.sc thesis has been submitted for examination with my approval as thesis advisor supervisor.

Name: Dr. Tewodiros Yirgashewa

Signature:————-

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List of Abbreviations

OPA(O)	Optical Parametric Amplifier(Oscillator)
NOPA(O)	Nondegenerate Optical Parametric Amplifier(Oscillator)
DOPA(O)	Degenerate Optical Parametric Amplifier(Oscillator)
CV	Continous Variable
TMGS	Two Mode Gaussian State
PPT	Positive Partial Transpose
EPR	Einstein-Podolsky-Rosen
LOCC	Local Operation and Classical Communication
GHZ	Greenberger-Horne-Zeilinger
TSMGS	Two Singl-Mode Gaussian States

ACKNOLEGMENT

A special thank to the almighty God, a secret not only to accomplish this research work but also being I am still alive. I would like to express my deep gratitude to my advisor Dr. Tewodros Yirgashewa for his unreserved guidance, comment and continues support to this research work. My thank and appreciation to Mekonen Bekele(Ph.D candidate) for sharing his knowledge and experience. And my warm thank to my friend Amanuel kebede for having unlimited discussions. I thank Adama Science and Technology University for sponsoring me to attend my M.Sc degree. Finally, I would like to extend my thank to all my families and friends who helped me in any case.

ABSTRACT

The quantum correlations such as entanglement, discord, steering, and quadrature squeezing investigated for a two mode cavity light produced by non-degenerate parametric oscillator. The master equation and the solutions of equation of evolution of expectation values derived employing the constracted Hamiltonian. Applying these solutions, the quadrature variance and quadrature squeezing calculated for the cavity mode. The results showed that at steady state and threshold there is maximum of 50% squeezing for the two mode cavity light. Furthermore the degree of entanglement quantified using logarithmic negativity criteria. Moreover, the discordant and steering were quantified for two mode gaussian states. The results obtained indicated that two modes are entangled and discordant but not steerable.

Contents

1

INTRODUCTION

1.1 Background of the Study

Quantum optics provided the tools to study the foundations of quantum mechanics with exquisite precision. The interaction of an isolated atom and a light field provided the ideal testbed for controversial ideas. For example, ”quantum jumps” are a manifestation of the discontinuity inherent in the quantum measurement process [1,2] which can be directly observed [3]. One of the key concepts of quantum optics is the coherent state, introduced by Schrodinger, and used by Glauber and Sudarshan to study high-order coherences of light [4,5]. Squeezed light [6], in which the quantum noise is reduced in one quadrature at the expense of the increased noise in the other quadrature, has also played a central role in quantum optics as it developed. From the beginning, quantum optics combined the study of fundamental physics with applications to technology. Squeezed light, for example, enables a new kind of precision measurement, with application, for instance, in gravitational wave detection [7] and for noiseless communications [8].

While quantum optics focuses on the physics of light and atoms, quantum information focuses on the properties and applications of the qubit. The counterpart to coherence in quantum optics is the quantum mechanical superposition of the qubit. Extended across multiple qubits, superposition leads to entanglement, and information can be represented and processed in a non-classical way [9]. Thus, a quantum computer [10] would be able to compute in an intrinsically different way to standard classical computers. Quantum computing became the subject of intense study after Shor invented an efficient quantum algorithm for factoring, a problem for which no efficient classical algorithm is known despite centuries of study [11]. Another application of quantum mechanics in information processing is found in sending secret messages. It is well-known that a one-time pad (a string of random numbers shared by two people) is the most secure way to send a secret message. In 1984, Bennett and Brassard [12] invented a scheme to create a one-time pad between distant partners using non-classical light fields prepared in superposition states.

In the quantum scenario, the information is stored in the quantum states of a physical system. The random variables are replaced by the observables and their possible values by the eigenvalues of the observables. The corresponding unit of information is the quantum bit or qubit. The qubit is a quantum system described by the state $|\psi\rangle$. Then, a qubit can be in the states $|0\rangle, |1\rangle$ or, and this is the crucial difference with the bit, a quantum coherent superposition of the orthogonal states $|0\rangle$ and $|1\rangle$, for example $|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$ [13].

The exploitation of the quantum laws in quantum information processing is now predicting many applications in secured communication and information processing [14]. After the local-realism theory of Einstein, Podolsky, and Rosen (EPR) [15], various remarkable attempts on quantum systems have been made to characterize and quantify quantum correlations, for instance, Bell non-locality, quantum steering, quantum entanglement, quantum discord, and quantum coherence [16, 17].

Quantum entanglement describes the non-classical property of composite quantum systems. The entanglement phenomenon is now shown to be a basic ingredient for successful implementation of various tasks of quantum information protocols [18, 19]. In response to the non-classical correlation addressed in the famous paper [15] by EPR, Schrodinger [20] defined the phenomenon of quantum steering between the two spatially separated parties shared an entangled state. According to this definition, the steering allows one party of the entangled state to perform a measurement on their side to influence the set of parameters of the other party. To quantify such steering effect in the discrete- and continuous- variable quantum systems [21, 22,] various mathematical inequalities have been theorized and implemented.

Quantum steering has various practical applications in quantum information processing such as one-side device-independent quantum key distribution, sub-channel discrimination and others [23, 24]. Among theoretical and experimental researchers, the Bell inequality [25], has been a great topic of interest to understand physics of the local and non-local correlations. This inequality can be formulated for the measurement of EPR state of a composite system in terms of Wigner function by measuring the joint probability of the classical correlations. On contrary, the correlations become non-classical when the system is subjected to the measurement of quantum mechanical parity [26]. In this connection, a scheme has been proposed to test quantum non-locality for the field-state of two spatially separated cavities using parity measurements and displacement operation [27].

On the other side, disentangled (separable) states traditionally assigned to classically correlated states may also exhibit a quantum behavior with useful applications to quantum information science [28, 29]. One such behavior for a composite quantum system is known as quantum discord [30, 31, 32]. Such measurements provide us with the amount of mutual information and account almost all the quantum correlations of multipartite systems that is not locally accessible. Quantum discord has been widely studied in many systems such as spins in quantum dots [33], cavity quantum electrodynamics [34] and quantum spin chains [35].

In quantum information theory, Gaussian states are the special forms of continuous variable states which offer many significant advantages among the other bipartite states. For example, the properties of symmetrical [36] and arbitrary type [37] of Gaussian quantum states have been studied in detail. The Gaussian type continuous variable states have also been demonstrated experimentally using techniques of two-mode squeezing effect [18], optical parametric oscillator [38], and parametric down converter [39]. Quantifying and characterizing the quantum correlations of such continuous variable [21] and discrete variable [17] states have been studied. Recently, [40] reported a relationship among the four kinds of quantum correlations using a linear beam splitter [41] which satisfy a strict hierarchy relation: quantum discord $\supseteq$ quantum entanglement $\supseteq$ quantum steering $\supseteq$ Bell non-locality, where M $\supseteq$ N represents M is the superset of N.

Entanglement, the phenomena whereby particles can be generated or interact in ways such that the quantum state of each particle cannot be described independently, which is a consequence of superposition principle, turns out to be an important resource for Quantum Information. Its most relevant application is the quantum teleportation [42]. In Quantum Information, one needs to quantify the amount of available resources for a given task. Thus, it is necessary to introduce the paradigm of measure of entanglement, say ε. For pure states, the entanglement is equal to the von Neumann entropy of one of the two subsystems, $\varepsilon(\rho_{pure}) = S(A) = S(B)$, while several functions have been adopted to measure quantitatively entanglement in mixed states [43].

As well-known light is the optimal conveyor of quantum information due to its high speed and weak interaction with the environment. The entangled optical fields are the necessary resources for performing continuous variable (CV) quantum information [44]. Degenerate and nondegenerate optical parametric amplifiers (DOPAs and NOPAs) have been widely applied in CV quantum information systems to be the necessary sources of squeezed and entangled states [45]. Tripartite Greenberger-Horne-Zeilinger-like (GHZ-like) and quadripartite entangled states of optical fields have been successfully generated by DOPAs and NOPAs and applied for CV quantum information implementations, such as teleportation networks, controlled dense coding, and quantum error correction via telecloning [46]. The main aim of this research is concerned in an investigation of Quantum correlations such as entanglement, steering, and, discord for nondegenerate optical parametric oscillator coupled to ordinary vacuum.

This paper is organized as follows in chapter 1 we discussed the introduction. In chapter 2 we review related literatures that have done previously by many researchers. In chapter three we investigate, using the pertinent master equation obtained applying the Hamiltonian, the steady state solutions of the cavity mode operators, the quadrature variance and quadrature squeezing. In chapter four the calculated and analyzed the quantum correlations such as, entanglement using logarithmic negativity criteria, Gaussian discord and finally Gaussian steering for fundamental modes were presented.

1.2 Statement of the Problem

Many researchers studied about quantum correlations and their criterion in different ways and separately but all the quantum correlations enumerated in specific objectives together have not yet been studied in such a system we select. In this research, the enumerated quantum correlations will be investigated together in a system; nondegenerate optical parametric oscillator coupled with vacuum reservoir.

1.3 Objectives of the Study

1.3.1 General Objective

The general objective of this work is to study quantum correlations for CV Gaussian state of a two mode light generated nondegenerate optical parametric oscillators coupled to a vacuum reservoir.

1.3.2 Specific Objectives

The specific objectives of this work in a system of Nondegenerate optical parametric oscillator coupled to vacuum reservoir are:

- constructing the Hamiltonian for non degenerate parametric oscillator
- deriving the master equation for NDPO
- investigating solutions of equation of evolution expectation values for cavity mode.
- calculating Gaussian entanglement using the criterion logarithmic negativity for NDPO.
- obtaining the quadrature variance and the quadrature squeezing in terms of amplitude proportional to the pump mode for NDPO.
- Determining Gaussian discord and Gaussian steering for NDPO.

