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SCHOOL OF APPLIED NATURAL SCIENCE,

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LASER SPECTROSCOPY STREAM

DETERMINATION OF THE DIFFUSION COEFFICIENT OF LEMON JUICE BY USING MOIRÉ DEFELECTOMETRY

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TABLE OF CONTENTS

CONTENTS **PAGE**

LIST OF TABLES

LIST OF FIGURES

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ABSTRACT

In determining the diffusion coefficient of lemon juice concentration we took temperature of the room at 27 ℃ and remain a constant throughout the experiment. The monochromatic light of wave length 632 nm from 35 mW He-Ne laser released to pass through two biconcave lenses of focal length 15 cm and 5cm respectively to expand the light from the laser. The light that passes through the sample and travels through first grating and by second grating the light were formed a moiré on the screen .There by a concave lens of focal length 15 cm moiré fringes formed on the screen made larger. The relative positions of adjacent fringes from vertical axis were measured. Then the photographs of the fringes on the screen were taken by using digital camera. The diffusion coefficient of concentration of lemon juice was computed based on Fick's law of diffusion. The determined values obtained as the result of fringe shift from vertical axis. The result revealed that the diffusion coefficient of lemon juice concentration ranges from 2.45 x $10^{-8}\,\frac{m^2}{s}$ to 6.31 x $10^{-8}\,\frac{m^2}{s}$. The average value of the diffusion coefficient of lemon juice concentration obtained due to moiré fringe shift is 4.48 x.$10^{-8}\,\frac{m^2}{s}$. In validating the result the error analysis technique has been employed. Thus the average error calculated is 1.19 %. This implies that the result has a god agreement with that stated in the literature review. [1]

Key words: Diffusion coefficient, Moiré defelectometry and Lemon juice concentration

1.0. INTRODUCTION

1.1. Back ground of the study

In this study we were used a high-precision laser-based technique to measure the refractive index of a liquid mixture or an aqueous solution, which includes the Moiré defelectometry technique for direct mapping of ray deflection due to change in refractive index of a medium. Moiré defelectometry technique as presented by Kafri and Glatt. is a well-known method for mapping of ray deflections introduced by an object placed within a collimated laser beam. If an object is placed in front of first grating the light deflected by the object yields the shifted images, and the resultant Moiré fringes show the deflection mapping, depending on the distribution of the refractive index of the object. Moiré defelectometry is a simple and powerful tool for measuring ray deflections within the paraxial approximation [2]. The main advantages of the Moiré deflectometry technique are its extreme experimental simplicity, low cost and low sensitivity to external disturbances in comparison with other interferometric methods that requires high prices and stable experimental set-up.The diffusion coefficient is determined as the result of moiré shift from vertical axis at the initial time and the ratio of fringe shift [3].

1.2. Objective

1.2.1 General objective

The General objective of this thesis is: To find the diffusion coefficient of transparent liquid solution by using moiré defelectometry.

1.1.2 Specific objective

The Specific objective of this experiment is: To determine the diffusion coefficient of lemon juice by using moiré defelectometry.

2.0 LITRATURE REVIEW

2.1 The diffusion process

Diffusion describes the spread of particles through random motion from regions of higher concentration to regions of lower concentration [3]. Diffusion is also the movement of molecules due to their thermal energies under the influence of a concentration gradient. The study of diffusion is important in fields as diverse as chemical engineering, biology, pollution control, separation of isotopes etc. Knowledge of diffusivity is necessary for the design of chemical equipments and for mass transfer studies. Diffusivity is therefore one of the most important fundamental property of a chemical system. The diffusion coefficient of a substance in a fluid can be predicted theoretically or found from empirical correlations.

Diffusion is the process by which matter is transported from one part of a system to another as a result of random molecular motions [4]. The composition of mixture is described by the concentration, defined as the ratio of the mass of one component to the total mass of the fluid in a given fluid volume element. In the course of time, the distributions of concentration through the fluid will general change. This change occurs in two ways. Firstly, when there is macroscopic motion of the fluid, any given small portion of it moves as the whole, its composition remaining unchanged. Secondly, a change in composition can occur by the molecular transfer of the components from one part of the fluid to another. The equalization of the concentration by this direct change of composition of every small portion of fluid is called diffusion [5]. Diffusion process may be divided into two types: (a) steady state and (b) non steady state. Steady state diffusion takes place at a constant rate that is, once the process starts the number of atoms (or molecules) crossing a given interface (the flux) is constant with time. This means that throughout the system $\frac{dC}{dx}$ = constant and $\frac{dC}{dt} = 0$. None steady state diffusion is a time dependent process in which the rate of diffusion is a function of tie. Thus $\frac{dC}{dx}$ varies with time and $\frac{dC}{dt} \neq 0$. Bothe types of diffusion are described quantitatively by Fick's laws of diffusion. The first law concerns both steady state and non-steady state diffusion [6].

2.1.1 The study of diffusion

There are many methods to determine diffusion coefficients of transparent liquid solutions. Optical methods are one of the most accurate methods used to determine diffusivity values. The optical method includes conventional interferometry, holographic interferometry and electronic speckle pattern interferometry. But the disadvantage of most of optical methods is that they impose stringent optical requirements, which are difficult to employ in industrial environments. So it becomes necessary to develop new methods for diffusivity measurement that are easy to implement and cheap. Laser-based techniques are among the most sensitive methods employed for measuring diffusion coefficient and thermal conductivity. They have several distinct advantages over their direct contact counterparts, including the non-invasive nature of the measurement, remote monitoring and location of test/analysis equipment, imperviousness to harsh and corrosive environments, very high spatial precision, fast response times and high reliability and repeatability. Wave front comparison methods such as optical and holographic interferometry techniques are the most widely used techniques for diffusivity studies [3]. The average effective diffusivity values varied from 2.81 x $10^{-8}\ \frac{m^2}{s}$ to 5.98 x$10^{-8}\frac{m^2}{s}$ for lemon slices [1].

2.1.2 Medicinal uses of lemon

Lemons have health and medicinal properties, It is cultivated mainly for its alkaloids, which are having anti cancer activities and the anti bacterial potential in crude extracts of different parts of lemon against clinically significant bacterial strains [7]. Lime is so famous as a cure for scurvy, the disease which is caused due to deficiency of vitamin C and characterized by frequent infections with cough and cold, cracked lips and lip corners; ulcers in tongue and mouth; spongy, swollen and bleeding gums [8]. Citric acid in lemon makes urine less favorable for the formation of stones. A half-cup (40 ounces) of pure lemon juice per day or 32 ounces of prepared lemonade provide about the same amount of citric acid as does pharmacological therapy [7].

2.2 Diffusion and diffusion equation

2.2.1 Basic hypothesis of Mathematical theory

Transfer of heat by conduction like diffusion is a random process in which from one Part to another by random molecular motions and there is an obvious analogy between the two processes. This was recognized by Fick's (1855) ,who first put diffusion on a quantitative basis by adopting the mathematical equation of heat conduction derived some years earlier by Fourier (1822). The mathematical theory of diffusion in isotropic substances is therefore based on the hypothesis that the rate of transfer o diffusing substance through unit area of a section is proportional to the concentration gradient measured normal to the section [4, 6].

$$F=-D\ \frac{\partial C}{\partial X} \qquad 2.2.2.1$$

Where: F is the rate of transfer per unit area of section, C is the concentration of diffusing substance, X is the space coordinate measured normal to the section and D is the diffusion coefficient. Where -Ve sign arise diffusion occurs in the direction opposite to that of concentration. In some cases e.g. Diffusion in dilute solution (D) can reasonably be taken as a constant, while in other diffusion in high polymers, it depends very markedly on Concentration. If F the amount of material diffusing and C the concentration are both expressed in terms of the some unit of quantity, gram/gram molecules, that D is independent of this unit and has dimensions $(Length)^2\ (time)^{-1}$. It is usually illustrated by the classical experiment in which a tall cylindrical vessel has its lower part filled with iodine solution. For example a column of clear water is poured on top carefully and slowly, so that no convection currents are set up. At first the colored part is separated from the clear by a sharp well-defend boundary. Later it is found that the upper part becomes colored, the colors getting fainter toward the top, while the lower part becomes correspondingly less intensely colored. After sufficient time the whole solution appears uniformly colored. Therefore a transfer of iodine molecules from the lower to the upper part of the vessel taking place in the absence of convection currents. The iodine is said to have diffused into the water [4, 6]. If it were possible to watch individual molecules of iodine and this can be done effectively by replacing them by particles small enough to share the molecular motion by just large enough to be visible under the microscope, it would be found that the motion of each

molecule is the random one. In a dilute solution each molecule of iodine behave independently of others, which it seldom meets and each is constantly undergoing collision with solvent molecules as a result of which collisions it moves sometimes toward a region of higher, sometimes of lower, concentration, having no preferred direction of motion towards one/or the other. The motion of a single molecule can be described in terms of the familiar ”random walk.” Consider any horizontal section in the solution and two thin, equal, elements of volume one just below and one just above the section. Though it is not possible to say which way any particular iodine molecule will more in a given interval of time, it can be said that on an average a definite fraction of molecules in the lower element of volume will cross the section from below and a fraction of molecules in the upper element will cross the section from above a given time. This occurs simply because there are more iodine molecules in the lower element than in the upper one, there is a net transfer from the lower to the upper side of the section as a result of random molecular motions.

2.2.2 Differential equation of diffusion

The fundamental differential equation of diffusion in an isotropic medium is derived from equation (2.2.2.1) as follows. Consider an element of volume in the form of a rectangular parallelepiped whose sides are parallel to the axes of coordinates and are of lengths 2dx, 2dy, 2dz . Let the centre of the element be at P (x, y, z), where the concentration of diffusing substance is *C.* Let ABCD and A'B'C'D' be the faces perpendicular to the axis of x as in Figure.2.2.2.1. Then the rate at which diffusing substance enters the element through the face ABCD in the plane

$x - dx$ is given by

$$\text{Rate of diffusion to substance} = 4dydz\left(F_x - \frac{\partial F_x}{\partial x}\right) \qquad 2.2.2.2$$

where *Fx* is the rate of transfer through unit area of the corresponding plane through *P.* Similarly the rate of loss of diffusing substance through the face A'B'C'D' is given by

$$\text{Rat e of loss of diffusing} = 4dydz\left(F_x + \frac{\partial F_x}{\partial x}\right) \qquad 2.2.2.3$$

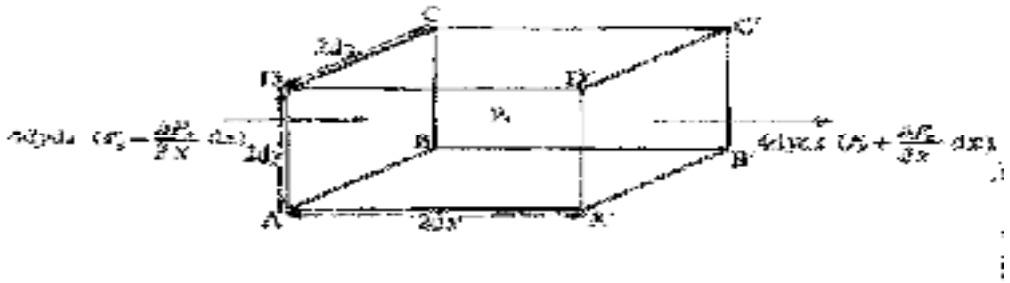


Figure 2.2.2.1 Element of Volume

The contribution to the rate of increase of diffusing substance in the element from these two faces is thus equal to -8dxdydz$\frac{\partial F_x}{\partial x}$. Similarly from the other faces we obtain -8dxdydz$\frac{\partial F_y}{\partial y}$

