

Fuzzy Sliding Mode Controller for Chattering Reduction on Position Control of 2-DOF Helicopter



Lemma Getachew Zenebe

A Thesis Submitted to the department of Electrical Power and Control
Engineering

School of Electrical Engineering and Computing

Presented in Partial Fulfillment of the Requirement for the Degree of Master's
in Electrical Power and Control Engineering (Control Engineering)

Office of Graduate Studies
Adama Science and Technology University

March, 2024

Adama, Ethiopia

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DECLARATION

I hereby declare that this Master Thesis entitled “**Fuzzy Sliding Mode controller for Chattering reduction on position control of 2-DOF Helicopter**” is my own work and has not been submitted to any university for similar purpose. The references used in this work are duly recognized by proper citations.

Lemma Getachew Zenebe

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ACRONYMS

Abbreviations	Description
DC	Direct Current
DOF	Degree of Freedom
DSC	Dynamic Surface Control
IAE	Integral Absolute Error
LQR	Linear Quadratic Regulator
MIMO	Multi Input Multi output
PID	Proportional Integral Derivatives
PSO	Particle Swarm Optimization
SISO	Single Input Single Output
SMC	Sliding Mode Controller
STA	Super Twisting Algorithm
TRMS	Twin Rotor MIMO System
UAV	Unmanned Aerial vehicles

ABSTRACT

A two-degree-of-freedom helicopter typically refers to a helicopter with two rotational degrees of freedom, usually pitch and yaw. 2-DOF helicopter model is a highly non-linear Twin Rotor Multi-Input Multi-Output System (TRMS). It consists of two rotors, vertical and horizontal, whose positions need to be controlled with the help of suitable controller. When creating control rules, nonlinearities, and external disturbances that affect helicopter control should be taken into account. In the case of two-degree-of-freedom helicopter models, chattering can arise due to the simplified control system and limitations in control authority. The control inputs may switch rapidly between different values, resulting in undesired oscillations. Mitigating chattering in control systems is an active area of research. This thesis presents a fuzzy sliding mode control-based control approach for yaw and pitch position control of a 2-DOF helicopter by removing the chattering. The method proposed involves a dynamic modeling for the 2-DOF helicopter to determine the position of the 2-DOF helicopter tracks the given trajectory by removing the disadvantage of sliding mode controller that is chattering. Sinusoidal trajectory was given for the system as a reference input so that it will track it by the provision of proper controller design parameters. The basic parameters of the controllers are tuned with help of a global metaheuristic optimization algorithm called PSO method. As simulation result shows in sinusoidal trajectory the steady state error is 0.402037. In general, the proposed algorithm control will function well, according to the simulation findings, simulation results will be presented to show how the suggested controller may significantly reduce the chattering effect and external disruption while maintaining accurate trajectory tracking in the Two-DOF helicopter system.

Keywords: TRMS; sliding mode control; fuzzy rule; nonlinear system; dynamic modelling.

CHAPTER ONE

1 INTRODUCTION

1.1 Background of the Study

Air vehicle Such as helicopters and unmanned aerial vehicles (UAVs) have gained significant attention and utilization in various fields. Their versatility and accessibility make them valuable assets in applications such as transportation, search and rescue operations, law enforcement, military operations, aerial photography and filming, and even recreational use. To effectively perform their intended tasks, these air vehicles require sophisticated control systems. The challenges associated with controlling these systems arise from their nonlinear behavior, coupling between different dynamics, parameter variations, model uncertainties, and external disturbances. These factors necessitate the development and research of advanced control systems to enhance their stability, reliability, and safety. In recent years, there has been a growing focus on advancing control techniques for air vehicles. Researchers and engineers are actively working on developing control strategies that can handle the complexities and uncertainties associated with these systems. These efforts aim to improve maneuverability, enhance stability in adverse flight conditions, and ensure the safe operation of air vehicles. Advanced control systems for air vehicles often involve the use of nonlinear control techniques, adaptive control algorithms, robust control methods, and intelligent control approaches. These approaches enable the control system to adapt to changing conditions, compensate for uncertainties, and mitigate disturbances effectively. The research and development of advanced control systems for air vehicles are crucial for addressing the evolving demands of various applications. By improving the control capabilities of these vehicles, it becomes possible to enhance their performance, expand their operational capabilities, and ensure their safe and reliable operation in challenging environments (Nilsen, 2017).

helicopters are highly adaptable and flexible platforms, capable of vertical takeoff and landing, as well as hovering in a stationary position. These characteristics make helicopters particularly suitable for a wide range of applications. The advancements in autonomous technology have further expanded the potential uses of small helicopters in both civil and military domains. The deployment of autonomous small helicopters is driven by their ability to perform tasks such as bridge and building inspections, air pollution monitoring, search

and rescue missions, area mapping, agricultural applications, and traffic surveillance. These platforms offer the advantage of high mobility and accessibility, as they can reach areas that are difficult to access by other means. However, to ensure the safe and efficient performance of these tasks, robust and adaptable controllers are crucial. The controllers used in autonomous small helicopters need to exhibit high robustness against disturbances and modeling errors. This robustness ensures that the helicopters can maintain stability and performance even in the presence of external factors or inaccuracies in the model used for control. The ability to handle uncertainties and disturbances is essential for the safe and reliable operation of these helicopters in various applications. In the pursuit of strong flight control solutions, the twin rotor multi-input multi-output system (TRMS) has attracted considerable interest. The TRMS is an aerodynamic system that shares similarities with helicopters and exhibits complex nonlinear dynamics. It is characterized by a strong coupling effect between its two propellers, which adds to the challenge of control design. Efforts are being made to develop advanced control techniques specifically tailored for TRMS and similar systems. These techniques often involve nonlinear control strategies, adaptive control algorithms, and robust control methods. The goal is to design controllers that can effectively handle the strong coupling effect and nonlinearity of the TRMS, while also providing robustness against disturbances and modeling uncertainties. By addressing the unique control challenges posed by systems like TRMS, researchers and engineers aim to enhance the performance, maneuverability, and safety of autonomous small helicopters. This, in turn, will contribute to their wider adoption and utilization in various civil and military applications (Ying Xin, Zhi-Chang Qin, 2019).

1.2 Statement of the Problem

The helicopter system with two degrees of freedom (DOF) represents a complex control system. It is characterized by its higher-order dynamics, multi-variable nature, nonlinearity, and strong coupling between the system's variables. Additionally, uncertainties in system parameters and unidentified external disturbances further complicate the control problem. To effectively address these challenges, it is necessary to develop sophisticated control algorithms specifically tailored for such complex systems. One approach that has been widely used for regulating nonlinear systems including helicopters, is sliding mode control. Sliding mode control is known for its robustness against model uncertainties and disturbances, making it suitable for systems with parametric uncertainty and external perturbations. However, a main drawback of sliding mode control is the chattering phenomenon and controlling a two-degree-of-freedom (2-DOF) helicopter can indeed present significant computational burden and complexity due to the inherent dynamics and control requirements of the system. However, there are controller design techniques that can help address these challenges and enable quicker and more precise computation. Chattering refers to high-frequency switching of control actions that lead to abrupt changes in the control inputs, the vibrations can introduce wear and tear, reduce the lifespan of the components, and potentially lead to mechanical failures. Also Chattering can introduce high-frequency noise into the control signal, which can propagate to the helicopter's actuators and affect their performance. This noise can deteriorate the control system's overall stability and accuracy, potentially leading to degraded control performance which can lead to excessive stress on actuators and degrade overall system performance. It is desirable to mitigate or eliminate chattering while preserving the benefits of sliding mode control. Therefore, the focus of this thesis is to address the issues of control and accuracy proposed fuzzy sliding mode control techniques to remove chattering and smooth the control input and reduce computational burden and complexity. These techniques aim to retain the robustness of sliding mode control while minimizing the adverse effects of chattering.

1.3 Objectives of the Study

There are two categories into which the objectives of this thesis work can be divided: general and specific.

1.3.1 General Objective

The primary goal of this thesis project is model an effective and robust controller for 2-DOF helicopter with external disturbances.

1.3.2 Specific Objectives

The specific objectives of this study are listed below:

- Build a mathematical model of 2-DOF helicopter system
- Design control algorithms for the system
- Design and simulate the proposed technique on MATLAB/SIMULINK
- Draw relevant conclusions and recommendations for the system from the results of the investigation.

1.4 Scope of the Study

The use of an ideal control method for a 2-DOF helicopter is covered in this thesis. Investigations are conducted into the control structures, system efficiency and effectiveness, and command and stability augmentation system. The 2-DOF helicopter system's mathematical model is introduced, and the system's parameters are listed. The suggested fuzzy SMC controller and their simulation findings are presented. Additionally, the system was mathematically designed and simulated using MATLAB/SIMULINK in order to show how the system's many components interact and to compare the suggested controllers.

1.5 Study Limitations

Due to the cost and time constraints involved in obtaining the necessary materials deploying is difficult, this thesis will primarily have limited to focus on the mathematical design and simulation of the system using MATLAB software.

1.6 Significance of the Study

The control of 2-DOF helicopters in the context of UAVs has driven the development and utilization of laboratory platforms. These platforms offer controlled environments for testing, analysis, and education, enabling the exploration and refinement of control algorithms and strategies before their deployment on actual UAVs. They play a crucial role in advancing the field of UAV control and promoting the safe and efficient operation of these aerial systems.

1.7 Thesis Format

The thesis is organized with in six chapters, which is briefly reviewed below.

The introduction, background, goals, problem statements, scope, importance of the study, and technique used in the thesis work are all presented in the first chapter. Furthermore, it offers the thesis's outline. The theoretical foundations of resilient control systems and a survey of the literature on two-degree-of-freedom helicopters are covered in the second chapter. In chapter three of the thesis, the mathematical model of the systems is designed and then the proposed control techniques are designed based on the collected data in chapter four. Chapter five presents the outcomes of the simulation studies, and this chapter provides logical explanations for the findings. Ultimately, chapter six discusses the findings, suggestions, and further study to be done. A summary of the thesis works' findings is provided, along with pertinent conclusions.

CHAPTER TWO

2 THEORETICAL BACKGROUND AND LITERATURE REVIEW

2.1 Theoretical Background

Two degrees of freedom (2-DOF) helicopters are widely used as modeling and control platforms in research and teaching environments. These helicopters are popular because they exhibit a highly nonlinear behavior, which provides a challenging and realistic testbed for developing and evaluating control algorithms. The term "two degrees of freedom" refers to the rotational motion of the helicopter around two axes: the pitch axis and the yaw axis. The pitch axis controls the forward and backward tilting motion of the helicopter, while the yaw axis controls the left and right turning motion. The nonlinear dynamics of a 2-DOF helicopter arise from several factors, including the complex aerodynamic interactions between the rotor blades, the gyroscopic effects due to the rotating blades, and the coupling between the pitch and yaw motions (Fouad Yacef, Nesrine Benkhalel, Lina Benhannouda, 2023). These nonlinearities make the control problem more difficult compared to linear systems, requiring the use of advanced control techniques. Researchers and educators often choose 2-DOF helicopters as experimental platforms because they provide a practical and accessible means to study nonlinear control strategies, such as adaptive control, robust control, and nonlinear control methods like feedback linearization and sliding mode control. Additionally, these helicopters can be relatively inexpensive and portable, making them suitable for educational purposes and laboratory experiments. By working with 2-DOF helicopters, researchers and students can gain hands-on experience in designing, implementing, and tuning control algorithms for nonlinear systems. The complexity and nonlinearity of these systems challenge learners to understand and apply advanced control concepts, which can enhance their understanding of control theory and its practical applications (Zeghlache, S., Bouguerra, A., & Ladjal, M, 2016).



Figure 2-1 An image of the USN-built helicopter model with two degrees of freedom
(Nilsen, 2017)

2.2 Physical system

The model for the helicopter is a Twin Rotor Multiple-inputs Multiple-outputs System. Figure 2.2 shows a representation of the physical model that will be used. The rotors in Figure 2.2 are of equal size and are labeled as yaw on the left and pitch on the right. The model has two degrees of freedom, with yaw shown in red and pitch shown in green. Additionally, the black incremental position sensor for the pitch position is shown (Nilsen, 2017).

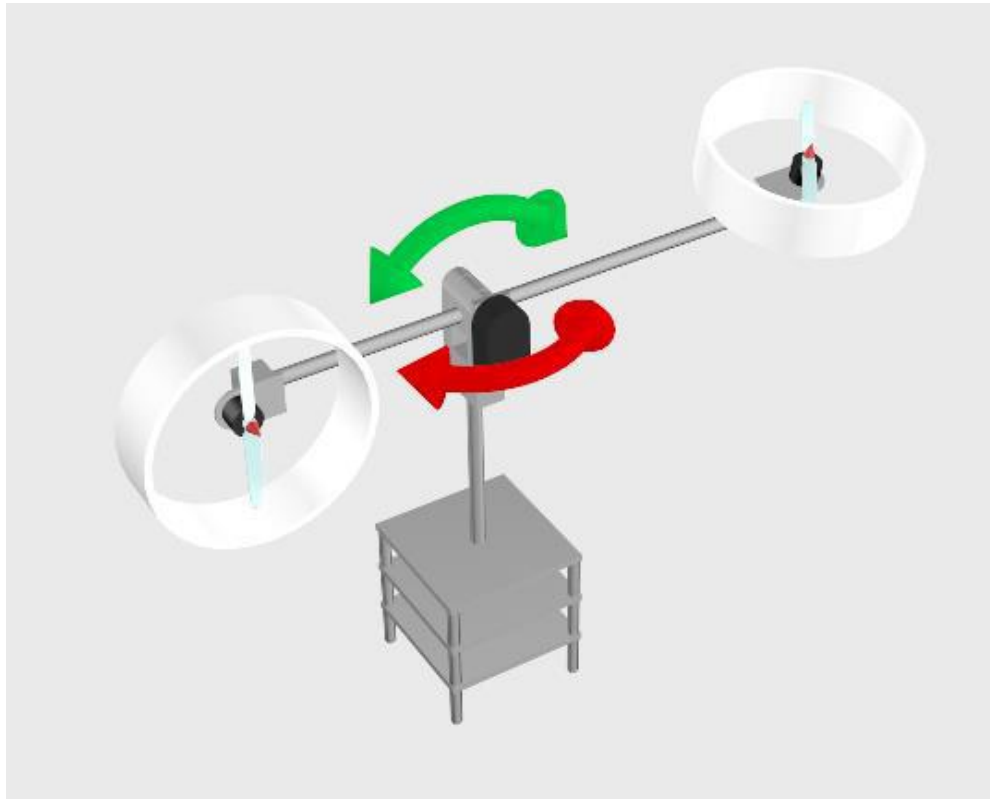


Figure 2-2 Two-degree-of-freedom helicopter model. Movement of green pitch and red yaw (Nilsen, 2017)

2.3 Factors Influencing the System

The system is being affected by a number of factors. friction forces, gravity, and the two propellers. The pitch rotor's forces are indicated by the red arrows in Figure 2.3, while the yaw rotor's forces are indicated by the blue arrow. The force of gravity that results on the helicopter model's arm is represented by the green arrow. The friction forces that oppose the direction of joint movement are not depicted (Soraya Bououden, Brahim brahmi and Mohammad Habibur Rahman, 2021).

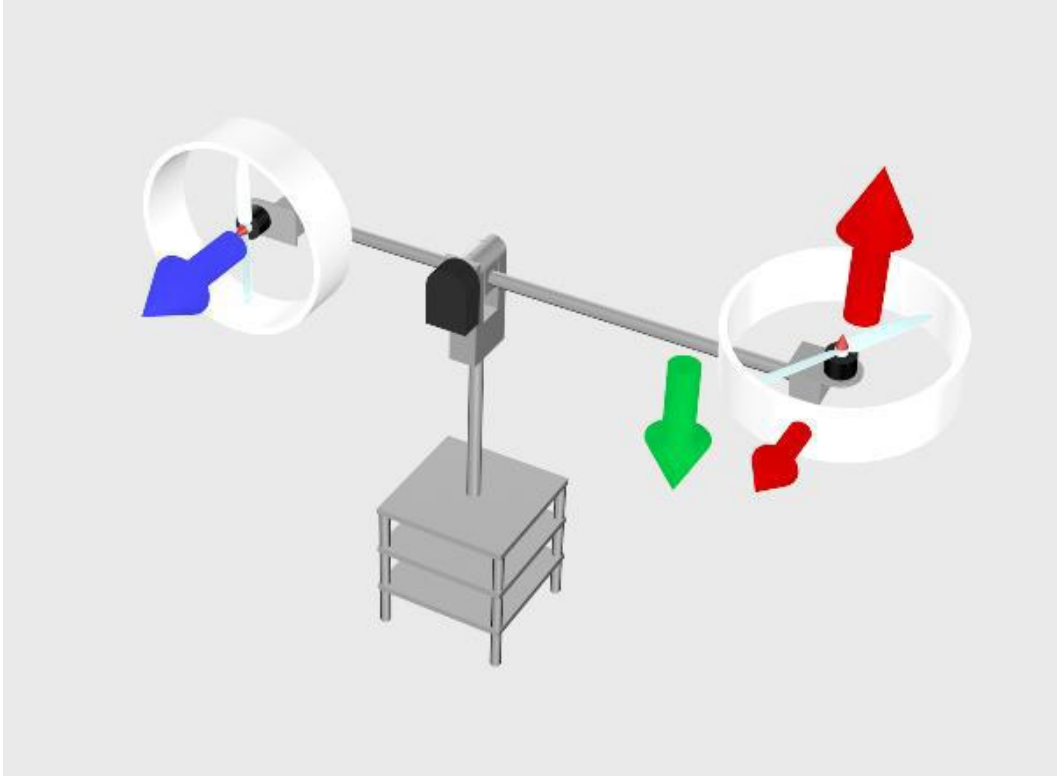


Figure 2-3 factors influencing the system. Gravity is represented by green, the yaw rotor's force is represented by blue, and the pitch rotor's force is represented by red. (Nilsen, 2017)

2.4 System Dynamics

Numerous factors influence the torques generated by the forces exerted on the system. The weight of the arm with the rotors, the pitch angle, and the separation between the rotational axis and the arm's center of gravity all affect how much torque is created by gravity. The distances between the axes and the rotors determine the torque generated by the force of the pitch and yaw rotors. There are moments of inertia in the system about the vertical and horizontal axes (S. Karthick, S. Kanthalakshmi, E. Vinodh Kumar, V. Joshi Kumar, A. Ezhil Kumaran, 2022). The system's angular acceleration is determined by the system's moments of inertia and the total torques that come from the forces acting on it. When the control inputs are maintained constant, the system exhibits severe oscillations and pitch angle instability.

2.5 Model Description of the 2-DOF Helicopter

Consider the schematic model of the 2-DOF helicopter depicted in Figure 2.4, where B represents a right-handed body-fixed frame with body-fixed axes x , y , and z . The helicopter system consists of two identical propellers, r_p and r_y , which are located at various distances

from the origin of the body-fixed frame. These propellers generate thrust forces, F_p and F_y , respectively.