1.4 Significance of the Study

Studying quantum correlations of two mode light generated by NDPO have uses in; addressing altogether the quantum correlations for the reader, motivating the researchers for further study on the area, to know the degree of quantum correlations for the system under consideration.

2

LITERATURE REVIEW

2.1 Overview of the Study

However, the beginning of studying quantum information is less than a century , a lot of researches have been addressed which assisted the technology forward beyond expected. The interest of this research is to have a deep understanding about quantum information which is the basis of technology in general and to investigate quantum correlations and their criterion in particular.

Quantum mechanics is one of the most precious scientific gifts of recent times with the theoretical background of an impressive range of applications and peculiar phenomena in complex systems. The burgeoning field of Quantum Information provides the tools for challenging the resilience of the quantum postulates and at the same time, for pushing technology over its inherent limits. The genuinely quantum speed-up in information processing and in energy transport mechanisms is believed to be due to quantum entanglement, i.e. peculiar correlations described by quantum laws which are shared, for example, between the sender and the receiver of a message. However, it has been recently shown that entanglement is not the most general form of quantum correlations there are others such as, steering and discord. Even unentangled states of compound systems have a quantum character, as their subsystems can share truly quantum correlations[13].

Beam splitters are important components in numerous tasks of quantum information protocols used either in simple or in an interferometric arrangement or together with other quantum systems[41]. Continuous variable states, as compared to discrete variable, are easy to manipulate experimentally[18]. The Gaussian type states which are one of the important class of the continuous variable states, usable at the input of a linear beam splitter, have been realized using techniques of optical parametric converter[47], parametric down converter[40] and two-mode squeezing effect[18]. The approach of quantifying the quantum correlations of continuous variable [18, 21] and discrete variable [17] states have been studied.

As the report [41] indicated the quantum correlations of photons retrieved via a linear beam splitter employing either pure or mixed TSMGSs of continuous variable system initially at the input. Boundaries of the quantum correlations of photons at the output obey quantum non-locality, steering, entanglement and discord in presence of the pure input states. On contrary, the Bell inequality does not violate, however, the quantum steering, entanglement and discord exist in some regime of angle $\theta$of the beam splitter when the input states are mixed. Against the thermal photon number, the quantum steering, entanglement and discord persist to some degrees whereas the Bell inequality is obeyed except in a very sharp regime [32]. In the case, [41] found that all the four kinds of the quantum correlations form a hierarchy (quantum coherence $\supseteq$ quantum discord $\supseteq$ quantum entanglement$\supseteq$ quantum steering $\supseteq$ Bell non-locality, where M $\supseteq$ N represent M is the superset of N etc).

2.2 Squeezing

The multipartite entangled states are the basic resources for transmitting quantum information or quantum states among distant stations or nodes in a quantum network, and a high entanglement degree is the elementary requirement for achieving information transferring and processing with high fidelity. In Refs. [48, 49] the tripartite and quadripartite entangled states are obtained by splitting or combining squeezed states of light on optical beam splitters. For constructing quadripartite cluster entangled states in Ref. [49], four squeezed states produced by four DOPAs (two NOPAs) are combining by the beam splitter network. To ensure highly coupling efficiency of squeezed states on each beam splitter, these squeezed states should be classically coherent, so all optical parametric amplifiers (OPAs) used in the system have to be pumped by a laser source and to be phase locked during the experiment[49]. For the practical applications in quantum information networks, it is desired to produce multipartite entangled states directly in a more compact device. Recently, Coelho et al.[50] presented an elegant experimental achievement on the generation of a three-color entangled state of light, in which the CV quantum entanglement among the pump, the signal, and the idler optical fields of an optical parameter oscillator was demonstrated first. For connecting different physical systems with respective resonance frequencies at the nodes of a quantum network, it is important to produce the multipartite entangled optical fields with different frequencies

[50]. In 2004 Pfister et al. presented a scheme of obtaining multipartite entanglement by the use of a single OPA without the necessity of beam splitters, which opens a hopeful avenue to prepare directly N-partite ($N > 2$) optical entangled states in a very compact device, and provides huge scaling potential [51].

On the other hand, for implementing a CV quantum information network, it is also necessary to find a scheme which can control and enhance the generated multipartite entangled states. In fact, due to the unavoidable intracavity losses in OPAs and the limitation of the effective nonlinear coefficient of a crystal, the squeezing and the entanglement of the quantum states produced by a single OPA is not high enough under usual conditions, thus the enhancement of the squeezing and entanglement is especially desired. Using a phase-sensitive DOPA (NOPA) the manipulation and the enhancement of a squeezed vacuum field (bipartite entangled optical beams) have been experimentally demonstrated, in which the manipulation is achieved via a second-order nonlinear interaction inside an optical cavity [52]. Yanagisawa et al. and Gough et al. proposed an enhancement scheme of optical field squeezing by placing a linear optical component in a loop in a simple coherent feedback mechanism involving a beam splitter [53].

2.3 Entanglement

Quantum entanglement is the term given to the phenomena whereby particles can be generated or interact in ways such that the quantum state of each particle cannot be described independently. In such cases the system of particles is said to be entangled and it is not correct to consider any of the individual particles in isolation from the others, but only as a single, entangled state[54]. A quantum state, described by density matrix ρ_s acting in H_s, is called separable if it can be represented as the tensor product of its two sub-systems, $\rho_s = \rho_A \bigotimes \rho_B$, where ρ_A acts in H_A and ρ_B acts in H_B such that $H_s = H_A \bigotimes H_B$. When the system is separable, the measurement results of the quantity A in H_A are not tied to the measurement results of the quantity B in H_B. The knowledge of ρ_A prescinds from knowledge of ρ_B. A nonseparable state is called entangled [20].

The concept of entanglement is usually traced back to the famous 1935 paper [15] by Albert Einstein, Boris Podolsky and Nathan Rosen where they presented what is now known as the EPR (Einstein-Podolsky-Rosen) paradox, which attempted to show that quantum mechanics was incomplete. Indeed Einstein later famously derided the idea of entanglement as "spooky action at a distance" as it seemed to violate the local realist view of causality contrary to the spirit of his theory of relativity.

One of the most exciting features of Quantum Mechanics has been discussed for the first time in the celebrated Einstein-Podolski-Rosen (EPR) paper [15, 55]. They argue that a physical theory must satisfy three properties: realism, which implies that an element of reality in a physical system is a quantity whose value can be known from the theory with certainty without perturbing the system itself; completeness, i.e. every element of the reality must be described by an object of the theory; and locality, which means that the outcomes of measurements made on a given system are independent of quantum operations on space-like separated systems. EPR pointed out that the quantum theory does not satisfy the three properties at the same time because it is not complete. Indeed, it paradoxically implies that incompatible physical quantities, mathematically represented by non-commuting observables, have simultaneous reality in the physical world[13]).

It was Erwin Schrodinger who in a letter to Einstein about the issues raised by EPR, first used the word ”Verschrankung”, that he later translated as ”entanglement”, to ”describe the correlations between two particles that interact and then separate, as in the EPR experiment”. Shortly thereafter, in a paper [20] where Schrodinger defined and discussed the notion of entanglement he stated, ”I would not call [entanglement] one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought”. Schrodinger also considered the possibility that resolution of the EPR paradox might lie in quantum mechanics breaking down for distant entangled systems via what is nowadays known as quantum decoherence. However, this is now known not to be the case[54].

Entanglement is the key resource in many quantum information processing protocols. Therefore it is of interest to develop a detailed understanding of its properties. In view of the resource character of entanglement it is of particular interest to be able to quantify entanglement [56]. Different entanglement measures have been introduced for mixed states. One of such measures is the logarithmic negativity of the state, which is easily computable and gives an upper bound to the distillable entanglement. Vidal and Werner proved that the logarithmic negativity is computable for general Gaussian states [57].

Any resource is intimately related to a constraint which the resource allows us to overcome. Therefore, the detailed character of entanglement and its quantification as a resource depends on the constraints that are being imposed on the set of operations. In a communication setting where two spatially separated parties aim to manipulate a joint quantum state it is natural to restrict attention to local quantum operations and classical communication (LOCC). In this case separable states are freely available while nonseparable states, which cannot be prepared by LOCC alone, become a resource. The phenomenon of bound entanglement i.e., nonseparable states that possess a positive partial transpose and are useless to distill pure maximally entangled states by LOCC [57], suggest that there are other natural restricted classes of operations.

A computable measure of entanglement, and thereby fill an important gap in the study of entanglement. It is based on the trace norm of the partial transpose ρ^{T_A} of the bipartite mixed state ρ a quantity whose evaluation is completely straightforward using standard linear algebra packages. It essentially measures the degree to which ρ^{T_A} fails to be positive, and therefore it can be regarded as a quantitative version of Peres' criterion for separability [57, 58].

2.4 Steering

Quantum steering, or Einstein- Podolsky-Rosen steering [15] is a fascinating phenomenon of quantum physics, yet has remained a topic of debate throughout the history of quantum mechanics. It indicates the ability of Alice to steer Bob's system into different ensembles from a distance. Though this description of quantum steering was known since the beginning of the debate [15, 20], it was not appreciated that quantum steering implies a precise classification of bipartite quantum states into steerable and unsteerable classes until recently. Quantum steerability has since formed a third distinct class of quantum non-locality, in addition to two other well-known classes, non-separability and Bell non-locality [25].

In the modern view, steering denotes the impossibility to describe the conditional states at one party by a local hidden state model. As such, steering denotes a quantum correlation situated between entanglement and Bell nonlocality. In the following years, the theory of steering developed rapidly. It was noted that steering provides the natural formulation for discussing quantum information processing, if for some of the parties the measurement devices are not well characterized. Also, the concept of steering helped to understand and answer open questions in quantum information theory. An important example is here the construction of counterexamples to the Peres conjecture, which states that certain weakly entangled states do not violate any Bell inequality. Finally, steering turned out to be closely related to the concept of joint measurability of generalized measurements in quantum mechanics. More precisely, measurements that are not jointly measurable are exactly the measurements that are useful to reveal the steering phenomenon. This has sparked interest in the question in which sense measurements in quantum mechanics can be considered as a resource[59]).

