THE EFFECT OF INJECTED THERMAL LIGHT IN CAVITY ON A DEGENERATE THREE LEVEL LASER COUPLED TO SQUEEZED VACUUM RESERVOIR

By

Daba Alemu Galmecha

A Thesis Submitted to
the Department of Applied Physics Program
School of Applied Natural Science

Presented in Partial Fulfilment of the Requirement for the Degree of Master's of science in Physics

Office of Graduate Studies
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Examination Committee Approval Sheet

This is to certify that the thesis prepared by **Daba Alemu:-The Effect of Injected Thermal Light in Cavity on a Degenerate Three Level laser Coupled to Squeezed Vacuum Reservoir** and submitted for partial fulfilment of the requirement for the degree of master of science in physics complies with regulation of university meets the accepted standard with respect to originality and ability.

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DECLARATION

I hereby declare that this Msc thesis is my original work and has not been presented for a degree in any other universities, and all sources of material used for the thesis have been duly acknowledged.

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This Msc thesis has been submitted to for examination with my approval as university advisor supervisor.

Name: Dr. Tewodros Yirgashewa

Signature: ____________________

Date of submission:......................

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Contents

Abstract

In this thesis, we studied the squeezing and statistical properties of light produced by degenerate three-level laser whose cavity initially in thermal light and the cavity mode is coupled to squeezed vacuum reservoir. We first obtained the master equation for the system under consideration. Using this master equation, we obtained c-number Langevin equation associated with the normal ordering and the correlation properties of the noise forces, with the aid of in which, we calculated the quadrature variance and the squeezing spectrum. It is found that the degree of squeezing increases with time and the effect of the squeeze parameter is to increase the degree of squeezing. It is also found that for $\omega = 0$ *the noise in the minus quadrature is completely suppressed. In addition, using the solutions of the c-number Langevin equations, we calculated the mean and the variance of the photon number. It so turns out that the injecting thermal light in cavity increases the mean and the variance of the photon number at the beginning and then it has no effect at time progresses.*

1

INTRODUCTION

1.1 Back ground of the study

Light has played a special role in our attempts to understand nature both classically and quantum mechanically. Classically light consists of waves with well-defined amplitude and phase such is not case when we treat the field quantum mechanically. Scientists gained technology and measure with great precision, strange phenomena was observed and this is birth of quantum physics. Recent development of technology greatly improved our ability to control individual quantum system. This brings quantum technology beyond academic research to control the concrete industrial programs.

Quantum optics deals mainly with quantum properties of the light generated by various quantum optical systems, such as:- laser and maser with their effect on the dynamics of atoms. The quantum properties of light are largely determined by the state of the mode and well known quantum states are; the number state, the coherent state, the chaotic state, and the squeezed state.

Squeezing is one of the interesting non-classical features of light that has been attracting attention and studied by many authors[1-23]. In squeezed light the noise in one quadrature is below the vacuum or coherent level at the expense of enhanced fluctuation in the other quadrature, with product of the uncertainties in the two quadrature's satisfying the uncertainty relation. Squeezed light can be generated as result of quantum optical process, such as:- sub-harmonic generation, second harmonic generation and a three-level laser under certain conditions.

Squeezed light can be generated by a three-level laser under certain conditions[2].It has been predicted that a three-level laser under certain conditions can produced squeezed light[6, 9 - 12]. In a cascade three-level laser, three-level atoms in a cascade configuration are injected into a cavity coupled to a squeezed vacuum reservoir via a single-port mirror. The injected atoms may initially be prepared in a coherent superposition of the top and bottom levels. When a three-level atom in a cascade configuration makes a transitions from the top to the bottom level via the intermediate level, two photons are generated. If two photons have the same frequency, the three-level atom is called degenerate otherwise it is called nondegenerate.

Squeezed light has potential applications in low-noise communication, weak signal detection, and optical amplification measurement [4-7]. The squeezing and entanglement of light produced by three-level lasers have been studied when the atoms are initially prepared in coherent superposition of the top and bottom levels[8].

1.2 Objective

1.2.1 General objectives

The general objective of this thesis is to study the effect of injected thermal light in cavity on a degenerate three-level laser coupled to squeezed vacuum reservoir.

1.2.2 Specific objectives

The specific objectives of this thesis are:-

• To construct the Hamiltonian of the system under consideration.

•To derive the master equation of a degenerate three-level laser coupled to squeezed vacuum reservoir.

• To find the c-number Langevin equation associated with the normal ordering and their solutions.

• To calculate the quadrature variance and squeezing spectrum.

• To determine the mean and variance of the photon number.

2

REVIEW OF THE LITERATURE

Some authors have studied the squeezing and statistical properties of light produced by three-level lasers in which the crucial role is played the superposition of the top and bottom levels [12-13]. Ansari[12] has studied the squeezing and Statistical properties of a degenerated three-level laser with the atoms initially prepared in coherent superposition of the top and bottom levels. He has predicted that cavity mode of such laser is in squeezed state, when there are more atoms initially in the bottom level than in the top level. Squeezing and entanglement properties of light has been studied by different authors using different methods [1-23]:

Fesseha has studied the squeezing properties of light by a degenerate three-level laser coupled to vacuum reservoir. He has found that the squeezing increases with the linear gain coefficient and the perfect squeezing can be achieved for large values of linear gain coefficient and for small values of η closed to zero.

Tesfa has studied the squeezing property of the cavity modes produced by a non-degenerate three-level laser applying the solutions of stochastic differential equations. He has found that the two-mode cavity radiation exhibits squeezing if the atoms are initially prepared with more atoms in the bottom level than in the upper level, and the degree of squeezing increases with linear gain coefficient.

Mekonnen has studied squeezing and entanglement of light by non-degenerate three-level laser with a strong driving coherent light and squeezed vacuum reservoir. He has found that the squeezing and entanglement of the cavity modes will be higher for large values of linear gain coefficient and the degree of entanglement is directly proportional to the degree of squeezing of the two-mode light but the degree of entanglement for the output modes will disappear for large values of linear gain coefficient. In other communication, the two-mode squeezed vacuum reservoir considerably increases the degree of squeezing as well as entanglement in the cavity.

Friew has studied that squeezing and statistical properties of the light generated by a non-degenerate three-level laser whose cavity contains two parametric amplifiers coupled to an ordinary vacuum reservoir. He has found that parametric amplifiers increase the degree of squeezing.

Abraham has studied the squeezing and entanglement of light produced by a non-degenerate three-level laser whose cavity contains two degenerate parametric amplifier coupled to squeezed vacuum reservoir. He has found that the degree of squeezed vacuum reservoir increases the decreases the degree of squeezing and entanglement. And the found that the squeezing of cavity mode is greater than output mode by some values. He also found that the maximum intra cavity of entanglement of the cavity mode and the output mode is 99.44 $\%$ and 59.55$\%$ respectively below coherent level.

Fesseha has analyzed the quadrature properties of light generated by a three-level laser couple to a vacuum reservoir. He show that the light generated by the three-level operating under the condition $\gamma_c > \frac{\gamma_a}{4}$ is in a squeezed state, with the maximum quadrature squeezing being 50$\%$ below the coherent state level.

Mulugeta has studied the quantum properties of the light generated by a coherently driven degenerate three-level atom in an open cavity coupled to a vacuum reservoir. Employing the master equation and the quantum Langiven equation. He found that the maximum quadrature squeezing of the light generated by the atom to be 43.42$\%$ for $\gamma = 0$ and 42.22$\%$ for $\gamma = 0.1$ below the coherent state level. Thus in the absence of spontaneous emission the mean, the variance and the quadrature squeezing of the presence light is greater than in the presence of spontaneous emission.

Yosef has studied that the squeezing and statistical properties of a degenerate three-level atom coupled to a coherent light and vacuum reservoir. Employing the Heseinberg and Quantum Langevin equation associated with the normal ordering noise operator. He found that the degenerate light exhibits squeezing under certain conditions. At steady state $\rho_{cc} > \rho_{aa}$. This implies there is more propability for finding the atom in the bottom level than in the upper or intermediate level. Moreover, in the absence of the coherent light there is no probability to find the atom either in the upper or intermediate level. In addition he found that for $\Omega < 2\gamma_c$, the light emitted by the atom is in a squeezed state and squeezing occurs in minus quadrature.

All the above authors studied their system by assuming the system to be initially in vacuum state.

On the other hand, S.Tesfa has studied the role of seeding thermal light in the cavity of a two-photon correlated emission laser coupled to vacuum reservoir. He has found that the degree of entanglement increases with the linear gain coefficient for value under consideration and he also found that the degree of two-mode squeezing is independent of the intensity of the thermal seeding light at large time scale.