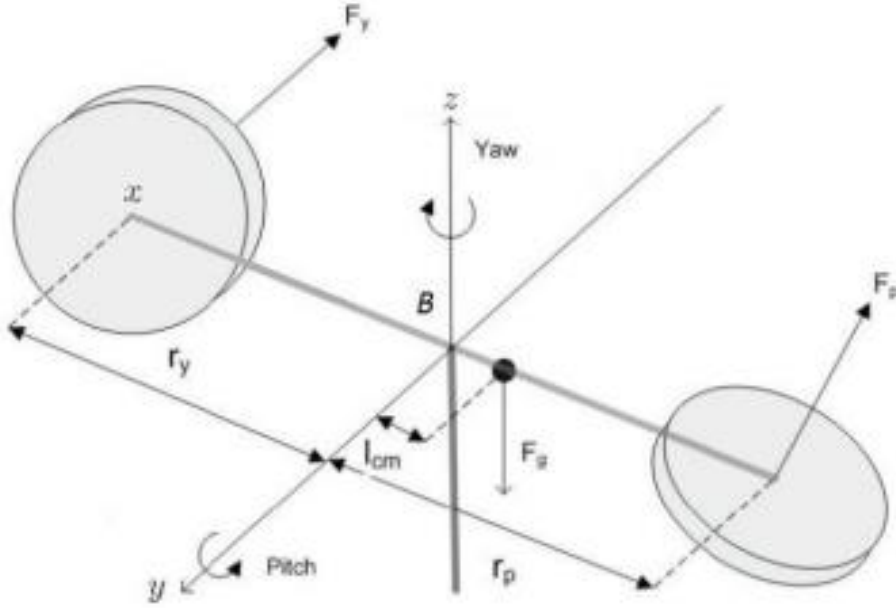


Figure 2-4 A two-degree-of-freedom helicopter system schematic (Peter Lambert, Mahmut Reyhanoglu, 2018)

When the first propeller generates a torque around the y-axis, it produces a pitch motion. Similarly, when the second propeller generates a torque around the z-axis, it creates a yaw motion. The pitch and yaw angles represent the system's outputs. These angles describe the orientation and rotation of the helicopter in the pitch and yaw directions, respectively. The input variables to the system are the voltages V_p and V_y applied to the DC motors that drive the propellers. These voltages control the thrust forces generated by the propellers. The helicopter system is highly nonlinear and exhibits a multi-input multi-output behavior due to the complex relationships between the input voltages and the resulting pitch and yaw angles. The system's mass is represented by the variable m , the center of gravity's action point is located at a specific distance from the body-fixed frame's origin along the pitch axis, the moments of inertia of the rotating beam about the pitch and yaw axes are denoted as J_y and J_p , respectively, the gravitational constant is represented by g , the viscous friction coefficients along the pitch and yaw axes are denoted as B_p and B_y , respectively and the

thrust torque constants that relate the motor-generated torques to the input voltages are represented by K_{pp} , K_{py} , K_{yp} , and K_{yy} .

2.6 Sliding Mode Controller

Sliding mode control techniques are indeed an approach to solving control problems and have gained increasing interest in the field of control systems. Sliding mode control is a robust control methodology that was introduced by Russian engineer and mathematician V.I. Utkin in the 1970s. It is particularly useful for controlling systems with uncertainties, disturbances, or nonlinear dynamics. The key idea behind sliding mode control is to create a sliding surface in the state space of the system, along which the system's trajectories are forced to slide. By properly designing the sliding surface and control law, sliding mode control can achieve robustness against parameter uncertainties and disturbances. The main advantage of sliding mode control is its ability to ensure finite-time convergence and robustness in the presence of uncertainties. The system trajectories are forced to stay on the sliding surface, which leads to the desired control objective being achieved within a finite time, regardless of the initial conditions or uncertainties. Moreover, sliding mode control is known for its insensitivity to modeling errors and disturbance rejection capabilities. However, sliding mode control also has some limitations. It can induce high-frequency chattering phenomena, which can cause control actuators to operate rapidly, leading to wear and tear. Additionally, the design of sliding mode controllers can be complex and requires a good understanding of the system dynamics. Despite these limitations, sliding mode control techniques have found applications in various fields, including robotics, power systems, automotive control, and aerospace. Researchers continue to explore and develop new variations and extensions of sliding mode control to address its limitations and improve its performance (Liu, Sliding mode control MATLAB simulation basic theory and design method, 2019).

Generally sliding mode techniques are a valuable approach to solving control problems, particularly for systems with uncertainties and disturbances. They offer robustness and finite-time convergence properties, although they also have some limitations that need to be considered in their application.

SMC has three main advantages:

1. **Tailoring dynamic behavior:** In sliding mode control, the system is designed to drive the system state to lie within a neighborhood of the switching function. The choice of the switching function allows for the tailoring of the dynamic behavior of the system. By carefully selecting the sliding surface and the control law, specific performance objectives can be achieved. This flexibility in shaping the system's behavior is a significant advantage of sliding mode control.
2. **Insensitivity to uncertainty:** Sliding mode control is known for its robustness against uncertainties. By enforcing the system trajectories to slide along the prescribed sliding surface, sliding mode control can make the closed-loop response insensitive to a particular class of uncertainty. This means that even when there are uncertainties or disturbances affecting the system, sliding mode control can maintain stability and achieve the desired performance.
3. **Direct performance specification:** Another advantage of sliding mode control is its ability to directly specify the desired performance. By designing the sliding surface and control law, the desired performance criteria can be incorporated directly into the control system design. This makes sliding mode control attractive from a design perspective, as it allows for a more intuitive and straightforward way to specify performance requirements.

It is worth noting that while sliding mode control offers these advantages, it also has some challenges and limitations, such as high-frequency chattering and complexity in controller design. Nevertheless, the ability to tailor system behavior, robustness against uncertainty, and direct performance specification make sliding mode control a valuable approach in control system design and have contributed to its increasing interest in the field (Boufadene, 2019).

Lyapunov's Direct Method

Lyapunov's direct method allows us to determine the stability of the system. An energy-like scalar function as a Lyapunov function is constructed to analyze the stability of the system.

Theorem 1: Lyapunov stability

The system is said to be stable if there is a scalar Lyapunov function $V(x)$ satisfying the following conditions exists.

1. $V(x) \geq 0$ for $\forall x$, and $V(x)=0$ for $x=0$.
2. $\dot{V}(x) \leq 0$ for $\forall x \neq 0$, and $\dot{V}(x)=0$ for $x=0$.

Chattering

High frequency switching between the two different control structures will let the trajectory cross the line repeatedly. This high frequency motion is commonly known as chattering. Chattering degrades the performance of the system and may even lead to instability and also leads to high wear of moving mechanical parts and high heat losses in electrical power circuits.

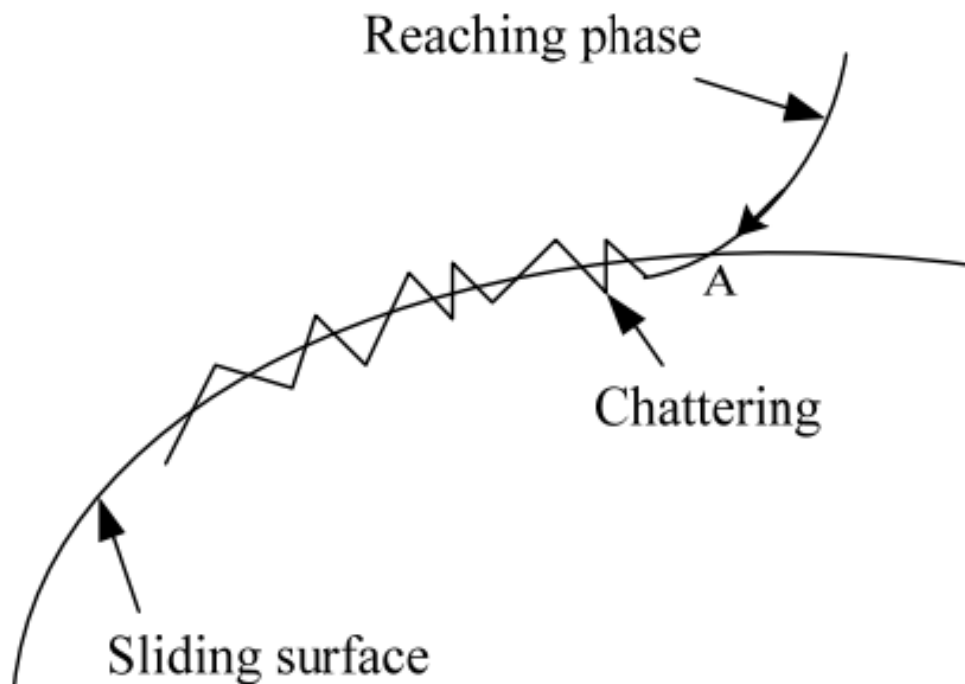


Figure 2-5 sliding surface and chattering phenomena

2.7 Introduction to Intelligent Control

Intelligent control refers to control techniques that leverage human intelligence and knowledge in a specific control domain. It implies that a knowledgeable human can effectively control a system using their expertise. This definition does not consider the limitations of human sensory and actuation capabilities or information processing speeds. Intelligent control often involves information abstraction and knowledge-based decision making, incorporating abstracted information. In this class, we study one of intelligent control part that is fuzzy logic and control.

2.7.1 Fuzzy Logic Controller (FLC)

Fuzzy Logic Controllers (FLCs) have been widely used and have proven to be a helpful approach in controller design since Mamdani's pioneering work on fuzzy theory in process control. FLCs offer a flexible and intuitive framework for handling complex and uncertain systems. The key advantage of FLCs lies in their ability to handle imprecise and uncertain information. Unlike traditional controllers that rely on precise mathematical models, FLCs can operate effectively even when the system dynamics are not fully known or accurately modeled. This makes them particularly useful in real-world applications where system behaviors may exhibit nonlinearities, uncertainties, or imprecise inputs. FLCs use linguistic variables and fuzzy sets to represent and reason about the system's inputs and outputs. By employing a set of linguistic rules, which capture the expert knowledge or heuristics about the system, FLCs can make decisions and generate control actions based on fuzzy logic reasoning. These linguistic rules are typically defined using "if-then" statements that map input combinations to appropriate control actions. One of the benefits of FLCs is their interpretability. The linguistic rules and fuzzy sets used in FLCs are easy to understand and interpret by human operators, which can be valuable in applications where transparency and human interaction are important. FLCs can handle nonlinearities and uncertainties, which involve fuzzification, rule evaluation, aggregation, and defuzzification steps. This allows FLCs to adaptively adjust control actions based on the system's current state and the knowledge captured in the fuzzy rule base. It's worth noting that FLCs have their own challenges, such as the tuning of membership functions and rule base, as well as the potential for high computational complexity. However, with advancements in fuzzy logic theory and computing technologies, these challenges can be mitigated (Juntao Fei, Jiahao Yang, 2023).

In general, Fuzzy Logic Controllers provide a valuable approach in controller design, enabling the control of complex systems with imprecise information and uncertainties. They have been successfully applied in a wide range of fields, including process control, robotics, and intelligent transportation systems, among others.

Structure of FLC

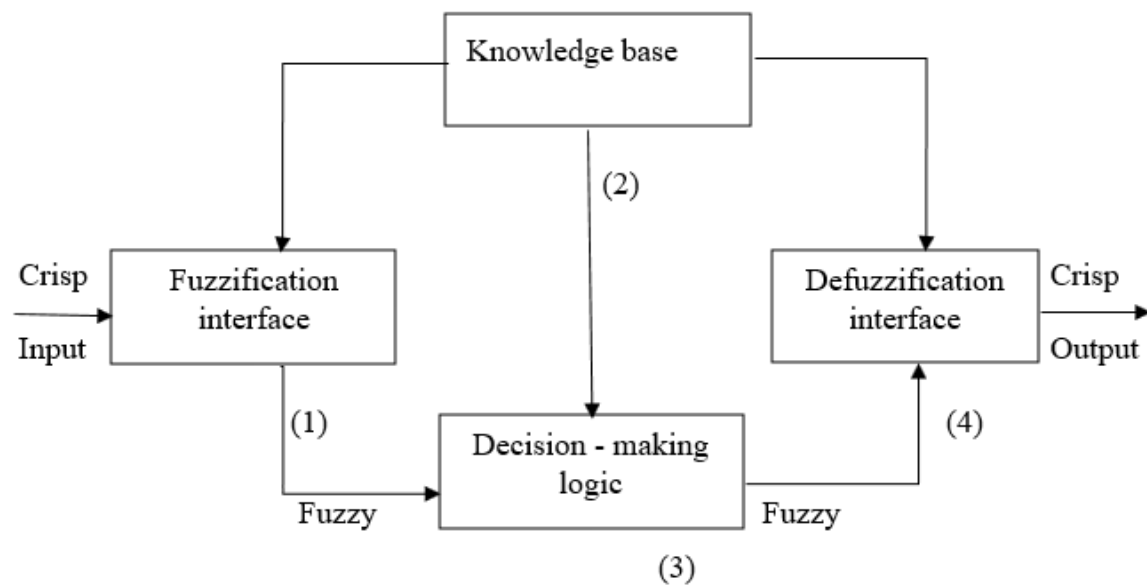


Figure 2-6 Structure of fuzzy logic controller

2.7.2 Fuzzification interface

In a Fuzzy Logic Controller (FLC), the fuzzification interface is a crucial component that converts crisp (numeric) inputs into fuzzy values. Fuzzification is the process of mapping precise inputs to fuzzy sets, which are linguistic representations of the input variables. The fuzzification interface takes the crisp input values and assigns membership degrees to relevant fuzzy sets. Membership degrees indicate the degree to which an input value belongs to a particular fuzzy set. The membership degrees can range from 0 to 1, representing the degree of membership or truthfulness of an input value to a fuzzy set. There are different methods for fuzzification, depending on the shape and characteristics of the fuzzy sets being used (Fouad Yacef, Nesrine Benkhaled, Lina Benhannouda, 2023).

Some common methods include:

- I. **Triangular membership function:** This method assigns a membership degree of 1 to the input value if it falls exactly on the triangular-shaped fuzzy set. If the input value falls between two adjacent fuzzy sets, the membership degrees are determined based on their proximity to each fuzzy set.
- II. **Trapezoidal membership function:** Like the triangular membership function, this method assigns membership degrees based on the trapezoidal-shaped fuzzy set. It

allows for a broader range of membership degrees when the input value falls within the trapezoidal shape.

- III. **Gaussian membership function:** This method uses a bell-shaped curve to assign membership degrees based on the distance between the input value and the mean of the fuzzy set. The closer the input value is to the mean, the higher the membership degree.
- IV. **Custom membership functions:** In certain cases, custom membership functions can be defined based on specific requirements or expert knowledge about the system. These functions provide more flexibility in representing the membership degrees based on the characteristics of the input values.

Once the fuzzification process is complete, the fuzzy values representing the inputs are passed to the fuzzy inference engine, where fuzzy rules are evaluated and control actions are determined. The fuzzification interface bridges the gap between crisp inputs and fuzzy logic reasoning, allowing the FLC to operate with linguistic variables and fuzzy sets.

2.7.3 Knowledge base

The knowledge base refers to the collection of linguistic rules and fuzzy sets that are used to make decisions and generate control actions based on the system's inputs.

The knowledge base in FLC consists of two main components:

Fuzzy Sets: Fuzzy sets define the linguistic variables that represent the inputs and outputs of the FLC. Each input and output variable is associated with one or more fuzzy sets, which partition the range of possible values into linguistic terms. For example, an input variable "temperature" could have fuzzy sets such as "low," "medium," and "high," each representing a different range of temperature values.

Linguistic Rules: Linguistic rules capture expert knowledge or heuristics about how the inputs and outputs of the system are related. These rules take the form of "if-then" statements and describe the control actions to be taken based on certain combinations of input values. For example, a linguistic rule in a temperature control system could be "IF temperature is low THEN increase heating."

The linguistic rules are typically defined using the linguistic variables and fuzzy sets from the knowledge base. The rules map the fuzzy inputs to fuzzy outputs, and they are evaluated in the fuzzy inference engine to determine the appropriate control actions based on the current state of the system. The knowledge base is developed through expert knowledge, system analysis, or data-driven approaches. Experts in the field provide their expertise to define the fuzzy sets and linguistic rules based on their understanding of the system and desired control objectives. In some cases, data-driven methods can be used to extract linguistic rules and fuzzy sets from observed input-output data. The knowledge base is a critical component of FLC, as it encapsulates the understanding and decision-making capabilities of the control system. It allows the FLC to reason and make control decisions based on fuzzy logic principles, handling uncertainties and imprecise information effectively.

2.7.4 Decision-making logic

The decision-making logic is based on fuzzy logic principles, which involve evaluating fuzzy rules and generating appropriate control actions based on the system's inputs and the knowledge captured in the fuzzy rule base.

The decision-making process in FLC typically involves the following steps:

- I. **Fuzzification:** The crisp (numeric) inputs to the FLC are converted into fuzzy values using the fuzzification process. Fuzzification assigns membership degrees to relevant fuzzy sets based on the input values, indicating the degree of membership or truthfulness of each input to a fuzzy set.
- II. **Rule Evaluation:** Fuzzy rules, defined in the knowledge base, are evaluated using the fuzzy values obtained from fuzzification. Each rule consists of antecedent (if) and consequent (then) parts. The antecedent of a rule specifies the condition based on the fuzzy inputs, while the consequent specifies the control action or output based on the fuzzy outputs. The fuzzy logic inference engine evaluates the rules by combining the membership degrees of the fuzzy inputs according to the logical operators (e.g., AND, OR) used in the rules.
- III. **Aggregation:** After evaluating the fuzzy rules, the next step is to aggregate the fuzzy outputs obtained from the rule evaluation. This involves combining the fuzzy outputs from multiple rules to obtain a comprehensive representation of the control action.

- IV. **Defuzzification:** The aggregated fuzzy output needs to be converted back into a crisp control action that can be applied to the system. Defuzzification is the process of determining a crisp output value based on the aggregated fuzzy output. Various defuzzification methods can be used, such as centroid, maximum membership, or weighted average, depending on the specific requirements of the control system.
- V. **Control Action:** The final step in the decision-making process is to apply the determined control action to the system. The crisp control action obtained from defuzzification is used to command the actuators or influence the behavior of the controlled system.

The decision-making logic in FLC involves fuzzification of crisp inputs, evaluation of fuzzy rules based on the fuzzy inputs, aggregation of fuzzy outputs, defuzzification to obtain crisp control actions, and applying the control actions to the system. This process allows FLC to reason and make decisions based on the linguistic knowledge and fuzzy logic principles, effectively handling uncertainties and imprecise information in control systems.

2.7.5 Defuzzification

The defuzzification interface is a vital component that converts the aggregated fuzzy output into a crisp control action that can be applied to the system. Defuzzification is the process of determining a single representative value from the fuzzy output, which is typically a fuzzy set or a combination of fuzzy sets. The defuzzification interface takes the aggregated fuzzy output, which represents the combined effect of all the fuzzy rules and transforms it into a crisp control action. The goal is to obtain a meaningful and interpretable value that can be used to influence the behavior of the controlled system.

There are several methods commonly used for defuzzification in FLCs:

- I. **Centroid Method:** This method calculates the center of gravity, or centroid, of the fuzzy output set. It computes a weighted average of the fuzzy output values, where the weights are determined by the membership degrees of the fuzzy output set. The centroid represents the "center" or the most representative value of the fuzzy output, and it is often used as the crisp control action.
- II. **Maximum Membership Method:** In this method, the crisp control action is determined by finding the output value with the highest membership degree in the

aggregated fuzzy output set. It selects the output value that has the strongest association with the control action.

- III. **Height Method:** This method selects the output value with the highest membership degree and uses its height or magnitude as the crisp control action. It does not consider the shape or distribution of the fuzzy output set but focuses solely on the highest membership degree.
- IV. **First Maximum Method:** This method identifies the first output value with a membership degree equal to 1 in the aggregated fuzzy output set. It assumes that the first maximum value encountered is the most significant and representative value.
- V. **Weighted Average Method:** This method calculates a weighted average of the output values in the aggregated fuzzy output set, where the weights are determined by the membership degrees. It considers both the membership degrees and the corresponding output values to compute the crisp control action.

The choice of defuzzification method depends on the specific requirements of the control system, the characteristics of the fuzzy output set, and the desired interpretation of the control action. Once the defuzzification process is complete, the crisp control action is obtained and can be applied to the system to influence its behavior.

The fuzzy sliding mode control (FSMC) method offers several advantages over classical sliding mode control (SMC) techniques. Here are some of the key advantages:

Robustness: FSMC has better robustness characteristics compared to classical SMC. It can effectively handle disturbances in system parameters, allowing it to keep the controlled system operating dynamically even when there are disruptions. This robustness is achieved through the combination of fuzzy control methodology and sliding mode control.

Reduced Chattering: Chattering refers to the undesirable high-frequency oscillations that can occur in sliding mode control systems. FSMC is designed to effectively reduce chattering compared to classical SMC methods. This leads to smoother control inputs and improved tracking performance.