2.5 Discord

Quantum correlations can be fundamentally different and stronger than classical correlations, and hence a resource for quantum computing and communication. The characterization and control of quantum correlations is thus of critical importance for quantum information applications. The most common measure of quantum correlations is entanglement. In two-qubit systems, entanglement is quantified by the concurrence [29] . However, entanglement is not necessarily sufficient to describe all quantum correlations in a system. An alternate measure of quantum correlations is quantum discord. Whereas both entanglement and discord are measures of quantum correlations they are not the same. For example discord can be nonzero even when there is zero entanglement. Discord is an important measure in quantum information because it can better quantify the quantum advantage in certain algorithms.

Quantum discord, introduced by [60], has been recognized as a possible candidate for those general quantum correlations [31, 60]. On the one hand, quantum discord can even exist in systems which are not entangled. On the other hand, it has been shown that the algorithm presented by Knill and Laflamme [61] exhibits nonvanishing amount of discord [28]. An even stronger statement has been made by Eastin, who showed that mixed state quantum computation with zero discord in each step can be simulated efficiently on a classical computer. In the light of these results, it is not surprising that an enormous amount of research has been devoted to this topic in the last few years. Three years after Zurek has proposed quantum discord as a new kind of quantum correlations beyond entanglement, he gave it an alternative thermodynamical interpretation [62]. He considered the amount of work which can be extracted from a quantum system by a classical and a quantum Maxwell's demon. He showed that the quantum demon is more powerful, since it can operate on the whole quantum state, while the classical demon is restricted to local subsystems only. Zurek concluded that more work can be extracted in the quantum case, and this quantum advantage is related to the quantum discord.

Quantum discord measures a very general form of non-classical correlation, which can be present in quantum systems even in the absence of entanglement. Since the first expositions of this concept is around two a decades ago [31], a substantial amount of research effort has been devoted to understanding the mathematical properties and physical meanings of discord and similar quantities. The study of discord presents many challenges and open questions. One major difficulty with discord-like quantities is that they are hard to compute or analyze. The canonical version of discord is defined to be the difference between quantum mutual information (total correlation) and the maximum amount of correlation that is locally accessible (classical correlation), which involves optimization over all possible local measurements. Such optimization renders the problem of studying discord and its variants (such as quantum deficit, geometric discord very difficult [63].

3

METHODOLOGY

In this chapter we first wish to construct the Hamiltonian for the system under consideration. Next we seek to obtain the master equation for the system by using the constructed Hamiltonian. And then we calculate the expectation values of the cavity mode operators.

3.1 Master Equation for the System Under Consideration

In a two-mode optical parametric oscillator a pump-photon of frequency ω_c is down converted into highly correlated signal and idler photons with frequency ω_a and ω_b such that $\omega_c = \omega_a + \omega_b$. Treating the pump mode classically the process of two-mode optical parametric oscillator is described by a Hamiltonian[64]

$$\hat{H} = i\varepsilon(\hat{a}\hat{b} - \hat{a}^{\dagger}\hat{b}^{\dagger}), \tag{3.1}$$

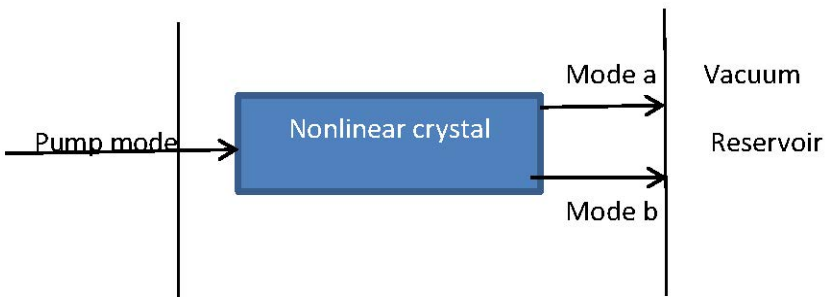

Fig. 3.1: Nondegenerate optical parametric oscillator coupled with vacuum reservoir

where $\hat{a}(\hat{b})$ is annihilation operator for the signal(idler) mode, $\varepsilon = \lambda\mu$ considered to be real and constant,is proportional to the amplitude of the pump mode. The master equation for a cavity modes coupled to vacuum reservoir can be written as,[64]

$$\frac{d\rho}{dt} = -i[\hat{H}_s, \rho] + \frac{\kappa}{2}(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + \frac{\kappa}{2}(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b}), \tag{3.2}$$

in which κ is the cavity damping constant and assumed to be equal for both modes. Assume that the operators $\hat{a}$ and $\hat{b}$ commute and satisfy the commutation relation[64, 65]

$$[\hat{a}, \hat{a}^\dagger] = [\hat{b}, \hat{b}^\dagger] = 1, [\hat{a}, \hat{b}] = [\hat{a}^\dagger, \hat{b}^\dagger] = 0. \tag{3.3}$$

So that on taking into account Eq. (3.1), the master equation for the system under consideration turns out to be

$$\begin{aligned} \frac{d\rho}{dt} &= \varepsilon(\hat{a}\hat{b}\hat{\rho} - \hat{\rho}\hat{a}\hat{b} + \hat{\rho}\hat{a}^\dagger\hat{b}^\dagger - \hat{a}^\dagger\hat{b}^\dagger\hat{\rho}) \\ &+ \frac{\kappa}{2}(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) \\ &+ \frac{\kappa}{2}(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b}). \end{aligned} \tag{3.4}$$

3.2 Operator Dynamics

Here the aim is to find, the expectation values of cavity mode operators, and their steady state solutions. Now employing the relation[65]

$$\frac{d}{dt}\langle\hat{A}\rangle = Tr(\frac{d\rho}{dt}\hat{A}), \tag{3.5}$$

along (3.4), we get

$$\frac{d}{dt}\langle\hat{a}(t)\rangle = -\frac{1}{2}\kappa\langle\hat{a}(t)\rangle - \varepsilon\langle\hat{b}^\dagger(t)\rangle, \tag{3.6}$$

$$\frac{d}{dt}\langle\hat{b}(t)\rangle = -\frac{k}{2}\langle b(t)\rangle - \varepsilon\langle\hat{a}^\dagger(t)\rangle. \tag{3.7}$$

In similar manner, we have

$$\frac{d}{dt}\langle\hat{a}^2(t)\rangle = -k\langle\hat{a}^2(t)\rangle - 2\varepsilon\langle\hat{a}(t)\hat{b}^\dagger(t)\rangle - \varepsilon, \tag{3.8}$$

$$\frac{d}{dt}\langle\hat{b}^2(t)\rangle = -k\langle\hat{b}^2(t)\rangle - 2\varepsilon\langle\hat{a}^\dagger(t)\hat{b}(t)\rangle - \varepsilon, \tag{3.9}$$

$$\frac{d}{dt}\langle\hat{a}(t)\hat{b}(t)\rangle = -\kappa\langle\hat{a}(t)\hat{b}(t)\rangle - \varepsilon\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle - \varepsilon\langle\hat{b}^\dagger(t)\hat{b}(t)\rangle - \varepsilon, \tag{3.10}$$

$$\frac{d}{dt}\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle = -\kappa\langle\hat{a}^\dagger(t)\hat{a}(t)\rangle - \varepsilon\langle\hat{a}(t)\hat{b}(t)\rangle - \varepsilon\langle\hat{a}^\dagger(t)\hat{b}^\dagger(t)\rangle, \tag{3.11}$$

$$\frac{d}{dt}\langle \hat{b}^\dagger(t)\hat{b}(t)\rangle = -\kappa\langle \hat{b}^\dagger(t)\hat{b}(t)\rangle - \varepsilon\langle \hat{a}(t)\hat{b}(t)\rangle - \varepsilon\langle \hat{a}^\dagger(t)\hat{b}^\dagger(t)\rangle. \tag{3.12}$$

So the steady-state solutions corresponding to Eqs. (3.6), (3.7), (3.10), (3.11), and (3.12) are

$$\langle \hat{a}\rangle = \frac{-2\varepsilon}{\kappa}\langle \hat{b}^\dagger\rangle, \tag{3.13}$$

$$\langle \hat{b}\rangle = \frac{-2\varepsilon}{\kappa}\langle \hat{a}^\dagger\rangle, \tag{3.14}$$

$$\langle \hat{a}^\dagger\hat{a}\rangle = -\frac{\varepsilon}{\kappa}\langle \hat{a}\hat{b}\rangle - \frac{\varepsilon}{\kappa}\langle \hat{a}^\dagger\hat{b}^\dagger\rangle, \tag{3.15}$$

$$\langle \hat{b}^\dagger\hat{b}\rangle = -\frac{\varepsilon}{\kappa}\langle \hat{a}\hat{b}\rangle - \frac{\varepsilon}{\kappa}\langle \hat{a}^\dagger\hat{b}^\dagger\rangle, \tag{3.16}$$

$$\langle \hat{a}\hat{b}\rangle = -\frac{\varepsilon}{\kappa}\langle \hat{a}^\dagger\hat{a}\rangle - \frac{\varepsilon}{\kappa}\langle \hat{b}^\dagger\hat{b}\rangle - \frac{\varepsilon}{\kappa}, \tag{3.17}$$

Considering Gaussian variable with zero mean Eqs. (3.13) and 3.14 reduced to

$$\langle \hat{a}\rangle = \langle \hat{b}\rangle = 0. \tag{3.18}$$

Furthermore, using Eqs. (3.15), (3.16), (3.17), we get

$$\langle \hat{b}^\dagger\hat{b}\rangle = \langle \hat{a}^\dagger\hat{a}\rangle = \frac{2\varepsilon^2}{\kappa^2 - 4\varepsilon^2}, \tag{3.19}$$

$$\langle \hat{a}^\dagger\hat{b}^\dagger\rangle = \langle \hat{a}\hat{b}\rangle = \frac{-\varepsilon\kappa}{\kappa^2 - 4\varepsilon^2}, \tag{3.20}$$

$$\langle \hat{a}^2\rangle = \langle \hat{b}^2\rangle = \langle \hat{a}^\dagger\hat{b}\rangle = 0. \tag{3.21}$$

Assume that,

$$\hat{c} = \hat{a} + \hat{b} \tag{3.22}$$

be the annihilation operator for superposition of light modes $\hat{a}$ and $\hat{b}$, produced by parametric oscillator. possibly one can arrive at

$$[\hat{c}, \hat{c}^\dagger] = 2. \tag{3.23}$$

This implies that the superposition of the two light modes, with different frequencies, constitutes a two-mode light.