3

METHODOLOGY

3.1 Description of the involved systems

3.1.1 Thermal light

Thermal light is a typical example of a light mode in a chaotic light. It is generated by a source in thermal equilibrium at some temperature T. We have to determine the density operator $\hat{\rho}$ for chaotic light using Lagrange multipliers and maximizing the entropy. The entropy is expressible as

$$S = -k_\beta Tr(\hat{\rho} \ln \hat{\rho}), \tag{3.1}$$

where k_β is the Boltzmann constant and Tr stands for trace operation and $\hat{\rho}$ is the density operator. Using the fact that at equilibrium entropy should be maximum and $Tr(\hat{\rho}) = 1$, the density operator for the thermal light is related to temperature by

$$\hat{\rho} = \frac{e^{\frac{-\hat{H}}{k_\beta T}}}{Tr(e^{\frac{-\hat{H}}{k_\beta T}})}, \tag{3.2}$$

where $\hat{H} = \omega \hat{a}^\dagger \hat{a}$ is the Hamiltonian of the radiation field when the zero point energy is shifted by $\frac{\omega}{2}$ and $\hbar = 1$ for convenience. The same density operator can be described in the number state applying the completeness relation for number state $\hat{I} = \sum_{n=0}^{\infty} |n\rangle\langle n|$ and assuming that the number state is orthonormal $\langle n|n\rangle = \delta_{nm}$ as

$$\hat{\rho} = \sum_{n=0}^{\infty} \frac{(\bar{n})^n}{(1+\bar{n})^{1+n}} |n\rangle\langle n|, \tag{3.3}$$

where $\bar{n}$ is the mean photon number for the chaotic light. The density operator for a single-mode thermal light is expressible as

$$\hat{\rho}_a = \sum_{n_a=0}^{\infty} \frac{(\bar{n}_a)^{n_a}}{(1+\bar{n}_a)^{1+n_a}} |n_a\rangle\langle n_a|, \tag{3.4}$$

where $\bar{n}_a$ is the mean photon numbers of a chaotic light. In other communication, the number state in general can be described interms of the vacuum state as

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n}}|0\rangle, \tag{3.5}$$

which can also be expressed interms of the coherent state using the power series expansion of the form $e^{\alpha\hat{a}^\dagger} = \sum_{n_a=0}^{\infty}\frac{(\alpha\hat{a}^\dagger)^n}{n!}$ as

$$|\alpha\rangle = e^{-\frac{1}{2}|\alpha|^2}\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n}!}|n\rangle. \tag{3.6}$$

Now applying the operator Eq.(3.4) along with the relation with coherent light Eq.(3.6) and definition of the mean value a $\langle\hat{0}\rangle = Tr(\hat{0}\hat{\rho})$ for any operator $\hat{0}$, it is possible to verify for single-mode chaotic light that

$$\langle\hat{a}^\dagger\hat{a}\rangle = \bar{n}, \tag{3.7}$$

$$\langle\hat{a}^2\rangle = \langle\hat{a}\rangle = 0. \tag{3.8}$$

Finally, the mean photon number of each state can be related to the frequency of the mode and temperature of the wall by

$$\bar{n}_i = \frac{1}{e^{\frac{\omega_i}{k_\beta T}} - 1} \tag{3.9}$$

3.1.2 Interaction between the cavity mode and the squeezed vacuum reservoir

To derive the master equation for a single-mode in an optical cavity coupled to a single-mode squeezed vacuum reservoir via of a single-port mirror. General, in the interaction picture, a quantum optical system coupled to a reservoir can be described by the Hamiltonian

$$\hat{H} = \hat{H}_S + \hat{H}_{SR}, \tag{3.10}$$

where $\hat{H}_S$ is the Hamiltonian in the cavity and $\hat{H}_{SR}$ is the Hamiltonian that describes the interaction of a cavity mode and a reservoir. Now let $\hat{\rho}_{SR}$ be the density operator for the system plus the reservoir in the interaction picture. We can then write the time evolution of this operator as

$$\frac{d\hat{\rho}_{SR}(t)}{dt} = -i[\hat{H}_S(t) + \hat{H}_{SR}(t), \hat{\rho}_{SR}(t)]. \tag{3.11}$$

In order to obtain the evolution in the cavity approach, which we are interested in, we trace $\hat{\rho}_{SR}(t)$ over the reservoir variables, that is, the density operator for the system, $\hat{\rho}(t) = Tr_R(\hat{\rho}_{SR}(t))$. In view of this, Eq.(3.11) takes the form

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}_S(t), \hat{\rho}(t)] - iTr_R[\hat{H}_{SR}(t), \hat{\rho}_{SR}(t)], \tag{3.12}$$

where the subscript R indicates that the expectation value is to be calculated using the reservoir density operator $\hat{R}$. In order to obtain a mathematically manageable equation of evolution of the reduced density operator, we need to adopt some approximation scheme. We thus seek to obtain this equation of evolution applying the approximately valid relation

$$\frac{d\hat{\rho}_{SR}(t)}{dt} = \frac{\hat{\rho}_{SR}(t) - \hat{\rho}_{SR}(t-d)}{d}, \tag{3.13}$$

where d is a small parameter. Now in view of Eq.(3.11) and (3.13), we see that

$$\hat{\rho}_{SR}(t) = \hat{\rho}_{SR}(t-d) - id[\hat{H}_{SR}(t) + \hat{H}_S(t), \hat{\rho}_{SR}(t)], \tag{3.14}$$

so that on substituting Eq.(3.14) into Eq.(3.12), we arrive at

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} &= -i[\hat{H}_S(t), \hat{\rho}(t)] - iTr_R[\hat{H}_{SR}(t), \hat{\rho}_{SR}(t-d)] - dTr_R[\hat{H}_{SR}(t), [\hat{H}_{SR}(t), \hat{\rho}_{SR}(t)]] \\ &- dTr_R[\hat{H}_{SR}(t), [\hat{H}_S(t), \hat{\rho}_{SR}(t)]],\end{aligned} \tag{3.15}$$

so as to proceed further, we need to introduce a certain approximation scheme. To this end, assuming that there is a weak interaction between the system and reservoir, we can write approximately valid relation

$$\hat{\rho}_{SR}(t) = \hat{\rho}(t)\hat{R}, \tag{3.16}$$

where $\hat{R}$ is the operator for the reservoir assumed to be constant in time. With the aid of Eq.(3.16) into Eq.(3.15), we obtain

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} &= -i[\hat{H}_S(t), \hat{\rho}(t)] - i[\langle\hat{H}_{SR}\rangle_R, \hat{\rho}(t-d)] - d[\langle\hat{H}_{SR}\rangle_R, [\hat{H}_S, \hat{\rho}(t)]] \\ &- dTr[\hat{H}_{SR}, [\hat{H}_{SR}, \hat{\rho}(t)\hat{R}]],\end{aligned} \tag{3.17}$$

Now proceed to obtain the equation of evolution of the reduced density operator, in short the master equation, for a cavity mode coupled to a squeezed vacuum reservoir via of a single-port mirror. We defined a squeezed vacuum by

$$|r\rangle = \hat{Y}(r)|0\rangle, \tag{3.18}$$

in which

$$\hat{Y}(r) = e^{\frac{r}{2}(\hat{a}_{in}^2 - \hat{a}_{in}^{\dagger 2})}, \tag{3.19}$$

where $\hat{a}_{in}$ being the annihilation operator for the squeezed vacuum and r is a squeezing parameter(real and positive). Using the definition

$$\hat{a}_{in}(r) = \hat{Y}^{\dagger}(r)\hat{a}_{in}\hat{Y}(r), \tag{3.20}$$

we find

$$\hat{a}_{in}(r) = \hat{a}_{in}\cosh(r) - \hat{a}^{\dagger}_{in}\sinh(r). \tag{3.21}$$

Now with the aid of Eq.(3.21), one can readily establish that for a squeezed vacuum

$$\langle \hat{a}_{in} \rangle = 0, \tag{3.22}$$

$$\langle \hat{a}^2_{in} \rangle = -\cosh(r)\sinh(r), \tag{3.23}$$

$$\langle \hat{a}^{\dagger}_{in}\hat{a}_{in} \rangle = \sinh^2(r). \tag{3.24}$$

Using the commutation relation

$$[\hat{a}_{in}, \hat{a}^{\dagger}_{in}] = 1, \tag{3.25}$$

along with Eq.(3.24), we get

$$\langle \hat{a}_{in}\hat{a}^{\dagger}_{in} \rangle = \sinh^2(r) + 1. \tag{3.26}$$