Improved Trajectory Tracking: The FSMC method ensures quick and precise trajectory tracking for the 2-DOF helicopter. It blends fuzzy control methods with the benefits of high

order sliding mode control, resulting in better tracking performance compared to classical SMC.

Smoother Control Input: FSMC not only reduces chattering but also makes the control input smoother. This leads to improved system behavior and less abrupt changes in control actions, which can be beneficial for various applications.

Adaptability: FSMC can handle uncertainties and variations in system parameters by utilizing fuzzy control techniques. It provides a more adaptive control approach compared to classical SMC, allowing the controller to adjust and optimize its behavior based on the system's needs.

It's important to note that the advantages mentioned above are specific to the context of the study and the application of FSMC to the trajectory tracking of a 2-DOF helicopter. The relative advantages of FSMC over classical SMC may vary depending on the specific system, control objectives, and operating conditions.

2.8 Review of Related Works

In the subject of control systems, controlling a helicopter with two degrees of freedom is a well-researched problem. To solve the issues with this kind of system, researchers have created a variety of control mechanisms. Several frequently employed control approaches for 2-DOF helicopters consist of: Proportional-Integral-Derivative (PID) Control: In numerous applications, PID control is a well-liked and frequently applied control technique. It entails modifying the control inputs in accordance with the discrepancy between the intended and actual states of the system. The 2-DOF helicopter may be stabilized and controlled with the help of PID control. Model Predictive Control, or MPC, is an advanced control technique that forecasts a system's future behavior by using a dynamic model of the system. The optimal control inputs that minimize a cost function while meeting system limitations are found by solving an optimization problem. For the 2-DOF helicopter, MPC can offer superior tracking and disturbance rejection capabilities. A reliable control method called sliding mode control (SMC) seeks to cause the system states to slide along a predetermined path. It is especially helpful for handling ambiguities and disruptions. The 2-DOF helicopter can benefit from SMC in order to achieve reliable tracking and stabilization. Adaptive Control: Using real-time measurements from the system, adaptive control

approaches modify the control parameters. These methods are helpful when there is uncertainty or variation in the system parameters. The 2-DOF helicopter control system's robustness and performance can be improved using adaptive control. These are just a few examples of the control strategies that researchers have developed for the 2-DOF helicopter. Each strategy has its own advantages and limitations, and the choice of control strategy depends on the specific requirements of the system and the desired performance objectives.

In (Ruobing, Changyi, Baiyang, Quanmin and Xicai, 2023) the study combining the DSM inverter and RBFNN, the control system can achieve specific performance requirements. The DSM inverter cancels out the system dynamics, simplifying the control design process and improving robustness, while the RBFNN approximates the system's behavior and uncertainties, enabling accurate control and compensation. This integrated approach leverages the strengths of both techniques to achieve precise and robust control of the 2-DOF helicopter system. But the adaptive UDSMC approach with the three adaptive laws aims to achieve convergence stability of the 2-DOF helicopter system using Lyapunov stability analysis. However, it does not explicitly address finite-time convergence, which refers to the property of a system reaching a desired state within a finite time duration and it does not efficiently remove the control input chattering.

In (Soraya Bououden, Brahim brahmi and Mohammad Habibur Rahman, 2021) the trajectory tracking of 2-DOF helicopter system using canonical normal form is designed and described the 2-DOF helicopter model serves as the basis for the nonlinear tracking control system. The objective is to create control laws that force the helicopter's locations to follow the intended values based on a predetermined time by utilizing 0-flat normal form, which is seen to be a particularly intriguing solution for nonlinear systems. However, in this study the control input is not smooth and did not take any outside disturbance into account.

In (M. Derakhshannia, S. B. F. Asl and S. S. Moosapour, 2021) describes the design and simulation of Back stepping Terminal Sliding Mode Control for a 2-DOF TRMS using MATLAB/SIMULINK. This study uses a fractional order back stepping terminal sliding mode controller to investigate resilient controllers. A back stepping control for trajectory tracking using virtual input is first provided, and then a proposal is made to combine this technique with a fractional-order terminal sliding mode control. Through numerical simulations, the controller's effectiveness is examined. It has been noted that stability

analysis and controller design are more difficult, and that control is not quicker or more precise.

In (Ayush, Hem Prabha, Rajul Kumar, 2020) design H_∞ mixed sensitivity controllers for the TRMS, aiming to achieve robust and satisfactory control performance for the system. It combines concepts from H_∞ control theory and loop shaping control for the design process and enhance the control system's performance in terms of stability, disturbance rejection, and tracking accuracy. But H_∞ control design is typically more complex. It involves solving optimization problems to find the controller that minimizes the H_∞ norm of the transfer function from disturbances to control errors, subject to certain constraints. This complexity can make the design and implementation of H_∞ control more challenging, especially for systems with complex dynamics or stringent performance requirements.

In (Patel, 2019) The back stepping controller with integral for controlling a 2-DOF helicopter is designed. It describes potential advantages such as good tracking performance, the ability to handle nonlinearities, and the incorporation of integral action for improved steady-state error handling. However, the authors did not take careful consideration to the specific requirements, constraints, and trade-offs associated with the control of a 2-DOF helicopter before selecting this control approach and Back stepping control relies on an accurate mathematical model. So impose a significant computational burden, especially for high-performance applications or systems with fast dynamics, limiting the real-time implementation feasibility on resource-constrained platforms.

In (Peter Lambert, Mahmut Reyhanoglu, 2018) The observer-based sliding mode controller for a 2-DOF helicopter is presented. This observer-based sliding mode controller can offer advantages such as robustness to disturbances and uncertainties, good tracking performance, and the ability to handle nonlinearities. But on the control input voltage have more chattering and the authors are not considered and try to eliminate this chattering.

In (Zeghlache, S., Bouguerra, A., & Ladjal, M, 2016), presents the control strategy for the twin rotor MIMO system (TRMS), which is based on sliding mode control with nonlinear sliding surfaces. The decision was supported by the algorithm's resilience to both modeling errors and outside interruptions. The main limitation of this study was that precise trajectory tracking was not maintained and the chattering effect was not considerably reduced.

In (Siri Marte Schlanbusch, Jing Zhou, 2020) presents a novel adaptive controller for a 2-DOF helicopter system. The controller is designed to handle input quantization and uncertain system parameters. The back stepping technique is employed to develop the adaptive control algorithm, which enables independent tracking of pitch and yaw position references. But a controller aims to minimize tracking error and closely follow a reference trajectory, it typically needs to provide larger control signals or voltages to the system actuators. These control signals exert more force or influence on the system, allowing it to track the reference trajectory more accurately. However, generating higher control signals requires a greater amount of energy or voltage.

In summary, centered on the above interpretation, the subsequent points represent the most common deficiencies observed in position controlling for 2-DOF helicopter system:

- In most papers when designing the controller, the focus is primarily on achieving good control performance rather than specifically addressing the chattering phenomenon.
- Often, trajectory tracking controllers for 2-DOF are based solely on kinematic models, neglecting the system's dynamic characteristics and potentially hindering accuracy improvements.
- Trajectory tracking typically involves a closed-loop simple control strategy, which may not be easily generalized for 2-DOF with multi-variable, nonlinear, and strongly coupled behaviors.
- While the FSMC algorithm has been a successful solution for smooth control input for trajectory tracking in 2-DOF helicopter, further verification is needed to determine if it satisfies the performance criteria for controlling the trajectory tracking of a 2-DOF helicopter system. All the journal papers mentioned above are related to the work in this thesis, and all of these important papers referred for the purpose of designing or simulation of the system. Apart from these mentioned papers, some more other journal papers were referred for the work that was helpful. So, to address the control problems described above and improve the accuracy of trajectory tracking for 2-DOF helicopter, this thesis proposes a suitable and efficient control method.

CHAPTER THREE

3 2-DOF HELICOPTER SYSTEM MODELING AND METHODOLOGY

3.1 Overview

This chapter mentions the designing process of the model (2-DOF Helicopter), the materials and procedures in the thesis are also presented.

3.2 Materials used

In this thesis we used Microsoft office (MS 2019) to writing the documentation whereas, Math Type 7.4.8 for writing the mathematical equations. Edraw-max website also used in drawing the flow charts and block diagrams. As well as we used the MATLAB/R2021a software to run, test and get the result of the system.

3.3 Methods or procedures followed

The flow and techniques followed throughout this thesis work to attain the objective mentioned back in chapter 1.

- Comprehensive review of work related to these topics which is 2-DOF Helicopter system was conducted and analyzed from many sources of papers such like conference papers, journals, books and so on.
- After gathering on the research design of the dynamic model of 2-DOF helicopter system developed.
- Afterward, on the fourth chapter controller design is considered which is SMC and FSMC are tuned.
- Then simulation result of the closed system is done using MATLAB R221a.
- Finally, comparison made between the chattering phenomena of the SMC and when apply fuzzy on SMC.

The below figure summarizes methods followed in the thesis work.

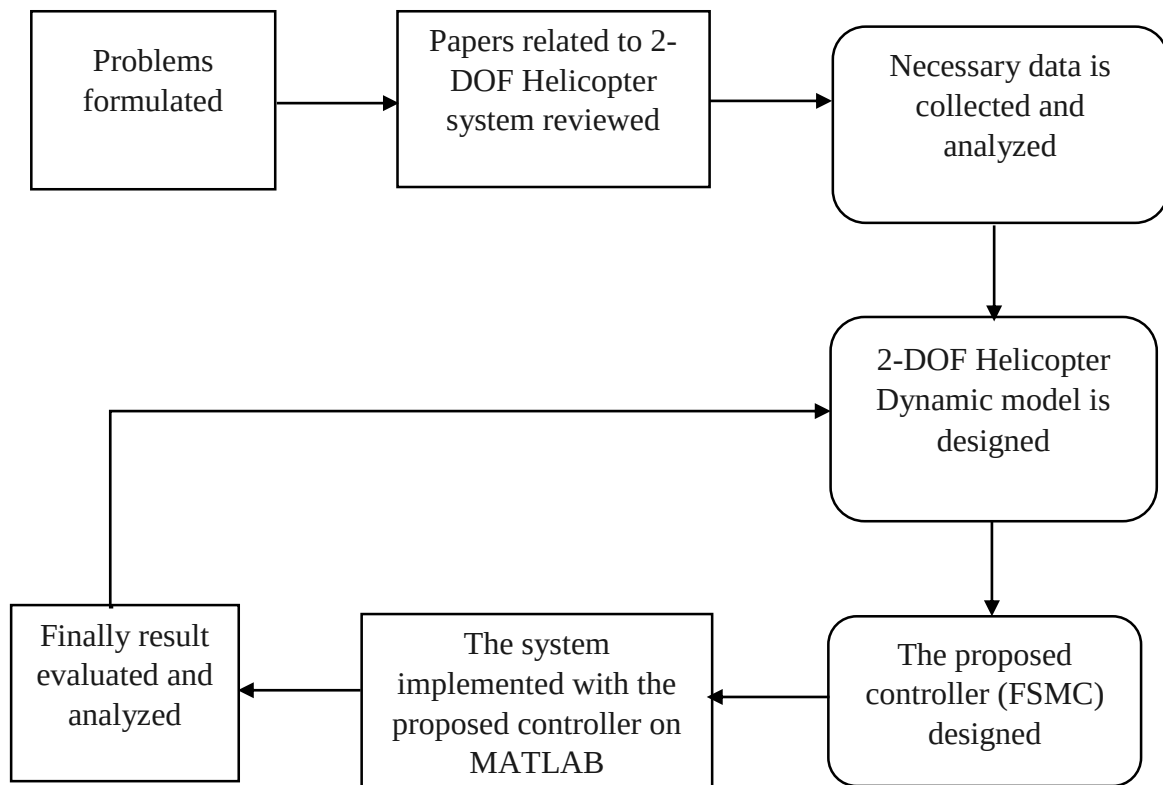


Figure 3-1 Flow Diagram for the Suggested Research Approach

3.4 Proposed controller and 2-DOF Helicopter system block diagram

The main contribution of this paper is applying the intelligent and robust controller on 2-DOF nonlinear Helicopter system, controlling the motion front and back of the helicopter system by maintaining the pitch and yaw angles error to zero and make the control input free of chattering. This controller (FSMC) effectiveness can be shown by comparing it with one of the best robust controllers (SMC) considering time varying sinusoidal of disturbance.

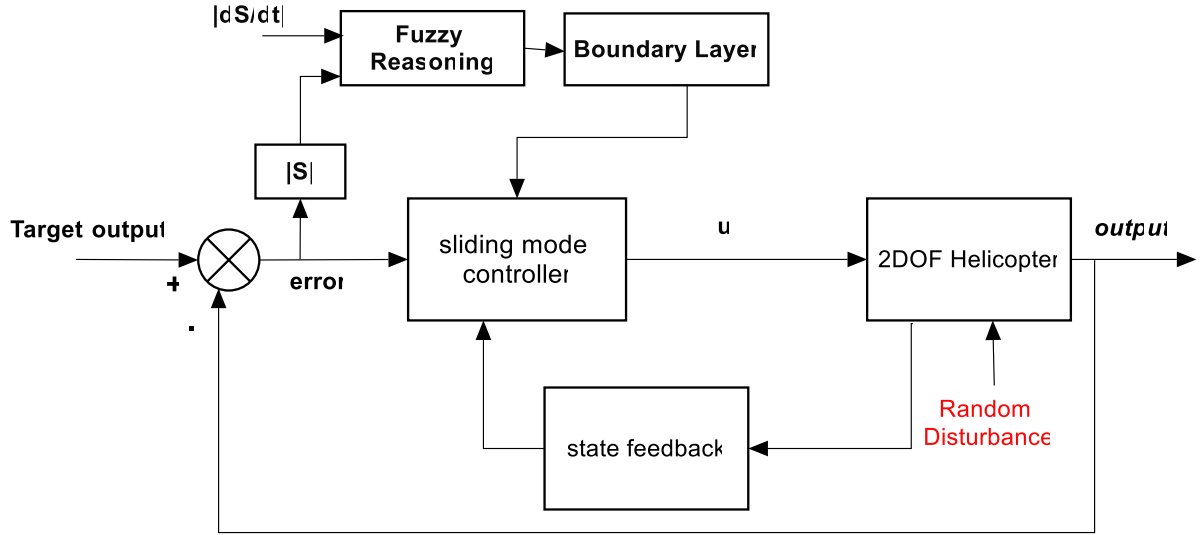


Figure 3-2 block diagram of 2-DOF Helicopter with proposed controller

3.5 Mathematical Modeling of 2-DOF Helicopter

Figure 3.3 represents a prototype of simple 2-DOF helicopter system. This system consists of two main rotor driven by direct current (DC) motors, positioned at the end of a beam. The beam is attached to a central pivot point, allowing it to rotate freely in both the vertical (pitch) and horizontal (azimuth) planes. The mathematical model of the helicopter system involves two degrees of freedom, represented by the angles θ and ϕ , which correspond to the pitch and yaw movements, respectively. In the system, there are two thrust forces: the pitch force (F_p) and the yaw force (F_y). The pitch force generates a torque around the pitch axis at a distance denoted as r_p , while the yaw force generates a torque around the yaw axis at a distance denoted as r_y . As depicted in Figure 3.3, the gravitational force (F_g) acts downward, causing the nose of the helicopter to be pulled down. This prototype serves as a simplified representation of a 2-DOF helicopter system, which can be further analyzed and modeled mathematically to develop control strategies and algorithms for stabilizing and maneuvering the helicopter.

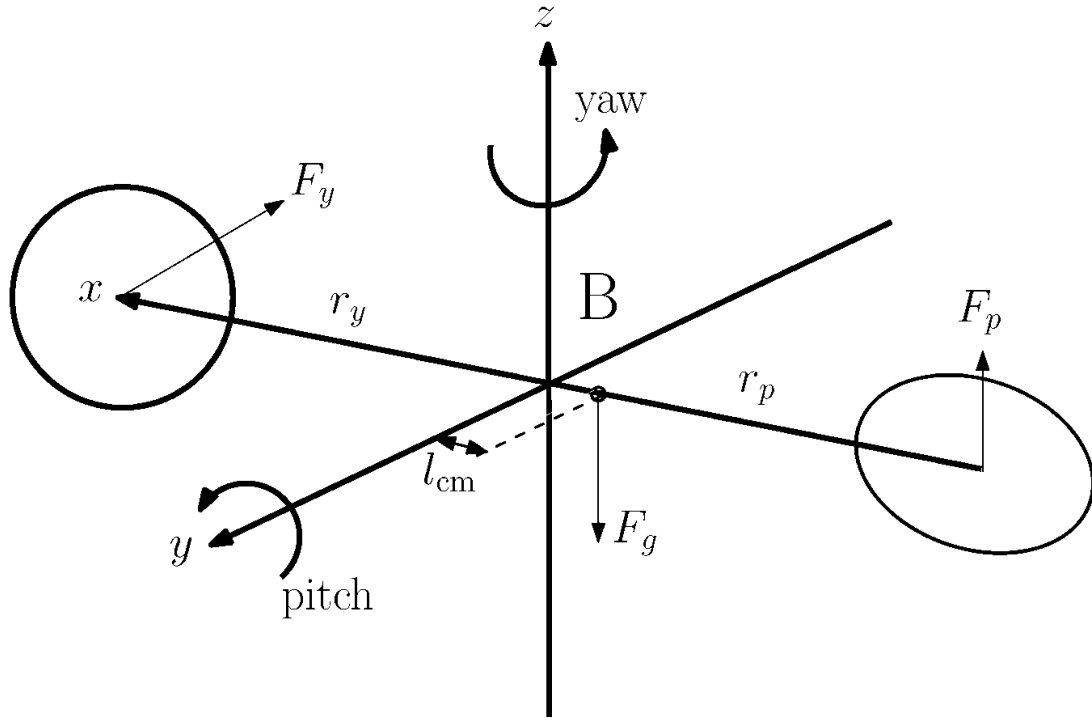


Figure 3-3 schematic of a two degree of freedom helicopter (Amjad J Humaidi and Alaq F Hasan, 2019)

Using yaw and pitch rotation matrices and coordinate transformations, the center of aircraft mass is given by

$$\begin{aligned} X_{cm} &= l_{cm} \cos \theta \cos \phi \\ y_{cm} &= l_{cm} \cos \theta \sin \phi \\ z_{cm} &= l_{cm} \sin \theta \end{aligned} \quad (3.1)$$

where the distance between the mass center and the point where the pitch and yaw axes connect is indicated by the symbol l_{cm} . Angles θ and ϕ can be used to express the mass center in Cartesian coordinates. As a result, the direct kinematic is represented by

$$l_{cm} = \frac{[(m_{m,p} + m_{shield})r_p + (m_{m,y} + m_{shield})r_y]}{m_{m,p} + m_{shield} + 2m_{shield}} \quad (3.2)$$

A. System Energy Analysis

Where m_{shield} is the mass of the propeller and shield, and $m_{m,p}$ and $m_{m,y}$ are the masses of the pitch and yaw motors, respectively. Potential energy was developed as a result of the gravitational force and was given by

$$V = m_{heli} g l_{cm} \sin \theta \quad (3.3)$$

The rotational kinetic energy can be expressed using the following formulas for pitch and yaw rotations

$$\begin{aligned} T_{r,p} &= \frac{1}{2} J_p \dot{\theta}^2 \\ T_{r,y} &= \frac{1}{2} J_y \dot{\phi}^2 \end{aligned} \quad (3.4)$$

where pitch and yaw are represented by moments of inertia J_p and J_y , respectively. The kinetic energy of translations is given by the formula:

$$T_t = \frac{1}{2} m (\dot{x}_m^2 + \dot{y}_m^2 + \dot{z}_m^2) \quad (3.5)$$

Where m is the mass of the helicopter (the entire mass). The total kinetic energy T of a helicopter can be expressed as the sum of the translational kinetic energy T_t and the rotational kinetic energies due to pitch $T_{r,p}$ and yaw $T_{r,y}$. Mathematically, it can be represented as

$$T = T_{r,p} + T_{r,y} + T_t \quad (3.6)$$

B. Nonlinear equation of Lagrange equation

The difference between the system's kinetic and potential energy, or the Lagrangian variable L is defined as

$$L = T - V \quad (3.7)$$

The following set of differential equations is used to develop the Euler-Lagrange equations of motion.

$$\begin{aligned}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}}\right) - \frac{\partial L}{\partial \theta} &= Q_1 \\ \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\phi}}\right) - \frac{\partial L}{\partial \phi} &= Q_2\end{aligned}\tag{3.8}$$

Consequently, the generalized forces listed below can be acquired.

$$\begin{aligned}Q_1 &= K_{pp}V_p + K_{py}V_y + B_p\dot{\theta} \\ Q_2 &= K_{yp}V_p + K_{yy}V_y + B_y\dot{\phi}\end{aligned}\tag{3.9}$$

Where K_{pp} stands for the pitch actuating motor's torque constant above the pitch axis, K_{py} for the yaw actuating motor above the pitch axis, K_{yp} for the pitch actuating motor above the yaw axis, and K_{yy} for the torque constant of the yaw actuating motor above the pitch axis. Pitch and yaw motor voltages are denoted by V_p and V_y , respectively. The rotating viscous friction operating on the pitch and yaw axes is denoted by B_p and B_y , respectively. The torques applied on the pitch and yaw axes are clearly cross-coupled, as shown by equation (3.9).