4

RESULTS AND DISCUSSION

4.1 Quadrature Variance and Quadrature Squeezing

In this chapter the quadrature variance and quadrature squeezing for two mode cavity light produced by non-degenerate parametric oscillator will be discussed.

4.1.1 Quadrature Variance of the cavity mode

We now seek to calculate the quadrature variance of a two-mode cavity light. To this end, we define the plus and minus quadrature operators for a two-mode light as[65]

$$\hat{c}_+ = (\hat{c}^\dagger + \hat{c}), \tag{4.1}$$

and

$$\hat{c}_- = i(\hat{c}^\dagger - \hat{c}). \tag{4.2}$$

with $\hat{c}$ beng the annihilation operator for the two-mode light. A two-mode light is said to be in a squeezed state if either $(\Delta c_+)^2 < 2$ and $(\Delta c_-)^2 > 2$ or $(\Delta c_+)^2 > 2$ and $(\Delta c_-)^2 < 2$ such that $\Delta c_+ \Delta c_- \geq 2$. A squeezed state for which the equality holds is called a minimum uncertainty state. We then note that the noise in one quadrature of a squeezed state is below the vacuum state level at the expense of enhanced noise in the other quadrature.

The variance of the plus and minus quadrature for a two-mode light is given by

$$(\Delta c_\pm)^2 = \langle c \pm^2 \rangle - \langle c \pm \rangle^2. \tag{4.3}$$

Taking into account Eqs.(3.18), and along with Eqs. (4.1), and (4.2) we see that

$$\langle \hat{c}_\pm \rangle = 0. \tag{4.4}$$

As a result Eq. (4.3) can be expressed as

$$(\Delta c_{\pm})^2 = \langle \hat{c}_{\pm}^2 \rangle. \tag{4.5}$$

In view of Eqs.(4.1) and (4.2) along with the commutation relation given by Eq. (3.23) the quadrature variance becomes

$$(\Delta c_{\pm})^2 = 2 + 2\langle \hat{c}^{\dagger}\hat{c} \rangle + \pm\langle \hat{c}^2 + \hat{c}^{\dagger 2} \rangle. \tag{4.6}$$

Using Eq. (3.22) we can expand Eq. (4.6) as

$$(\Delta c_{\pm})^2 = 2 + 2\langle (\hat{a}^{\dagger} + \hat{b}^{\dagger})(\hat{a} + \hat{b}) \rangle \pm \langle \hat{a}^2 + \hat{b}^2 + \hat{a}^{\dagger 2} + \hat{b}^{\dagger 2} + 2\hat{a}\hat{b} + 2\hat{a}^{\dagger}\hat{b}^{\dagger} \rangle. \tag{4.7}$$

Taking into account Eq. (3.21) we arrive at

$$(\Delta c_{\pm})^2 = 2 + 2\langle (\hat{a}^{\dagger}\hat{a} + \hat{b}^{\dagger}\hat{b}) \rangle \pm 2\langle (\hat{a}\hat{b} + \hat{a}^{\dagger}\hat{b}^{\dagger}) \rangle. \tag{4.8}$$

Finally considering Eqs. (3.19) and (3.20) the plus and minus quadrature variances take the form

$$(\Delta c_{+})^2 = 2 - \frac{4\varepsilon}{\kappa + 2\varepsilon}, \tag{4.9}$$

and

$$(\Delta c_{-})^2 = 2 + \frac{4\varepsilon}{\kappa - 2\varepsilon}. \tag{4.10}$$

In general

$$(\Delta c_{\pm})^2 = 2 \mp \frac{4\varepsilon}{\kappa \pm 2\varepsilon}. \tag{4.11}$$

From the above equations we can clearly see that the two mode subharmonic cavity light is in a squeezed state and the Squeezing occurs in the plus quadrature. At $\kappa = 2\varepsilon$ the variance for minus quadrature diverges. Then we take $\kappa = 2\varepsilon$ as the threshold condition and in this paper we perform everything below threshold condition.

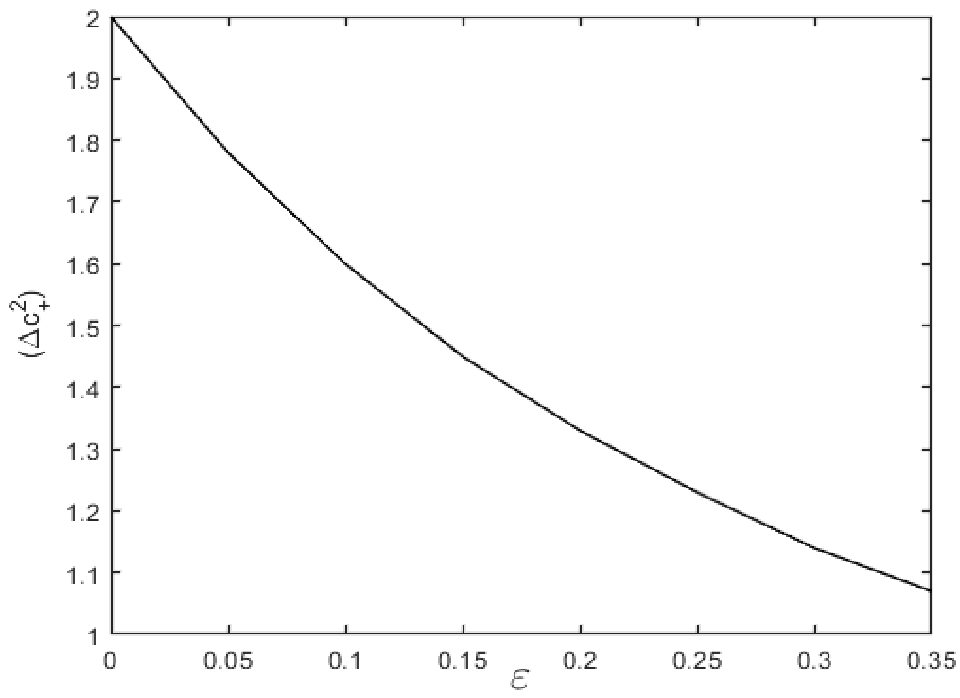

Fig. 4.1: Plot of quadrature variance (Eq.(4.9)) versus amplitude of the pump mode (ε) of the cavity radiation at steady state for $\kappa = 0.8$

From the plot of Fig. 4.1 we see that the quadrature variance decreases with increasing the amplitude of the pump mode and its maximum quadrature variance appears at $\varepsilon = 0$.

4.1.2 Quadrature Squeezing

Here we seek to determine the quadrature squeezing of the two-mode cavity light relative to the quadrature variance of the two-mode cavity vacuum states. Setting up on $\varepsilon = 0$ for equations (4.9) and (4.10) we see that

$$(\Delta c_\pm)_v^2 = 2. \tag{4.12}$$

For $\varepsilon = 0$ we observe that cavity light is in a two-mode vacuum state in which the uncertainities in the two quadratures are equal and satisfies minimum uncertainity relation. We therefore define the quadrature squeezing of the two-mode cavity light by

$$s = \frac{(\Delta c_\pm)_v^2 - (\Delta c_\pm)^2}{(\Delta c_\pm)_v^2}, \tag{4.13}$$

which is reduced to

$$s = \frac{2 - (\Delta c_+)^2}{2}. \tag{4.14}$$

Substituting Eq. (4.9) into (4.14) we get

$$s = \frac{2\varepsilon}{\kappa + 2\varepsilon}. \tag{4.15}$$

Equation4.15 represents the global quadrature Squeezing of subharmonic cavity light. We easily observe that at steady state and threshold there is a 50%quadrature Squeezing below the vacuum state level. As the plot of Fig. (4.2) indicates the quadrature squeezing increases

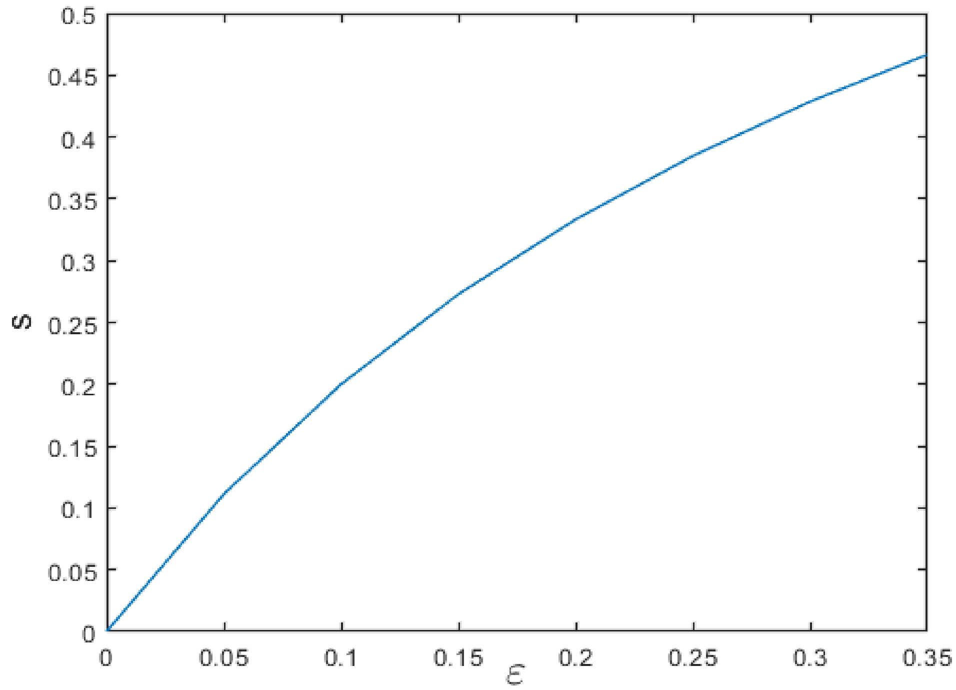

Fig. 4.2: Plots of quadrature squeezing (Eq.(4.15)) versus amplitude of the pump mode (ε) of the cavity radiation at steady state for $\kappa = 0.8$

with amplitude of the pump mode

Comparison of Figures (4.1) and (4.2) indicates that the quadrature variance and the quadrature squeezing are inversely proportional. The quadrature variance decreases with increasing amplitude related to the pump mode, while the quadrature squeezing increases with increasing amplitude proportional to the pump mode and reaches its maximum value at steady state and threshold.