The interaction of a cavity mode with a squeezed vacuum reservoir via of a single-port mirror can be described by the Hamiltonian

$$\hat{H}_{SR} = i\beta(\hat{a}^{\dagger}\hat{a}_{in} - \hat{a}^{\dagger}_{in}\hat{a}), \tag{3.27}$$

where β is the coupling constant between a cavity mode and reservoir. We then see that

$$\langle \hat{H}_{SR} \rangle_R = i\beta(\hat{a}^{\dagger}\langle \hat{a}_{in} \rangle_R - \hat{a}\langle \hat{a}^{\dagger}_{in} \rangle_R), \tag{3.28}$$

with the aid of Eq.(3.22) and Eq.(3.28), becomes

$$\langle \hat{H}_{SR} \rangle_R = 0, \tag{3.29}$$

so that on account of Eq.(3.29) and Eq.(3.17), one can easily show that

$$[\langle \hat{H}_{SR} \rangle_R, \hat{\rho}(t-d)] = 0, \tag{3.30}$$

$$[\langle \hat{H}_{SR} \rangle_R, [\hat{H}_S, \hat{\rho}(t)] = 0, \tag{3.31}$$

$$[\langle \hat{H}_{SR} \rangle_R, \hat{\rho}(t-d)] = [\langle \hat{H}_{SR} \rangle_R, [\hat{H}_S, \hat{\rho}(t)] = 0. \tag{3.32}$$

With the aid of Eq.(3.32) and Eq.(3.17), one can write as

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}_S(t), \hat{\rho}(t)] - dTrR(\hat{H}^2_{SR}\hat{\rho}(t)\hat{R} - d\hat{\rho}(t)Tr\hat{R}\hat{H}^2_{SR}) + 2dTrR(\hat{H}_{SR}\hat{\rho}(t)\hat{H}_{SR}). \tag{3.33}$$

Furthermore, using the Hamiltonian described by Eq.(3.27), we get

$$dTr\hat{H}_{SR}^{2}\hat{R} = d\beta^{2}[\langle \hat{a}_{in}\hat{a}_{in}^{\dagger}\rangle_{R}\hat{a}^{\dagger}\hat{a} + \langle \hat{a}_{in}^{\dagger}\hat{a}_{in}\rangle_{R}\hat{a}\hat{a}^{\dagger} - \langle \hat{a}_{in}^{\dagger 2}\rangle_{R}\hat{a}^{2} - \langle \hat{a}_{in}^{2}\rangle_{R}\hat{a}^{\dagger 2}]. \tag{3.34}$$

On account of Eqs.(3.23), (3.24), (3.26) and (3.34), we arrive at

$$dTr\hat{H}_{SR}^{2}\hat{R} = \frac{\kappa}{2}[(N+1)\hat{a}^{\dagger}\hat{a} + N\hat{a}\hat{a}^{\dagger} + M\hat{a}^{2} + M\hat{a}^{\dagger 2}], \tag{3.35}$$

where κ damping constant of the cavity mode and squeezed vacuum reservoir

$$\kappa = 2d\beta^{2}, \tag{3.36}$$

$$N = \sinh^{2}(r), \tag{3.37}$$

$$M = \cosh(r)\sinh(r). \tag{3.38}$$

Using the cyclic property of the trace operation of Eq.(3.35), one can easily show that

$$dTrR\hat{H}_{SR}\hat{R}\hat{\rho}(t)\hat{H}_{SR} = \frac{\kappa}{2}[(N+1)\hat{a}^{\dagger}\hat{\rho}\hat{a} + N\hat{a}\hat{\rho}\hat{a}^{\dagger} + M\hat{a}\hat{\rho}\hat{a} + M\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger}], \tag{3.39}$$

$$dTrR\hat{H}_{SR}^{2}\hat{R}\hat{\rho}(t) = \frac{\kappa}{2}[(N+1)\hat{a}^{\dagger}\hat{a})\hat{\rho} + N\hat{a}\hat{a}^{\dagger}\hat{\rho} + M\hat{a}^{2}\hat{\rho} + M\hat{a}^{\dagger 2}\hat{\rho}], \tag{3.40}$$

$$d\hat{\rho}(t)TrR\hat{H}_{SR}^{2}\hat{R} = \frac{\kappa}{2}[(N+1)\hat{\rho}\hat{a}^{\dagger}\hat{a} + N\hat{\rho}\hat{a}\hat{a}^{\dagger} + M\hat{\rho}\hat{a}^{2} + M\hat{\rho}\hat{a}^{\dagger 2}]. \tag{3.41}$$

Finally, substituting Eqs.(3.39), (3.40) and (3.41) into Eq.(3.33), we obtain the master equation for cavity mode coupled to a single-mode squeezed vacuum reservoir to be

$$\begin{aligned}\frac{d}{dt}\hat{\rho(t)} = &-i[\hat{H}_{s},\hat{\rho}(t)] + \frac{\kappa}{2}(N+1)(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}) + \frac{\kappa N}{2}[2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{a}\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^{\dagger}]\\ &+ \frac{\kappa M}{2}[2\hat{a}\hat{\rho}\hat{a} - \hat{a}^{2}\hat{\rho} - \hat{\rho}\hat{a}^{2} + 2\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}],\end{aligned} \tag{3.42}$$

This entails that the thermal light in the cavity does not contribute to the master equation[18].

3.1.3 The interaction between an atom and cavity mode

In this section we consider a degenerate three-level laser whose cavity contains initially thermal light and coupled to squeezed vacuum reservoir. Moreover, a three level atoms are injected into the cavity at a constant rate γ_a and removed from the cavity after a certain time T. We denote the top, middle, and bottom levels by $|a\rangle$, $|b\rangle$, and $|c\rangle$, respectively. In the addition, we assume the cavity mode to be at resonance with the two transitions $|a\rangle \rightarrow |b\rangle$ and $|b\rangle \rightarrow |c\rangle$, with direct transition between levels $|a\rangle$ and $|c\rangle$ to be electric dipole for bidden.

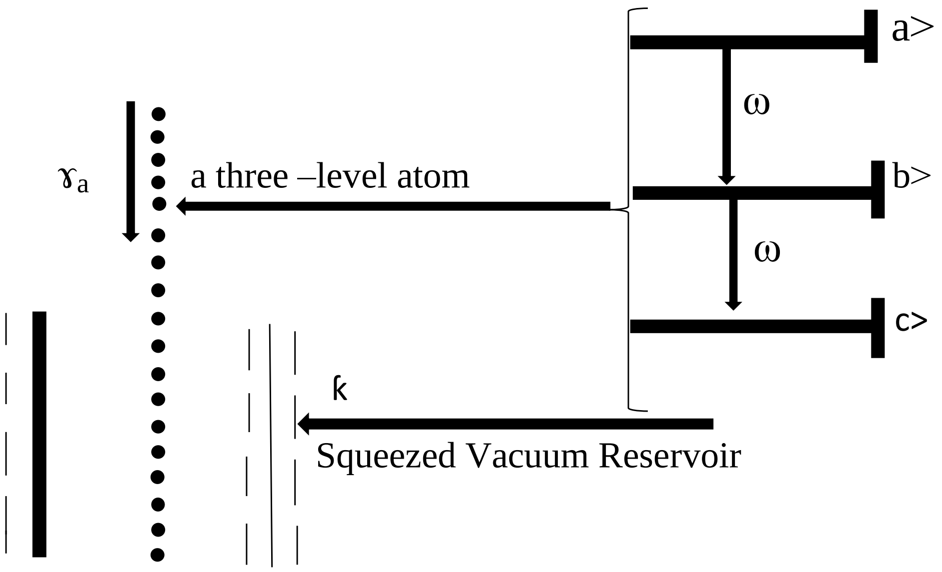

Fig. 3.1: Pictorial representation of a degenerate three-level laser coupled to squeezed vacuum reservoir

The interaction of a three-level atom with cavity mode can be described by the Hamiltonian

$$\hat{H}_s = ig[(|a\rangle\langle b| + |b\rangle\langle c|)\hat{a} - \hat{a}^{\dagger}(|b\rangle\langle a| + |c\rangle\langle b|)], \tag{3.43}$$

where g is the coupling constant between the cavity mode and a three-level atom and $\hat{a}$ is the annihilation operator for the cavity mode. Inserting Eq.(3.43) into Eq.(3.42), we have

$$\begin{aligned}\frac{d}{dt}\hat{\rho}(t) &= g[\hat{a}\hat{\rho}_{ba} - \hat{\rho}_{ba}\hat{a} + \hat{a}\hat{\rho}_{cb} - \hat{\rho}_{cb}\hat{a} - \hat{a}^{\dagger}\hat{\rho}_{ab} + \hat{\rho}_{cb}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{\rho}_{bc} + \hat{\rho}_{bc}\hat{a}^{\dagger}] \\ &+ \frac{\kappa}{2}(N+1)(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}) + \frac{\kappa N}{2}[2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{a}\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^{\dagger}] \\ &+ \frac{\kappa M}{2}[2\hat{a}\hat{\rho}\hat{a} - \hat{a}^{2}\hat{\rho} - \hat{\rho}\hat{a}^{2} + 2\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}].\end{aligned} \tag{3.44}$$