Motion can be obtained by expanding the Euler-Lagrange expressions of equation (3.8) in terms of the forces given by equation (3.9).

$$(J_p + ml_{cm}^2)\ddot{\theta} = K_{pp}V_p + K_{py}V_y - B_p\dot{\theta} - mgl_{cm}\cos\theta - ml_{cm}^2\sin\theta\cos\theta\tag{3.10}$$

$$(J_y + ml_{cm}^2\cos^2\theta)\ddot{\phi} = K_{yp}V_p + K_{yy}V_y - B_y\dot{\phi} + 2mgl_{cm}^2\dot{\theta}\dot{\phi}\sin\theta\cos\theta\tag{3.11}$$

By adding uncertainties to the parameters B_p and B_y , the aforementioned equations can be changed as follows:

$$\ddot{\theta} = \frac{K_{pp}V_P + K_{py}V_y - (B_p\dot{\theta} + \varphi_1(t)) - mgl_{cm}\cos\theta - ml_{cm}^2\sin\theta\cos\theta}{J_p + ml_{cm}^2} \quad (3.12)$$

$$\ddot{\phi} = \frac{K_{yp}V_P + K_{yy}V_y - (B_y\dot{\phi} + \varphi_2(t)) + 2mgl_{cm}\dot{\theta}\sin\theta\cos\theta}{J_y + ml_{cm}^2\cos^2\theta} \quad (3.13)$$

Open loop Step response 2DOF helicopter pitch and yaw angles are shown in figures 3.3 and 3.4.

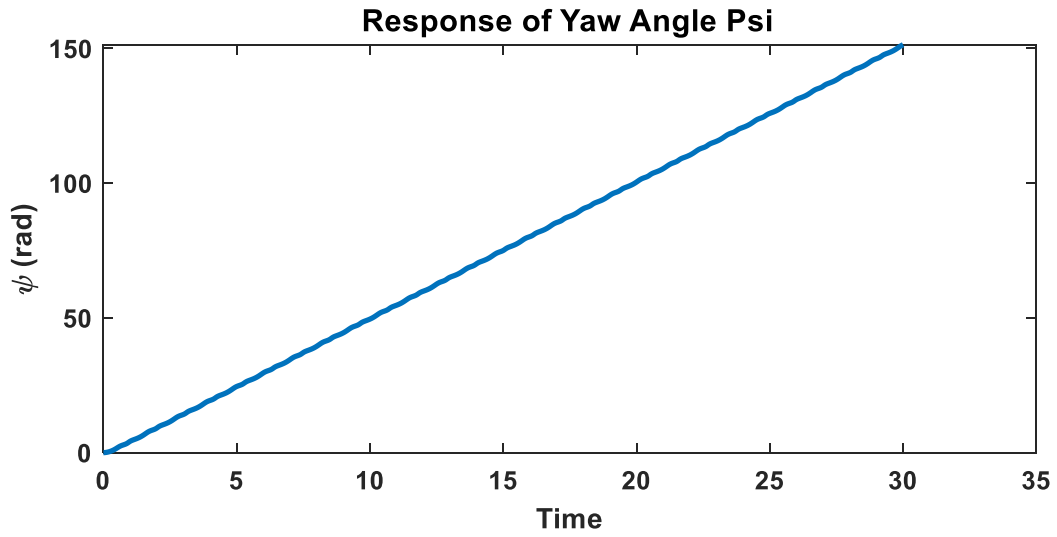


Figure 3-4 Open loop Step response of Pitch angle

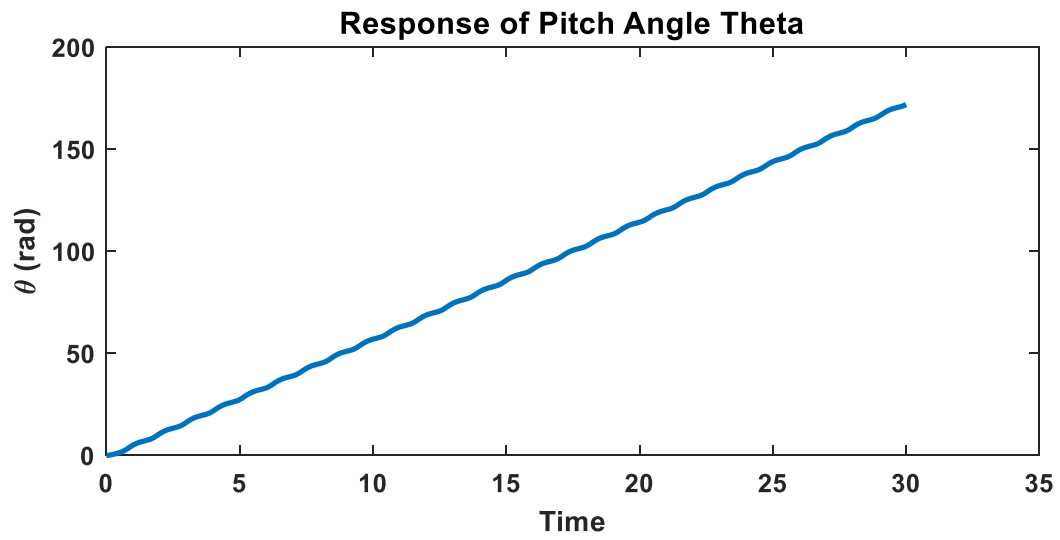


Figure 3-5 Open loop Step response of yaw angle

From the above shown figures, we see that we need to design controllers as the step response obtained is unstable.

CHAPTER FOUR

4 CONTROLLER DESIGN

4.1 Sliding mode controller

The nonlinear control method known as sliding mode control (SMC) is notable for its accuracy, resilience, and simplicity of tuning and application. The purpose of SMS systems is to force the system states onto specific sliding surface in the state space. Sliding mode control maintains the states in the immediate vicinity of the sliding surface after it has been reached. As a result, the sliding mode control has a two-component architecture. Designing a sliding surface to ensure that the motion complies with design requirements is the first step. The choice of a control rule that will make the switching surface appealing to the system state is the focus of the second (Utkin, 2019).

The benefits of sliding mode control are twofold. First, the specific sliding function selection may be used to modify the system's dynamic behavior. Second, in response to some specific uncertainty, the closed loop reaction becomes completely insensitive. This principle also applies to bounded nonlinearity, disturbance, and uncertainties in model parameters. Practically speaking, SMC makes it possible to manage nonlinear systems that are influenced by outside shocks and significant model uncertainty (Biao Luo, Huai-Ning Wu, Tingwen Huang, 2018).

4.2 Design SMC for 2-DOF helicopter

The block diagram of SMC control system is shown in figure 4.1. the control target is to limit the error between θ and θ_d and between ϕ and ϕ_d to a certain range, where θ , θ_d , ϕ and ϕ_d are the output pitch angle, target pitch angle, output yaw angle and target yaw angle respectively.

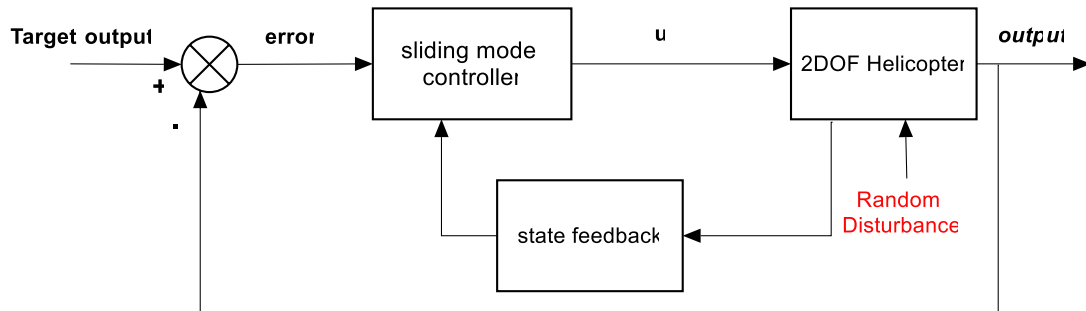


Figure 4-1 Block diagram of SMC control system

The first step is to establish the pitch channel with θ signals sliding mode control design. The sliding surface is given by

$$S_\theta = \dot{e}_\theta + C_1 e_\theta \quad (4.1)$$

Where C_1 is an arbitrary positive design parameter and the equation calculates the error e_θ by subtracting the actual position (θ), from the desired angular position (θ_d).

$$e_\theta = \theta_d - \theta \quad (4.2)$$

Equation (4.1)'s time derivative provides

$$\dot{S}_\theta = \ddot{e}_\theta + C_1 \dot{e}_\theta = \ddot{\theta}_d - \ddot{\theta} + C_1 \dot{e}_\theta \quad (4.3)$$

Equation (3.12) is substituted into equation (4.3), yielding the following result.

$$\dot{S}_\theta = \ddot{\theta}_d - \frac{[K_{pp} * V_p + K_{py} * V_y - (B_p * \dot{\theta} + \phi_1) + a_2]}{a_1} + C_1 * \dot{e}_\theta \quad (4.4)$$

The pitching control signal is comprised of the following comparable and switching control laws:

$$V_p = \left(\frac{a_1}{K_{pp}} \right) (u_{eq1} + u_{sw1}) \quad (4.5)$$

The defined of the corresponding equivalent control law is

$$u_{eq1} = \ddot{\theta}_d - \left(\frac{K_{pp}}{J_p + ml_{cm}^2} \right) V_y + \left(\frac{B_p}{J_p + ml_{cm}^2} \right) \dot{\theta} - \left(\frac{-mgl_{cm} \cos \theta - ml_{cm}^2 \sin \theta \cos \theta}{J_p + ml_{cm}^2} \right) + C_1 \dot{e}_\theta \quad (4.6)$$

Conversely, the STA serves as the foundation for the switching control law's construction. This algorithm determines how the switching control legislation should be configured.

$$u_{sw1} = k_1 \sqrt{|S_\theta|} \operatorname{sgn}(S_\theta) + k_2 \int \operatorname{sgn}(S_\theta) dt \quad (4.7)$$

Equations (4.6) and (4.7), where k_1 and k_2 are positive design parameters, can be substituted into equation (4.5) to produce equation (4.8).

$$V_p = \left(\frac{J_p + ml_{cm}^2}{K_{pp}} \right) \left(\ddot{\theta}_d - \left(\frac{K_{pp}}{J_p + ml_{cm}^2} \right) V_y + \left(\frac{B_p}{J_p + ml_{cm}^2} \right) \dot{\theta} - \left(\frac{-mgl_{cm} \cos \theta - ml_{cm}^2 \sin \theta \cos \theta}{J_p + ml_{cm}^2} \right) + \right. \\ \left. c_1 \dot{e}_\theta + k_1 \sqrt{|S_\theta|} \operatorname{sgn}(S_\theta) + k_2 \int \operatorname{sgn}(S_\theta) dt \right) \quad (4.8)$$

After that, the following is derived by substituting equation (4.8) into equation (4.4).

$$\dot{s}_\theta = -k_1 \sqrt{|s_\theta|} \operatorname{sgn}(s_\theta) - k_2 \int \operatorname{sgn}(s_\theta) dt + \varphi_1(t) \quad (4.9)$$

To build the SMC the yaw channel with $\emptyset$ signals sliding mode control design. The sliding surface is given by

$$s_\phi = \dot{e}_\phi + c_2 e_\phi \quad (4.10)$$

The switching control law's component is supplied by and the equation calculates the error $e_\emptyset$ by subtracting the actual position ($\emptyset$), from the desired angular position ($\emptyset_d$).

$$e_\phi = \phi_d - \phi \quad (4.11)$$

Equation (4.10)'s time derivative provides

$$\dot{s}_\phi = \ddot{e}_\phi + c_2 \dot{e}_\phi = \ddot{\phi}_d - \ddot{\phi} + c_2 \dot{e}_\phi \quad (4.12)$$

Equation (3.13) is substituted into equation (4.12), yielding the following result

$$\dot{S}_\phi = \ddot{\phi}_d - \frac{[K_{yp} * V_p + K_{yy} * V_y - (B_p * \dot{\phi} + \varphi_2) + 2mgl_{cm}^2 \dot{\theta} \dot{\phi} \sin \theta \cos \theta]}{J_y + ml_{cm}^2 \cos^2 \theta} + c_2 * \dot{e}_\phi \quad (4.13)$$

After that, the following is derived by substituting equation (4.8) to equation (4.13).

$$\begin{aligned} \dot{S}_\phi = \ddot{\phi}_d - \frac{K_{yp} J_p + ml_{cm1}^2}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} & \left(\ddot{\theta}_d - \left(\frac{K_{py}}{J_p + ml_{cm}^2} \right) V_y + \left(\frac{B_p}{J_p + ml_{cm}^2} \right) \dot{\theta} - \right. \\ & \left. \left(\frac{-mgl_{cm} \cos \theta - ml_{cm}^2 \sin \theta \cos \theta}{J_p + ml_{cm}^2} \right) \right) - \\ \frac{K_{yp} J_p + ml_{cm1}^2}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} & \left(c_1 \dot{e}_\theta + k_1 \sqrt{|s_\theta|} \operatorname{sgn}(s_\theta) + k_2 \int \operatorname{sgn}(s_\theta) dt \right) - \left(\frac{K_{yy}}{J_y + ml_{cm}^2 \cos^2 \theta} \right) V_y \\ + \frac{B_y \dot{\phi} + \varphi_2(t)}{J_y + ml_{cm}^2 \cos^2 \theta} - \frac{2mgl_{cm}^2 \dot{\theta} \dot{\phi} \sin \theta \cos \theta}{J_y + ml_{cm}^2 \cos^2 \theta} & + c_2 \dot{e}_\phi \end{aligned} \quad (4.14)$$

Equivalent and switching control rules make up the two components of the yaw control signal V_y . Consequently, the definition of this yaw control signal is

$$V_y = \frac{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}}{K_{yy} K_{pp} - K_{yp} K_{py}} (u_{eq2} + u_{sw2}) \quad (4.15)$$

The defined of the corresponding equivalent control law is

$$\begin{aligned}
u_{eq2} = & \ddot{\phi}_d - \left(\frac{K_{yp}J_p + ml_{cm}^2}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \ddot{\theta}_d - \left(\frac{K_{yp}B_p}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \dot{\theta} + \left(\frac{K_{yp} - mgl_{cm} \cos \theta -}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \\
& \left(\frac{K_{yp}J_p + ml_{cm}^2 c_1}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \dot{\epsilon}_\theta - \left(\frac{K_{yp}J_p + ml_{cm}^2}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} k_1 \sqrt{|s_\theta|} \right) \text{sgn}(s_\theta) \\
& - \left(\frac{K_{yp}J_p + ml_{cm}^2 k_2}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \int \text{sgn}(s_\theta) dt + \left(\frac{B_y}{J_y + ml_{cm}^2 \cos^2 \theta} \right) \dot{\phi} - \frac{2mgl_{cm}^2 \dot{\theta} \dot{\phi} \sin \theta \cos \theta}{J_y + ml_{cm}^2 \cos^2 \theta} + c_2 \dot{\epsilon}_\phi
\end{aligned} \tag{4.16}$$

The switching control law's component is supplied by

$$u_{sw2} = k_3 \sqrt{|s_\phi|} \text{sgn}(s_\phi) + k_4 \int \text{sgn}(s_\phi) dt \tag{4.17}$$

where k_3 and k_4 are positive design parameters. Substituting equation (4.16) and (4.17) in to equation (4.15), the following is obtained

$$\begin{aligned}
V_y = & \frac{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}}{K_{yy}K_{pp} - K_{yp}K_{py}} (\ddot{\theta}_d - \left(\frac{K_{yp}J_p + ml_{cm}^2}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \ddot{\theta}_d - \left(\frac{K_{yp}B_p}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \dot{\theta} + \\
& \left(\frac{K_{yp} - mgl_{cm} \cos \theta -}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \left(\frac{K_{yp}J_p + ml_{cm}^2 c_1}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \dot{\epsilon}_\theta - \left(\frac{K_{yp}J_p + ml_{cm}^2}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} k_1 \sqrt{|s_\theta|} \right) \\
& \text{sgn}(s_\theta) - \left(\frac{K_{yp}J_p + ml_{cm}^2 k_2}{J_y + ml_{cm}^2 \cos^2 \theta K_{pp}} \right) \int \text{sgn}(s_\theta) dt + \left(\frac{B_y}{J_y + ml_{cm}^2 \cos^2 \theta} \right) \dot{\phi} - \left(\frac{2mgl_{cm}^2 \dot{\theta} \dot{\phi} \sin \theta \cos \theta}{J_y + ml_{cm}^2 \cos^2 \theta} \right) \\
& + c_2 \dot{\epsilon}_\phi + k_3 \sqrt{|s_\phi|} \text{sgn}(s_\phi) + k_4 \int \text{sgn}(s_\phi) dt)
\end{aligned} \tag{4.18}$$

Substituting equation (4.18) into equation (4.14), $\dot{s}_\phi$ reduces as follows

$$\dot{s}_\phi = -k_3 \sqrt{|s_\phi|} \text{sgn}(s_\phi) - k_4 \int \text{sgn}(s_\phi) dt + \varphi_2(t) \tag{4.19}$$

The Lyapunov approach is used to analyze the stability of a 2-DOF helicopter operated by an SMC controller. A quadratic Lyapunov candidate function is selected with respect to the pitch and yaw surfaces. Through an analysis of its derivative with respect to time, this Lyapunov candidate function is utilized to evaluate the stability of the system.

$$v(s_\theta, s_\phi) = \frac{1}{2}(s_\theta^2 + s_\phi^2) \quad (4.20)$$

The Lyapunov function's time derivative can be calculated as

$$\dot{v} = s_\theta \dot{s}_\theta + s_\phi \dot{s}_\phi \quad (4.21)$$

Using equation (4.9) and (4.19)

$$\dot{v} = s_\theta \left(-k_1 \sqrt{|s_\theta|} \operatorname{sgn}(s_\theta) - k_2 \int \operatorname{sgn}(s_\theta) dt + \varphi_1(t) \right) + s_\phi \left(-k_3 \sqrt{|s_\phi|} \operatorname{sgn}(s_\phi) - k_4 \int \operatorname{sgn}(s_\phi) dt + \varphi_2(t) \right) \quad (4.22)$$

$$\dot{v} \leq -k_1 \sqrt{|s_\theta|} |s_\theta| - |s_\theta| \int k_2 dt + |\varphi_1(t) s_\theta| - k_3 \sqrt{|s_\phi|} |s_\phi| - |s_\phi| \int k_4 dt + |\varphi_2(t) s_\phi| \quad (4.23)$$

$$\dot{v} \leq -k_1 \sqrt{|s_\theta|} |s_\theta| - |s_\theta| \int k_2 dt + |s_\theta| \int \varphi_1(t) dt - k_3 \sqrt{|s_\phi|} |s_\phi| - |s_\phi| \int k_4 dt + |s_\phi| \int \varphi_2(t) dt \quad (4.24)$$

$$\dot{v} \leq -k_1 \sqrt{|s_\theta|} |s_\theta| - |s_\theta| \int k_2 dt + |s_\theta| \int \delta_1 dt - k_3 \sqrt{|s_\phi|} |s_\phi| - |s_\phi| \int k_4 dt + |s_\phi| \int \delta_2(t) dt \quad (4.25)$$

$$\dot{v} \leq -k_1 \sqrt{|s_\theta|} |s_\theta| - |s_\theta| \left(\int k_2 dt + \int \delta_1 dt \right) - k_3 \sqrt{|s_\phi|} |s_\phi| - |s_\phi| \left(\int k_4 dt + \int \delta_2(t) dt \right) \leq 0 \quad (4.26)$$

To guarantee the stability of the 2-DOF helicopter, it is necessary that the values of and meet the requirements of k_2 and k_4 must satisfy $k_2 > \delta_1 > |\dot{\varphi}_1(t)|$ and $k_4 > \delta_2 > |\dot{\varphi}_2(t)|$. The sliding variables (s_θ and s_ϕ) will initially converge to zero as $t \rightarrow \infty$, as it is evident that is a positive definite and that is negative definite for all $t \geq 0$.