And-8dxdydz$\frac{\partial F_z}{\partial z}$. But the rate at which the amount of diffusing substance in the element increases is also given by 8dxdydz $\frac{\partial C}{\partial t}$ and hence we have immediately

$$\frac{\partial C}{\partial t} + \frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z} = 0 \qquad 2.2.2.4$$

If the diffusion coefficient is constant, Fx, Fy, Fz are given by (2.2.2.1), and (2.2.2.2) Becomes

$$\frac{\partial C}{\partial t} = D\left(\frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}\right) \qquad 2.2.2.5$$

Reducing simply to

$$\frac{\partial C}{\partial t} = D\,\frac{\partial^2 C}{\partial X^2} \qquad 2.2.2.6$$

If the diffusion is along one-dimensional i..e, if there is a gradient of concentration only along the x-axis, expressions (2.2.2.1) and (2.2.2.4) are usually referred to as Fick's first and second laws of diffusion, since they were first formulated by Fick (1855) by direct analogy with the equations of heat conduction. In many systems, e.g. the inter diffusion of metals or the diffusion of organic vapors in high-polymer substances, *D* depends on the concentration of diffusing

substance C. In this case, and also when the medium is not homogeneous so that *D* varies from point to point, equation (2.2.2.2) becomes

$$\frac{\partial C}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial C}{\partial x}\right) + \frac{\partial}{\partial y}\left(D\frac{\partial C}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial C}{\partial z}\right) \qquad 2.2.2.7$$

where *D* may be a function of x, *y,* z, and C. If *D* depends on the time during which diffusion has been taking place but not on any of the other variables, i.e.

$$D = f(x) \qquad 2.2.2.8$$

then on introducing a new time-scale *T* such that

$$DT = f(x)\, dt \qquad 2.2.2.9$$

the diffusion equation becomes

$$\frac{\partial C}{\partial T} = \frac{\partial^2 C}{\partial X^2} + \frac{\partial^2 C}{\partial y^2} + \frac{\partial^2 C}{\partial z^2}, \qquad 2.2.2.10$$

Which is the same as (2.2.2.3) for a constant diffusion coefficient equal to unity [10].

2.2.3 The Empirical laws of Diffusion

The phenomenological description of diffusion is embodied in Fick's two laws, which are empirical statements that relate the diffusive flow of matter to concentration gradients. To illustrate these laws, consider a single-phase dilute binary alloy in which the constituents are in homogeneously distributed and let C be the concentration of the minor constituent. The concentration is function of position and time and the concentration gradient induces a flow of matter [4, 9].

Fick's First law: "States that the flux of the diffusing species in a given direction has a magnitude proportional to the concentration gradient in that direction". That is if J_1 the flux along the x_1 axis, then

$$J_1 = -D_1 \frac{\partial C}{\partial x_1}, \qquad 2.2.3.11$$

where: D_1 is proportionality factor called diffusion coefficient. The negative sign expresses the fact that diffusion occurs from the region of high concentration to region of law concentration [6]. If x_1, x_2, x_3 are the points in a Cartesian coordinate system with unit vectors (i_1, i_1, i_1)

then we would expect that an equation similar to (Eq.2.2.3.11) hold for each of the three directions along the coordinate axis. For isotropic systems such as gases and liquids, this turns out to be the case. Further more, for such systems the diffusing coefficient is experimentally found to be the same in all directions. In nonhomogeneous systems, however it is not the same in all directions. Also the flux in a one direction can depend on the flux in other directions [5, 11]. These results can be described by a generalized Fick's first law, as

$$J_1 = D_{11} \frac{\partial C}{\partial x_1} - D_{12} \frac{\partial C}{\partial x_2} - D_{13} \frac{\partial C}{\partial x_3} \qquad 2.2.3.12$$

Similarly the fluxes in other two directions of the Cartesian coordinate system are given by

$$J_2 = D_{21} \frac{\partial C}{\partial x_1} - D_{22} \frac{\partial C}{\partial x_2} - D_{23} \frac{\partial C}{\partial x_3} \qquad 2.2.3.13$$

$$J_3 = D_{31} \frac{\partial C}{\partial x_1} - D_{32} \frac{\partial C}{\partial x_2} - D_{33} \frac{\partial C}{\partial x_3} \qquad 2.2.3.14$$

These equations can be written in a compact form by defining the flu vector Jas

$$J = J_1 i_1 + J_2 i_2 + J_3 i_3 \qquad 2.2.3.15$$

And the diffusion D as

$$D = D_{11} i_1 i_1 + D_{12} i_1 i_2 + D_{13} i_2 i_1 + D_{21} i_1 i_1 + D_{22} i_2 i_2 + D_{23} i_2 i_3 +$$

$$D_{31} i_3 i_1 + D_{32} i_3 i_2 D_{33} i_3 i_3 \; . \qquad 2.2.3.16$$

The equations 2.2.2.12 - 2.2.3.14 give the generalized Fick's First law as

$$J = -D \nabla C \qquad 2.2.3.17$$

Fick's Second law "states that the time rate of change of concentration in volume element of membrane within the differential filed is proportional to the rate of change of flux gradient at that point in the field" i.e.

$$\frac{\partial c}{\partial t} + \nabla . J = 0 \qquad 2.2.3.18$$

This is just an expression of law of conservation of matter and states that any change in concentration in a volume element is the result of the difference in matter flow in and out of the

volume element [2, 9]. Combining Equations 2.2.3.16 and 2.2.3.17 gives a generalized form of Fick's second law:

$$\frac{\partial C}{\partial t} = \nabla \text{ D.}\nabla\text{C} \qquad 2.2.3.19$$

In isotropic and cubic systems this takes a particularly simple form since

$D_1 = D_2 = D_3 = \text{D}$ [5, 14]. If D is a constant independent of position and time the simplification is even greater and Fick's laws become.

$$\frac{\partial c}{\partial t} = \nabla^2\text{C} \qquad 2.2.3.20$$

2.2.4 diffusion coefficient and mean free path

The average distance an atom or a molecule travels between collisions is called its mean free path. The mean free path depends upon the size of the atom or molecule. For typically small molecule in the gas phase at 300 K,the mean free path is about 8 nm. Taking an average speed of 400 m/s the average time collisions is about $2\times10^{-10}\ \frac{m^2}{se}$, this means this small molecules makes 5×10^{-9} collisions per second. As the molecules undergo this seemingly random motion induced by collisions with other atoms or molecules they migrate through the solution. If there are initially larger proportions of molecules of particular type in one region of the solution then over the time the molecules will spread out and become evenly distributed throughout the solution. This in the process of diffusion for an initial concentration gradient of a gas or liquid media the rate at which the molecules spread out is proportional to the difference in the concentration gradient [4, 15].

$$\text{Diffusion Rate} = \text{D} \times \text{Concentration gradient} \qquad 2.2.4.21$$

Where: D-is diffusion coefficient gradient in $(length)^2$/ time. It is a measure of the number of molecules moving through a particular cross-sectional area per unit of time. Experimental measurement of typical diffusion coefficient in water at 300 K yields values in the range of $10^{-6}cm^2$/s. Diffusion coefficient is related to the mean free path that a molecule travels via Einstein-Smoluchowsky equation.

$$D = \frac{\phi\lambda}{2\tau} \qquad 2.2.4.22$$

Where λ is the mean free path, τ is the average time between collisions. Assuming that the average time between the collisions can be calculated from the average speed and mean free path, the time between the collisions can be expressed as

$$\tau = \frac{\lambda}{v_{av}} \qquad 2.2.4.23$$

Where: V_{av} is the average speed of molecule.

Combining Equation 2.2.1.22 and 2.2.1.23 gives

$$D = \frac{\lambda V_{av}}{2} \qquad 2.2.4.24$$

Using the gas phase value of 80 nm for the mean free path and 400 m/s as the average speed the predicated diffusion coefficient for a small molecule is 1.6x $10^{-6} cm^2$/s, which is much larger that the typical value for the diffusion coefficient of liquid phase $10^{-9} m^2$ [13, 8].

2.2.5 Diffusion Coefficient and Viscosity

The transport property which is most important for the fluid to flow is the viscosity. Viscous forces appear only when adjacent part of fluid is moving with different velocities. Thus in the flow of liquid down a pipe the liquid layer which is usually in contact with the pipe is stationary and the axial region is moving with maximum velocity. Common experience is that high viscosity liquid will flow more slowly than a low viscosity liquid, like water for a given applied pressure difference. Poiseuille's formula for the flow of liquid in a capillary of radius R and length l is

$$\frac{dV}{dt} = \frac{(p_1^2 - p_2^2)}{16\lambda\eta p}, \qquad 2.2.5.25$$

Where v is the volume of flowing, p_1 and p_2, are the pressure at each end of the tube, P is the pressure at which the when Volume is measured and ,η is the coefficient of viscosity [16]. Diffusion in the fluid medium is related to a measurable property called viscosity. Viscosity is the measure of the resistance to the fluid flow. Liquid such as water that flow easily have a relatively low viscosity compared to thicker fluids. The viscosity of liquid is also its measure to

transport momentum. Its unit is poise P where 1P = 0.1 $\frac{kg}{m.s}$. Typical viscosities are in the range of 10^{-4} P for gas and 10^{-2} P for liquids. For a gas kinetic theory predict that viscosity is independent of pressure and proportional to the square root of the absolute temperature. For liquid viscosities have an exponential dependence on the temperature as described by the formula

$$\eta = A \exp\left[\frac{\Delta Ev}{RT}\right] \qquad 2.2.5.26$$

Where: ΔEV -an activation barrier. R-is the radius of the molecule. T-is the temperature. The relation between diffusion coefficient and viscosity is given by Einstein Stokes equation

$$D = \frac{KT}{6\pi\eta R} \qquad 2.2.5.27$$

Where K is Boltzmann's constant.

The equation is derived by assuming that the molecule is spherical in shape. The Einstein-Stokes equation that the viscosity and diffusion coefficient are inversely related to D_0 is also related to temperature via the equation [8, 14].

$$D = D_0 \exp\left[-E_A/RT\right] \qquad 2.2.5.28$$

Where D_0 is the diffusion coefficient at infinite temperature (maximum diffusion coefficient).and E_A is activation energy.

2. 2. 6 Random Walk as a statically Model for Diffusion

One-Dimensional treatment process such as diffusion in which a single molecule moves through a solution a huge number of particles, can be models statically. Even though the presence of other particles can affect the dynamics of the particle of interest through collision and interactions we can consider the motion of particle as a random walk. The diffusion as a motion of random walk simulation can be shown by calculating the average square distance from the origin. To do this, many individual walks simulation is carried out, and the average is taken over many individual walks. Let us consider a distribution where the concentration depends on only one coordinate, be the x-coordinate (one-dimensional diffusion). The concentration of the marked (atom) particle is then a function of X and of time't'. Let it be C (X, t), in order to calculate, the diffusion flow we need to count the number of particles which pass through a

surface of unit area, perpendicular to the axis, in the plane x_0 during time 't' [17]. The particles migrate through the lattice by making a sequence of jumps owing to the presence of defects.