We take the initial state of a three-level atom to be

$$|\psi_A(0)\rangle = C_a(0)|a\rangle + C_c(0)|c\rangle, \tag{3.45}$$

the initial density operator for a three-level atom can then be written as

$$\rho_A^{(0)} = \rho_{aa}^{(0)}|a\rangle\langle a| + \rho_{ac}^{(0)}|a\rangle\langle c| + \rho_{ca}^{(0)}|c\rangle\langle a| + \rho_{cc}^{(0)}|c\rangle\langle c|, \tag{3.46}$$

in which

$$\rho_{aa}^{(0)} = |C_a|^2, \rho_{ac}^{(0)} = C_a C_c^*, \rho_{ca}^{(0)} = C_c C_a^*, \rho_{cc}^{(0)} = |C_c|^2$$

It proves to be more convenient to introduced new parameter η defined by[9]

$$\rho_{aa}^{(0)} = \frac{1-\eta}{2}, \tag{3.47}$$

where $-1 \leq \eta \leq 1$. Using the fact that

$$\rho_{aa}^{(0)} + \rho_{cc}^{(0)} = 1, \tag{3.48}$$

along with

$$|\rho_{ac}^{(0)}|^2 = \rho_{aa}^{(0)}\rho_{cc}^{(0)}, \tag{3.49}$$

one easily finds

$$\rho_{cc}^{(0)} = \frac{1+\eta}{2}, \tag{3.50}$$

and

$$|\rho_{ac}^{(0)}| = \frac{1}{2}(1-\eta^2)^{\frac{1}{2}}. \tag{3.51}$$

We note that the parameter η describes the initial preparation of a three-level atom. Upon setting

$$\rho_{ac}^{(0)} = |\rho_{ac}^{(0)}|e^{i\theta}, \tag{3.52}$$

expression Eq.(3.46) can be put in the form

$$\begin{aligned} \rho_A^{(0)} &= (\frac{1-\eta}{2})|a\rangle\langle a| + \frac{1}{2}(1-\eta^2)^{\frac{1}{2}}e^{i\theta}|a\rangle\langle c| \\ &+ \frac{1}{2}(1-\eta^2)^{\frac{1}{2}}e^{-i\theta}|c\rangle\langle a| + (\frac{1+\eta}{2})|c\rangle\langle c|. \end{aligned} \tag{3.53}$$

Suppose $\hat{\rho}_{AR}(t, t_j)$ is the density operator for a single atom plus the cavity mode at time t, with the atom injected at time t_j, such that $(t-\tau) \leq t_j \leq t$. The density operator for all atoms in the cavity plus the cavity mode at time t can then be written as

$$\hat{\rho}_{AR}(t) = \gamma_a \sum_j \hat{\rho}_{AR}(t, t_j)\Delta t_j, \tag{3.54}$$

where $\gamma_a \Delta t_j$ represents the number of atoms injected into the cavity in a time Δt_j. Now converting the summation into integration in the limit $\Delta t_j \to 0$, we have

$$\hat{\rho}_{AR}(t) = \gamma_a \int_{t-\tau}^{t} \hat{\rho}_{AR}(t, t_j)dt'. \tag{3.55}$$

Differentiating with respect to t, and taking into account the Leibnitz' rule

$$\frac{d}{dx}(\int_{u(x)}^{v(x)} f(x,x')dx') = (f(x,v)\frac{dv(x)}{dx} - f(x,u)\frac{du(x)}{dx}) + \int_{u(x)}^{v(x)} \frac{\partial f(x,x')}{\partial x}dx', \tag{3.56}$$

there follows

$$\frac{d}{dt}\hat{\rho}_{AR}(t) = \gamma_a[\hat{\rho}_{AR}(t,t) - \hat{\rho}_{AR}(t, t-\tau)] + \gamma_a \int_{t-\tau}^{t} \frac{\partial}{\partial t}\hat{\rho}_{AR}(t,t')dt'. \tag{3.57}$$

We observe that $\hat{\rho}_{AR}(t,t')$ is the density operator for the cavity mode plus an atom injected at time t. This operator can be expressed as

$$\hat{\rho}_{AR}(t,t) = \hat{\rho}_{AR}(t)\hat{\rho}(t), \tag{3.58}$$

with $\hat{\rho}(t)$ being the density operator for the cavity mode alone. We also note that $\hat{\rho}_{AR}(t,t-\tau)$ represents the density operator for an atom plus the cavity mode at time t, with the atom removed from the cavity at this time. This operator can also be put in the form

$$\hat{\rho}_{AR}(t,t-\tau) = \hat{\rho}_{A}(t-\tau)\hat{\rho}(t). \tag{3.59}$$

Now in view of Eqs.(3.58) and (3.59), one can write Eq.(3.57) as

$$\frac{d}{dt}(\hat{\rho}_{AR}(t)) = \gamma_a[\hat{\rho}_A(t) - \hat{\rho}_A(t-\tau)\hat{\rho}(t)] + \gamma_a\int_{t-\tau}^{t}\frac{\partial}{\partial t}\hat{\rho}_{AR}(t,t')dt'. \tag{3.60}$$

In the absence of the damping of the cavity mode by squeezed vacuum reservoir, the density operator $\hat{\rho}_{AR}(t,t')$ evolves in time according to

$$\frac{d}{dt}\hat{\rho}_{AR}(t,t') = -i[\hat{H},\hat{\rho}_{AR}(t,t')], \tag{3.61}$$

so that using this and taking into account Eq.(3.55), one can put Eq.(3.60) in the form

$$\frac{d}{dt}\hat{\rho}_{AR}(t) = \gamma_a[\hat{\rho}_A(t) - \hat{\rho}_A(t-\tau)]\hat{\rho}(t) - i[\hat{H},\hat{\rho}_{AR}(t)]. \tag{3.62}$$

Furthermore, tracing over the atomic variables and taking into account the damping of the cavity mode by the squeezed vacuum reservoir, we have

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= -iTr_A[\hat{H},\hat{\rho}_{AR}(t)] + \frac{\kappa}{2}(N+1)(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + \frac{\kappa N}{2}[2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger]\\ &\quad + \frac{\kappa M}{2}[2\hat{a}\hat{\rho}\hat{a} - \hat{a}^2\hat{\rho} - \hat{\rho}\hat{a}^2 + 2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}],\end{aligned} \tag{3.63}$$

where we have used the fact

$$Tr\hat{\rho}_A(t) = Tr\hat{\rho}_A(t-\tau) = 1. \tag{3.64}$$

The master equation associated with the interactions described by the Hamiltonian Eq.(3.43) can be put in the form

$$\begin{aligned}\frac{d}{dt}\hat{\rho}(t) &= g[\hat{a}\hat{\rho}_{ba} - \hat{\rho}_{ba}\hat{a} + \hat{a}\hat{\rho}_{cb} - \hat{\rho}_{cb}\hat{a} - \hat{a}^\dagger\hat{\rho}_{ab} + \hat{\rho}_{cb}\hat{a}^\dagger - \hat{a}^\dagger\hat{\rho}_{bc} + \hat{\rho}_{bc}\hat{a}^\dagger]\\ &\quad + \frac{\kappa}{2}(N+1)(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + \frac{\kappa N}{2}[2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger]\\ &\quad + \frac{\kappa M}{2}[2\hat{a}\hat{\rho}\hat{a} - \hat{a}^2\hat{\rho} - \hat{\rho}\hat{a}^2 + 2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}],\end{aligned} \tag{3.65}$$

in which the matrix element $\hat{\rho}_{\alpha\beta}$ is defined by

$$\hat{\rho}_{\alpha\beta} = \langle\alpha|\hat{\rho}_{AR}|\beta\rangle, \tag{3.66}$$

with $\alpha, \beta = a, b, c$

On the other hand, we see from Eq.(3.62) that

$$\begin{aligned}\frac{d}{dt}\hat{\rho}_{\alpha\beta} &= \gamma_a(\langle\alpha|\hat{\rho}_0|\beta\rangle - \langle\alpha|\hat{\rho}(t-\tau)|\beta\rangle)\hat{\rho}_{(t)} \\ &- i(\langle\alpha|\hat{H}\hat{\rho}_{AR}|\beta\rangle - \langle\alpha|\hat{\rho}_{AR}\hat{H}|\beta\rangle) - \gamma\hat{\rho}_{\alpha\beta},\end{aligned} \tag{3.67}$$

where the last term is included to account for the decay of the atoms due to spontaneous emission. Here γ, considered to be the same for all the three-levels, is the atomic decay constant. We assume that the atoms are removed from the cavity after they have decayed to a level other than the middle or bottom level. We then see that

$$\langle\alpha|\hat{\rho}(t-\tau)|\beta\rangle = 0, \tag{3.68}$$

and hence Eq.(3.67) reduced to

$$\frac{d}{dt}\hat{\rho}_{\alpha\beta} = \gamma_a(\langle\alpha|\hat{\rho}_0|\beta\rangle)\hat{\rho}_{(t)} - i(\langle\alpha|\hat{H}\hat{\rho}_{AR}|\beta\rangle - \langle\alpha|\hat{\rho}_{AR}\hat{H}|\beta\rangle) - \gamma\hat{\rho}_{\alpha\beta}. \tag{3.69}$$