4.3 Chattering Reduction

Chattering in the context of control systems refers to high-frequency oscillations or rapid switching between control modes near the sliding surface. In the case of a 2-DOF helicopter, chattering can occur when applying sliding mode control or other control techniques. Chattering is characterized by rapid and frequent switching of the control signal or control action as the system transitions between different control modes, cause undesirable vibrations in the mechanical components of the 2-DOF helicopter, such as the actuators, joints, or other parts of the system and requires high control efforts or rapid switching between control modes to drive the system towards the sliding surface. Then Incorporate chattering reduction techniques within the FSMC framework is necessary. This can include techniques such as boundary layer design, saturation functions, or filters to smooth out the control action near the sliding surface and reduce chattering. These techniques help minimize the adverse effects of chattering on the mechanical components and control system performance.

4.4 Fuzzy Sliding Mode Controller

4.4.1 Role of membership functions

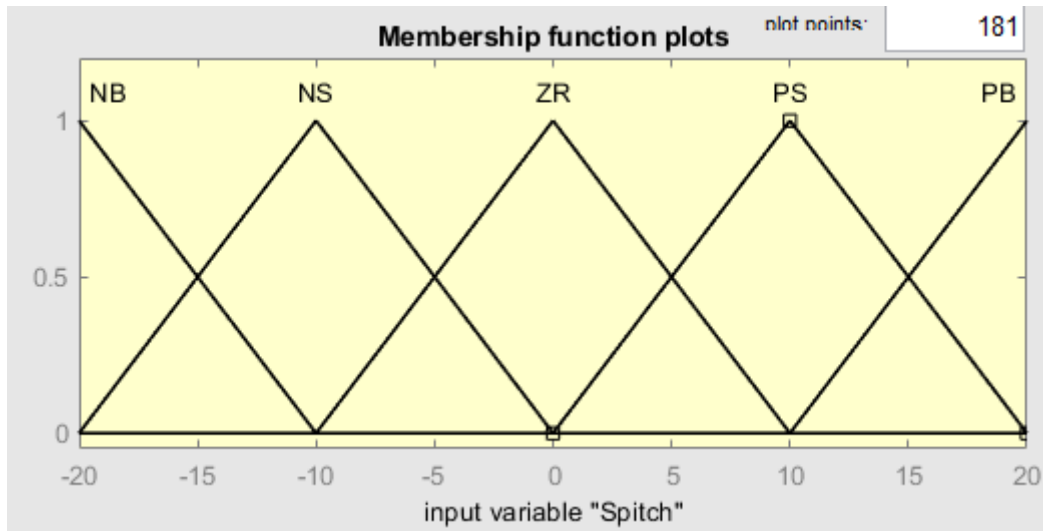
Membership functions (MFs) are essential components of fuzzy logic systems and have a significant impact on the overall performance of fuzzy representations. MFs determine the degree of membership or fuzziness of an element to a fuzzy set. The shape of the membership function is crucial as it influences the behavior and characteristics of the fuzzy inference system. Different shapes can be used to model different types of uncertainty and capture specific characteristics of the problem domain. Here are a few commonly used shapes for membership functions such as triangular, trapezoidal, Gaussian and so forth.

the choice of membership function (MF) shapes often involves a trial-and-error process. This is because there is no exact method or universal rule for determining the best MF shapes for a given problem. The selection of MF shapes depends on several factors, including the problem domain, the nature of the input data, and the expert's knowledge or belief about the linguistic variables. Since fuzzy logic is based on human reasoning and linguistic terms, the choice of MF shapes often relies on the subjective interpretation of the problem by experts or domain specialists. They use their knowledge and intuition to determine the appropriate shapes that capture the desired characteristics of the problem.

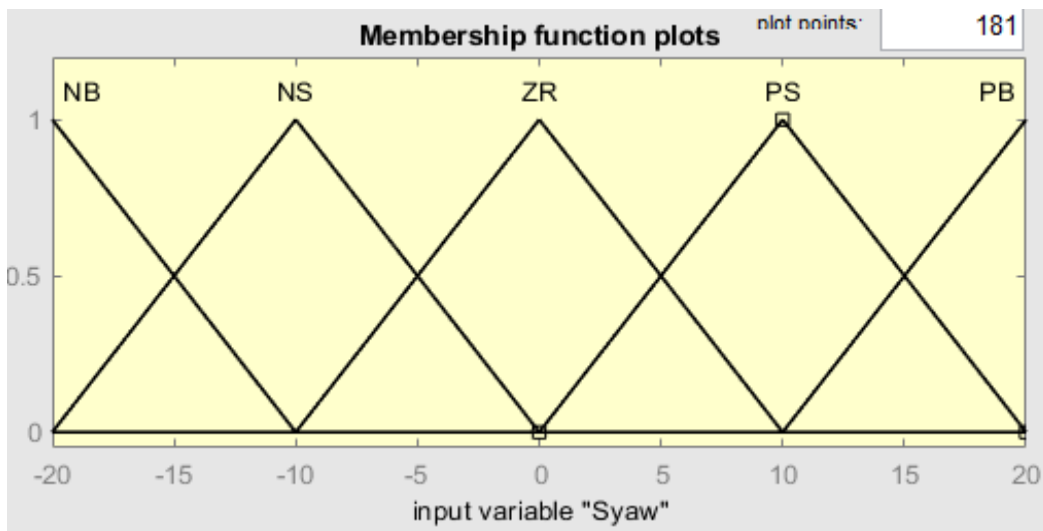
Triangular membership functions (MFs) are indeed one of the most commonly encountered MFs in practice. They are widely used due to their simplicity and ease of interpretation. Triangular MFs are formed using straight lines to define the boundaries of the membership function. The function has three parameters: the left boundary, the peak (also called the center), and the right boundary. The membership value starts at zero at the left boundary, increases linearly to reach its maximum value at the peak, and then decreases linearly to zero at the right boundary.

By adjusting the boundary layer's thickness in real time, the Fuzzy Inference Controller can considerably lessen chattering in the vicinity of equilibrium. Instead of moving down the sliding surface when the conditions are not optimum, the system moves in a chattering manner. Since chattering is an undesirable occurrence, fuzzy logic will be used to fuzzify the sliding surface. To start sliding as quickly as possible, the value of must be larger the greater the distance. To avoid introducing excessive chatter, the gain must be reduced the smaller the distance.

The switching K for both pitch and yaw angle fuzzified into five fuzzy sets named PB, PS, ZR, NS and NB respectively for positive big, positive small, zero, negative small and negative big. Approximate sketch of the fuzzy sets using fuzzy logic toolbox for the input variables S_{pitch} and S_{yaw} are shown in figure 4.3:



(a)



(b)

Figure 4-2 approximate shapes of the fuzzy sets for the normalized variable a, Spitch and b, Syaw

The following rules are applied when mapping the input and output fuzzy sets:

For Pitch angle control: -

- rule 1:* if $s_{pitch}(t)$ is NB then $k(t)$ is PB
rule 2: if $s_{pitch}(t)$ is NS then $k(t)$ is PS
rule 3: if $s_{pitch}(t)$ is ZR then $k(t)$ is ZR
rule 4: if $s_{pitch}(t)$ is PS then $k(t)$ is NS
rule 5: if $s_{pitch}(t)$ is PB then $k(t)$ is NB
- (4.27)

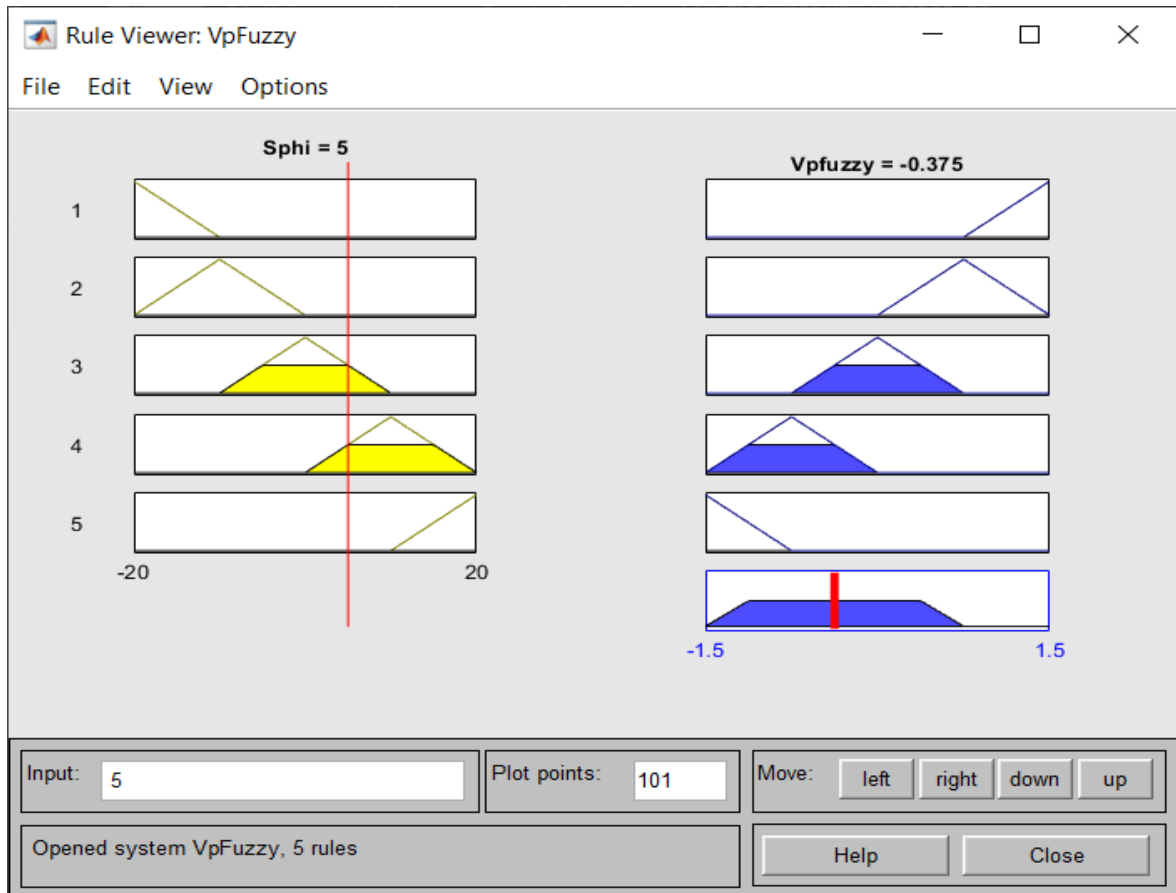


Figure 4-3 Pitch angle fuzzy rule view in MATLAB

For yaw angle control: -

- rule 1:* if $s_{yaw}(t)$ is NB then $k(t)$ is PB
rule 2: if $s_{yaw}(t)$ is NS then $k(t)$ is PS
rule 3: if $s_{yaw}(t)$ is ZR then $k(t)$ is ZR
rule 4: if $s_{yaw}(t)$ is PS then $k(t)$ is NS
rule 5: if $s_{yaw}(t)$ is PB then $k(t)$ is NB
- (4.28)

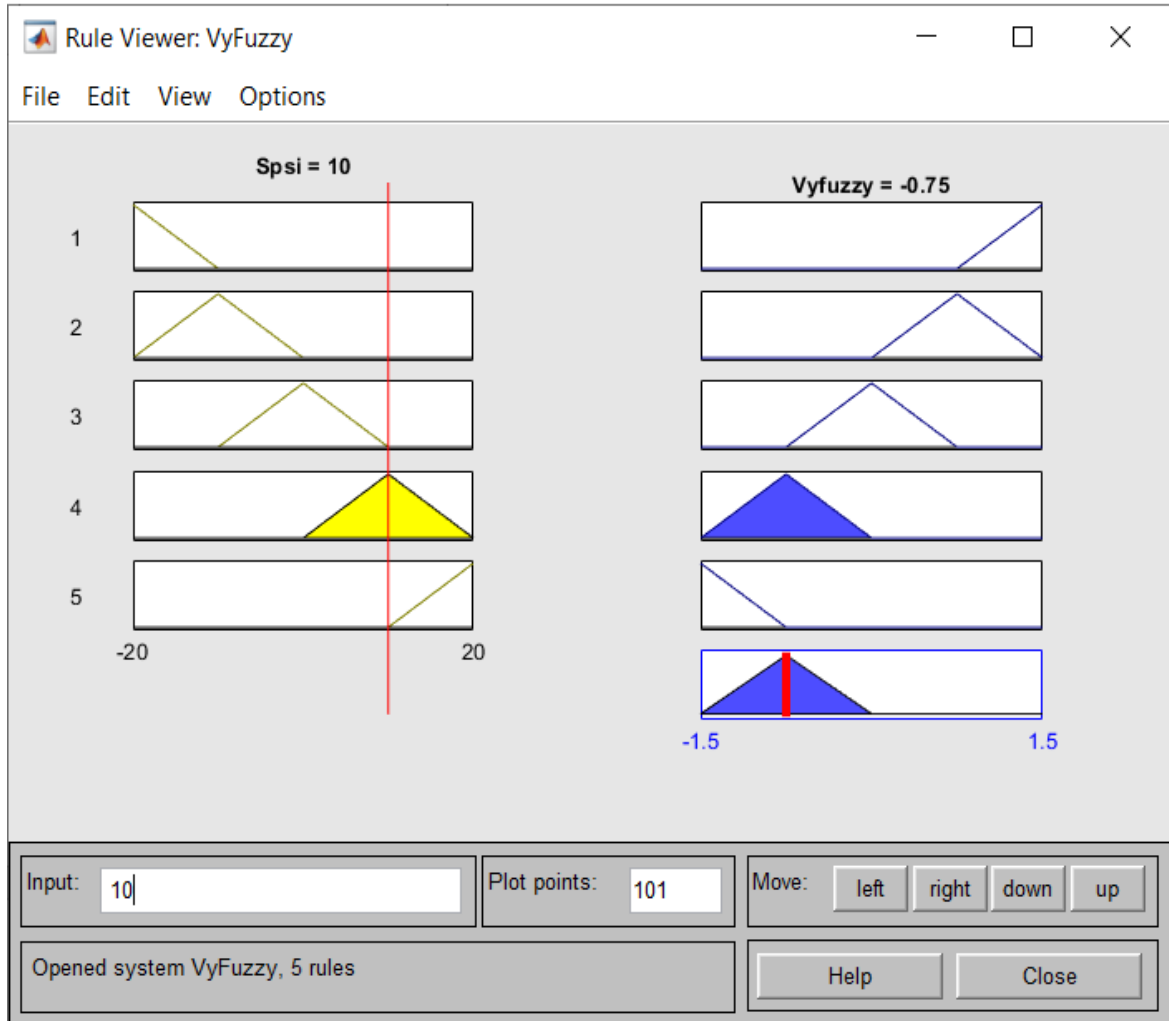


Figure 4-4 Yaw angle fuzzy rule view in MATLAB

4.5 Tuning Parameters of Controllers

Tuning of the controller's parameters is an important activity to meet objective of this work which is obtaining high trajectory tracking accuracy for the 2-DOF helicopter. These parameters can be tuned in two ways: manual and automatic tuning ways. In this thesis work

the controller's parameters are tuned automatically with a metaheuristic optimization algorithm called particle swarm optimization algorithm.

4.5.1 Particle Swarm Optimization Algorithm

Particle Swarm Optimization (PSO) is an optimization technique inspired by nature and developed by Kennedy and Eberhart. It is a powerful and simple algorithm that has been successfully applied in various scientific and engineering fields. At the start, the PSO system comprises a population of solution values randomly chosen from a distribution. Each solution value is called a particle, and it has a randomly selected velocity. Additionally, each particle has a position known as its best position (P_{best}), and particles move while keeping track of their P_{best} . For each P_{best} , there is a corresponding fitness value, and the particle with the highest fitness value is referred to as the global best (G_{best}).

There are two important main equations in the metaheuristic algorithm of the PSO, position and velocity vectors respectively shown below in equations (4.15) and (4.16):

$$v_{p,q}(t+1) = wv_{p,q}(t) + r_1c_1(p_{p,q}(t) - x_{p,q}(t)) + r_2c_2(g_q(t) - x_{p,q}(t)) \quad (4.29)$$

$$x_{p,q}(t+1) = x_{p,q}(t) + v_{p,q}(t+1) \quad (4.30)$$

Here, r_1 and r_2 are two random vectors whose values range between 0 and 1. The parameters c_1 and c_2 represent learning parameters. The variable x represents the position, while the variable v represents the velocity.

4.5.1.1 PSO Algorithm

The PSO algorithm comprises the following steps:

1. Create an array of individuals with randomly assigned positions and velocities that satisfy the inequality constraints.
2. Check whether the equality constraints are satisfied and adjust the solution as necessary.
3. Evaluate the fitness function for each individual.
4. Compare the current fitness value with the individual's previous best value (p_{best}). If the current fitness value is lower, update p_{best} with the current coordinates (positions).

5. Determine the current global minimum fitness value among the current positions.
6. Compare the previous global minimum (g_{best}) with the current global minimum. If the current global minimum is better than g_{best} , update g_{bestx} with the current coordinates (positions).
7. Update the individual's position and velocity according to the PSO algorithm.
8. Repeat Steps 2-7 until the optimization is satisfied or the maximum number of iterations is reached.

The objective function considered for the particle swarm optimization algorithm is Integral of the Absolute Magnitude of the Error (IAE) is determined by equation for all controllers in this thesis:

$$IAE = \int_0^T |e(t)| dt \quad (4.31)$$

Table 4-1 PSO algorithm parameters

PSO parameters	Values
Number of iteration	200
Population size	50

The flow chart of PSO algorithm is illustrated as follows:

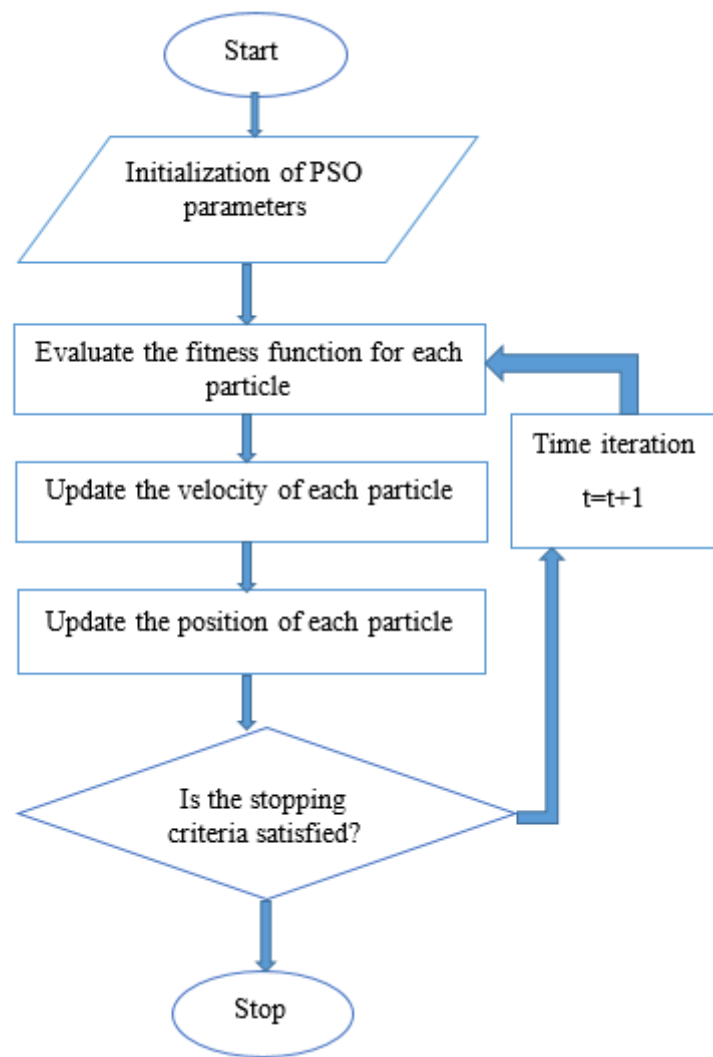


Figure 4-5 Flow chart of PSO algorithm

In this thesis work, the particle swarm MATLAB function from global optimization toolbox is used for tuning of both controllers' parameters.