We have chosen the case of tracer-diffusion with no external applied force, so that the probability of displacement is the same along both positive and negative direction will be called X. Let use define the distribution function f(X, t)dt as the probability of displacement with a projection between X and X+ dX. It follows from this definition that

$$\int_{-\infty}^{\infty} f(x,t)dx = 1 \qquad 2.2.6.29$$

More over since the jumps along both +X and -X direction occur with the same probability one has f(x ,t)= f(-x , t) Considering a large number of particles the projection of the mean displacement $\langle x \rangle$ may then be written as [9, 11].

$$\langle x \rangle = \int_{-\infty}^{\infty} x f(x,t)dx \qquad 2.2.6.30$$

It follows from Equation 2.2.6.30 and every $\langle x^{2n}+1 \rangle$ is zero. The most important parameter in diffusion is the mean square displacement,$\langle x2 \rangle$ the projection of which is then

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 f(x,t)dx \qquad 2.2.6.31$$

$$\langle x^2 \rangle \alpha \ D.t \qquad 2.2.6.32$$

Where t is the time t (proportional to the number of steps) D is the diffusion coefficient

2.3 basic physical optics

2.3.1 Interference

Optical interference corresponds to the interaction of two or more light waves yielding a resultant irradiance that deviates from the sum of the component irradiances [18]. Figure 2.3.1.2 shows the general setup for producing interference with coherent light from two slits S1 and S_2. The source S_0 is a monochromatic point source of light whose spherical wave fronts (circular in the drawing) fall on the two slits to create secondary sources S_1 and S_2. Spherical waves radiating out from the two secondary sources S_1 and S_2 maintain a fixed phase relationship with each other as they spread out and overlap on the screen, to produce a series of alternate bright

and dark regions. The alternate regions of bright and dark are referred to as interference fringes [19].

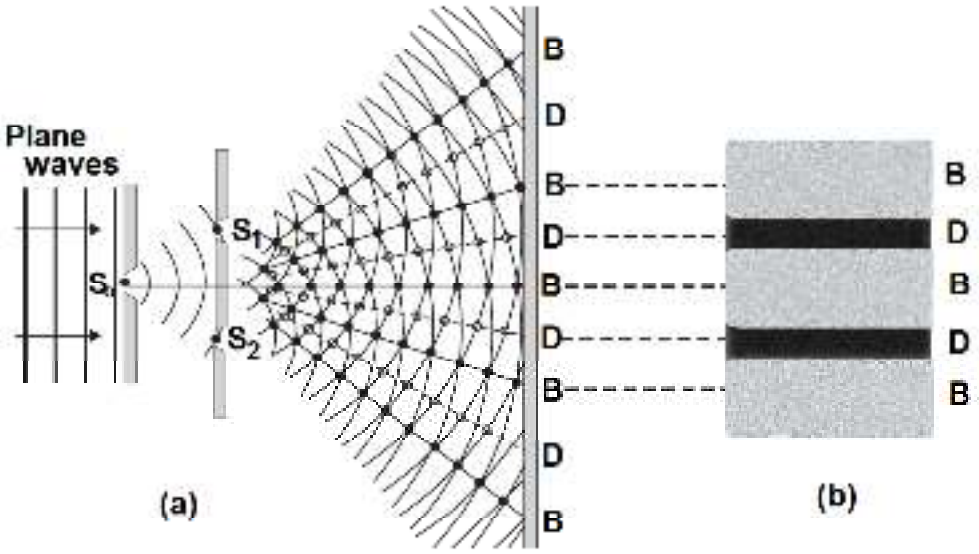


Figure 2.3.1.2 schematic diagram of Young's double-slit interference experiment [19].

With the help of the principle of superposition, it is possible to calculate the positions of the alternate maxima (bright regions) and minima (dark regions) shown in figure 2.3.1.2 using the following conditions on figure 2.3.1.3 shown below:

(a) Light from slits S_1 and S_2 is coherent; that is, there exists a fixed phase Relationship between the waves from the two sources.

(b) Light from slits S_1 and S_2 is of the same wavelength.

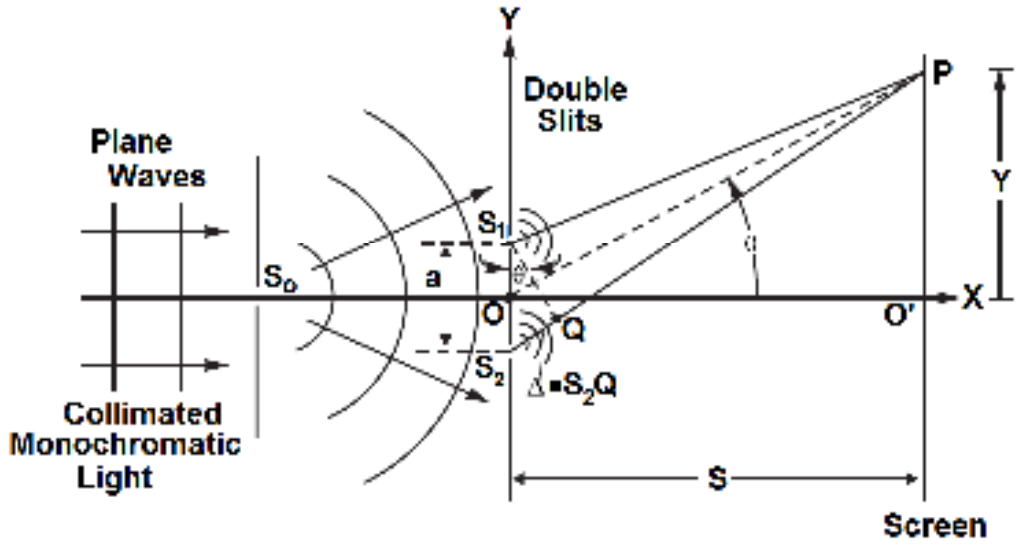


Figure 2.3.1.3 Young's double-slit interference experiment arrangement.

Source S_0 is generally a small hole or narrow slit; sources $S_2$and $S_2$are generally long, narrow slits perpendicular to the page [19]. In figure 2.3.1.3 light waves from S_1 and S_2 spread out and overlap at an arbitrary point P on the screen. If the overlapping waves are in phase, we expect a bright spot at P; if they are out of phase, we expect a dark spot. S_0 the phase difference between the two waves arriving at point P is a key factor in determining what happens there. We shall express the phase difference in terms of the path difference, which we can relate to the wavelength λ. For clarity, figure 2.3.1.3 is not drawn to scale. It will be helpful in viewing the drawing to know that, in practice, the distance S from the slits to the screen is about one meter, the distance between slits is less than a millimeter, so that the angle θ in triangle S_1S_2Q, or triangle OPO', is quite small. And on top of all this, the wavelength of light is a fraction of a micrometer. The path difference between S_1P and S_2P, as seen in figure 2.3.1.3 is given by equation (2.3.1.33), since the distances PS_1 and PQ are equal and since $\sin\theta = \frac{\Delta}{a}$ in triangle S_2S_2Q.

$$\Delta = S_2P - S_2P = S_2Q \qquad 2.3.1.33$$

If the path difference Δ is equal to λ or some integral multiple of , the two waves arrive at P in phase and a bright fringe appears there (constructive interference). The condition n for bright (B)

fringes is, then, The number is called the order number. The central bright fringe at $\theta = 0$ (point O' on the screen) is called the zeroth-order maximum (m=0). The first maximum on either side, for which m = ± 1 , is called the first-order maximum, and so on. If, on the other hand, the path difference at P is an odd multiple of $\frac{\lambda}{2}$, the two waves arrive out of phase and create a dark fringe (destructive interference). The condition for dark (D) fringes is given by equation (2.3.1.34).

$$\Delta_D = a\sin\theta = \left(m + \frac{1}{2}\right)\lambda \qquad 2.3.1.34$$

Where, m=0,± 1,± 2

Since the angle θ exists in both triangles S_1S_2Q and OPO', we can find an expression for the positions of the bright and dark fringes along the screen, because θ is small, as mentioned above, we know that, $\sin\theta \cong \tan\theta$ for triangle OPO' we can write

$$\sin\theta \cong \tan\theta = \frac{Y}{\lambda S} \qquad 2.3.1.35$$

Combining equation (2.3.1.35) with equations (2.3.1.33) and (2.3.1.34) in turn, by substituting for in each, we obtain expressions for the position Y of bright and dark fringes on the screen.

$$Y_B = \frac{\lambda S}{a}\mathrm{m} \qquad 2.3.1.36$$

$$Y_D = \frac{\lambda S}{a}\left(\mathrm{m} + \frac{1}{2}\right) \qquad 2.3.1.37$$

Where m=0, ± 1, ± 2 by representing the two separate electric fields at point P, the one coming from S1 can be written as

$$E_1 = E_0\sin\left(\frac{2\pi}{T}t\right) \qquad 2.3.1.38$$

And the one from as S_2also can be written as

$$E_2 = E_0\sin\left(\frac{2\pi}{T}t + \sigma\right) \qquad 2.3.1.39$$

The waves are assumed to have the same amplitude E_0. Here σ is the phase angle difference between the two waves arriving at P. The path difference Δ is related to the phase angle σ by the relationship

$$\frac{\sigma}{\Delta} = \frac{2\pi}{\lambda} \qquad 2.3.1.40$$

So that if $\Delta = \lambda$, $\sigma = 2\pi rad = 360^0$, if $\Delta = \frac{\lambda}{2}$, $\sigma = \pi rad = 180^0$, and so on. Then, by using the principle of superposition, we can add the two electric fields at point P to obtain $E_{Res} = E_1 + E_2$. (Carrying out this step involves some trigonometry, the details of which can be found in most optics texts.) Since the intensity of the light goes as the square of the electric field , we square and average the result over one cycle of wave oscillation at P, obtaining, finally, an expression for the average intensity, I_{AV} [18].

$$I_{AV} = E_0 cos^2 \frac{\sigma}{2} \qquad 2.3.1.41$$

Here σ,is the critical phase angle difference at point P. For all points P for which

σ = 0, 2π, 4 π,and so on, corresponding to Δ=0, λ, 2λ, etc., $cos^2 \frac{\sigma}{2} = 1$ and $I_{AV} = I_0$, the maximum possible "brightness." At these points, bright fringes form. For σ =0, 3π,5 π, and so on, corresponding to $\Delta = \frac{\lambda}{2}, \frac{3\lambda}{2}, \frac{5\lambda}{2}$,etc, $cos^2 \frac{\sigma}{2} = 0$ and dark fringes form. The maximum intensity I_0 is equal to $(E_0 + E_0)^2$ or $4E_0^2$, since each wave has amplitude E_0. Further, from equations (2.3.1.38) and (2.3.1.39), we see that

$$\sigma = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} a \, Sin\theta \qquad 2.3.1.42$$

So that the phase angle σ is connected clearly through the angle θ to different points P on the screen. Going one step further, replacing $Sin\theta$ by $\frac{y}{s}$ in equation (2.3.1.42), we have the connection between σ and any position on the screen, such that:

$$\sigma = \frac{2\pi a}{\lambda s} y \qquad 2.3.1.43$$

With equation (2.3.1.42) and $I_0 = 4E_0^2$ we can rewrite equation (2.3.1.41) in a form that relates I_{AV} directly to a position on the screen [11].

$$I_{AV} = 4E_0^2 \cos^2\left(\frac{\pi a}{\lambda s} y\right) \qquad 2.3.1.44$$

Where: I_{AV} = intensity of light along screen at position

E_0 = amplitude of light wave from S1 or S2

s = distance from the plane of the double slit to the screen

a = slit separation

λ = wavelength of monochromatic light

y = distance above (or below) central bright fringe on the screen

2. 3.2 Diffraction

The ability of light to bend around corners, a consequence of the wave nature of light, is fundamental to both interference and diffraction. Diffraction is simply any deviation from geometrical optics resulting from the obstruction of a wave front of light by some obstacle or some opening. Diffraction occurs when light waves pass through small openings, around obstacles, or by sharp edges. Several common diffraction patterns for He-Ne laser light are shown in figure 2.3.2.4 below [19].