We next proceed to determine the matrix elements $\hat{\rho}_{\alpha\beta}$ involved in Eq.(3.43) and (3.46), we can write

$$\begin{aligned}\frac{d}{dt}\hat{\rho}_{\alpha\beta} &= \gamma_a[\rho^{(0)}_{aa}\delta_{\alpha a}\delta_{a\beta} + \rho^{(0)}_{ac}\delta_{\alpha a}\delta c\beta + \rho^{(0)}_{ca}\delta_{\alpha c}\delta_{a\beta} + \rho^{(0)}_{cc}\delta_{\alpha c}\delta_{c\beta}]\hat{\rho} \\ &+ g[\hat{a}\hat{\rho}_{b\beta}\delta_{\alpha a} + \hat{a}\hat{\rho}_{c\beta}\delta_{\alpha b} - \hat{a}^\dagger\hat{\rho}_{a\beta}\delta_{\alpha b} - \hat{a}^\dagger\hat{\rho}_{b\beta}\delta_{\alpha c} - \hat{\rho}_{\alpha a}\hat{a}\delta_{b\beta} \\ &- \hat{\rho}_{\alpha b}\hat{a}\delta_{c\beta} + \hat{\rho}_{\alpha b}\hat{a}^\dagger\delta_{a\beta} + \hat{\rho}_{\alpha c}\hat{a}^\dagger\delta_{b\beta}] - \gamma\hat{\rho}_{\alpha\beta}.\end{aligned} \tag{3.70}$$

Using Eq.(3.70), we obtain

$$\frac{d}{dt}\hat{\rho}_{ab} = g[\hat{a}\hat{\rho}_{bb} - \hat{\rho}_{aa}\hat{a} + \hat{\rho}_{ac}\hat{a}^\dagger] - \gamma\hat{\rho}_{ab}, \tag{3.71}$$

$$\frac{d}{dt}\hat{\rho}_{bc} = g[\hat{a}\hat{\rho}_{cc} - \hat{a}^\dagger\hat{\rho}_{ac} - \hat{\rho}_{bb}\hat{a}] - \gamma\hat{\rho}_{bc}, \tag{3.72}$$

$$\frac{d}{dt}\hat{\rho}_{ba} = g[\rho_{bb}\hat{a}^\dagger - \hat{a}^\dagger\rho_{aa} + \hat{a}\hat{\rho}_{ca}] - \gamma\hat{\rho}_{ba}, \tag{3.73}$$

$$\frac{d}{dt}\hat{\rho}_{bb} = g[\rho_{bc}\hat{a}^\dagger + \hat{a}\rho_{cb} - \hat{a}^\dagger\rho_{ab} - \rho_{ba}\hat{a}] - \gamma\hat{\rho}_{bb}, \tag{3.74}$$

$$\frac{d}{dt}\hat{\rho}_{ac} = \gamma_a \rho_{ac}^{(0)}\hat{\rho} + g[\hat{a}\hat{\rho}_{bc} - \hat{\rho}_{ab}\hat{a}] - \gamma\hat{\rho}_{ac}, \tag{3.75}$$

$$\frac{d}{dt}\hat{\rho}_{ca} = \gamma_a \rho_{ca}^{(0)}\hat{\rho} + g[\hat{\rho}_{cb}\hat{a}^\dagger - \hat{a}^\dagger\hat{\rho}_{ba}] - \gamma\hat{\rho}_{ca}, \tag{3.76}$$

$$\frac{d}{dt}\hat{\rho}_{cc} = \gamma_a \rho_{cc}^{(0)}\hat{\rho} - g[\hat{a}^\dagger\hat{\rho}_{bc} + \hat{\rho}_{cb}\hat{a}] - \gamma\hat{\rho}_{cc}, \tag{3.77}$$

$$\frac{d}{dt}\hat{\rho}_{aa} = \gamma_a \rho_{aa}^{(0)}\hat{\rho} + g[\hat{\rho}_{ab}\hat{a}^\dagger + \hat{a}\hat{\rho}_{ba}] - \gamma\hat{\rho}_{aa}. \tag{3.78}$$

We confine our selves to linear analysis and this can be achieved by dropping the g-terms in Eqs.(3.74), (3.75), (3.77) and (3.78). We then see that

$$\frac{d}{dt}\hat{\rho}_{bb} = -\gamma\hat{\rho}_{bb}, \tag{3.79}$$

$$\frac{d}{dt}\hat{\rho}_{ac} = \gamma_a \rho_{ac}^{(0)}\hat{\rho} - \gamma\hat{\rho}_{ac}, \tag{3.80}$$

$$\frac{d}{dt}\hat{\rho}_{cc} = \gamma_a \rho_{cc}^{(0)}\hat{\rho} - \gamma\hat{\rho}_{cc}, \tag{3.81}$$

$$\frac{d}{dt}\hat{\rho}_{aa} = \gamma_a \rho_{aa}^{(0)}\hat{\rho} - \gamma\hat{\rho}_{aa}. \tag{3.82}$$

In addition, imposing the good cavity limit ($\kappa \ll \gamma$) in which the atomic variables reach steady state in a relatively short period of ($\frac{1}{\gamma}$), we can take the limit derivatives of such variables to be zero, while keeping the zero order atomic and cavity mode variables at time t. This procedure may be referred as adiabatic approximation scheme, we get from Eqs.(3.79), (3.80), (3.81) and (3.82), We then see that

$$\hat{\rho}_{bb} = 0, \tag{3.83}$$

$$\hat{\rho}_{ac} = \frac{\gamma_a \rho_{ac}^{(0)}}{\gamma}, \tag{3.84}$$

$$\hat{\rho}_{cc} = \frac{\gamma_a \rho_{cc}^{(0)}}{\gamma}, \tag{3.85}$$

$$\hat{\rho}_{aa} = \frac{\gamma_a \rho_{aa}^{(0)}}{\gamma}. \tag{3.86}$$

Now combination of Eqs.(3.70), (3.83), (3.84) and (3.86) as well as (3.71), (3.83), (3.84) and (3.85) leads to

$$\frac{d}{dt}\hat{\rho}_{ab} = \frac{g\gamma_a}{\gamma}(\rho^{(0)}_{ac}\hat{\rho}\hat{a}^\dagger - \rho^{(0)}_{aa}\hat{\rho}\hat{a}) - \gamma\hat{\rho}_{ab}, \tag{3.87}$$

$$\frac{d}{dt}\hat{\rho}_{bc} = \frac{g\gamma_a}{\gamma}(\rho^{(0)}_{cc}\hat{a}\hat{\rho} - \rho^{(0)}_{ac}\hat{a}^\dagger\hat{\rho}) - \gamma\hat{\rho}_{bc}. \tag{3.88}$$

Using once more the first-order approximation scheme, we easily find

$$\hat{\rho}_{ab} = \frac{g\gamma_a}{\gamma^2}(\rho^{(0)}_{ac}\hat{\rho}\hat{a}^\dagger - \rho^{(0)}_{aa}\hat{\rho}\hat{a}), \tag{3.89}$$

$$\hat{\rho}_{bc} = \frac{g\gamma_a}{\gamma^2}(\rho^{(0)}_{cc}\hat{a}\hat{\rho} - \rho^{(0)}_{ac}\hat{a}^\dagger\hat{\rho}). \tag{3.90}$$