CHAPTER FIVE

5 RESULT AND DISCUSSION

5.1 Simulation Results

In this section, present and evaluate the effectiveness of the proposed controller by utilizing simulated results within the MATLAB. The proposed controller performed and discussed and provide a comprehensive performance comparison with the SMC through simulation studies. The analysis and discussion of the results contribute to evaluating the effectiveness and potential benefits of the proposed controller in practical applications.

The initial value of variables θ , ϕ , $\dot{\theta}$ and $\dot{\phi}$ which are used to initialized for both sliding mode controller and fuzzy sliding mode controller were set as follow

$$[\theta(0), \phi(0), \dot{\theta}(0), \dot{\phi}(0)]^T = [0,0,0,0]^T$$

The model parameter values of the 2-DOF helicopter system are listed in Table 5.1.

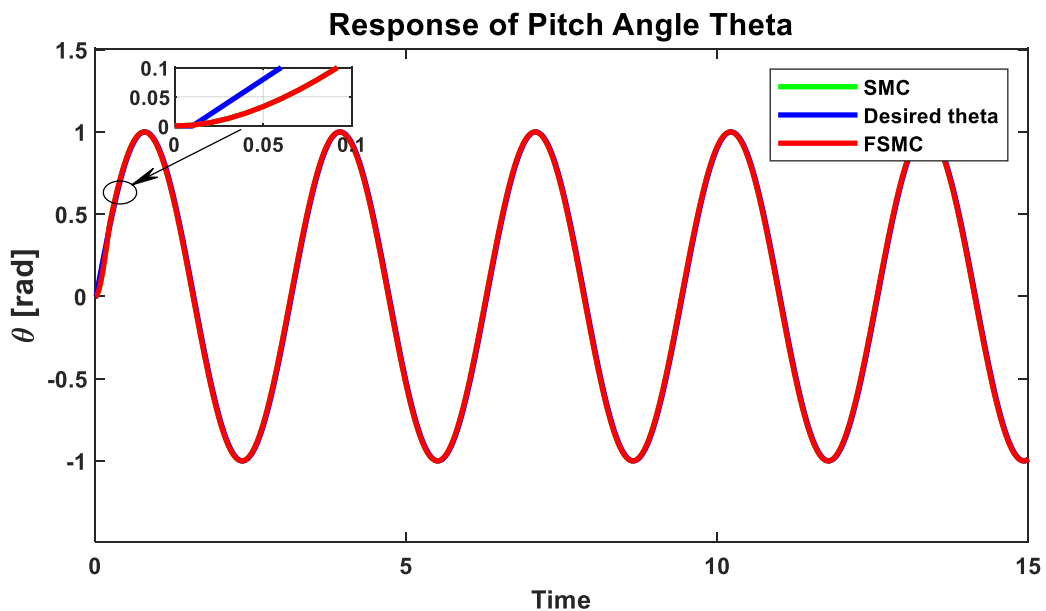
Table 5-1 The 2-DOF helicopter system parameters' numerical values (Amjad J Humaidi and Alaq F Hasan, 2019).

Parameters	Symbols	Values	Units
Total inertia moment about the pitch axis	Jp	0.0384	Kg m ²
Total inertia moment about the yaw axis	Jy	0.0432	Kg m ²
Comparable viscous damping around the pitch axis	Bp	0.800	N/V
Comparable viscous damping around the yaw axis	By	0.318	N/V
Pitch axis thrust force constant from pitch motor/propeller	Kpp	0.204	Nm/V
Pitch axis thrust force constant from yaw motor/propeller	Kyy	0.072	Nm/V
Thrust torque constant from the yaw motor/propeller operating on the pitch axis	Kpy	0.0068	Nm/V
Thrust torque constant from the pitch motor/propeller operating on the yaw axis	Kyp	0.0219	Nm/V
The helicopter's total moving mass	m	1.3872	Kg

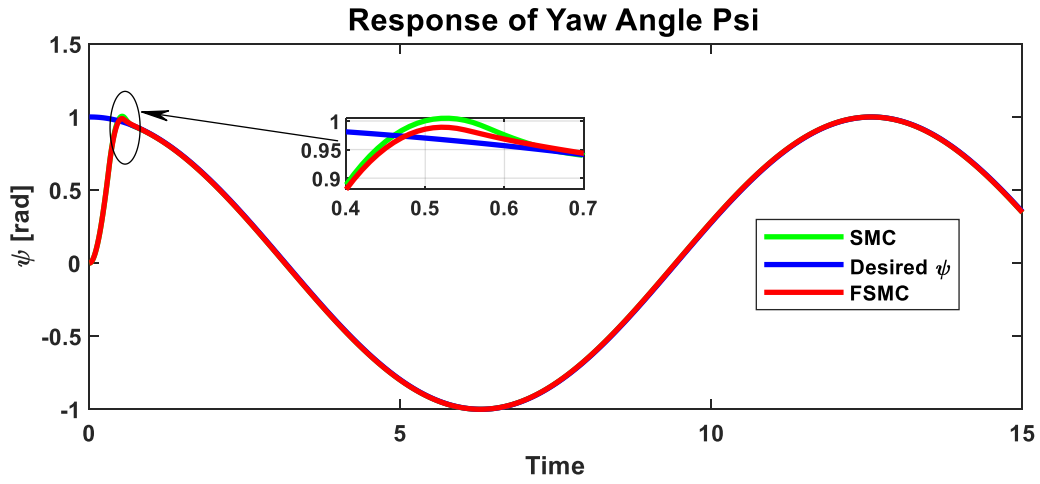
Center of mass length from pitch axis along the helicopter body	lcm	0.186	M
Acceleration due to gravity	g	9.81	m/s ²
voltage of the pitch motor	V _p	+25	V
voltage of the yaw motor	V _y	±15	V

Figure 5.1 and 5.2 displays the regulated system's dynamic reaction. In order to present the tracking performance of the 2-DOF helicopter system based on both conventional and fuzzy SMCs, with the same initial conditions, Figure 5.1(a) shows the response of pitch angle, Figure 5.1(b) shows the response of yaw angle, Figure 5.2(a) shows the response of angular velocity yaw angle, and Figure 5.2(b) shows the response of angular velocity pitch angle.

The system is simulated for 15 seconds. Following the simulation, system angle graphs were generated, comparing the actual and reference angles during the simulation. The behavior of the pitch behavior is represented by the θ angle, following the reference desired trajectory. The ϕ angle represents the yaw angle behavior, following the reference desired trajectory. Figure 5-1 depicts how the actual system angle accurately and rapidly tracks the provided reference angle.



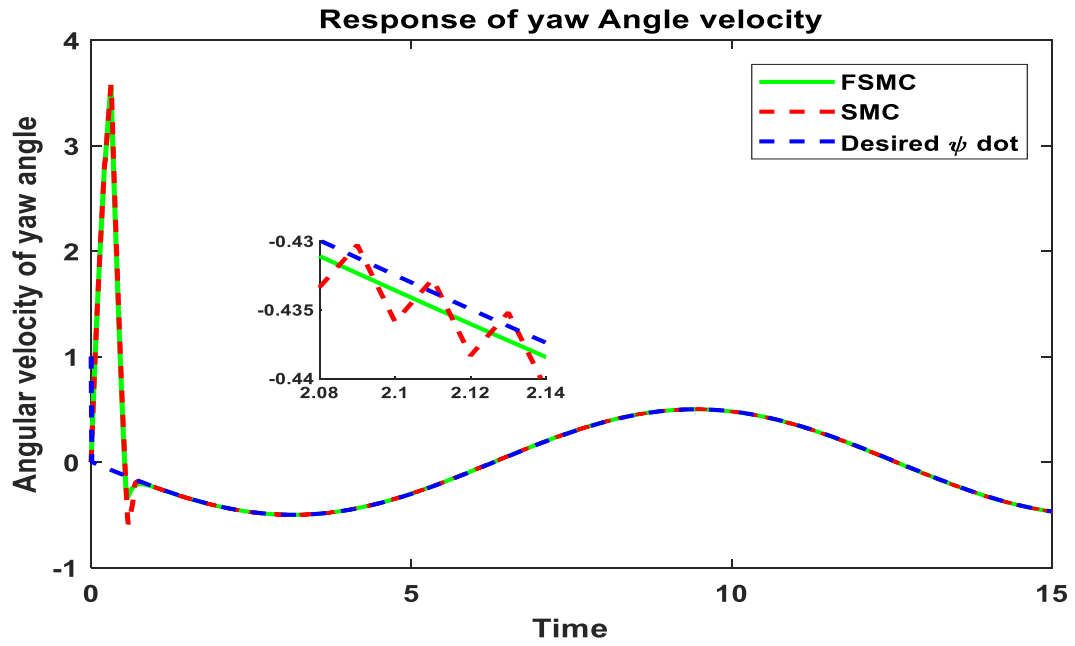
(a)



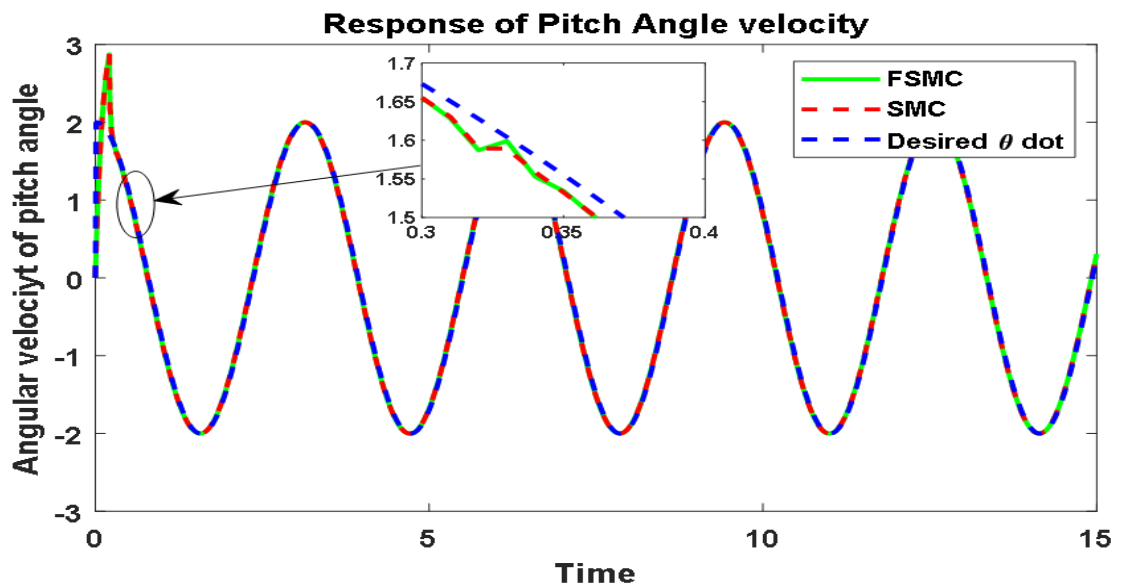
(b)

Figure 5-1 Dynamic responses of the (a) pitch angle, (b) yaw angle

During simulation, the yaw angular velocity was represented by $\dot{\phi}$, its reference value is desired trajectory, as it can be seen in Figure 5-2(a) the yaw angle simulated by both SMC and FSMC it is clearly seen that the actual or real value corresponding to SMC has more chattering and the actual or real value of the system corresponding to FSMC tracked the given reference accurately and gave a fast response without chattering. Similarly, the angular velocity observed during simulation corresponds to the pitch angular velocity, as it follows the reference desired trajectory. Likewise, the angular velocity corresponding to SMC and FSMC observed during simulation, it denotes the rate of change of pitch angle, as it follows the intended reference path, just like illustrated in the figure below.



(a)

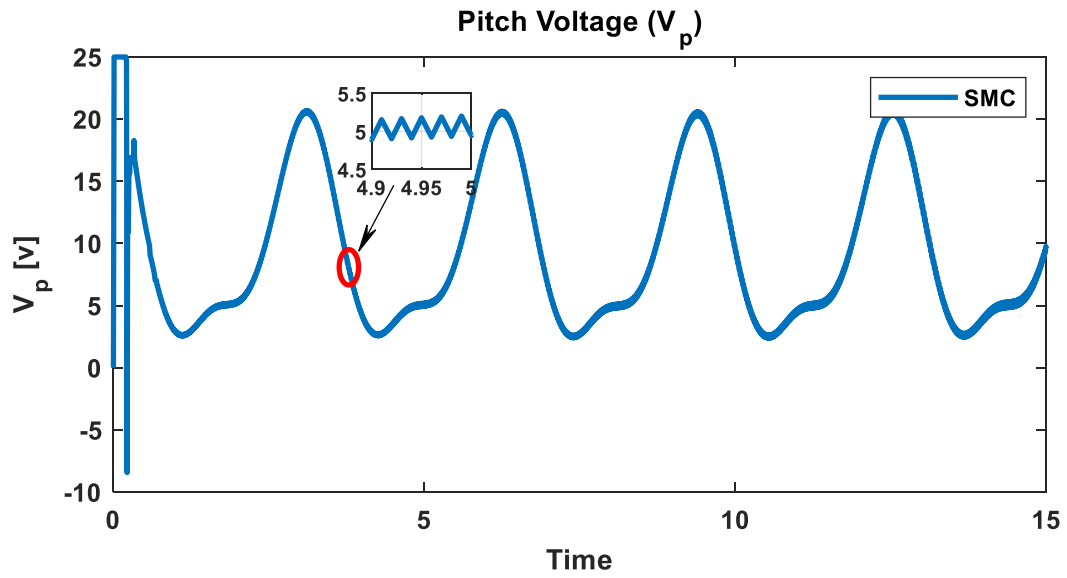


(b)

Figure 5-2 Dynamic response of the (a) angular velocity of pitch angle and (b) angular velocity of yaw angle

In Figure 5.3 (a), and (b), the corresponding control signals for pitch angle resulting from each controller are displayed. Simulation shown that, the controller for pitch angle resulting

from SMC is more chattering and the controller for pitch angle resulting from FSMC is smooth. In comparison to the controller resulting from classical SMC, the fuzzy SMC can not only assure speedy and exact trajectory tracking of a 2-DOF helicopter, but can also effectively eliminate chattering and make the control input smooth. Chattering is a phenomenon commonly associated with sliding mode control. It refers to high-frequency oscillations or rapid switching between control modes near the sliding surface. Chattering that came from the SMC lead to undesirable vibrations and can be detrimental to the mechanical components of the helicopter, such as actuators and joints. It can also introduce noise into the control system, affecting the stability and performance of the helicopter. So, to mitigate the disadvantages of sliding mode control in 2-DOF helicopters, another approach is to combine SMC with fuzzy logic control. This hybrid approach, known as Fuzzy Sliding Mode Control (FSMC), can help reduce chattering while maintaining robustness.



(a)

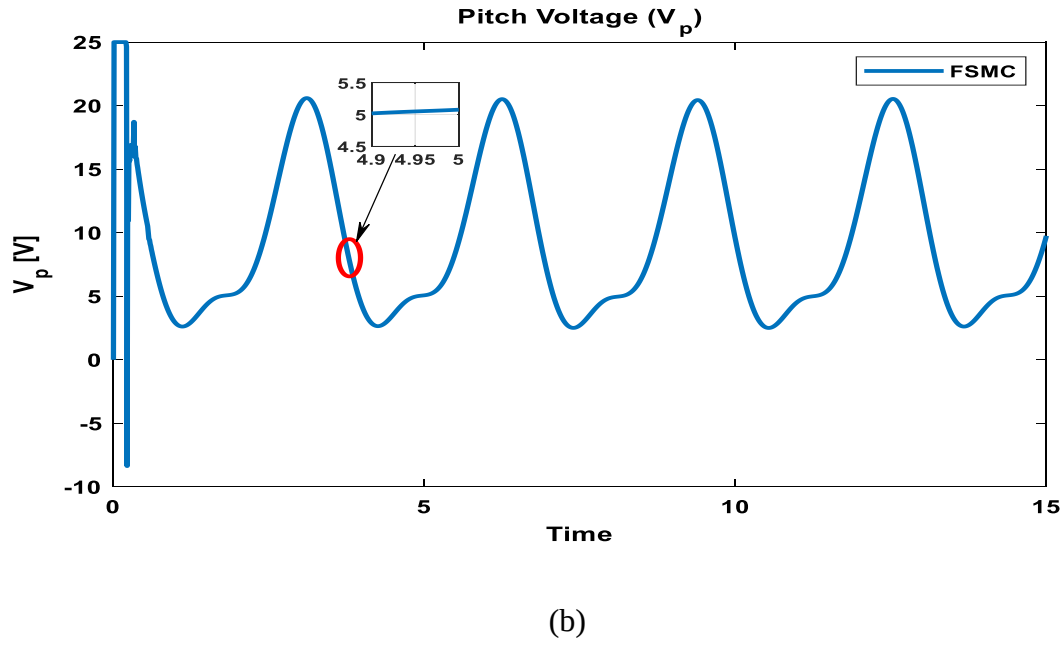
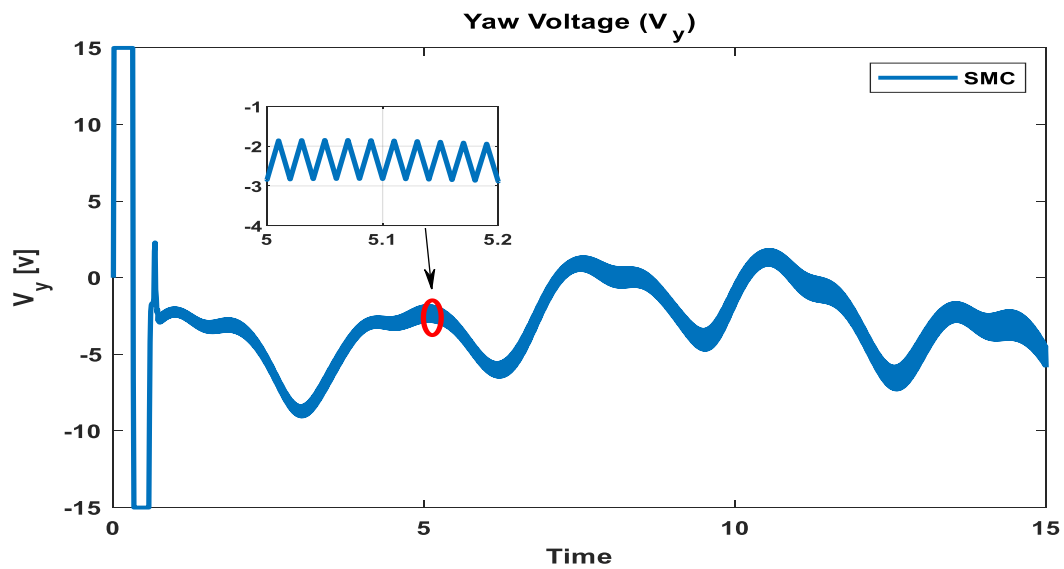
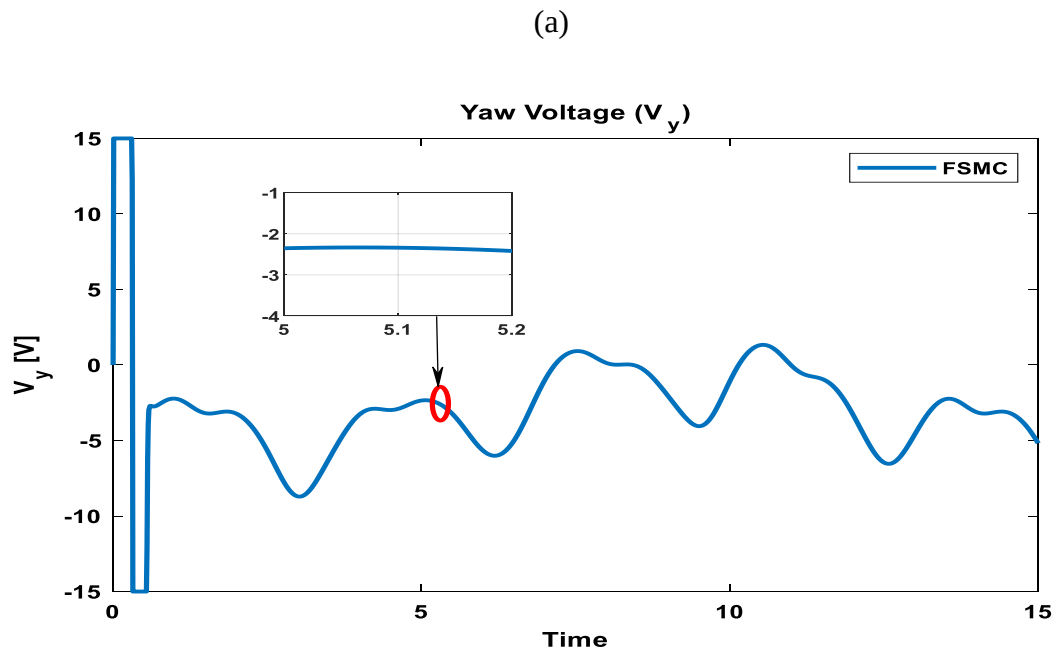