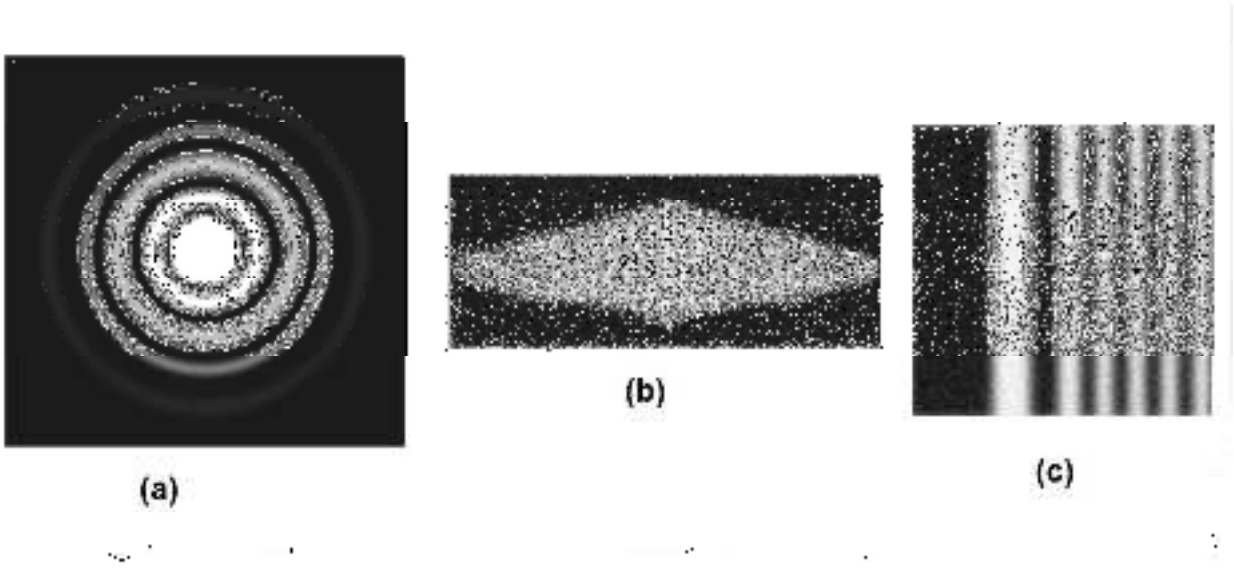


Figure 2.3.2.4 A typical diffraction pattern showing, light wave [19].

(a) Passing through a circular pinhole (b) Passing through a narrow (vertical) slit and (c) A typical pattern for diffraction by a sharp edge.

2.3.3 Diffraction Grating

An aperture with thousands of adjacent slits is called transmission diffraction grating. The width of a single slit the opening is given by d, and the distance between slit centers is given by l(see figure 2.3.2.5). For clarity, only a few of the thousands of slits normally present in a grating are shown. Note that the spreading of light occurs always in a direction perpendicular to the direction of the long edge of the slit opening that is, since the long edge of the slit opening is vertical in figure 2.3.2.5 the spreading is in the horizontal direction along the screen.

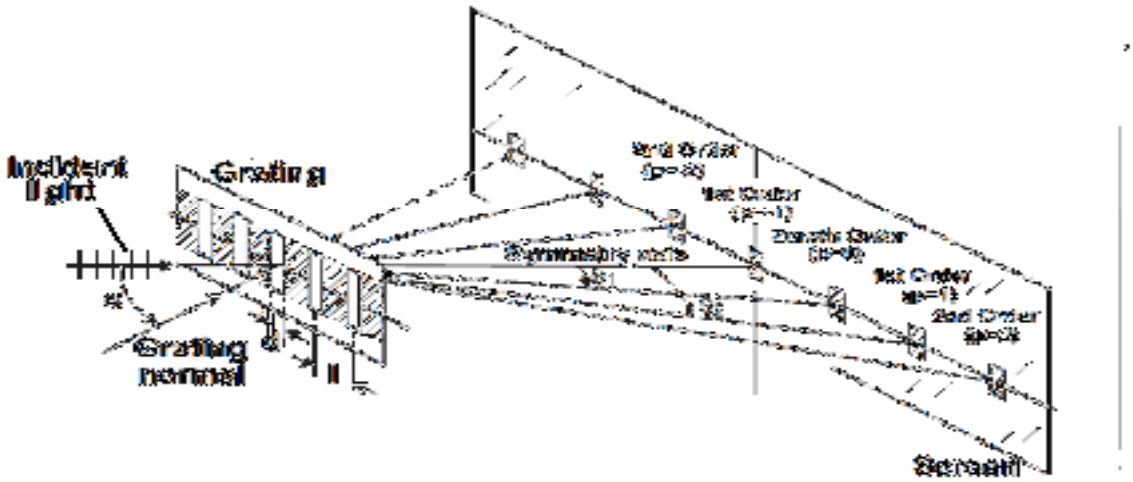


Figure 2.3.2.5 Diffraction of light through a grating [19].

The resulting diffraction pattern is a series of sharply defined, widely spaced fringes, as shown. The central fringe, on the symmetry axis, is called the zeroth-order fringe. The successive fringes on either side are called 1^{st}order, 2^{nd} order, etc., respectively. They are numbered according to their positions relative to the central fringe, as denoted by the letter p, the intensity pattern on the screen is a superposition of the diffraction effects from each slit as well as the interference effects of the light from all the adjacent slits. The combined effect is to cause overall cancellation of light over most of the screen with marked enhancement over only limited regions, as shown in

figure 2.3.2.5 the location of the bright fringes is given by the following expression, called the grating equation, assuming that Fraunhofer conditions hold [18].

$$l(\sin\alpha+\sin\theta_p) = p\lambda \qquad 2.3.3.45$$

Where p=0,$\pm1, \pm2$

l = distance between slit centers

$\boldsymbol{\alpha}$ = angle of incidence of light measured with respect to the normal to the grating surface

θ_p= angle locating the p^{th}-order fringe

p = an integer taking on values of 0, 1,2,.... , etc.

λ = wavelength of light

Note that, if the light is incident on the grating along the grating normal ($\boldsymbol{\alpha}$=0),

the grating equation, equation (2.3.1.44), reduces to the more common form shown in equation (2.3.3.45).

$$l(\sin\theta_p) = p\lambda \qquad 2.3.3.46$$

If, for example, you shine a He-Ne laser beam perpendicularly onto the surface

of a transmission grating, you will see a series of brilliant red dots, spread out as shown in figure.(2.3.2.5) A complete calculation would show that less light falls on each successively distant red dot or fringe, the p =0 or central fringe being always the brightest. Nevertheless, the location of each bright spot, or fringe, is given accurately by equation (2.3.3.46) for either normal incidence ($\boldsymbol{\alpha}$=0) or oblique incidence ($\boldsymbol{\alpha}$=0). If light containing a mixture of wavelengths (white light, for example) is directed onto the transmission grating, equation (2.3.3.46), holds for each component color or wavelength. So each color will be spread out on the screen according to equation (2.3.3.46), with the longer wavelengths (red) spreading out farther than the shorter wavelengths (blue). In any case, the central fringe (p = 0) always remains the same color as the incident beam, since all wavelengths in the p =0 fringe have θ_p, hence all overlap to re-form the "original" beam and therefore the original "color" [19].

2.3.4 Reflection and Refraction

Reflection and refraction are the most common phenomenon when light and matter inter- acts each other. Light intensity will be reflected or/and refracted because of a refractive index mismatch between the air and the sample boundary. The intensity of both reflected and refracted light doesn't only depend on the difference in refractive index mismatch but also depend on polarization, angle of incidence, wavelength of light and structure & shape of the surface reflection and refraction are strongly related to each other by Fresnel's law. Reflectance or .The fraction of reflected intensity R, for un polarized light is defined as the ratio of the reflected intensity from the medium to the incident intensity. This can be expressed as Fresnel's law [20].

$$R=\frac{1}{2}\left(\left[\frac{n_{1}\cos\theta_{i}-n_{2}\cos\theta_{t}}{n_{1}\cos\theta_{i}+n_{2}\cos\theta_{t}}\right]^{2}+\left[\frac{n_{2}\cos\theta_{i}-n_{1}\cos\theta_{t}}{n_{2}\cos\theta_{i}+n_{1}\cos\theta_{t}}\right]^{2}\right) \quad 2.3.4.47$$

where n_1 and n_2 are the refractive indices for the external medium (air) and material respectively. θ_i and θ_t are the incident and the transmitted angle respectively. using Snell's law of refraction.

$$n_1 sin\theta_{\ i} = n_2 sin\theta_t \quad 2.3.4.48$$

$$\theta_t = \sin^{-1}\left(\frac{n_1}{n_2}\sin\theta_i\right) \quad 2.3.4.49$$

When the light is incident in the direction normal to the medium surface, i.e. $\theta_i = \theta_i = 0°$, and if the external medium is air ($n_1 = 1$), and n_2 = n, equation (2.3.4.47) becomes [20].

$$R = \left(\frac{n-1}{n+1}\right)^{2} \quad 2.3.4.50$$

2.4 Moiré projection techniques and defelectometry

2.4.1 What is Moiré?

The term “Moiré” is not the name of a person; in fact, it is a French word referring to “an wavy finish usually produced on a fabric by pressing between engraved rollers”. In optics it refers to a beat pattern produced between two gratings of approximately equal spacing. It can be seen in everyday things such as the overlapping of two window screens, the rescreening of a half-tone picture, or with a striped shirt seen on television. The use of moiré for reduced sensitivity testing was introduced by Lord Rayleigh in 1874. Lord Rayleigh looked at the moiré between two identical gratings to determine their quality even though each individual grating could not be resolved under a microscope. Moiré patterns are extremely useful to help understand basic interferometer and interferometric test results. Figure 2.4.1.6 shows the moiré pattern (or beat pattern) produced by two identical straight-line gratings rotated by a small angle relative to each other. A dark fringe is produced where the dark lines are out of step one-half period, and a bright fringe is produced where the dark lines for one grating fall on top of the corresponding dark lines for the second grating. If the angle between the two gratings is increased, separation between the

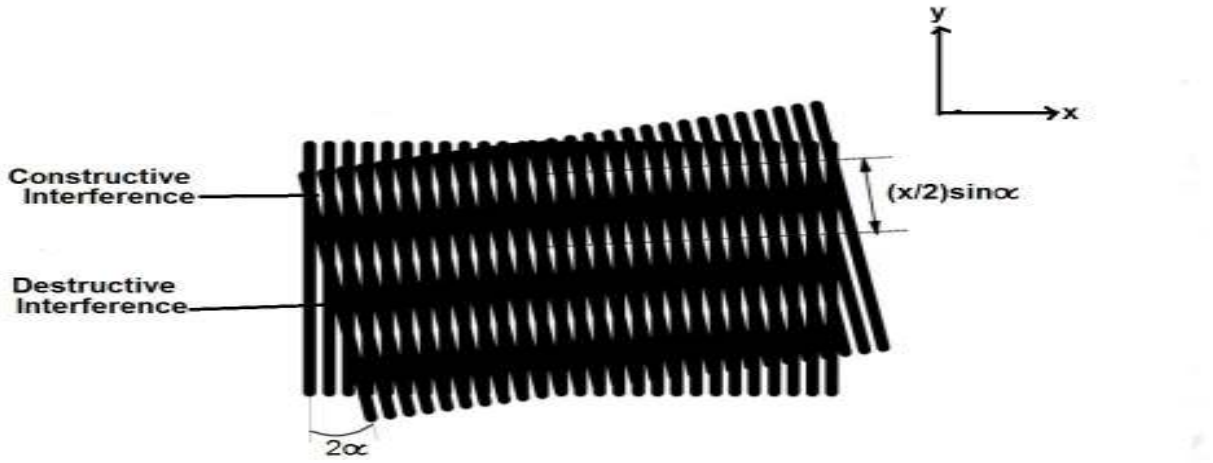


Figure2.4.1.6 Moiré between two straight-line gratings of the same pitch at an angle 2α with respect to one another.