4.2 Gaussian Entanglement

In this section we use the logarithmic negativity to determine the entanglement of a system with two-modes, $\hat{a}$ and $\hat{b}$. Quantum entanglement is a physical phenomenon that occurs when pairs or groups of particles cannot be described independently- instead, a quantum state may be given for the system as a whole. A bipartite state of a system is said to be separable if and only if it expressed as a product of the density operators of the individual constituents[13]. Mathematically,

$$\hat{\rho}_{ab} = \sum_i p_i \hat{\rho}_i^{(a)} \otimes \hat{\rho}_i^{(b)}, \tag{4.16}$$

where $p_i \geq 0$, and $\sum_i p_i = 1$ to verify the normalization of the combined density states. And $\hat{\rho}(a)$ and $\hat{\rho}^{(b)}$ stands for density matrices for mode a and mode b respectively. If the density matrix of a state can not be written as in Eq.(4.16), then it is said to be entangled. In synonymous expression if a state is not separable, then it is said to be entangled. Entanglement is a truly quantum feature of a state. The criteria to detect either a given state is entangled or not are in the form of some sort of inequality and in most instances provide sufficient condition of inseparability. Even if, there are different criterion for detecting entanglement here in our work we are interested the one so called logarithmic negativity.

4.2.1 Entanglement for the Cavity Mode

Here we are going to calculate the entanglement of the cavity modes. A state of the system is entangled, if it satisfies the criteria of logarithmic negativity (E_N) which is among the sufficient criteria for a composite system inseparable state for a continuous variable(CV). It can be expressed as in the form[37]

$$E_N = max[0, -\log_2 V_s], \tag{4.17}$$

where V_s is the smallest eigen value of symplectic matrix which provide a powerful tool to access essential information of the properties of gaussian state and given by

$$V_{s\pm} = [\frac{\gamma \pm \sqrt{\gamma^2 - 4\det\sigma}}{2}]^{\frac{1}{2}}, \tag{4.18}$$

where

$$\gamma = \det A + \det B - 2\det C_{AB}., \tag{4.19}$$

in which A and B are covariance matrices describing each modes separately, C_{AB} are intermodal correlations. The 2x2 block form of covariance matrices(σ) can be expressed as

$$\sigma = \begin{pmatrix} A & C_{AB} \\ (C_{AB})^T & B \end{pmatrix}, \tag{4.20}$$

where

$$A = \begin{pmatrix} m & 0 \\ 0 & m \end{pmatrix}, \tag{4.21}$$

$$B = \begin{pmatrix} n & 0 \\ 0 & n \end{pmatrix}, \tag{4.22}$$

and

$$C_{AB} = \begin{pmatrix} c & 0 \\ 0 & -c \end{pmatrix}, \tag{4.23}$$

with

$$\sigma_{ij} = \frac{1}{2}\langle \hat{x}_i\hat{x}_j + \hat{x}_j\hat{x}_i\rangle - \langle \hat{x}_i\rangle\langle \hat{x}_j\rangle. \tag{4.24}$$

And the corresponding quadrature operators are

$$\hat{x_1} = \hat{a}^\dagger + \hat{a}, \tag{4.25}$$

$$\hat{x_2} = i(\hat{a}^\dagger - \hat{a}), \tag{4.26}$$

$$\hat{x_3} = \hat{b}^\dagger + \hat{b}, \tag{4.27}$$

$$\hat{x_4} = i(\hat{b}^\dagger - \hat{b}). \tag{4.28}$$

In order to assess the contribution of the involved cross- and autocorrelations, the covariance matrix is expanded as

$$\sigma = \begin{pmatrix} m & 0 & c & 0 \\ 0 & m & 0 & -c \\ c & 0 & n & 0 \\ 0 & -c & 0 & n \end{pmatrix}, \tag{4.29}$$

where

$$\begin{aligned} m = {} & 1 + 2\langle \hat{a}^\dagger\hat{a}\rangle + \langle \hat{a}^2\rangle + \langle \hat{a}^{\dagger 2}\rangle \\ & - [\langle \hat{a}^2\rangle + \langle \hat{a}^{\dagger 2}\rangle + 2\langle \hat{a}\rangle\langle \hat{a}^\dagger\rangle], \end{aligned} \tag{4.30}$$

$$\begin{aligned} n = {} & 1 + 2\langle \hat{b}^\dagger\hat{b}\rangle + \langle \hat{b}^2\rangle + \langle \hat{b}^{\dagger 2}\rangle \\ & - [\langle \hat{b}^2\rangle + \langle \hat{b}^{\dagger 2}\rangle + 2\langle \hat{b}\rangle\langle \hat{b}^\dagger\rangle], \end{aligned} \tag{4.31}$$

$$c = \langle \hat{a}\hat{b} \rangle + \langle \hat{a}^\dagger \hat{b}^\dagger \rangle + \langle \hat{a}\hat{b}^\dagger \rangle + \langle \hat{a}^\dagger \hat{b} \rangle$$
$$- [\langle \hat{a} \rangle \langle \hat{b} \rangle + \langle \hat{a}^\dagger \rangle \langle \hat{b}^\dagger \rangle + \langle \hat{a} \rangle \langle \hat{b}^\dagger \rangle + \langle \hat{a}^\dagger \rangle \langle \hat{b} \rangle]. \tag{4.32}$$

Considering Eq.(3.21) and Eqs. (4.27), (4.28), (4.29) reduced to

$$m = 1 + 2\langle \hat{a}^\dagger \hat{a} \rangle, \tag{4.33}$$

$$n = 1 + 2\langle \hat{b}^\dagger \hat{b} \rangle, \tag{4.34}$$

$$c = \langle \hat{a}\hat{b} \rangle + \langle \hat{a}^\dagger \hat{b}^\dagger \rangle. \tag{4.35}$$

Using Eqs. (4.20), (4.21), (4.22),and (4.23) with Eqs.(4.30), (4.31), (4.32), one can get

$$\det A = m^2 = 1 + 4\langle \hat{a}^\dagger \hat{a} \rangle [\langle \hat{a}^\dagger \hat{a} \rangle + 1], \tag{4.36}$$

$$\det B = n^2 = 1 + 4\langle \hat{b}^\dagger \hat{b} \rangle [\langle \hat{b}^\dagger \hat{b} \rangle + 1], \tag{4.37}$$

$$\det C_{AB} = (c)(-c) = -4\langle \hat{a}\hat{b} \rangle^2, \tag{4.38}$$

and

$$\det \sigma = (mn - (c)^2)(mn - (-c)^2) = 16[\frac{1}{4} + \frac{\langle \hat{a}^\dagger \hat{a} \rangle + \langle \hat{b}^\dagger \hat{b} \rangle}{2} + \langle \hat{a}^\dagger \hat{a} \rangle \langle \hat{b}^\dagger \hat{b} \rangle - \langle \hat{a}\hat{b} \rangle^2]^2. \tag{4.39}$$

The two-mode Gaussian state would be entangled if and only if $E_N = -\log_2 V_s$ is positive which implies that $\log_2 V_s$ is negative[66] otherwise $E_N = 0$, the state is separable. As a result $Vs < 1$ can be taken as a sufficient entanglement condition. taking into account Eqs. (3.19), (3.20), and along with Eqs. (4.19) and (4.36) we can get the value of V_s and check $\log_2 V_s$ is negative or not in order to detect the degree of entanglement for a two mode gaussian state.