Figure 5-3 The corresponding control signals: (a) control signal V_y due to sliding mode controller and (b) control signal V_y due to fuzzy sliding mode controller

Similarly, In Figure 5.4 (a), and (b), the corresponding control signals for yaw angle resulting from each controller are displayed. Simulation shown that, the controller for yaw angle resulting from SMC is more chattering and the controller for yaw angle resulting from FSMC is smooth. In comparison to the controller resulting from classical SMC, the fuzzy SMC can not only assure speedy and exact trajectory tracking of a 2-DOF helicopter, but can also effectively eliminate chattering and make the control input smooth.





(b)

Figure 5-4 The corresponding control signals: (a) control signal V_p due to sliding mode controller, (b) control signal V_p due to fuzzy sliding mode controller

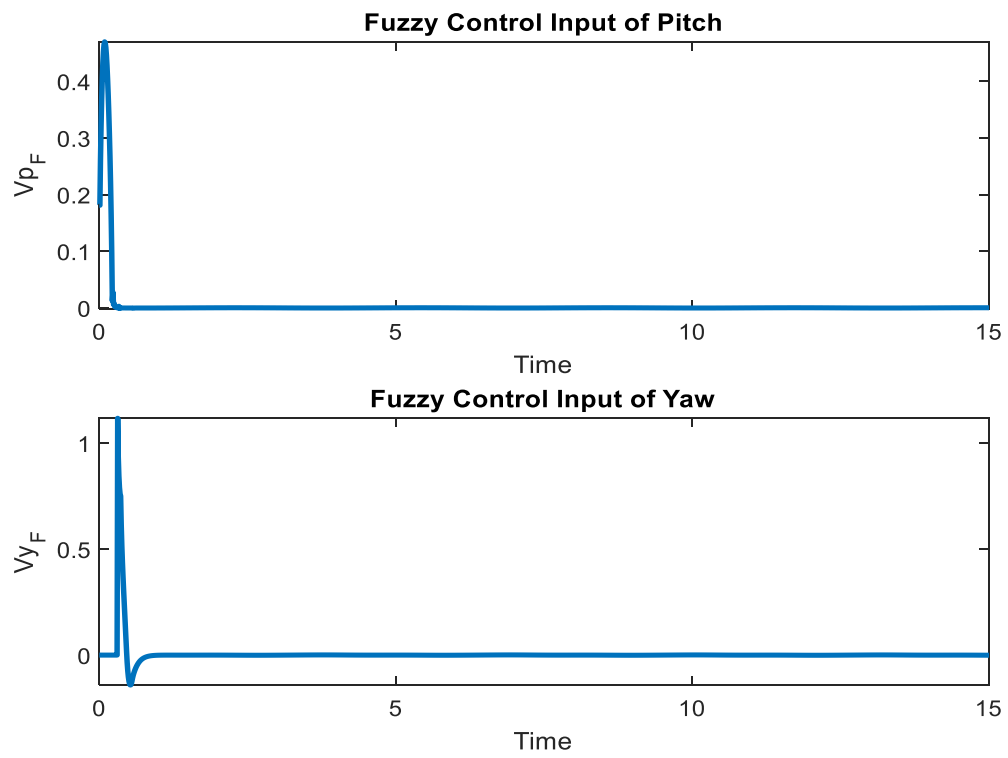


Figure 5-5 fuzzy control input for both pitch and yaw voltage without disturbance

Figure 5.3 and Figure 5.4 displays the control signals due to both controllers. This figure shows that the response of the control signal based on classical SMC still exhibits the chattering phenomena. On the other hand, the response of the control signal based on fuzzy SMC has significantly reduced chattering. Consequently, the fuzzy controller has improved chattering more than the non-fuzzy controller.

When considering a two-degree-of-freedom (2-DOF) helicopter model, the addition of sinusoidal disturbances, such as wind, can have an impact on its behavior. The presence of wind as a disturbance introduces external forces and moments that can affect the helicopter's stability and control. The controller tries to address the impact of wind disturbances on a 2-DOF helicopter.

The sinusoidal disturbance applies to the system considering as wind is:

UP1 represents the disturbance added to the pitch motion and UP2 represents disturbance added to the yaw motion (Liu, Intelligent control design and MATLAB simulation, 2018)

$$\begin{aligned} UP1 &= \sin(t) \\ UP2 &= \cos(t) \end{aligned} \tag{5.1}$$

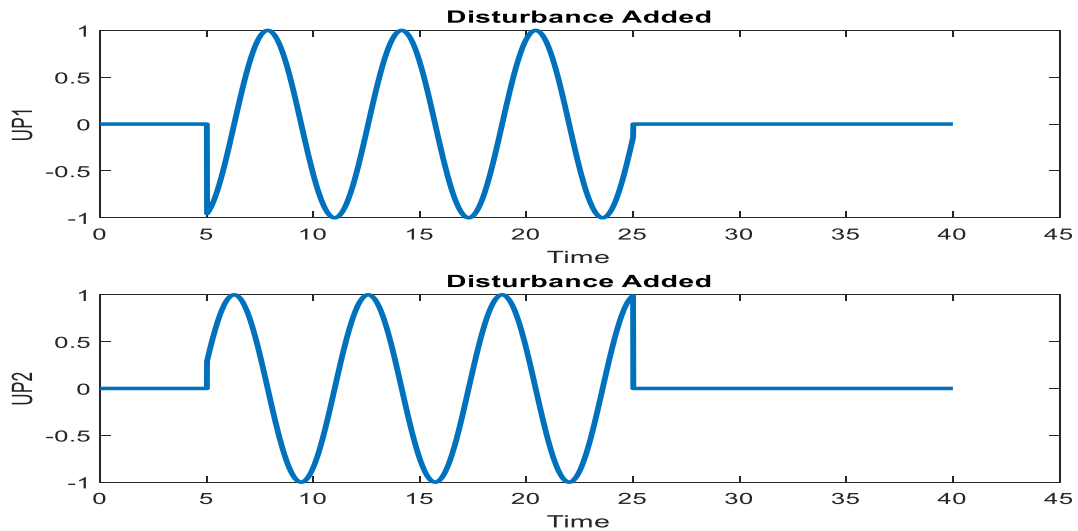
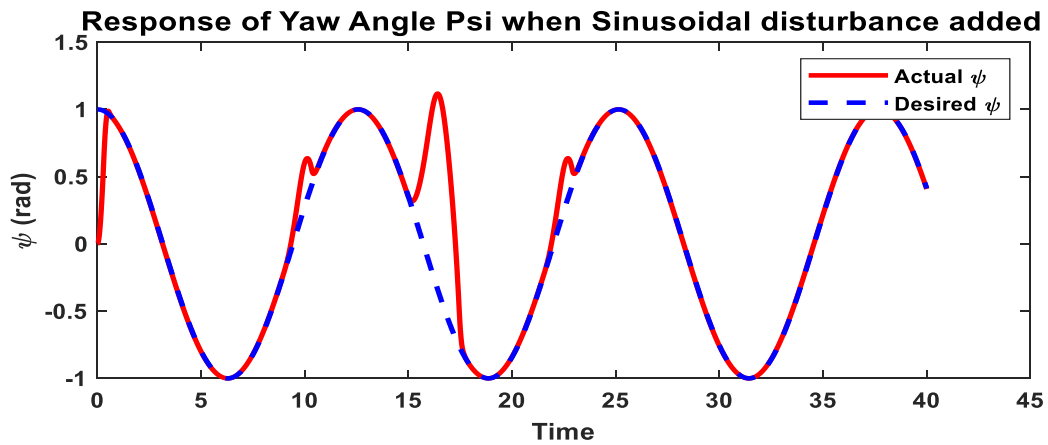


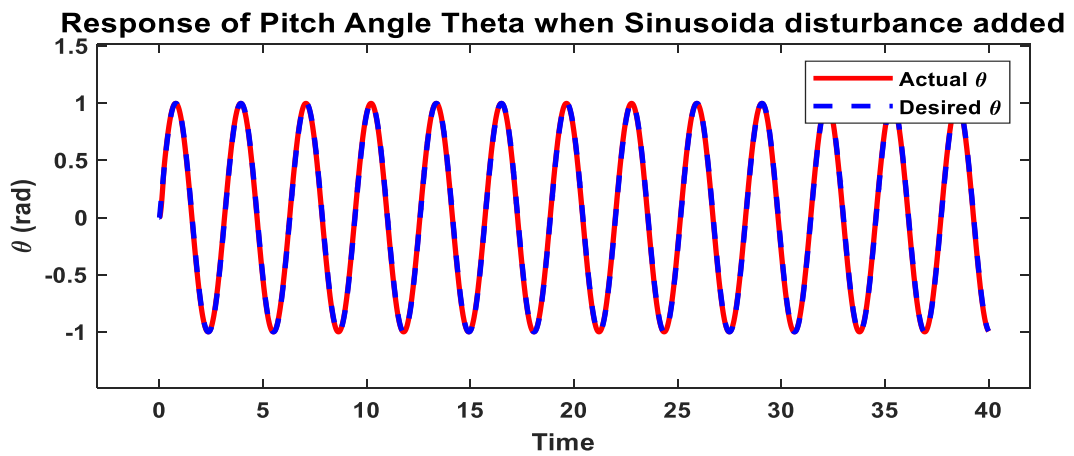
Figure 5-6 sinusoidal disturbance added both pitch and yaw

A fuzzy sliding mode controller is a type of controller that combines the concepts of fuzzy logic and sliding mode control. It is designed to handle systems disturbances by adjusting the control action in a robust manner. By combining fuzzy logic reasoning, sliding mode control and disturbance estimation the fuzzy sliding mode controller is able to mitigate the effects of the sinusoidal disturbance and return the pitch angle and yaw angle of the system to their desired reference values with smooth control action.

Figure 5.7 (a) and (b) shows when sinusoidal disturbance is added the controller actively reject or minimize the impact of the sinusoidal disturbance on the system's response. This is typically achieved by adjusting the control inputs based on the disturbance information.



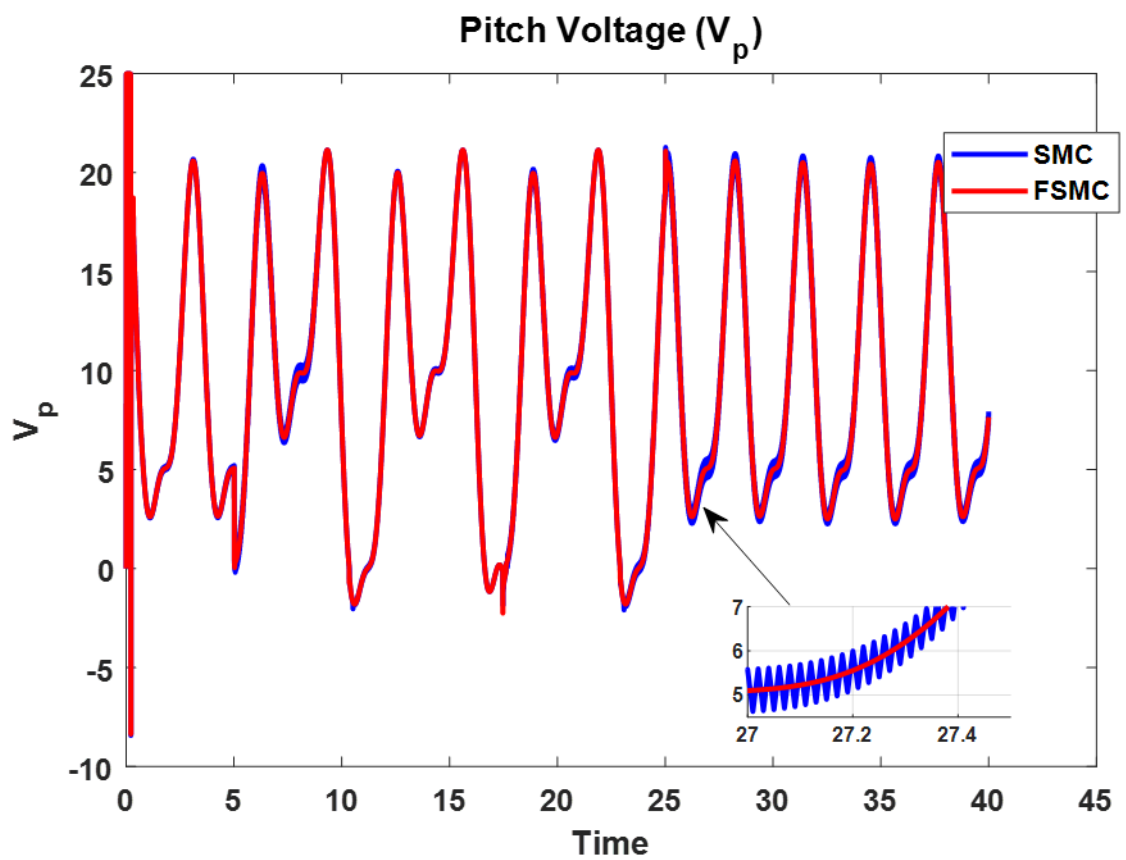
(a)



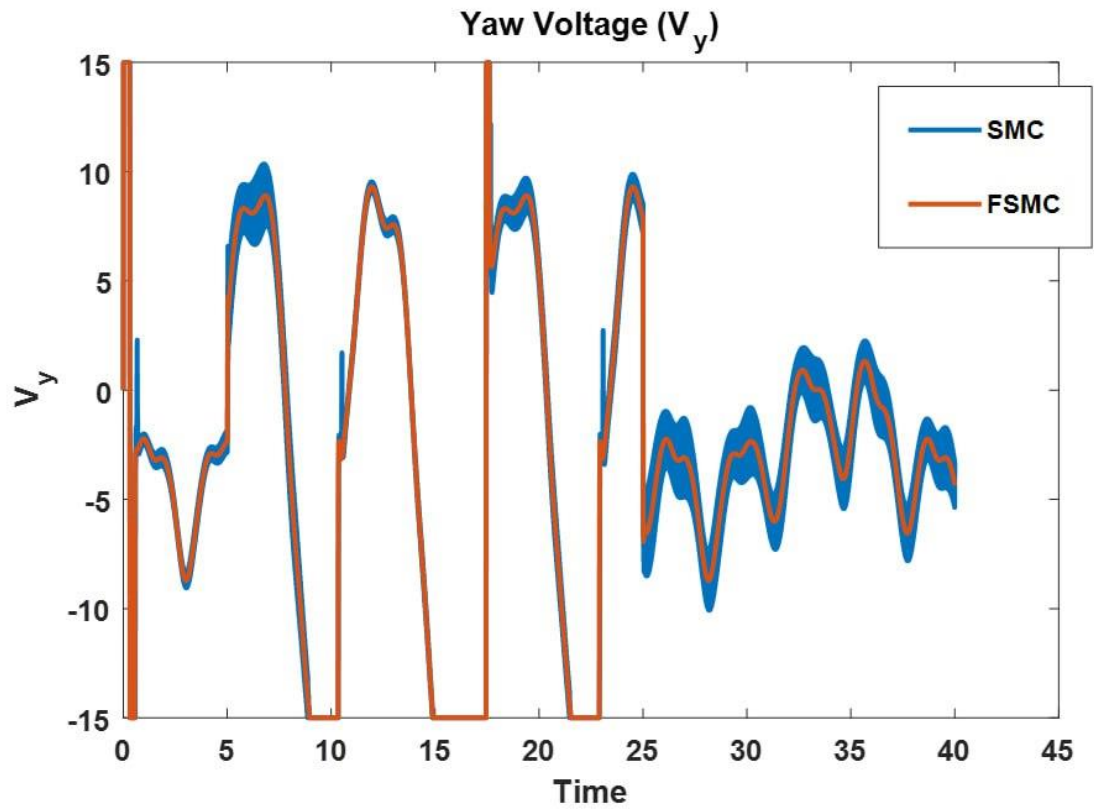
(b)

Figure 5-7 Dynamic response of the (a) yaw angle with disturbance (b) pitch angle with disturbance

In Figure 5.8 shows that the control input for both pitch and yaw angle in the presence of disturbance for both the sliding mode control (SMC) and fuzzy sliding mode control (FSMC) methods. It can be clearly shown that the FSMC method effectively removes or reduces chattering in the presence of disturbances, resulting in a smoother control input compared to the SMC method. This suggests that the FSMC method is more suitable for controlling the 2-DOF helicopter system in the presence of disturbances.