If the gratings are not identical straight-line gratings, the moiré pattern (bright and dark fringes) will not be straight equi-spaced fringes. The following analysis shows how to calculate the moiré

pattern for arbitrary gratings. Let the intensity transmission function for two gratings f_1 (x , y) and f_2(x ,y) be given by

$$f_1(x, y) = a_1 + \sum_{n=1}^{\infty} b_{1n} \cos[n\Phi_1(x, y)] \quad 2.4.1.51$$

$$f_2(x, y) = a_2 + \sum_{n=1}^{\infty} b_{2m} \cos[m\Phi_2(x, y)] \quad 2.4.1.52$$

where Φ(x,y) is the function describing the basic shape of the grating lines. For the fundamental frequency, at the center of each bright line

$$\Phi(x, y) = 2\pi n \quad 2.4.1.53$$

and at the center of each dark line

$$\Phi(x, y) = 2\pi \left(n + \frac{1}{2}\right) \quad 2.4.1.54$$

The b coefficients determine the profile of the grating lines (i.e., square wave, triangular, sinusoidal, etc.). For a sinusoidal line profile b_{i1}, is the only non zero term. When these two gratings are superimposed, the resulting intensity transmission function is given by the product.

$$F_1(x, y)\, f_2(x, y) = a_1 a_2 + a_1 \sum_{m=1}^{\infty} b_{2m} cos[m\Phi_2(x, y) + a_2 \sum_{n=1}^{\infty} b_{1n} cos[n\Phi_2(x, y)]$$

$$+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b_{1n} b_{2m} cos[n\Phi_2(x, y)] \cos[m\Phi_2(x, y)] \quad 2.4.1.55$$

The first three terms of equation (2.4.1.55) provide information that can be determined by looking at the two patterns separately. The last term is the interesting one, and can be rewritten as:

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b_{1n} b_{2m} cos[n\Phi_1(x, y)] \cos[m\Phi_2(x, y)] = \frac{1}{2} b_{11} b_{21} \cos[\Phi_1(x, y) -$$

$$\Phi_2(x, y)] + \frac{1}{2} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} b_{1n} b_{2m} cos[n\Phi_1(x, y)] - m\Phi_2(x, y)]; \; n = m \neq \quad 2.4.1.56$$

This expression shows that by superimposing the two gratings, the sum and difference between the two gratings is obtained. The first term of equation (2.4.1.56) represents the difference between the fundamental pattern masking up the two gratings. It can be used to predict the moiré pattern shown in figure 2.4.6. Assuming that two gratings are oriented with an angle 2α between them with the y-axis of the coordinate system bisecting this angle, the two grating functions, Φ_1(x, y) and Φ_2 (x, y) can be written as:

$$\left.\begin{aligned}\Phi_1(x,y) &= \frac{2\pi}{\lambda_1}(xcos\alpha + ysin\alpha)\\ \Phi_2(x,y) &= \frac{2\pi}{\lambda_2}(xcos\alpha + ysin\alpha)\end{aligned}\right\} \quad 2.4.1.57$$

where λ_1 and λ_2 are the line spacing's of the two gratings and equation (2.4.1.57) can be rewritten as

$$\Phi_1(x,y) - \Phi_1(x,y) = \frac{2\pi}{\lambda_{beat}}x\cos\alpha + \frac{4\pi}{\lambda}y\sin\alpha \quad 2.4.1.58$$

Where $\langle\lambda\rangle$ is the average line spacing, λ_{beat} and is the beat wavelength between the two gratings given by

$$\lambda_{beat} = \frac{\lambda_1\lambda_2}{\lambda_2-\lambda_1} \quad 2.4.1.59$$

Using equation (2.4.1.56), the moiré or beat will be lines whose centers satisfy the equation

$$\Phi_1(x,y) - \Phi_2(x,y) = M2\pi \quad 2.4.1.60$$

Three separate cases for moiré fringes can be considered. When, $\lambda_1 = \lambda_2 = \lambda_1 = \lambda$

the first term of equation (2.4.1.56) is zero, and the fringe centers are given by

$$m\lambda_1 = 2y\sin\alpha \quad 2.4.1.61$$

Where m is an integer corresponding to the fringe order. So, equation(2.4.1.6) is the equation of equi-spaced horizontal lines as seen in figure 2.4.1.6The other simple case occurs when the gratings are parallel to each other with α=0.This makes the second term of equation (2.4.1.60) vanish. The moiré will then be lines that satisfy

$$m\lambda_{1beat} = X \quad 2.4.1.62$$

These fringes are equally spaced, vertical lines parallel to the y axis. For the more general case where the two gratings have different line spacing's and the angle between the gratings is nonzero, the equation for the moiré fringes will now be

$$m\lambda_1 = x\cos\alpha + 2y\sin \quad 2.4.1.63$$

This is the equation of straight lines whose spacing and orientation is dependent on the relative difference between the two grating spacings and the angle between the gratings. Figure 2.4.67 shows moiré patterns for these three cases.

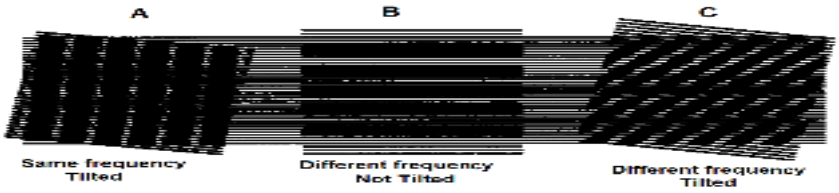


Figure2.4.1.7 Moiré patterns caused by two straight-line gratings with;

(A) The same pitch tilted with respect to one another,

(B) Different frequencies and no tilt, and

(C) Different frequencies tilted with respect to one another

The orientation and spacing of the moiré fringes for the general case can be determined from the geometry shown in figure 2.4.1.7. The distance AB can be written in terms of the two grating spacing's:

$$\overline{AB} = \frac{\lambda_1}{\sin(\theta-\alpha)} = \frac{\lambda_2}{\sin(\theta+\alpha)} \qquad 2.4.1.64$$

Where θ is the angle the moiré fringes make with the y axis. After rearranging, the fringe orientation angle θ is given by:

$$\text{Tan } \theta = \tan \alpha \left(\frac{\lambda_1+\lambda_2}{\lambda_1-\lambda_2}\right) \qquad 2.4.1.65$$

When α=0 and $\lambda_1 \neq \lambda_2$, 90^0 and when$\lambda_1=\lambda_2$ with α≠ 0,θ=90^o as expected. The fringe spacing perpendicular to the fringe lines can be found by equating quantities for the distance DE;

$$\overline{DE} = \frac{\lambda_1}{sin2\alpha} = \frac{\lambda_1}{\sin(\theta-\alpha)} \qquad 2.4.1.66$$

Where the fringe spacing or contour can be rearranged to yield C

$$C = \Lambda_1[\frac{\sin(\theta+\alpha)}{sin2\alpha}] \qquad 2.4.1.67$$

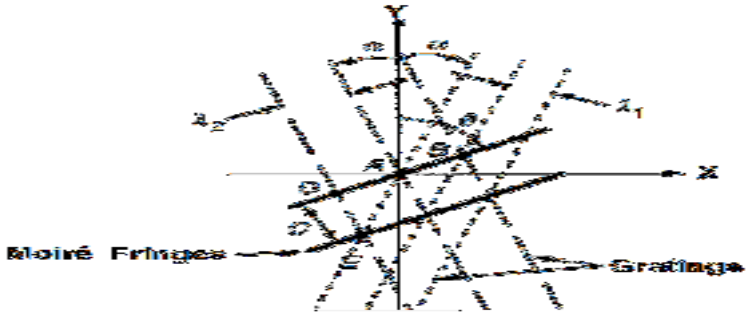


Figure2.4.1.8 Geometry used to determine spacing and angle of moiré fringes between two gratings of different frequencies tilted with respect to one another.

By substituting for the fringe orientationθ, the fringe spacing can be found in terms of the grating spacing's and angle between the gratings;

$$C = \frac{\lambda_{1\lambda_2}}{\sqrt{\lambda_{22}\sin^2 2\alpha+(\lambda_2\cos 2\alpha-\lambda_1)^2}} \qquad 2.4.1.68$$

In the limit that α=0, $\lambda_1 \neq \lambda_2$ and the fringe spacing equals λ_{beat}, and in the limit that $\lambda_1 = \lambda_2$=λ and $\alpha \neq 0$, the fringe spacing equals $\frac{\lambda}{2sin\alpha}$. It is possible to determine $\lambda_2$and α from the measured fringe spacing and orientation as long as λ_2 is known [11].

2.4.2 Shadow moiré

A simple method of moiré interferometry for contouring objects uses a single grating placed in front of the object as shown in Fig. 2.4.1.9. The grating in front of the object produces a shadow on the object that is viewed from a different direction through the grating. A low-frequency beat or moiré pattern is seen.

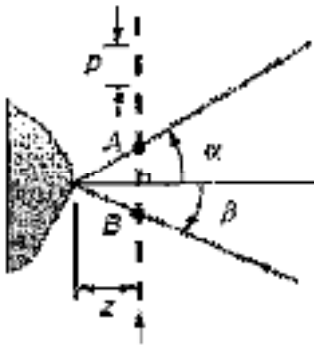


Figure 2.4.2.9 Geometry for shadow moiré with illumination and viewing at infinity, i.e., parallel illumination and viewing. Illuminate View Grating & Reference Plane

This pattern is due to the interference between the grating shadows on the object and the grating as viewed. Assuming that the illumination is collimated and that the object is viewed at infinity or through a telecentric optical system, the height z between the grating and the object point can be determined from the geometry shown in Fig. 2.4.2.9. This height is given by

$$Z=\frac{N_P}{tan\alpha+tan\beta} \qquad 2.4.2.69$$

where α is the illumination angle, β is the viewing angle, p is the spacing of the grating lines, and N is the number of grating lines between the points A and B see figure (2.4.2.9).