The complex conjugate of Eqs.(3.89) and (3.90), can be written as

$$\hat{\rho}_{ba} = \frac{g\gamma_a}{\gamma^2}(\rho^{(0)}_{ca}\hat{a}\hat{\rho} - \rho^{(0)}_{aa}\hat{a}^\dagger\hat{\rho}), \tag{3.91}$$

$$\hat{\rho}_{cb} = \frac{g\gamma_a}{\gamma^2}(\rho^{(0)}_{cc}\hat{\rho}\hat{a}^\dagger - \rho^{(0)}_{ca}\hat{\rho}\hat{a}). \tag{3.92}$$

Finally, on account of Eqs.(3.89), (3.90), (3.91) and (3.92), the master equation for the cavity mode given by Eq.(3.44), takes the form

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= \frac{1}{2}(\kappa N + A\rho^{(0)}_{aa})[2\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{a}^\dagger\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^\dagger] + \frac{1}{2}(\kappa(N+1) + A\rho^{(0)}_{cc})[2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}] \\ &+ \frac{1}{2}(\kappa M - A\rho^{(0)}_{ac})[2\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}] + \frac{1}{2}(\kappa M - A\rho^{(0)}_{ca})[2\hat{a}\hat{\rho}\hat{a} - \hat{a}^2\hat{\rho} - \hat{\rho}\hat{a}^2],\end{aligned} \tag{3.93}$$

where

$$A = \frac{2\gamma_a g^2}{\gamma^2} \tag{3.94}$$

is the linear gain coefficient. It is worth mentioning that the quantum properties of the light generate by the three-level laser are determined by the master equation (3.93). It is easily to observe that with $\rho^{(0)}_{aa} = 1$ and $\rho^{(0)}_{ac} = \rho^{(0)}_{cc} = 0$, this equation reduced to the master equation for a two-level laser operating below threshold.

3.1.4 c-number Langevin equation

The dynamics of a cavity mode coupled to the squeezed vacuum reservoir can be also described using the c-number Langevin equation. In this section we determine c-number Langevin equations for the cavity mode employing the master equation in Eq.(3.93) to determine the equation of the evaluation for the expectation value of the normally ordered cavity mode operators. The time evolution of the expectation value of an operator $\hat{A}$ in Schödinger picture can be expressed

$$\frac{d}{dt}\langle A(t)\rangle = Tr(\frac{d}{dt}\hat{\rho}(t)\hat{A}), \tag{3.95}$$

along with the master equation (3.93), we get

$$\frac{d\langle\hat{a}\rangle}{dt} = \frac{1}{2}(\kappa N + A\rho_{aa}^{(0)})Tr[2\hat{a}^{\dagger}\hat{\rho}\hat{a} - \hat{a}\hat{a}^{\dagger}\hat{\rho} - \hat{\rho}\hat{a}\hat{a}^{\dagger}] + \frac{1}{2}(\kappa(N+1) + A\rho_{cc}^{(0)})Tr[2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}]$$
$$+ \frac{1}{2}(\kappa M - A\rho_{ac}^{(0)})Tr[2\hat{a}^{\dagger}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger 2}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger 2}] + \frac{1}{2}(\kappa M - A\rho_{ca}^{(0)})Tr[2\hat{a}\hat{\rho}\hat{a} - \hat{a}^{2}\hat{\rho} - \hat{\rho}\hat{a}^{2}]. \tag{3.96}$$

On account of the cyclic property trace operation along Boson commutation relation, we get

$$\frac{d}{dt}\langle a(t)\rangle = \frac{-1}{2}\mu\langle a(t)\rangle, \tag{3.97}$$

where

$$\mu = \kappa + A(\rho_{cc}^{(0)} - \rho_{aa}^{(0)}). \tag{3.98}$$

We also note that

$$\frac{d}{dt}\langle\hat{a}^{\dagger}(t)\rangle = \frac{-1}{2}\mu\langle\hat{a}^{\dagger}(t)\rangle - \frac{1}{2}\nu\langle\hat{a}(t)\rangle, \tag{3.99}$$

where $\nu = \kappa M - A\rho_{ca}^{(0)}$. In the same fashion, one can show that

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}(t)\rangle = -\mu\langle\hat{a}^{2}(t)\rangle + A\rho_{ca}^{(0)} - \kappa M, \tag{3.100}$$

and

$$\frac{d}{dt}\langle\hat{a}^{\dagger}(t)\hat{a}(t)\rangle = -\mu\langle\hat{a}^{\dagger}(t)\hat{a}(t)\rangle + A\rho_{aa}^{(0)} + \kappa N. \tag{3.101}$$

We note that the c-number equation corresponding to Eqs.(3.97), (3.99), (3.100) and (3.101) are

$$\frac{d}{dt}\langle\alpha(t)\rangle = \frac{-1}{2}\mu\langle\alpha(t)\rangle, \tag{3.102}$$

$$\frac{d}{dt}\langle\alpha^{*}(t)\rangle = \frac{-1}{2}\mu\langle\alpha^{*}(t)\rangle - \frac{1}{2}\nu\langle\alpha\rangle, \tag{3.103}$$

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = -\mu\langle\alpha^{2}(t)\rangle + A\rho_{ca}^{(0)} - \kappa M, \tag{3.104}$$

$$\frac{d}{dt}\langle\alpha^{*}(t)\alpha(t)\rangle = -\mu\langle\alpha^{*}(t)\alpha(t)\rangle + A\rho_{aa}^{(0)} + \kappa N. \tag{3.105}$$

The steady state solution of the c-number equations are

$$\langle\alpha(t)\rangle_{ss} = 0, \tag{3.106}$$

$$\langle\alpha^2(t)\rangle_{ss} = \frac{A\rho_{ca}^{(0)} - \kappa M}{\mu}, \tag{3.107}$$

$$\langle\alpha^*(t)\alpha(t)\rangle_{ss} = \frac{A\rho_{aa}^{(0)} + \kappa N}{\mu}. \tag{3.108}$$

On the basis of Eq.(3.102), c-number Langevin equations, one can write

$$\frac{d}{dt}\alpha(t) = \frac{-1}{2}\mu\langle\alpha(t)\rangle + f(t), \tag{3.109}$$

where f(t) is a noise force whose correlation properties remain to be determined. Taking the expectation value of Eq.(3.109) and compering it with Eq.(3.102), we see that the

$$\langle f(t)\rangle = 0. \tag{3.110}$$

As $\alpha(t)$ is simply a c-number function, ordering does not affect the c- number Langevin equations. Therefore, we see that

$$\frac{1}{2}\frac{d}{dt}(\alpha(t)\alpha(t)) = \alpha(t)\frac{d}{dt}(\alpha(t)). \tag{3.111}$$

Upon multiplying Eq.(3.109) by $\alpha(t)$, we find

$$\alpha(t)\frac{d}{dt}\alpha(t) = -\frac{1}{2}\mu\alpha^2(t) + \alpha(t)f(t), \tag{3.112}$$

or

$$\frac{1}{2}\frac{d}{dt}(\alpha(t)\alpha(t)) = -\frac{1}{2}\mu\alpha^2(t) + \alpha(t)f(t). \tag{3.113}$$

From which follows

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = -\mu\langle\alpha^2(t)\rangle + 2\langle\alpha(t)f(t)\rangle. \tag{3.114}$$

In view of Eqs.(3.104) and (3.114), we see that

$$\langle\alpha(t)f(t)\rangle = \frac{A\rho_{ac}^{(0)} - \kappa M}{2}. \tag{3.115}$$

The formal solution of Eq.(3.109) can be written as

$$\alpha(t) = \alpha(0)e^{-\mu t/2} + \int_0^t e^{-\mu(t-t')/2} f(t')dt'. \tag{3.116}$$

Upon multiplying Eq.(3.89) by f(t) and taking the expectation value, there follows

$$\langle\alpha(t)f(t)\rangle = \langle\alpha(0)f(t)\rangle e^{-\mu t/2} + \int_0^t e^{-\mu(t-t')/2}\langle f(t)f(t')\rangle dt'. \tag{3.117}$$

Noting the fact that a noise force at a certain time t should not affect system variables at earlier time, we get

$$\langle\alpha^*(0)f(t)\rangle = \langle\alpha(0)\rangle\langle f(t)\rangle = 0, \tag{3.118}$$

$$\langle\alpha(t)f(t)\rangle = \int_0^t e^{-\mu(t-t')/2}\langle f(t)f(t')\rangle dt'. \tag{3.119}$$

From Eqs.(3.115), we see that

$$\int_0^t e^{-\mu(t-t')/2}\langle f(t)f(t')\rangle dt' = \frac{A\rho_{ac}^{(0)} - \kappa M}{2}. \tag{3.120}$$