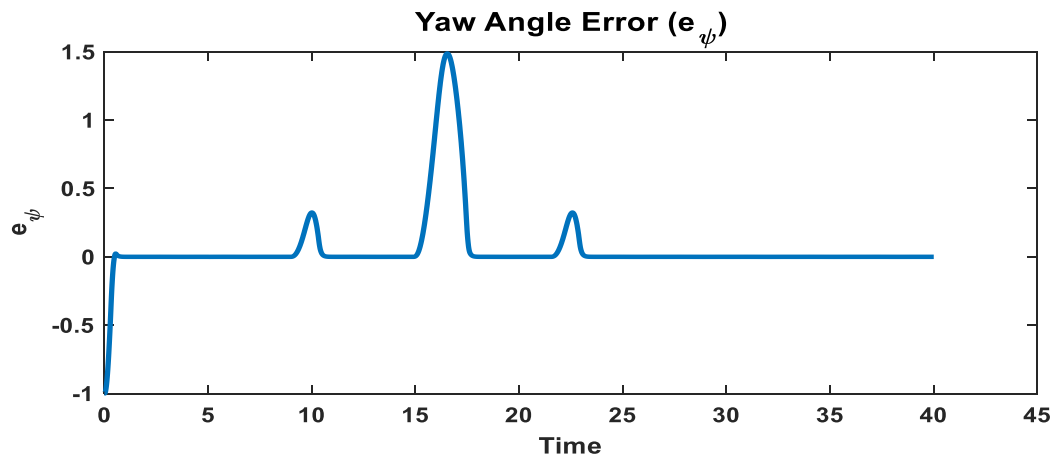
(a)



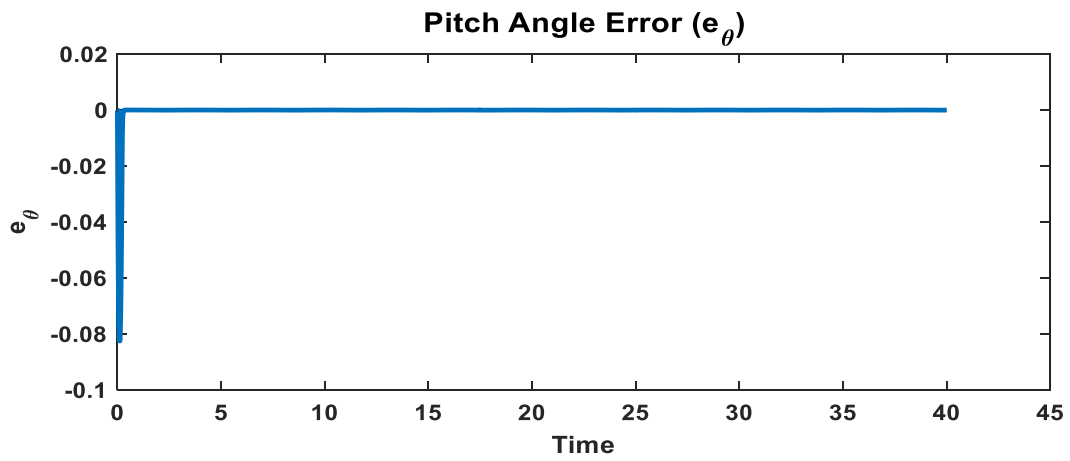
(b)

Figure 5-8 control input response with sinusoidal disturbance is added (a) pitch angle control input, (b) yaw angle control input

Figure 5.9 shows Yaw and pitch angle error refer to the difference between the desired or reference angles and the actual angles of yaw and pitch, the controller continuously adjusts the control inputs based on the error between the desired reference and the system's actual output to drive the error towards zero.



(a)



(b)

Figure 5-9 (a) yaw angle error (b) pitch angle error

Below figure 5.10 shows control inputs used in conjunction with fuzzy rules and fuzzy inference systems to determine the appropriate control actions based on the system's current state and the desired control objective.

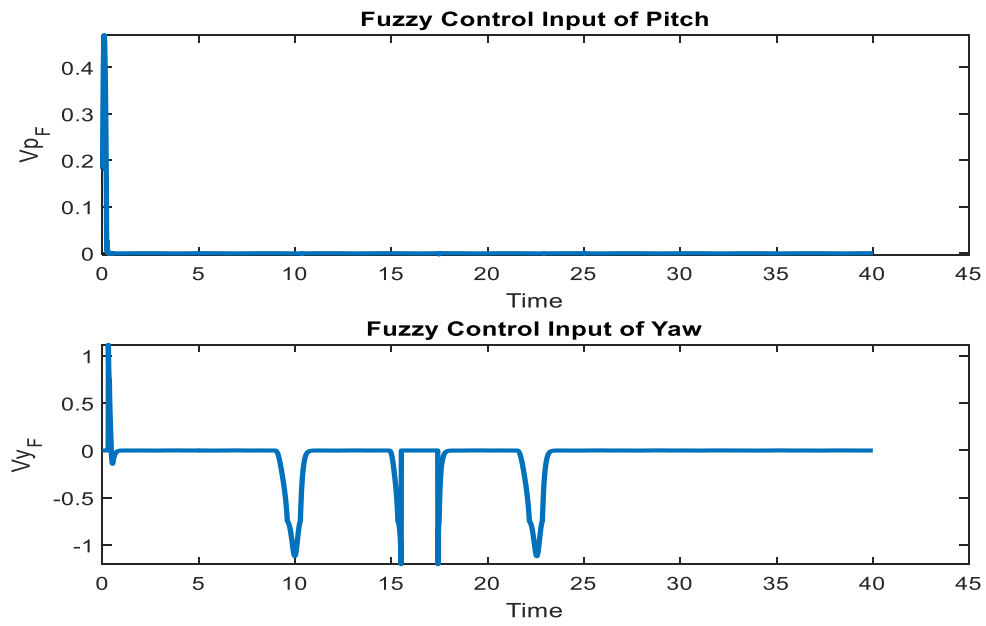


Figure 5-10 Fuzzy control input for pitch and yaw angle motion when disturbance added

The objective function minimized to obtain optimal parameter values of SMC controller is graphed as shown below Figure 5.11.

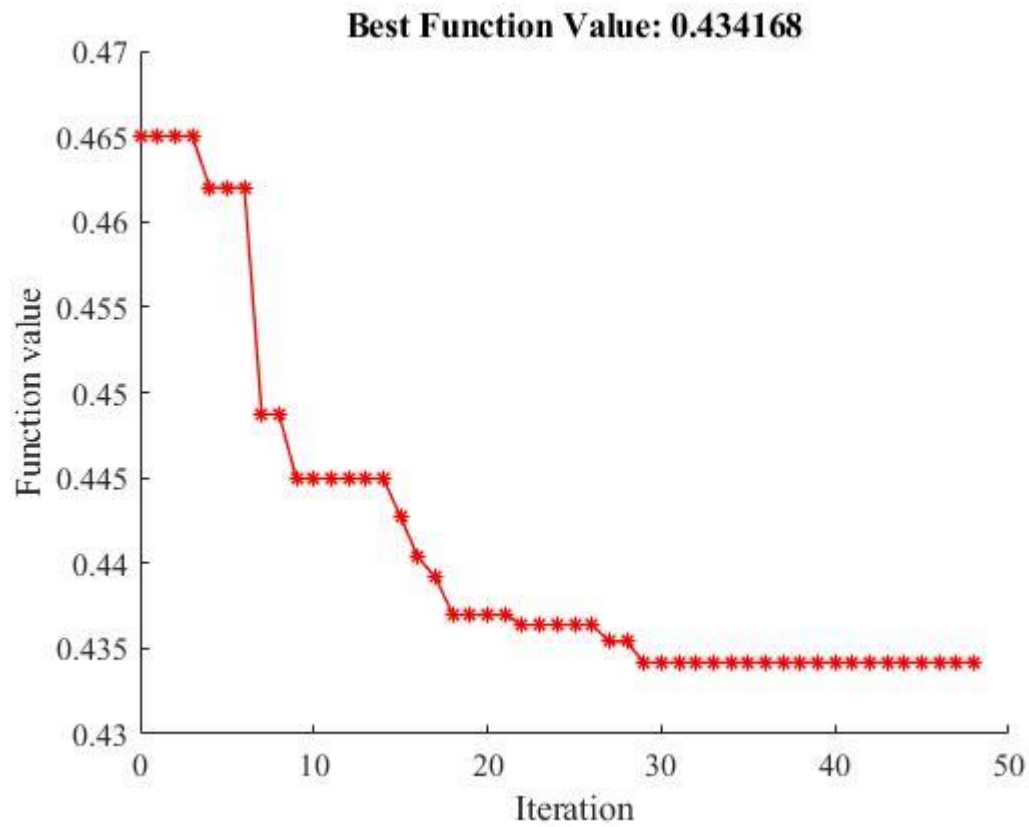


Figure 5-11 Objective function minimized for optimal parameters in pitch and yaw controller of SMC

Table 5-2 Upper and lower bounds of PSO parameters for SMC controller

Parameter	Lower bound	Upper bound
C1	0	50
C2	0	50
K1	0	50
K2	0	50
K3	0	50
K4	0	50

Table 5-3 Optimal design parameter values for SMC

Designed parameters	Value
C1	48.886
C2	8.0911
K1	50
K2	46.2291
K3	50
K4	9.9101

The objective function minimized to obtain optimal parameter values of FSMC controller is graphed as shown below Figure 5.12.

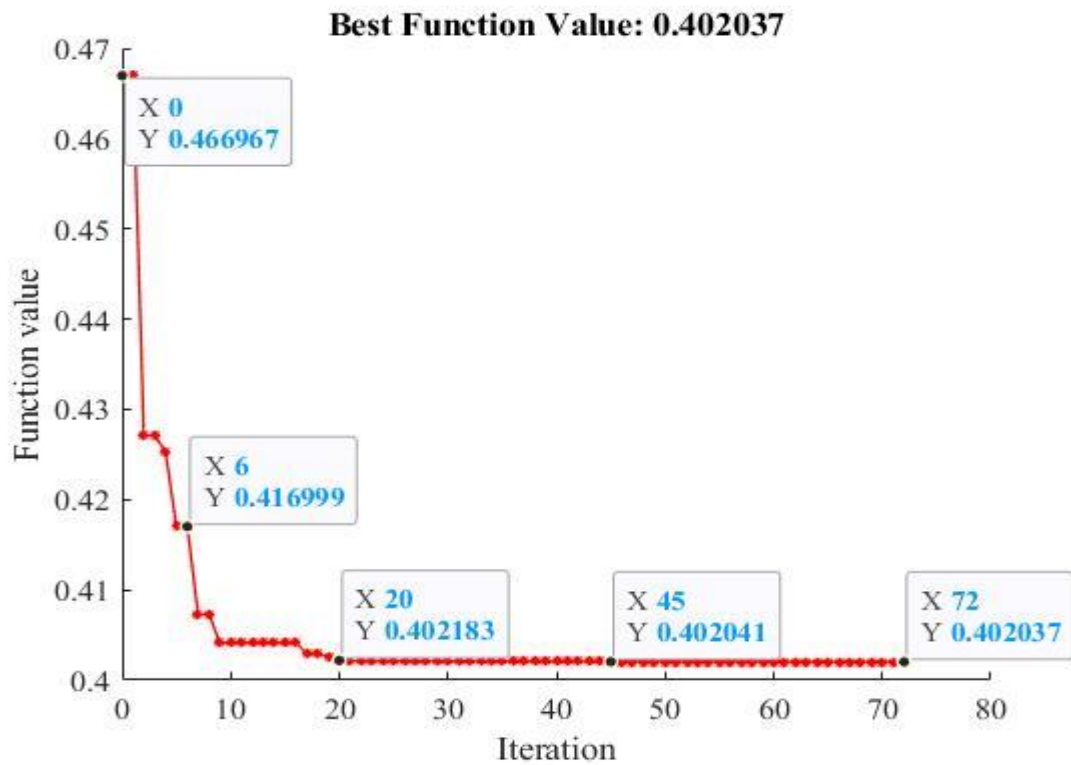


Figure 5-12 Objective function minimized for optimal parameters in pitch and yaw controller of FSMC

Table 5-4 Upper and lower bounds of PSO parameters for FSMC controller

Parameter	Lower bound	Upper bound
C1	0	200
C2	0	200
K1	0	200
K2	0	200
K3	0	200
K4	0	200

Table 5-5 Optimal design parameter values for FSMC

Designed parameters	Value
C1	78.1498
C2	58.5526
K1	86.2629
K2	0.0019
K3	11.3589
K4	10.4467

Table 5-6 Performance of 2-DOF helicopter system based on controller

Controller type	IAE
SMC	0.4342
FSMC	0.4020

6 CONCLUSION AND FUTURE RECOMMENDATION

6.1 Conclusion

In this thesis describes the development and application of a fuzzy sliding mode control method for trajectory tracking of a 2-DOF helicopter. This method combines the advantages of sliding mode control (SMC) and fuzzy control. Sliding mode control is a control technique that aims to drive the system's state onto a predefined sliding surface, where the system dynamics are known and can be controlled effectively. However, SMC can result in chattering, which is a high-frequency oscillation of the control input. Chattering can lead to wear and tear on mechanical systems and can also introduce instability. FSMC method was found to make the control input smoother, which is desirable in control systems as it reduces the abrupt changes in the control signal. The gain parameters of both controllers have been tuned by a metaheuristic algorithm called PSO. As simulation result shows in sinusoidal trajectory the steady state error is 0.402037. In this study also mentions that the FSMC method exhibited better chattering suppression characteristics even in the presence of disturbances. This suggests that the FSMC method is robust and can handle external disturbances while maintaining smooth control. Based on the simulated results, the FSMC method showed improved performance in terms of chattering reduction, and control input smoothness compared to the SMC method for the 2-DOF helicopter system. The gain parameters of both controllers have been tuned by a metaheuristic algorithm called PSO. As simulation result shows in sinusoidal trajectory the steady state error is 0.402037.

6.2 Future Recommendations

In this thesis, the designed controllers are used for the system in analytical approach. Numerous areas of opportunity exist for testing the designed controllers on a real helicopter. Here are some recommendations to make the robot more attractive:

- Design a real 2-DOF helicopter and implementing a controller.
- Consider mass of payload in the dynamics of the system.
- Use estimation techniques like state observer, kalman filter for estimating the states of the helicopter dynamic system.

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APPENDIX

Appendix A

Developed dynamic model of the helicopter:

$$\frac{d}{dt} \begin{bmatrix} \hat{\phi} \\ \hat{\theta} \\ \hat{\psi} \\ \hat{x} \\ \hat{y} \\ \hat{z} \\ \hat{w}_x \\ \hat{w}_y \\ \hat{w}_z \\ \hat{\dot{x}} \\ \hat{\dot{y}} \\ \hat{\dot{z}} \\ \hat{g} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \cos(\varphi) & -\sin(\varphi) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \sin(\varphi) & \cos(\varphi) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \hat{\phi} \\ \hat{\theta} \\ \hat{\psi} \\ \hat{x} \\ \hat{y} \\ \hat{z} \\ \hat{w}_x \\ \hat{w}_y \\ \hat{w}_z \\ \hat{\dot{x}} \\ \hat{\dot{y}} \\ \hat{\dot{z}} \\ \hat{g} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \\ -\frac{c_1\sigma_2}{\sigma_1} & -\frac{c_1\sigma_3}{\sigma_1} & -\frac{a_1\sigma_2}{\sigma_1} + \frac{b_1\sigma_3}{\sigma_1} & \dots & -\frac{c_n\sigma_2}{\sigma_1} & -\frac{c_n\sigma_3}{\sigma_1} & -\frac{a_n\sigma_2}{\sigma_1} + \frac{b_n\sigma_3}{\sigma_1} \\ \frac{c_1\sigma_4}{\sigma_1} & \frac{c_1\sigma_2}{\sigma_1} & -\frac{b_1\sigma_2}{\sigma_1} - \frac{a_1\sigma_4}{\sigma_1} & \dots & \frac{c_n\sigma_4}{\sigma_1} & \frac{c_n\sigma_2}{\sigma_1} & -\frac{b_n\sigma_2}{\sigma_1} - \frac{a_n\sigma_4}{\sigma_1} \\ -\frac{b_1}{I_{zz}} & \frac{a_1}{I_{zz}} & 0 & \dots & -\frac{b_n}{I_{zz}} & \frac{a_n}{I_{zz}} & 0 \\ \frac{1}{m} & 0 & 0 & \dots & \frac{1}{m} & 0 & 0 \\ 0 & \frac{1}{m} & 0 & \dots & 0 & \frac{1}{m} & 0 \\ 0 & 0 & \frac{1}{m} & \dots & 0 & 0 & \frac{1}{m} \\ 0 & 0 & 0 & \dots & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_{1x} \\ f_{1y} \\ f_{1z} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ f_{nx} \\ f_{ny} \\ f_{nz} \end{bmatrix}$$

Where

$$\begin{aligned}\sigma_1 &= I_{xx}I_{yy}\cos(\varphi)^4 + 2I_{xx}I_{yy}\cos(\varphi)^2\sin(\varphi)^2 + I_{xx}I_{yy}\sin(\varphi)^4 \\ \sigma_2 &= I_{xx}\cos(\varphi)\sin(\varphi) - I_{yy}\cos(\varphi)\sin(\varphi) \\ \sigma_3 &= I_{yy}\cos(\varphi)^2 + I_{xx}\sin(\varphi)^2 \\ \sigma_4 &= I_{xx}\cos(\varphi)^2 + I_{yy}\sin(\varphi)^2\end{aligned}$$

Appendix B

Controller mat lab code

```
Kth = 78.1498; Kps = 58.5526; %Optimized
Hth = 86.2629; Hps = 11.3589;

%%%% Load Fuzzy Control
fis1= readfis('VpFuzzy');
fis2= readfis('VyFuzzy');

%%%% Desired Angles Sinusoidal
th_des = sin(2*t); ps_des = cos(0.5*t);
th_des_dot = 2*cos(2*t); ps_des_dot = -0.5*sin(0.5*t); %first derivative of desired
values

%%%% Error
eth = x(1) - th_des;
eps = x(3) - ps_des;

%%%% Error Derivatives
eth_dot = x(2) - th_des_dot;
eps_dot = x(4) - ps_des_dot;

%%%% Sliding Surface
Sth = Kth*eth + eth_dot;
Sps = Kps*eps + eps_dot;

Vp_Fuzzy= evalfis(fis1,Sth);
Vy_Fuzzy= evalfis(fis2,Sps);

buf= [t x' th_des ps_des u' eth eps Vp_Fuzzy Vy_Fuzzy];

for i=1:loop
    %%%%% Desired Angles Sinusoidal
    th_des = sin(2*t); ps_des = cos(0.5*t);
    th_des_dot = 2*cos(2*t); ps_des_dot = -0.5*sin(0.5*t); %first derivative of desired
    values
```

```
th_des_2dot= -4*sin(2*t); ps_des_2dot= -0.25*cos(0.5*t); %second derivative of
desired values
```

```
%%% WIND DISTURBUNCES OF SIGNUM FUNCTION
```

```
if (t<5)
    UP1= 0; UP2= 0;
end
if(t>5&& t<25)
    UP1= sin(t); UP2= cos(t);
end
if (t>25)
    UP1= 0; UP2= 0;
end
```

```
a1 = Jp + m*lcm^2;
a2 = -m*g*lcm*cos(x(1)) - m*lcm^2*x(4)^2*sin(x(1))*cos(x(1));
b1 = Jy + m*lcm^2*cos(x(1))*cos(x(1));
b2 = 2*m*lcm^2*x(2)*x(4)*sin(x(1))*cos(x(1));
```

```
%%% Error
```

```
eth = x(1) - th_des;
eps = x(3) - ps_des;
```

```
%%% Error Derivatives
```

```
eth_dot = x(2) - th_des_dot;
eps_dot = x(4) - ps_des_dot;
```

```
%%% Sliding Surface
```

```
Sth = Kth*eth + eth_dot;
Sps = Kps*eps + eps_dot;
```

```
%%% Maximum and Minimum value of the Input Voltages
```

```
Vp_max = 25; Vp_min= -25;
Vy_max = 15; Vy_min= -15;
```

```
%%% Fuzzy Approximations
```

```
Vp_Fuzzy= evalfis(fis1,Sth);
Vy_Fuzzy= evalfis(fis2,Sps);
```

```
%%% Sliding Mode based Control Inputs
```

```
Vp = (a1/Kpp)*(th_des_2dot - Kth*eth_dot - Hth*Sth - Vp_Fuzzy - (Kpy/a1)*u(2) +
(1/a1)*(Bp*x(2) + UP1) - a2/a1);
Vy = (b1/Kyy)*(ps_des_2dot - Kps*eps_dot - Hps*Sps - Vy_Fuzzy - (Kyp/b1)*u(1) +
(1/b1)*(By*x(4) + UP2) - b2/b1);
```

```
%%%Control Input Limits
```

```

Vp = max(min(Vp,Vp_max),Vp_min);
Vy = max(min(Vy,Vy_max),Vy_min);

u= [Vp;Vy];
x   = RK4(@HelicModel,t,x,u,h);
t   = t+h;
buf = [buf;t x' th_des ps_des u' eth eps Vp_Fuzzy Vy_Fuzzy];
end

```