The contour interval in a direction perpendicular to the grating will simply given by

$$C=\frac{P}{tan\alpha+tan\beta} \qquad 2.4.2.70$$

Again, the distance between the moiré fringes in the beat pattern depends on the angle between the illumination and viewing directions. The larger the angle; the smaller the contour interval. If the high frequencies due to the original grating are filtered out; then only the moiré interference term is seen. The reference plane will be parallel to the grating. Note that this reference plane is tilted with respect to the reference plane obtained when fringes are projected onto the subject. Essentially, the shadow moiré technique provides a way of removing the tilt" term and repositioning the reference plane. Figure 2.4.2.10 shows a geometry where the distance between the illumination source and the viewing camera is given by w, and the distance between these and the grating is 1. The grating is assumed to be close enough to the object surface so that diffraction effects are negligible [11].

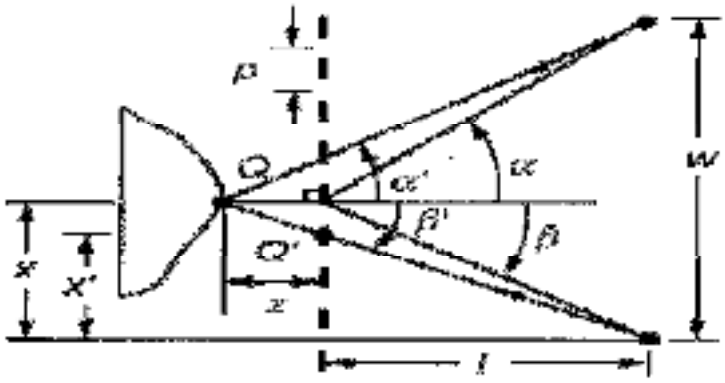


Figure 2.4.2.10 Geometry for shadow moiré with illumination and viewing at finite distances. The height between the object and the grating is given by

$$Z = \frac{N_P}{tan\alpha' + tan\beta'} \qquad 2.4.2.71$$

where $\alpha'$and $\beta'$are the illumination and viewing angles at the object surface. These angles change for every point on the surface and are different from and in figure (2.4.2.10). Where and are the illumination and viewing angles at the grating (reference) surface. The surface height can also be written as

$$Z = NC\ (Z) = \frac{N_{P(l+z)}}{W} = \frac{N_P l}{W - N_P} \qquad 2.4.2.72$$

This equation indicates that the height is a complex function depending on the position of each object point. Thus, the distance between contour intervals is dependent on the height of the surface and the number of fringes between the grating and the object. Individual contour lines will no longer be planes of equal height. They are now surfaces of equal height. The expression for height can be simplified by considering the case where the distance to the source and viewer is large compared to the surface height variations, $1 \gg z$. Then the surface height can be expressed as

$$Z = \frac{N_P l}{W} = \frac{N_P}{tan\alpha + tan\beta} \qquad 2.4.2.73$$

Even though the angles and vary from point-to-point on the surface, the sum of their tangents remain equal to w/l for all object points as long as $1 \gg z$. Because of the finite distances, there is also distortion due to the viewing perspective. A point on the surface Q will appear to be at the location Q' when viewed through the grating. By similar triangles, the distances x and x' from a

line perpendicular to the grating intersecting the camera location can be related using where x and x' are defined in Fig. 2.4.2.10 Equation (2.4.2.68) can be rearranged to yield the actual coordinate x in terms of the measured coordinate x' and the measurement geometry,

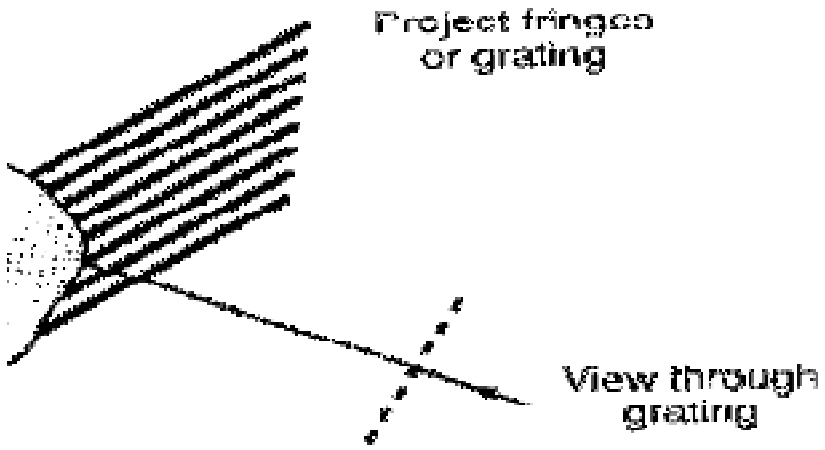


Figure 2.4.2.11 Projection moiré where fringes or a grating are projected onto a surface and viewed through a second grating.

$$x = x' \left(1 + \frac{z}{l}\right) \qquad 2.4.2.74$$

Likewise, the y coordinate can be corrected using

$$Y = y' \left(1 + \frac{z}{l}\right) \qquad 2.4.2.75$$

This enables the measured surface to be mapped to the actual surface to correct for the viewing perspective. These same correction factors can be applied to fringe projection [11].

2.4.3 Fringe Projection

A simple approach for contouring is to project interference fringes or a grating onto an object and then view from another direction.

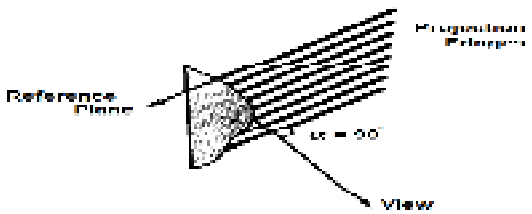


Figure 2.4.3.12 Maximum sensitivity for fringe projection with a 90° angle between projection and viewing.

Figure 2.4.3.12. Projection of fringes or grating onto objects and viewed at an angle α. p is the grating pitch or fringe spacing and C is the contour interval. Set up for this measurement. Assuming a collimated illumination beam and viewing the fringes with a telecentric optical system, straight equally-spaced fringes are incident on the object producing equally-spaced contour intervals. The departure of a viewed fringe from a straight line shows the departure of the surface from a plane reference surface. When the fringes are viewed at an angle α relative to the projection direction, the spacing of the lines perpendicular to the viewing direction will be

$$D = \frac{p}{cos\alpha} \qquad 2.4.3.76$$

The contour interval C (the height between adjacent contour lines in the viewing direction) is determined by the line or fringe spacing projected onto the surface and the angle between the projection and viewing directions;

$$C = \frac{p}{sin\alpha} = \frac{d}{tan\alpha} \qquad 2.4.3.77$$

These contour lines are planes of equal height and the sensitivity of the measurement is determined by α.The larger the angle α, the smaller the contour interval. If α=90^0, then th contour interval is equal to p, and the sensitivity is a maximum. The reference plane will be parallel to the direction of the fringes and perpendicular to the viewing direction as shown in .Figure 2.4.3.12. Even though the maximum sensitivity can be obtained, at 90^0 angle between the projection and viewing directions will produce a lot of unacceptable shadows on the object. These shadows will lead to areas with missing data where the object cannot be contoured. When α = 0, the contour interval is infinite, and the measurement sensitivity is zero. To provide the best results, an angle no larger than the largest slope on the surface should be chosen. When interference fringes are projected onto a surface rather than using a grating, the fringe spacing p is determined by the geometry shown in Figure 2.4.3.12 and is given by

$$P = \frac{\lambda}{2sin\Delta\theta} \qquad 2.4.3.78$$

Figure 2.4.3.13 Fringes produced by two interfering beams

where λ is the wavelength of illumination and $2\Delta\theta$is the angle between the two interfering beams. Substituting the expression for p into Eq. (2.4.3.78), the contour interval becomes

$$C = \frac{\lambda}{2(sin\Delta\theta)sin\alpha} \qquad 2.4.3.79$$

If a simple interferometer such as a Twyman–Green is used to generate projected interference fringes, tilting one beam with respect to the other will change the contour interval. If the source and the viewer are not at infinity, the fringes or grating projected onto the object will not be composed of straight, equally-spaced lines. The height between contour planes will be a function of the distance from the source and viewer to the object. There will be a distortion due to the viewing of the fringes as well as due to the illumination. This means that the reference surface will not be a plane. As long as the object does not have large height changes compared to the illumination and viewing distances, a plane reference surface placed in the plane of the object can be measured first and then subtracted from subsequent measurements of the object. This enables the mapping of a plane in object space to a surface which will serve as a reference surface. If the object has large height variations, the plane reference surface may have to be measured in a number of planes to map the measured object contours to real heights. Finite illumination and viewing distances will be considered in more detail with shadow moire´ [11]

2. 4.4 Talbot effect

By illuminating a grating or other periodic structure with a spatially coherent light beam, exact images and many other images of the structure can be found at finite distances from the grating. This self-imaging phenomenon is called the Talbot effect. The regular distance between the grating and the imaging plane is called the Talbot length, $Z_{k=1} = \frac{2d_2}{\lambda}$, where d and λ are the period of the grating and the wavelength of the incident light, respectively. The effect has found interesting applications in image processing and synthesis, photolithography, optical testing, optical metrology, and spectrometry [21].

2. 4.5 Moiré Defelectometry

A moiré defelectometry consists of two gratings, in which the second grating is installed at the plane of one of the self-images of the first grating and the object under inspection is placed in front the first grating. The moiré defelectometry measures ray deflections in the paraxial approximation induced by the phase object. The resulting moiré fringe pattern is a map of ray deflections corresponding to the optical properties of the inspected object. Generally, when the image-forming light propagates in a perturbed medium the self-image is distorted and the distortion is magnified by the moiré pattern. A schematic set-up of a moiré defelectometry is shown in Figure 2.4.5.14

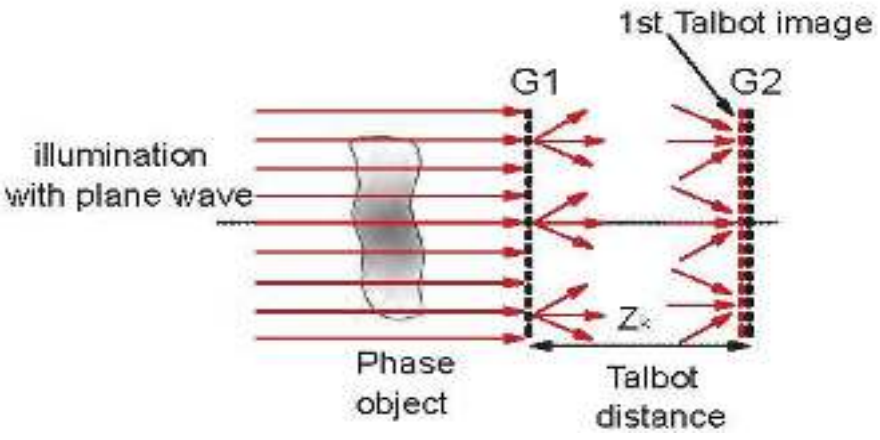


Figure2.4.5.14 Schematic set-up of a moiré defelectometry for the study of phase objects [21].