Now on the basis of the relation

$$\int_0^t e^{-\mu(t-t')/2}\langle f(t)g(t')\rangle dt' = E, \tag{3.121}$$

we assert that

$$\langle f(t)g(t')\rangle = 2E\delta(t-t'), \tag{3.122}$$

where E is a constant or some function of time t. In view of Eq.(3.95), we easily see that

$$\langle f(t)f(t')\rangle = (A\rho_{ac}^{(0)} - \kappa M)\delta(t-t'). \tag{3.123}$$

Furthermore, taking the sum of equations obtained on multiplying Eq.(3.109) by $\alpha^*(t)$ from the left and its complex conjugate by $\alpha(t)$ from the right, we find

$$\alpha(t)\frac{d}{dt}\alpha^*(t) + \frac{d}{dt}\alpha(t)\alpha^*(t) = -\mu\alpha^*(t)\alpha(t) + \alpha(t)f^*(t) + \alpha^*(t)f(t), \tag{3.124}$$

we then see that

$$\frac{d}{dt}\langle\alpha(t)\alpha^*(t)\rangle = -\mu\alpha^*(t)\alpha(t) + \alpha(t)f^*(t) + \alpha^*(t)f(t). \tag{3.125}$$

By comparing Eqs.(3.105) and (3.123), we find

$$\langle(\alpha(t)f^*(t) + \alpha^*(t)f(t))\rangle = A\rho_{aa}^{(0)} + \kappa N. \tag{3.126}$$

In the same way from Eq.(3.116) and its complex conjugate, one can easily get

$$\langle(f^*(t)\alpha(t)\rangle = \int_0^t e^{-\mu(t-t')/2}\langle f^*(t)f(t')\rangle dt', \tag{3.127}$$

$$\langle\alpha^*(t)(f(t)\rangle = \int_0^t e^{-\mu(t-t')/2}\langle f^*(t)f(t')\rangle dt'. \tag{3.128}$$

Using Eqs.(3.127) and (3.128) together with the assumption that

$$\langle f^*(t)f(t')\rangle = \langle f(t')f^*(t)\rangle. \tag{3.129}$$

With the aid of Eqs.(3.126), (3.127), (3.128) and (3.129), we arrive at

$$2\int_0^t e^{-\mu(t-t')/2}\langle f^*(t)f(t')\rangle dt' = A\rho_{aa}^{(0)} + \kappa N, \tag{3.130}$$

from which we assert that

$$\langle f^*(t)f(t')\rangle = (A\rho_{aa}^{(0)} + \kappa N)\delta(t-t'). \tag{3.131}$$

It is worth mentioning that Eqs.(3.110), (3.123) and (3.131) represent the correlation properties of the noise force f(t) associated with the normal ordering. In order to obtain the solution of Eq.(3.109), we introduce a new variable defined by

$$\alpha_\pm(t) = \alpha^*(t) \pm \alpha(t), \tag{3.132}$$

$$\frac{d}{dt}\alpha_\pm(t) = \frac{d}{dt}\alpha^*(t) \pm \frac{d}{dt}\alpha(t). \tag{3.133}$$

Using Eq.(3.109) and its complex conjugate, we get

$$\frac{d}{dt}\alpha\pm(t) = \frac{-\mu}{2}\alpha\pm(t) + f^*(t) \pm f(t). \tag{3.134}$$

The solution of Eq.(3.134) can be written as

$$\alpha\pm(t) = \alpha\pm(0)e^{-\mu t/2} + \int_0^t e^{-\mu(t-t')/2}(f^*(t)\pm f(t))dt'. \tag{3.135}$$

In view of Eq.(3.135), we find

$$\alpha(t) = A_+(t)\alpha(0) + B_+(t) - B_-(t), \tag{3.136}$$

where

$$A_+(t) = e^{-\mu t/2}, \tag{3.137}$$

and

$$B_\pm(t) = \frac{1}{2}\int_0^t e^{-\mu(t-t')/2}(f^*(t)\pm f(t))dt'. \tag{3.138}$$

It is worth noting that $\alpha(0)$ along with their corresponding complex conjugate represents the thermal light. But if the source of the chaotic light remains in the cavity through out the lasing operation, the effect of the thermal light should also be included in the correlations of the noise.

Employing the resulting solution of the c-number Langevin equation along with the correlation relations of the noise forces, we will study the quadrature variance and squeezing spectrum for the a single-mode cavity radiation. In addition, we determine the mean photon number and the variance of the photon number.

4

RESULT AND DISCUSSION

4.1 Quadrature fluctuations

We next proceed to determine the quadrature variance and squeezing spectrum for the cavity radiation produced by the effect of injected thermal light in cavity a degenerate three-level laser coupled to squeezed vacuum reservoir.

4.1.1 Quadrature variance of the cavity mode

The squeezing properties of a single mode light is described by two quadrature operators

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a}, \tag{4.1}$$

and

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}). \tag{4.2}$$

Using the commutation relation

$$[\hat{a}, \hat{a}^\dagger] = 1, \tag{4.3}$$

with the aid of Eqs.(4.1), (4.2) and (4.3), one can readily verify that

$$[\hat{a}_+, \hat{a}_-] = 2i. \tag{4.4}$$

A single - mode light is said to be in a squeezed state if either $\Delta\hat{a}_+ < 1$ and $\Delta\hat{a}_- > 1$ or $\Delta\hat{a}_+ > 1$ and $\Delta\hat{a}_- < 1$ such that $\Delta\hat{a}_+\Delta\hat{a}_- \geq 1$. The quadrature variance is defined by

$$\Delta\hat{a}^2_\pm(t) = \langle\hat{a}^2_\pm(t)\rangle - (\langle\hat{a}_\pm(t)\rangle)^2. \tag{4.5}$$

On account of Eqs.(4.1), (4.2) and (4.5) can be written in the normal order as

$$\Delta\hat{a}^2_\pm(t) = 1 \pm (\langle\hat{a}^2(t) + \hat{a}^{\dagger 2}(t) \pm 2\hat{a}^\dagger(t)\hat{a}(t)\rangle \mp (\langle\hat{a}^\dagger(t) \pm \hat{a}(t)\rangle)^2), \tag{4.6}$$

$$\Delta\hat{a}_{\pm}^{2}(t) = 1 \pm (\langle \hat{a}_{\pm}^{2}(t)\rangle - \langle \hat{a}_{\pm}(t)\rangle^{2}). \tag{4.7}$$

This can be expressed in terms of c-number variables associated with the normal ordering as

$$\Delta\alpha_{\pm}^{2}(t) = 1 \pm \langle \alpha^{*2}(t) + \alpha^{2}(t) \pm 2\alpha^{*}(t)\alpha(t)\rangle \mp \langle \alpha^{*}(t) \pm \alpha(t)\rangle^{2}, \tag{4.8}$$

$$\Delta\alpha_{\pm}^{2}(t) = 1 \pm (\langle \alpha_{\pm}^{2}(t)\rangle - \langle \alpha_{\pm}(t)\rangle^{2}), \tag{4.9}$$

where $\alpha(t)$ is the c-number corresponding to the operator $\hat{a}(t)$. Introducing a new variable defined by

$$\alpha_{\pm}(t) = \alpha^{*}(t) \pm \alpha(t). \tag{4.10}$$

Eq.(4.9), can be write

$$\Delta\alpha_{\pm}^{2}(t) = 1 \pm \langle \alpha_{\pm}(t), \alpha_{\pm}(t)\rangle. \tag{4.11}$$

The c-number equation corresponding to Eqs.(4.1) and (4.2) is expressible as

$$\alpha_{+}(t) = \alpha^{*}(t) + \alpha(t), \tag{4.12}$$

$$\alpha_{-}(t) = i(\alpha^{*}(t) - \alpha(t)). \tag{4.13}$$

On account of Eq.(3.109), we obtain

$$\frac{d}{dt}\alpha(t) = \frac{-1}{2}\mu\alpha(t) + f(t). \tag{4.14}$$

With the help of Eq.(4.10), one can easily get

$$\frac{d}{dt}\alpha_{\pm}(t) = \frac{d}{dt}\alpha^{*}(t) \pm \frac{d}{dt}\alpha(t). \tag{4.15}$$

Employing Eq.(4.14) and its complex conjugate along with Eq.(4.14), we obtain

$$\frac{d}{dt}\alpha_{\pm}(t) = \frac{-1}{2}\mu\alpha_{\pm}(t) + f^{*}(t) \pm f(t), \tag{4.16}$$

the solution of Eq.(4.16), can be written as

$$\alpha_{\pm}(t) = \alpha_{\pm}(0)e^{-\mu t/2} + \int_{0}^{t} e^{-\mu(t-t')/2}(f^{*}(t) \pm f(t))dt'. \tag{4.17}$$