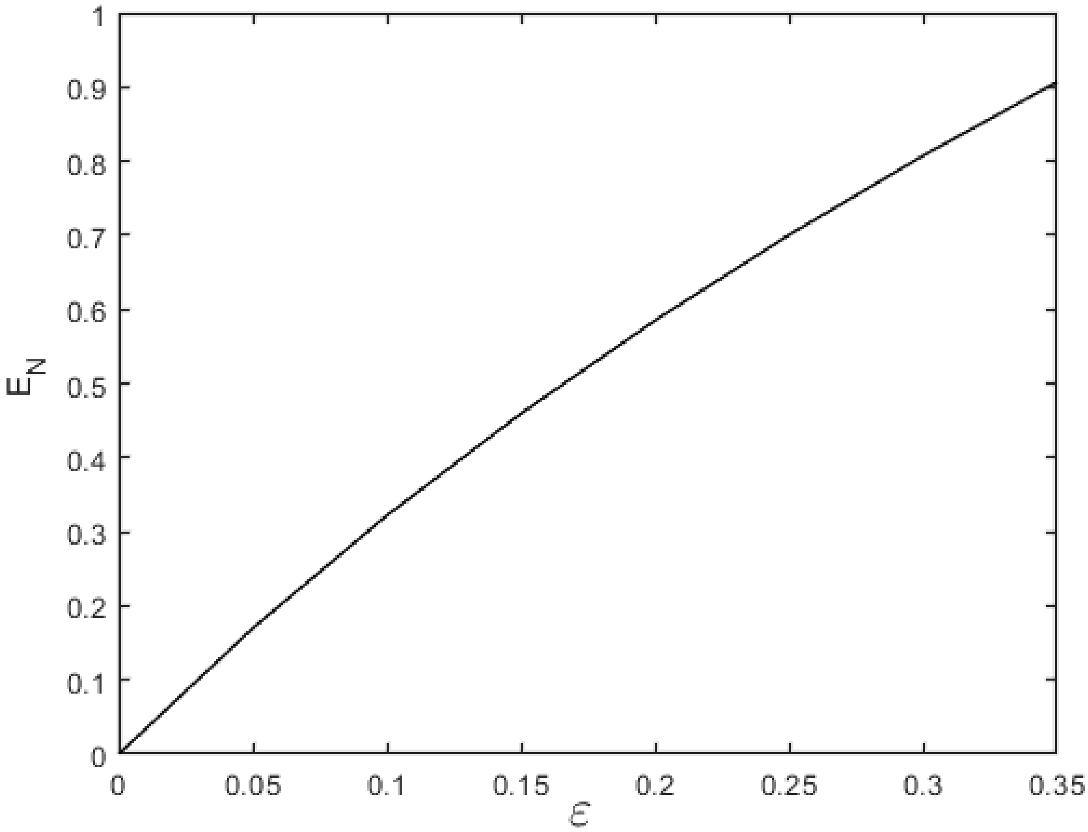

Fig. 4.3: Plots of logarithmic negativity (E_N)(Eq. (4.17)) versus amplitude of the pump mode (ε) of the cavity radiation at steady state for $\kappa = 0.8$

The plot in Fig. (4.3) indicate that the degree of entanglement of the cavity modes for the system under consideration increase with the amplitude of the pump mode.$E_N = 0$ indicates that there is no entanglement between the modes (i.e the state is separable). Hence we get $E_N = 0$ when the amplitude proportional to the pump mode vanishes.

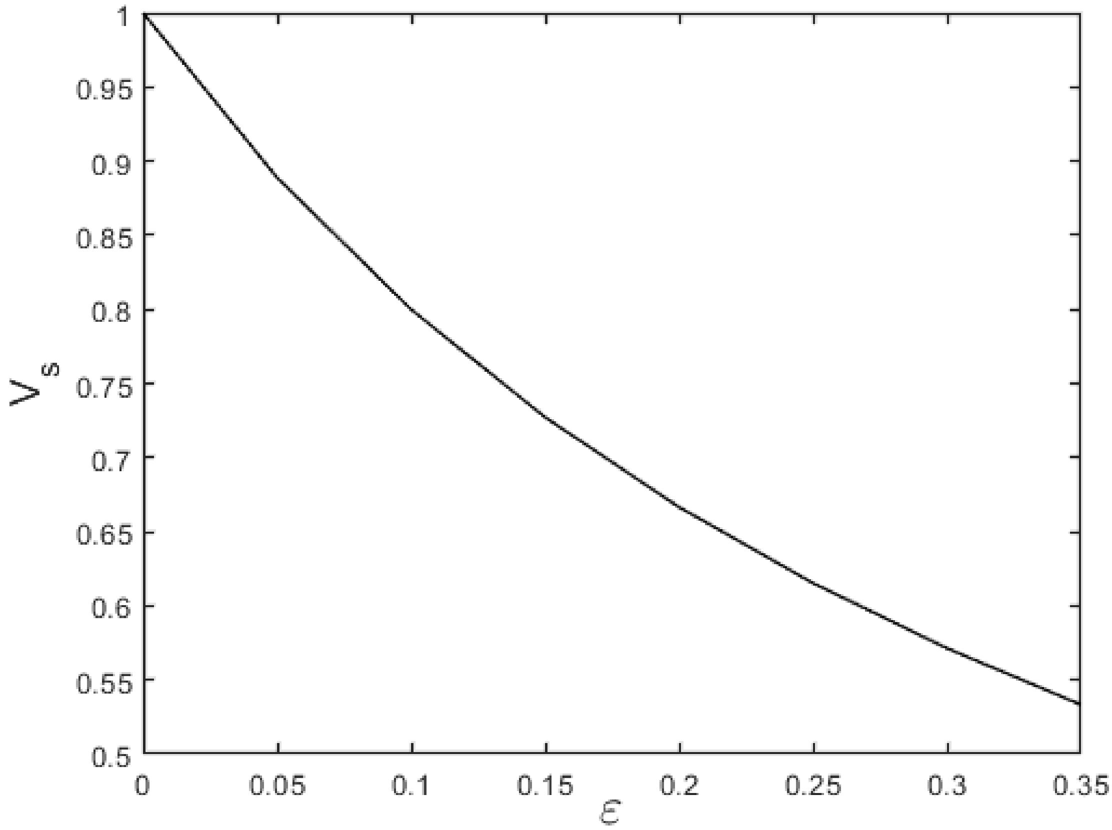

Fig. 4.4: Plots of the smallest eigenvalue (Vs) of a symplectic matrix (Eq. (4.18)) versus amplitude of the pump mode of the cavity radiation at steady state for $\kappa = 0.8$

As the plot in Fig. (4.4) indicate the smallest eigenvalues of the symplectic matrix decreases with increasing amplitude of the pump mode of the cavity modes for the system under consideration. In our case the value of Vs is in between 0.5 and 1 because of our solution works below threshold.

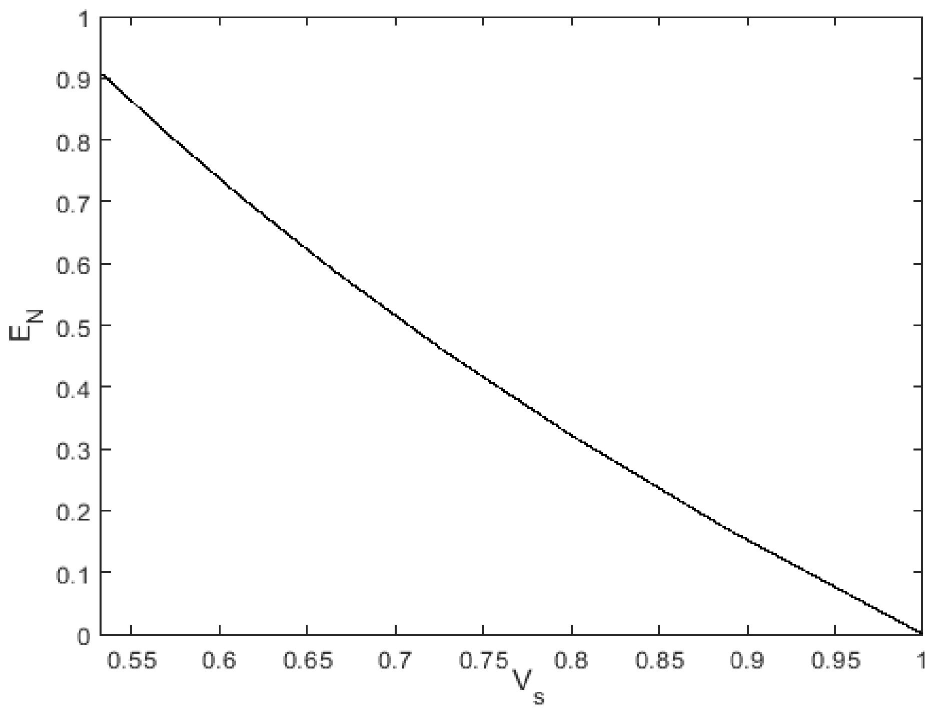

Fig. 4.5: Plots of logarithmic negativity (E_N) versus the smallest eigenvalue (Vs) of a symplectic matrix of the cavity radiation at steady state (Eq.(4.17)) for $\kappa = 0.8$

According to the plot in Fig. (4.5) when V_s approaches to 0.5 we can say the system is maximally entangled and when V_s approaches to one we can say the system is minimally entangled. In general the logarithmic negativity(E_N) and the smallest eigenvalue of the symplectic matrix (Vs) are inversely proportional and hence the smaller V_s the more entanglement is encoded in the corresponding Gaussian state.