2.5 diffusion coefficient and moiré' fringe shift

The Moiré Defelectometry experimental set-up Used to measure the diffusion coefficient of transparent solution. The deflected laser beam due to changes in refractive index in the diffusion cell yields the shifted self-image on gratings and resultant Moiré' fringes show a deviation. For an isothermal mixture, without generation and recombination, the physical problem associated with the experiment described above can be formulated mathematically. A diffusion column of constant cross-section, extended in the x-direction, and the concentration varying with the position coordinate x only [22, 23]. Thus, the diffusion process can be formulated in terms of one-dimensional problem. The free diffusion process is governed by Fick's second law, and for one dimensional diffusion along x-axis it is given by

$$\frac{\partial C(X,t)}{\partial t} = D\frac{\partial 2}{\partial x_2}C\ (X,\ t) \qquad 2.5.80$$

Where C (x, t) is the concentration at position x at time t, D is the diffusion coefficient, assumed to be independent of concentration, and x is the direction of diffusion. If there is the net external force, F (e.g. gravitational force), acting in the x-direction up on the dissolved molecules and if μ be the mobility of the molecules under consideration (i.e. the steady velocity, acquired under the action of unit force) the steady velocity of these molecules would be μ. F and the resulting flux of matter would be μ. FC (x, t). Consequently the rate of concentration changes would then become

$$\frac{\partial c}{\partial t} = D\frac{\partial^2}{\partial x^2}C(X,t)\ -\mu F\frac{\partial C(X,t)}{\partial x} \qquad 2.5.81$$

It is even possible to reduce the problem of equation 2.5.79 to that of ordinary diffusion equation 2.5.80 by the means of following transformation

$$C(x,\ t) = c^*(x,\ t)\ \exp\left[\frac{-\mu Fx}{2D}\ -\frac{\mu^2F^2t}{4D}\right] \qquad 2.5.82$$

Where c^* is the function that satisfies equation 2.5.82, If the steady velocity of the dissolved molecules (μ, F) is very slow the current due to the external forces can be neglected and equation 2.5.82 reduce to equuation2.5.82.The solution of equation 2.5.82 for two binary liquid mixtures initially (t=0) separated at (x=0) with concentration C_o and is C_1an error function given by

$$C(x,t) = \frac{C_o - C_1}{2}\ + \frac{C_o - C_1}{2}\,\mathrm{erf}\left[\frac{X}{2\sqrt{Dt}}\right] \qquad 2.5.83$$

Where the error function is defined as

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2}\, dt \qquad 2.5.84$$

On the other hand, if we consider the solution of equation2.5.83 in equation2.5.84 is replaced by x-μgt. In this case the influence of gravity causes a change in the central diffusion location, that is another layer as the second center of diffusion will formed [3, 24].In a transparent mixture, a change in the concentration produces a change of refractive index, which for the small concentration gradient can be considered a linear function between refractive index and concentration. The refractive index gradient is given by

$$\frac{\partial n(x,t)}{\partial x} = \frac{n_o - n_1}{2\sqrt{\pi D t}} \exp\left[\frac{-X^2}{4Dt}\right] \qquad 2.5.85$$

Where n(x, t) is the local index at time t in the diffusion cell,n_0 the refractive index of high concentration solution and n_1 that of low concentration solution. If the refractive index changes the rays contributing to the formation of the Moire' fringes pattern will be deflected locally according to refractive index distribution, accordingly the Moire' fringes will also change. The incident parallel laser beam passing through the diffusion cell, bends by angle $\Psi(x, t)$ depending on the distribution of the refractive index on the cell. Because the refractive index (concentration) depends on position (X) and time (t),accordingly, the deflection angle also would be position and time dependent. Due to the deflection of the collimated beam, imaging on the defelectometry system, the Fourier image of the first grating shifts on the second grating [25]. For small bending of light rays, the amount of Fourier image shift would be Ψ (x, t) Zt. The displacement of the Fourier image of the first grating on the second one, the Moire' fringe to shift with respect to the original locations. The amount of fringe shift $\Delta\xi(x, t)$ is given by

$$\Delta\xi(x, t) = \frac{Z(t)}{\theta} \Psi(x, t) \qquad 2.5.86$$

Where θ a small tilt angle between the intersection of the grating lines and Z_t is The tth talbot distance between gratings. The desired information from diffusion process is contained in $\Psi(x, t)$ .On the other hand, the relation between the integrated deflection angle and the refractive index n(x,t) is

$$\Delta\Psi(x, t) = \frac{1}{n_a} \int_0^l \left[\frac{\partial n(x,y,z,t)}{\partial x}\right]_{y=cons} dz \qquad 2.5.87$$

Where n_a is the ambient refractive index along the x-axis diffusion, the direction and L represents the thickness of diffusion cell measured along the propagation axis(z axis). The diffusion process is assumed to occur only along the x-direction. Thus, equation 2.5.87 becomes

$$\bigcup(x, t) = \frac{1}{n_a}\frac{\partial n(x,t)}{\partial x} \quad 2.5.88$$

Taking into account Equation 2.5.81 and Equation 2.5.84 can be written as

$$\Delta\xi(x, t) = \frac{Z_t}{n_a\theta}\frac{\partial n(x,t)}{\partial x} \quad 2.5.89$$

.Equation.(2.5.89) shows the Moiré Fringe deformation is proportional to the refractive index gradient introduced by the diffusion process [24,25]. By replacing Equation 2.5.86 and Equation2.4.5.85 the corresponding local shift of moiré Fringes Results

$$\Delta\xi(x, t) = \frac{Z_t d(n_o - n_1)}{n_a\theta 2\sqrt{\pi Dt}}\exp[\frac{-X^2}{4Dt}] \quad 2.5.90$$

Where for simplicity,Δ_0 defined as

$$\Delta_0 = \frac{Z_t d(n_o - n_1)}{n_a\theta 2\sqrt{\pi Dt}} \quad 2.5.91$$

$$\Delta\xi(x, t) = \frac{\Delta_0}{\sqrt{t}}\exp[\frac{-X^2}{4Dt}] \quad 2.5.92$$

The diffusion coefficient can be obtained from a single Moiré' pattern recorded at time instant. The local shift of Moiré' fringes would be:

$$\Delta_1 = \frac{Z_t L(n_o - n_1)}{n_a\theta 2\sqrt{\pi Dt_o}}\exp[\frac{-X_1^2}{4Dt_o}] \quad 2.5.93$$

$$\Delta_2 = \frac{Z_t L(n_o - n_1)}{n_a\theta 2\sqrt{\pi Dt_o}}\exp[\frac{-X_2^2}{4Dt_o}] \quad 2.5.94$$

The ratio of two local shifts is

$$\eta = \frac{\Delta_1}{\Delta_2} = \exp[\frac{-X_1^2 - X_2^2}{4Dt_o}] \quad 2.5.95$$

Then the diffusion coefficient can be defined as

$$D = [\frac{x_2^2 - X_1^2}{4t_o \ln \eta}] \quad 2.5.96$$

3.0 MATERIALS AND METHODS

In this chapter, the materials and methods used in this work are presented. The first section of this chapter describes the samples used to carry out this research. In the second section of this chapter the methods applied to determine the diffusion coefficient of lemon juice have been reported.

3.1 Materials (apparatus) of the experiment

Materials used in this experiment are carried out as follows; a rectangular beaker of area 1.4cm by1.6cm used to carry the sample, lemon juice used to prepare the sample, syringe used to measure the sample, meter ruler used to measure the horizontal fringe shift from the vertical axis and pencil used to point position of the fringe on the screen were provided. The samples were collected from respective area. The laboratory apparatus and instruments used for this work are the following; He-Ne laser to produce monochromatic light, two ranchi ratings to produce moiré fringes, two biconvex lens, opal screen to observe fringes and digital camera to capture fringes on opal screen.

3.2 Methods

3.2.1 Lemon - juice preparation

The samples were produced from local vegetable market, and fresh lemon where brought from it. The lemon were cleaned before it is used for experiment and allowed to attain in a room temperature. The lemon solutions were distillated by using distillation paper and the samples were prepared by volume-volume ratio of water and lemon juice.

3.2.2 Optical measurement procedures

The experimental work was focused on the measurement of diffusion coefficient of water- lemon juice concentration by using moiré defelectometry. In determining the diffusion coefficient of water-lemon juice concentration we took temperature of the room at 27℃ and remain a constant throughout the experiment and the experimental set up for this work is shown in figure 3.15 below. The monochromatic light of wave length 632nm from35mW He-Ne laser released to pass through two biconcave lenses of focal length 15cm and 5cm respectively to expand the light from the laser. Then the light that passes through the sample and travels through first grating and by second grating the light were formed a moiré on the screen. There by a concave lens of focal

length 15 cm moiré fringes formed on the screen made larger. The relative position of adjacent fringes from vertical axis were measured manually. Then the photographs of the fringes on the semi-transparent screen were captured at the back of the screen by using digital camera. The algebraic calculations are used to show the relationship between various quantities. The relative position of adjacent fringe from vertical axis and the time taken to carry out the experiment were used to measure diffusion coefficient of water-lemon juice concentration. The concentration of water-lemon juice was varied from 0.4% to 2% in volume ratio for which the volume of water was kept at a constant.

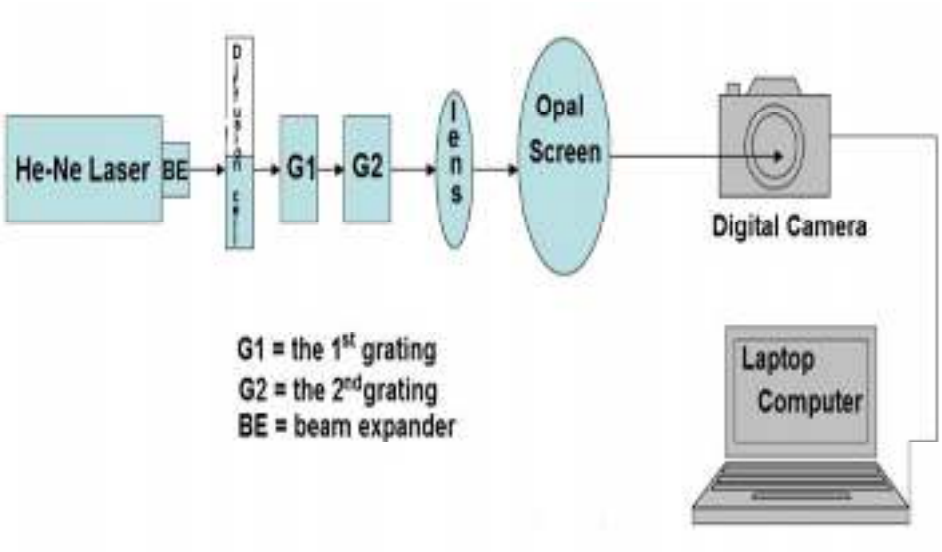
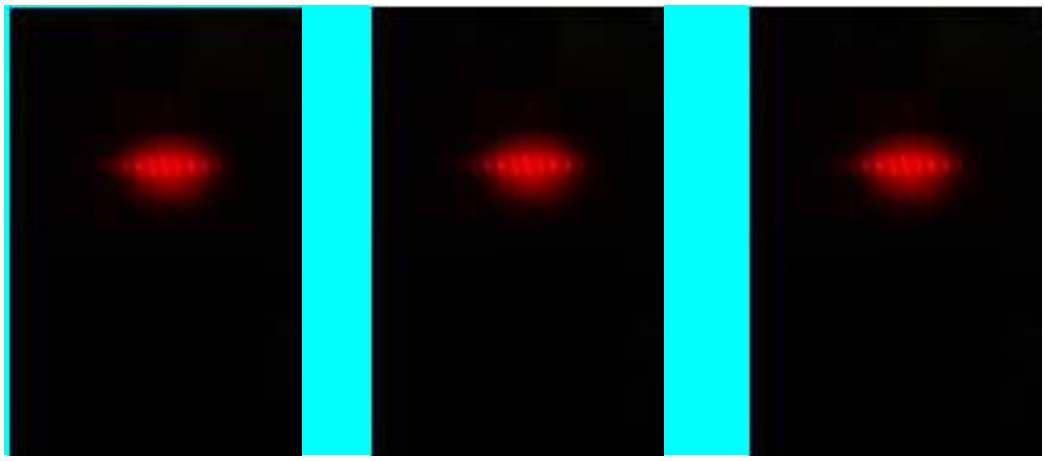


Figure 3.15 Schematic diagram of the expermental set-up

4.0 RESULTS AND DISCUSSION

In this chapter the results from the experiment were presented. The analysis was formed by Fick's law of diffusion. According to equation 2.5.96 a number of determinations of diffusion coefficient can be made from a single moiré pattern recorded at a desired time t_0 .The following photograph were taken by digital camera at different time. Figure 4.16 (a), (b) and (c) shows the shift in fringes for 0.4% concentration of lemon juice at a time of 6hr,18hr and 24hr respectively.