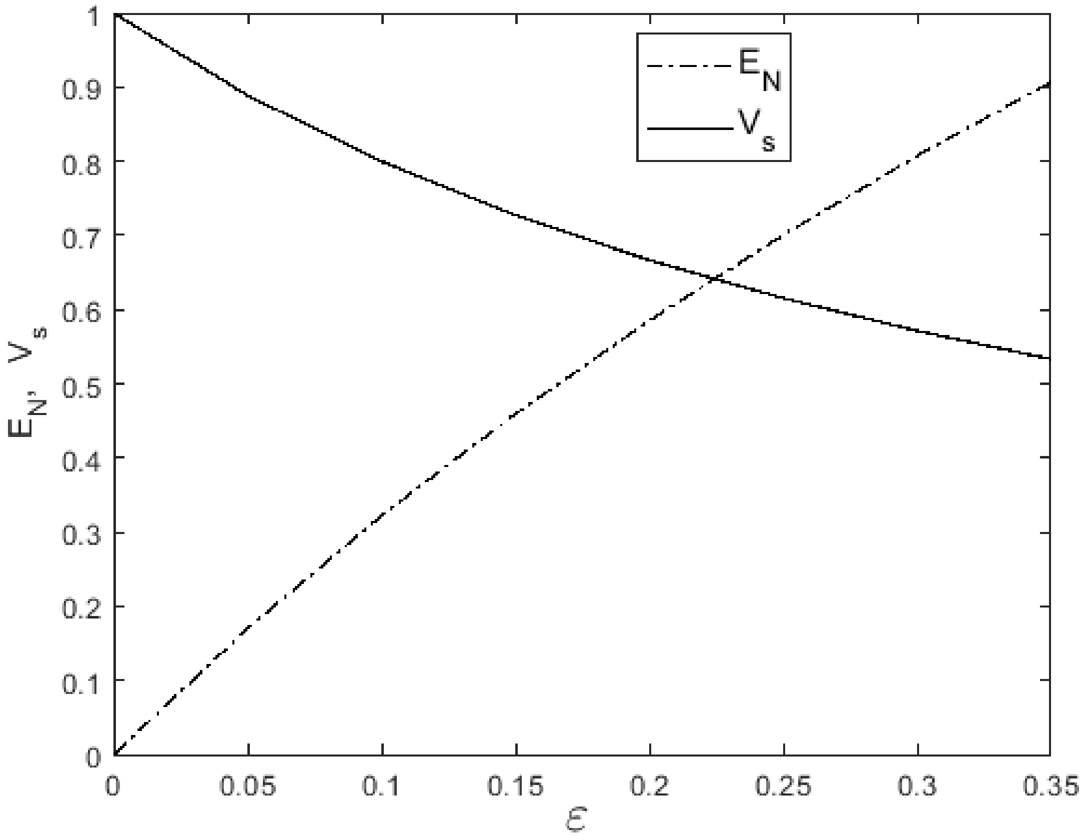

Fig. 4.6: Plots of logarithmic negativity (E_N) (Eq.(4.17)) and the smallest eigenvalue (Vs) of a symplectic matrix (Eq.(4.18)) versus amplitude of the pump mode of the cavity radiation at steady state for $\kappa = 0.8$

Fig. (4.6) shows that the logarithmic negativity increases with the amplitude proportional however, the smallest eigenvalue (Vs) of a symplectic matrix decreases.

4.3 Gaussian Discord

Quantum correlations play a central role as a resource in quantum information processing and quantum communication tasks that cannot be done efficiently classically. Previously, any correlation in the absence of entanglement was thought to be purely classical as they can be prepared with local operations and classical communications. However, there were reasons to believe this was not the whole story; for example, there are quantum computational models with no or little entanglement, which can efficiently perform tasks that are believed to be classically hard [61, 63].

A state is said to be separably correlated if it can be prepared via LOCC. And a state is said to be classically correlated if it can be fully determined without disturbing it with the aid of local measurements and classical communication. The state of classically correlated system can be written as

$$\rho_{AB} = \sum_{ab} P_{ab}|ab\rangle\langle ab| \tag{4.40}$$

$|ab\rangle$ forms orthogonal basis. Quantum discord [40] measures the non-classical correlations which goes beyond quantum entanglement in a quantum state of a composite system. The Gaussian quantum discord can be measured in the domain of generalized Gaussian measurements on a bipartite quantum system of two-mode covariance matrix σ. A state is said to be discordant if it cannot be fully determined with out disturbing it with LOCC i.e

$$\rho_{AB} \neq \sum_{ab} P_{ab}|ab\rangle\langle ab| \tag{4.41}$$

Some separable states have quantum discord that means quantum discord can be generated by LOCC.

Quantum discord is also a conditional-entropy-based measure for bipartite systems, and quantifies quantum correlations for a specific direction from one subsystem to another. For a bipartite system in quantum state ρ_{AB}, quantum discord from subsystem B to A is defined in terms of classical and quantum conditional entropies as

$$D(B \to A) = H_L^{(min)}(A|B) - S(A|B). \tag{4.42}$$

where $S(A|B)$ is the quantum conditional entropy,

$$S(A|B) = S(A, B) - S(B). \tag{4.43}$$

Here $S(A, B) = -Tr[\rho_{AB} \ln \rho_{AB}]$ is the von Neumann entropy of the joint state, and $S(B) = -Tr[\rho_B \ln \rho_B]$ is the von Neumann entropy of marginal state of subsystem B,$\rho_B = Tr_A[\rho_{AB}]$. The classical conditional entropy in Eq (4.42)is defined as the minimum conditional entropy that can be measured using local measurements

$$H_L^{(min)}(A|B) = min_{\Pi_b,\Pi_a} \sum_b p(b)H(A|B). \tag{4.44}$$

where $p(b) = Tr[\rho_B \Pi_b]$ and $H(A|B)$ is the Shannon entropy of the conditional probability distribution $p(a|b) = p(a, b)/p(b)$,

$$H(A|b) = -\sum_a p(a|b) \ln p(a|b). \tag{4.45}$$

The strategy for obtaining the classical conditional entropy is: a measurement with POVM elements Π_b is performed on subsystem B, then the conditional states

$$\rho_{A|b} = Tr_B[\rho_{AB}\Pi_b]/p(b). \tag{4.46}$$

are measured to obtain $H(A|b)$; all the measurements are chosen such that the minimum conditional entropy is achieved.

Both classical and total mutual information is defined as

$$I(a:b) = H(a) + H(b) - H(ab) \rightarrow classicalinformation \tag{4.47}$$

$$I(A:B) = S(A) + S(B) - S(AB) \rightarrow totalmutualinformation \tag{4.48}$$

For classical mutual information using Eq.(4.47) can be defined as

$$J(b/a) = H(b)) - \sum_a p_a H(b/a) \tag{4.49}$$

mutual information is the measuring of information stored in the joint state (i.e correlation).

The Gaussian quantum discord can be measured in the domain of generalized Gaussian measurements on a bipartite quantum system of two-mode covariance matrix δ . By using the set of positive operator valued measurements on subsystem β, the general expression of Gaussian quantum discord [31] is obtained as

$$D(\rho_{AB}) = f(\sqrt{\det B}) - f(Vs_+) - f(Vs_-) + f(\sqrt{Em}), \tag{4.50}$$

with

$$f(x) = \left(x + \frac{1}{2}\right)\log_2\left(x + \frac{1}{2}\right) - \left(x - \frac{1}{2}\right)\log_2\left(x - \frac{1}{2}\right)$$

,

$$Em = (\frac{\sqrt{n^2} + 2\sqrt{m^2n^2} + 2(c)(-c)}{1 + 2\sqrt{n^2}}),$$

$$\gamma^{\sim} = \det A + \det B + 2\det C_{AB}.$$

When $D_{B/A} = 0$ it shows classical behavior and $D_{B/A} > 0$ indicates quantumness (quantum behavior).

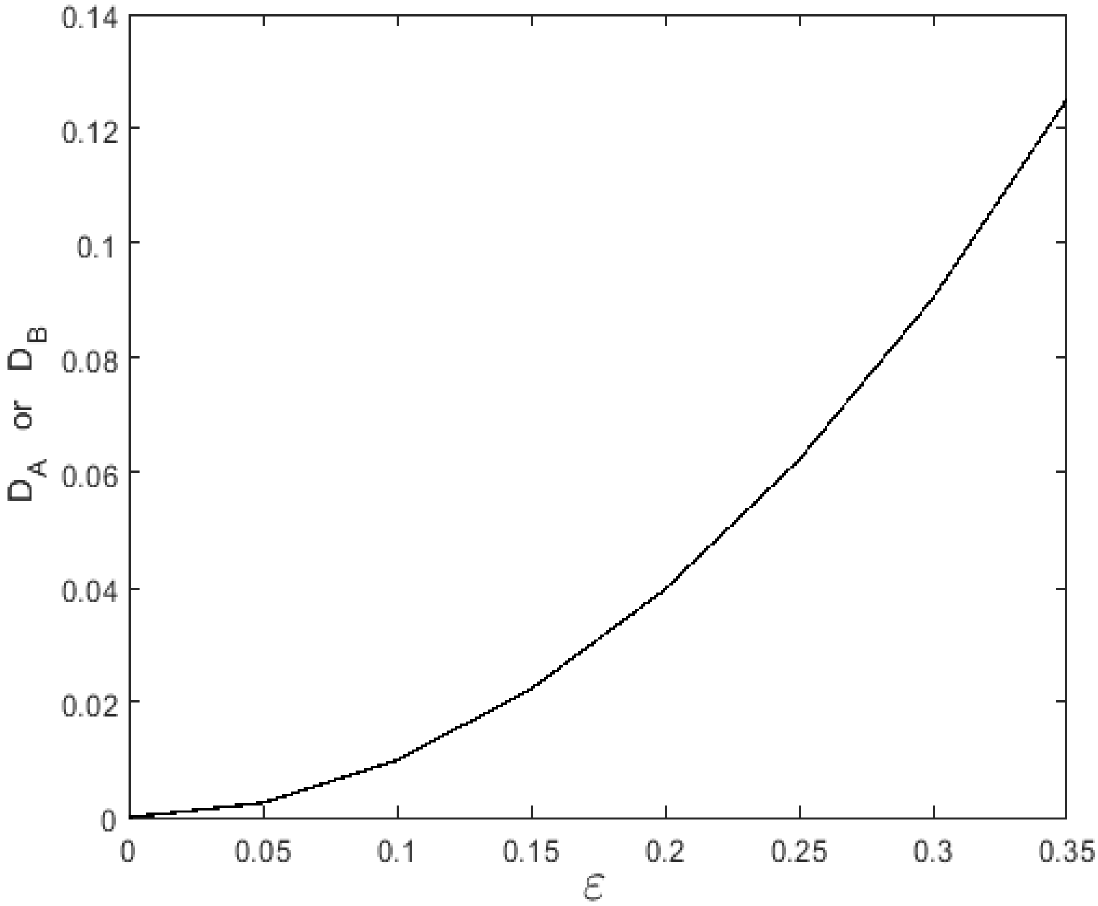

Fig. 4.7: plots of discord(Eq.(4.47)) versus amplitude of the pump mode (ε)of the cavity radiation at steady state for $\kappa = 0.8$

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Here as shown in Fig. (4.7) the discord increases with increasing the amplitude proportional to the pump mode. The discord from A to B and B to A are identical, since the determinants of sub matrices A and B are the same.

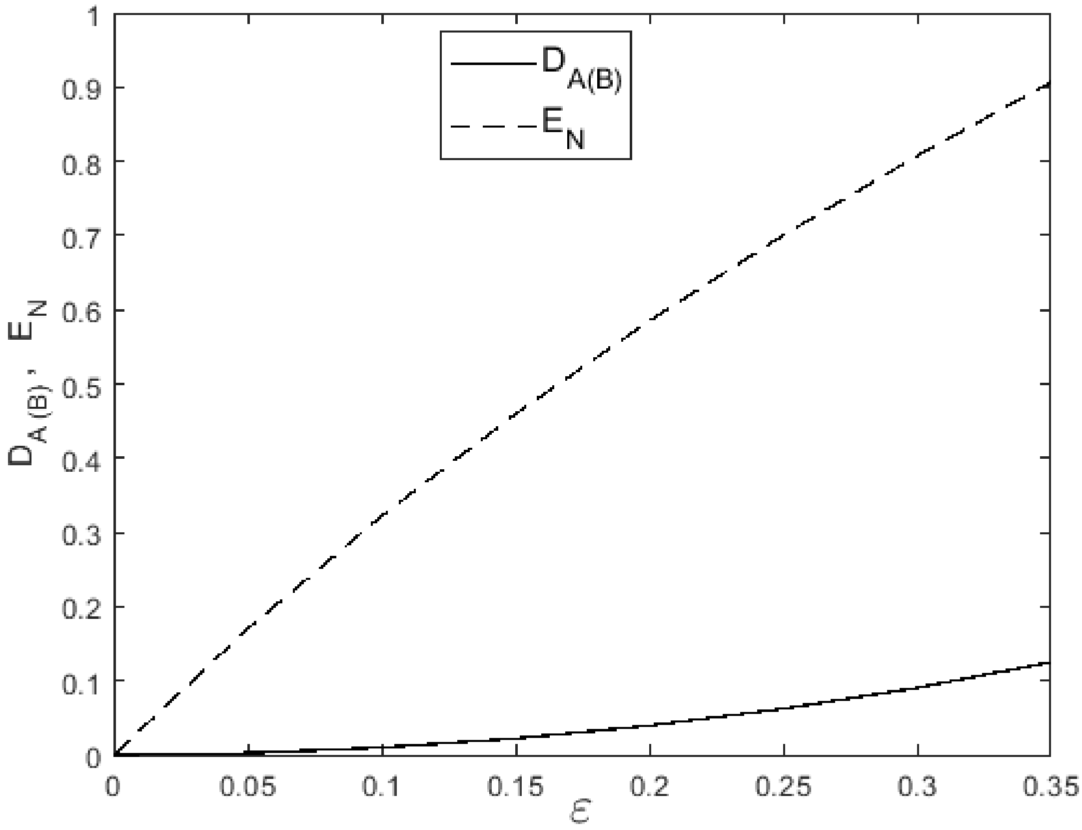

Fig. 4.8: Plots of logarithmic negativity (E_N)(Eq.(4.17) and the discord(Eq.(4.47)) versus amplitude of the pump mode of a cavity radiation at steady state for $\kappa = 0.8$

In the above, Fig. (4.8) the degree of entanglement employing the criterion of logarithmic negativity (Eq. (4.17)) and the corresponding quantum discord(Eq. (4.47)) versus amplitude proportional to the pump mode are ploted. As indicated both entanglement and discord increases with amplitude proportional to the pump mode, but the discord increases slowly compared to entanglement.

4.4 Gaussian Steering

Quantum steering[21] is an interesting quantum mechanical phenomenon which fills the space between entanglement and Bell non-locality on the part of the quantum correlation effect in a system. The steering provides one party with the ability to perform a measurement on their side of an entangled quantum system. As a response, the set of parameters of other party at a non-local distance are influenced. Steering is strictly stronger than entanglement, but does not entail a violation of a Bell inequality. Wiseman et al. showed that violation of the Reid criteria [66] for existence of the EPR paradox is a demonstration of steering for Gaussian systems. Wiseman et al. also raised the very interesting question of whether there could be asymmetric states which could be steerable by one party but not the other.

The possibility of asymmetric steering or the violation of the Reid criteria for the existence of the EPR paradox arises from an intrinsic asymmetry in the description which is not present in the usual criteria for establishing the inseparability of a bipartite quantum state. Asymmetric steering arises when Alice can convince Bob that the state is entangled but, after a reversal of the roles, Bob cannot convince Alice of the same thing. As both Alice and Bob are both looking at the same overall system, but get different answers, this is an interesting, although, different example of the role of the observer in quantum mechanics.

The EPR criteria developed by Reid [66] and violated experimentally [67] by Ou et al. are the criteria needed for demonstrating steering in a continuous variable system for Gaussian measurements. The Reid criteria give the optimal Gaussian measurement has also been shown more recently by Jones et al. [68], who have shown that if the inferred quadrature variances do not violate the appropriate inequality, the system is not steerable by any Gaussian measurements. The Reid criteria depend on the predictability of observables on one half of the system from measurements made on the other half. Reid defines inferred quadrature variances as;

$$V_{inf}(\hat{x_i}) = V(\hat{x_i}) - \frac{[V(x_i,\hat{x_j})]^2}{V(\hat{x_j})}, \tag{4.51}$$

$$V_{inf}(\hat{y_i}) = V(\hat{y_i}) - \frac{[V(y_i,\hat{y_j})]^2}{V(\hat{y_j})}, \tag{4.52}$$

in which $V(\hat{M},\hat{N}) = \langle\hat{M}\hat{N}\rangle - \langle\hat{M}\rangle\langle\hat{N}\rangle$. When $V_{inf}(\hat{x_i})V_{inf}(\hat{y_i}) < 1$ one can have an example of j steering i for Gaussian measurements, whereas an observation of $V_{inf}(\hat{x_i})V_{inf}(\hat{y_i}) \geq 1$ can be explained classically. This therefore means that a manifestation of

$$V_{inf}(\hat{x_i})V_{inf}(\hat{y_i}) < 1 \leq V_{inf}(\hat{x_j})V_{inf}(\hat{y_j}) \tag{4.53}$$

would be an example of asymmetric steering for the case of Gaussian measurements. The most significant one way states were qualified by the Reid criterion giving 0.908 ± 0.003 for the direction from Alice to Bob and 1.206 ± 0.004 for the reverse direction, where the normalization was chosen such that below 1 steering is certified[69].

Quantum steering is asymmetric in general i.e. $S^{A\to B} \neq S^{A\to B}$ where steering from A to B can be quantified with[21]

$$G^{A\to B} = max[0, \frac{1}{2}ln(\frac{\det A}{\det \sigma})]. \tag{4.54}$$

Where $\det A$ and $\det \sigma$ are given in Eqs. (4.36) and (4.39) respectively.

When quantifying the steering in our system using Eq.(4.54) above, its maximum value is zero for all ε values taken in our solution. And this indicates that steering from $A \to B$ and or $B \to A$ vanishes, which means there is no steering at all in this system. We can also qualify and quantify a steering in terms of in an interplay between the global purity $\mu = \frac{1}{\sqrt{\det \sigma}}$ and the two marginal purities $\mu_{A(B)} = \frac{1}{\sqrt{\det A(B)}}$. Since as it is from Eq (4.29) the standard form of states with fixed marginal purities always satisfies $\mu_A = \frac{1}{m}$, $\mu_B = \frac{1}{n}$. Introducing the ratio $\eta = \frac{\mu_A \mu_B}{\mu_\sigma}$, all physical two-mode Gaussian states live in the region $\eta_0 \leq \eta \leq 1$, where $\eta_0 = \mu_A\mu_B + |\mu_A - \mu_B|$ [21]. States with $\eta_s \leq \eta \leq 1$ where $\eta_s = \mu_A + \mu_B - \mu_A\mu_B$ are necessarily separable, states with $\eta_e \leq \eta < \eta_s$ where $\eta_e = [\mu_A^2 + \mu_B^2 - \mu_A^2\mu_B^2]^{\frac{1}{2}}$ can be entangled or separable (coexistence region), while states with $\eta_0 \leq \eta \leq \eta_e$ are necessarily entangled [21]. states with $\eta \geq \mid \mu_A, \mu_B \mid$ are nonsteerable, states with $\eta < \mu_B$ are $A \to B$ steerable; states with $\eta < \mu_A$ are $B \to A$ steerable. All the above situation allows us to classify the separability and steerability (by Gaussian measurements) of all two-mode Gaussian states in the(μ_A, μ_B, η) space.

We did some useful calculations based on the above expressions and get the result which satisfies the situations $\eta_0 \leq \eta \leq \eta_e$ and $\eta \geq \{\mu_A, \mu_B\}$ that shows a system necessarily entangled but not steerable.

5

CONCLUSION

In this thesis we have studied the various quantum correlations of the two mode light generated by non-degenerate parametric oscillator. Applying the steady state solutions of the equation of evolution of the expectation values of the cavity mode operation, we have calculated the quadrature variance and quadrature squeezing. Furthermore, employing the same solutions along with the logarithmic negativity criteria, we have quantified the entanglement for the cavity mode. In addition, applying the same solutions, we quantified the discordant and steering for the cavity mode. Our results showed that the light produced is in squeezed state, the degree of squeezing being 50% below coherent level at threshold. The degree of squeezing increases with the amplitude of the pump mode. It so turned out that the light modes are entangled and discordant but not steerable. We have found that the degree of entanglement increases with increasing the amplitude proportional to the pump mode but decreases with increasing the smallest eigne value of the symplectic matrix. Moreover, we have shown that the discordant value increases with increasing amplitude proportional to the pump mode. On the other hand it so happened that steering for the two mode cavity light (i.e the signal and idler modes) has zero value.

ACKNOWLEDGMENT

This research project is funded by Adama Science and Technology University under the grant number ASTU/SM-R/024/19, Adama, Ethiopia.

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