(a) (b) (c)

Figure 4.16 (a), (b) and (c) the shift in fringes for 0.4% concentration of lemon juice at a time of 6hr,18hr and 24hr respectively

As it has been presented in figure 4.16 the moiré fringe shift along vertical direction as compared to along the horizontal direction is very small. Since the major variation of moiré fringe shift is along the horizontal direction emphasis has been given to the variation of moiré fringe shift along the horizontal direction for our discussion. However, the variation of moiré fringe shift along the vertical direction is very small; its variation hasn't been ignored. Instead, the average value of the moiré fringe shift along the horizontal direction has been considered throughout our discussion. Thus, throughout this thesis the discussion is based on the average value of diffusion coefficient obtained based on moiré fringe shift along the horizontal direction. Figure 4.16: the shift in fringes for 0.4% concentration of lemon juice the relative position of adjacent fringe from vertical axis was measured and the raw data of measurements were presented in table 4.16.

Table 4.1: Fringe shift for a concentration of 0.4% lemon-juice solution

Time	t_o=6hr	t_1=18hr	t_2=24hr
Shift	x_o=0.34cm	x_1=1.32cm	x_2=1.42hr

The diffusion coefficient due to moiré fringe shift has been solved by using Fick's law of diffusion. And the solution taken from the table above has been calculated accordingly:

The change in horizontal fringe shift is:

$$\Delta_1 = x_1\text{-}x_0 = 1.2\text{cm-}0.32\text{cm} = 1.98\text{cm}$$

$$\Delta_2 = x_2\text{-}x_0 = 1.4\text{cm-}0.32\text{cm} = 1.08\text{cm}$$

Then the ratio of the horizontal fringe shift is:

$$\text{Ŋ} = \frac{\Delta_1}{\Delta_2} = \frac{1.88cm}{1.08cm} = 0.92$$

Then by using equation 2.5.96 the diffusion coefficient of 0.4% the water-lemon juice concentration is :

$$\text{D} = [\frac{x_2^2 - X_1^2}{4t_o ln\text{ŋ}}]$$

$$\text{D} = \frac{(1.42CM)^2 - (0.34CM)^2}{4X6x60x60ln0.82}$$

$$\text{D} = 2.45 \text{ x} 10^{-8} \frac{m^2}{s}$$

The simulation result for concentration of 0.6%, 1% and 2% lemon juice is described below. The results have been presented in the subsequent sections in the form of photographs and numerical solutions. The photograph of moiré fringe shift has been captured from the back side of the screen by keeping the digital camera at affixed point. Similarly the photograph of moiré fringe shift taken for a concentration 0.6% of 1%and2% lemon juice are presented in figure 4.17 (d), (e), (f), (g), (h) (i), (j), (k) and (l) respectively.

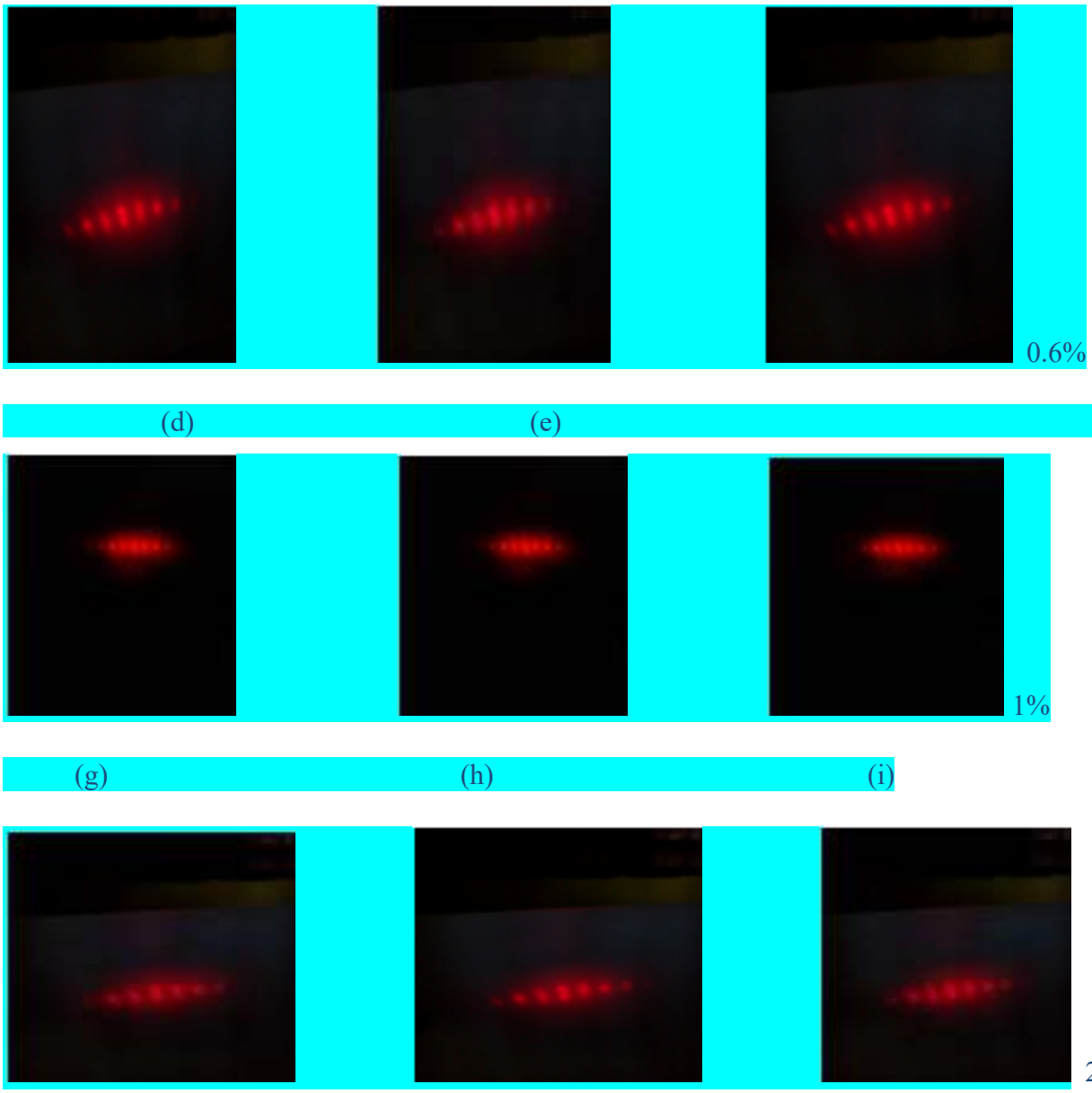


Figure4.17: Fringe shift taken for a concentration of 0.6%, 1% and 2% lemon juice

To validate the result of the moiré fringe shift, horizontal moiré fringe shift has been measured manually. Then diffusion coefficient due to moiré fringe has been solved by using Fick's law of diffusion. The respective calculations of Δ, η and D of fringe shift shown in figure above are obtained by following the process under table 4.16 above and the summary of these were given in table 4.17 below.

Table 4.2: Fringe shift and diffusion for a concentration of 0.6%, 1% and 2% lemon-juice solution.

Concentration	Time	Shift	Δ_1	Δ_2	η	D
0.6% Of Lemonjuice solution	t_0=9hr	x_0=0.2cm	2.2cm	2.4cm	0.92	6.31 x $10^{-8}\frac{m^2}{s}$
	t_1=22hr	x_1=2.4cm				
	t_2=28hr	x_2=2.6cm				
1% of lemon juice solution	t_0=7hr	x_0=0.2cm	1.8cm	2cm	0.9	4.52 x $10^{-8}\frac{m^2}{s}$
	t_1=24hr	x_1=2cm				
	t_2=28hr	x_2=2.2cm				
2%of lemon juice solution	t_0=8hr	x_0=0.41cm	1.8cm	2cm	0.91	4.61 x $10^{-8}\frac{m^2}{s}$
	t_1=48hr	x_1=2.2cm				
	t_2=63hr	x_2=2.4cm				

From the raw data values in the table 4.1and 4.2, we see that; Taking temperature of the room at 27℃ and by using the values obtained for the horizontal fringe shift from vertical axis; the magnitude of the diffusion coefficient of different concentration of water-lemon juice was computed based on Fick's law of diffusion .The result revealed that the diffusion coefficient of water- lemon juice concentration ranges from 2.45 x $10^{-8}\frac{m^2}{s}$ to 6.31 x $10^{-8}\frac{m^2}{s}$.From this the average value of diffusion coefficient for different concentration of water-lemon concentration is to 4.48 x $10^{-8}\frac{m^2}{s}$. Thus, the discussion is based on the average value of diffusion coefficient. And whether the result is valid or not the results should be checked by other experimental works. Based on this, the results that have been obtained in this study are compared with other results obtained from literature [31] .In validating the result the error analysis technique has been employed. Thus the error calculated is 1.19%

5.0 CONCLUSION, RECOMMENDATION AND FUTURE PLAN

5.1 Conclusion

Moiré defelectometry technique method is useful for determination of the diffusion coefficient of transparent liquids. Taking temperature of the room at 27℃ and by using the values obtained for the horizontal fringe shift from vertical axis; the magnitude of the diffusion coefficient of different concentration of lemon juice was computed based on Fick's law of diffusion. The methods for determination of diffusion coefficient of lemon juice have been developed in this experimental research. The result revealed that the diffusion coefficient of lemon juice concentration ranges from 2.45 x $10^{-8}\frac{m^2}{s}$ to 6.31 x $10^{-8}\frac{m^2}{s}$.The average value of the diffusion coefficient of lemon juice concentration due to moiré fringe shift is 4.48 x.$10^{-8}\frac{m^2}{s}$. The result has a good agreement with that stated in the literature review with average error of 1.19% [1].

5.2 Recommendation and future plan

The laser-moiré defelectometry for determination of diffusion coefficient of transparent liquid is simple, cheap and valid. Thus laser-moiré defelectometry is recommended for determination of diffusion coefficient of transparent fluids. Considering all the above explanations the following points are recommended as a future work.

1. The model can be extended for other transparent medium
2. One can use US-SCOAN IT-Software to measure the horizontal moiré fringe shift.

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