Now in view of Eq.(3.110), Eq.(4.17), we have

$$\langle \alpha_{\pm}(t)\rangle = \langle \alpha_{\pm}(0)\rangle e^{-\mu t/2}. \tag{4.18}$$

Further more, applying the relation

$$\frac{d}{dt}\alpha_{\pm}^2(t) = 2\alpha_{\pm}(t)\frac{d}{dt}\alpha_{\pm}(t), \tag{4.19}$$

by substitution Eq.(4.14) into Eq.(4.19), we obtain

$$\frac{d}{dt}(\alpha_{\pm}^2(t)) = -\mu\langle\alpha_{\pm}^2(t)\rangle + 2\langle\alpha_{\pm}(t)f^*(t)\rangle \pm 2\langle\alpha_{\pm}(t)f(t)\rangle. \tag{4.20}$$

Upon multiplying Eq.(4.17) by $f(t')$ and taking expectation value, we gets

$$\langle\alpha_{\pm}(t)f(t')\rangle = \langle\alpha_{\pm}(0)f(t')\rangle e^{-\mu t/2} + \int_0^t e^{-\mu(t-t')/2}(\langle f^*(t)f(t')\rangle \pm \langle f(t)f(t')\rangle)dt'. \tag{4.21}$$

Taking into account Eqs.(3.110), (3.123) and (4.21), noting the fact that a noise force at time t should not affect system variables at earlier time, we get

$$\langle\alpha_{\pm}(t)f(t')\rangle = (A\rho_{ac}^{(0)} - \kappa M)\int_0^t e^{-\mu(t-t')/2}\delta(t-t')dt'. \tag{4.22}$$

Now on the basis of the relation

$$E\int_0^t e^{-\mu(t-t')/2}\delta(t-t')dt' = E/2, \tag{4.23}$$

we can easily arrive at

$$\langle\alpha_{\pm}(t)f(t')\rangle = \frac{A\rho_{ac}^{(0)} - \kappa M}{2}. \tag{4.24}$$

In similar case, we have

$$\langle\alpha_{\pm}(t)f^*(t) = \frac{A\rho_{aa}^{(0)} + \kappa N}{2}. \tag{4.25}$$

Taking into account of Eqs.(4.24) and (4.25), we can write Eq.(4.21) as

$$\frac{d}{dt}(\alpha_{\pm}^2(t)) = -\mu\langle\alpha_{\pm}^2(t)\rangle + (A\rho_{aa}^{(0)} + \kappa N) \pm (A\rho_{ac}^{(0)} - \kappa M). \tag{4.26}$$

By substituting Eq.(4.18) into Eq.(4.26), we obtain

$$\frac{d}{dt}\langle\alpha_{\pm}^2(t)\rangle = -\mu\langle\alpha_{\pm}^2(0)\rangle e^{-\mu t} + (A\rho_{aa}^{(0)} + \kappa N) \pm (A\rho_{ac}^{(0)} - \kappa M). \tag{4.27}$$

The solution of Eq.(4.27), can be written as

$$\langle\alpha_{\pm}^2(t)\rangle = -\mu\langle\alpha_{\pm}^2(0)\rangle e^{-\mu t} + (A\rho_{aa}^{(0)} + \kappa N) \pm (A\rho_{ac}^{(0)} - \kappa M)\int_0^t e^{-\mu(t-t')}\delta(t-t')dt'. \tag{4.28}$$

Carrying out the integration, we get

$$\langle\alpha_{\pm}^2(t)\rangle = -\mu\langle\alpha_{\pm}^2(0)\rangle e^{-\mu t} + \frac{1}{\mu}[(A\rho_{aa}^{(0)} + \kappa N) \pm (A\rho_{ac}^{(0)} - \kappa M)](1 - e^{-\mu t}). \tag{4.29}$$

On the other hand, using Eq.(4.18), one can write

$$\langle \alpha_{\pm}(t)\rangle^2 = \langle \alpha_{\pm}(0)\rangle^2 e^{-\mu t}. \tag{4.30}$$

Hence substitution of Eqs.(4.29) and (4.30) into Eq.(4.9), one can easily arrive at

$$\Delta\alpha_{\pm}^2(t) = 1 \pm \Delta\alpha_{\pm}^2(0)e^{-\mu t} + \frac{1}{\mu}[(A\rho_{aa}^{(0)} + \kappa N) \pm (A\rho_{ac}^{(0)} - \kappa M)](1 - e^{-\mu t}), \tag{4.31}$$

$$\Delta\alpha_{+}^2(t) = 1 + \Delta\alpha_{+}^2(0)e^{-\mu t} + \frac{1}{\mu}[(A\rho_{aa}^{(0)} + \kappa N) + (A\rho_{ac}^{(0)} - \kappa M)](1 - e^{-\mu t}), \tag{4.32}$$

$$\Delta\alpha_{-}^2(t) = 1 - \Delta\alpha_{-}^2(0)e^{-\mu t} + \frac{1}{\mu}[(A\rho_{aa}^{(0)} + \kappa N) - (A\rho_{ac}^{(0)} - \kappa M)](1 - e^{-\mu t}), \tag{4.33}$$

where

$$\Delta\alpha_{\pm}^2(0) = 1 \pm (\langle \alpha_{\pm}^2(0)\rangle - \langle \alpha(0)\rangle^2), \tag{4.34}$$

taking cavity mode to be initially in chaotic state, we obtain

$$\Delta\alpha_{+}^2(0) = 2\bar{n} + 2, \tag{4.35}$$

upon substituting Eq.(4.35) into Eq.(4.32), results in

$$\Delta\alpha_{+}^2(t) = 1 + (2\bar{n} + 2)e^{-\mu t} + \frac{1}{\mu}[(A\rho_{aa}^{(0)} + \kappa N) + (A\rho_{ac}^{(0)} - \kappa M)](1 - e^{-\mu t}). \tag{4.36}$$

Further more, it can also be established in a similar manner that

$$\Delta\alpha_{-}^2(0) = 2\bar{n} + 1, \tag{4.37}$$

upon substituting Eq.(4.37) into Eq.(4.33), we have

$$\Delta\alpha_{-}^2(t) = 1 - (2\bar{n} + 1)e^{-\mu t} + \frac{1}{\mu}[(A\rho_{aa}^{(0)} + \kappa N) - (A\rho_{ac}^{(0)} - \kappa M)](1 - e^{-\mu_- t}). \tag{4.38}$$

Taking into account of Eqs.(4.47) and (4.50), we can writes Eqs.(4.36) and (4.38) as

$$\begin{aligned}\Delta\alpha_{+}^2(t) = 1 + (2\bar{n} + 2)e^{-(\kappa + A\eta)t} + \frac{1}{2(\kappa + A\eta)}[A(1 - \eta) + 2\kappa N \\ + (A(1 - \eta^2)^{1/2}e^{i\theta} - 2\kappa M)](1 - e^{-(\kappa + A\eta)t}),\end{aligned} \tag{4.39}$$

$$\begin{aligned}\Delta\alpha_{-}^2(t) = 1 - (2\bar{n} + 1)e^{-(\kappa + A\eta)t} + \frac{1}{2(\kappa + A\eta)}[A(1 - \eta) + 2\kappa N \\ - (A(1 - \eta^2)^{1/2}e^{i\theta} - 2\kappa M)](1 - e^{-(\kappa + A\eta)t}),\end{aligned} \tag{4.40}$$

where

$$\mu = \kappa + A\eta. \tag{4.41}$$

N and M are the reservoir parameters given by

$$N = \frac{e^{2r} + e^{-2r} - 2}{4}, \tag{4.42}$$

$$M = \frac{e^{-2r} - e^{2r}}{4}. \tag{4.43}$$

With r being the squeezed parameter. Using the Eqs.(4.42) and (4.43) into Eqs.(4.39) and (4.30), the results in

$$\Delta\alpha_+^2(t) = 1+(2\bar{n}+2)e^{-(\kappa+A\eta)t}+\frac{1}{2(\kappa+A\eta)}[A(1-\eta)+(A(1-\eta^2)^{1/2}e^{i\theta})+\kappa(e^{2r}-1)](1-e^{-(\kappa+A\eta)t}), \tag{4.44}$$

$$\Delta\alpha_-^2(t) = 1-(2\bar{n}+1)e^{-(\kappa+A\eta)t}+\frac{1}{2(\kappa+A\eta)}[A(1-\eta)-(A(1-\eta^2)^{1/2}e^{i\theta})-\kappa(e^{-2r}-1)](1-e^{-(\kappa+A\eta)t}). \tag{4.45}$$

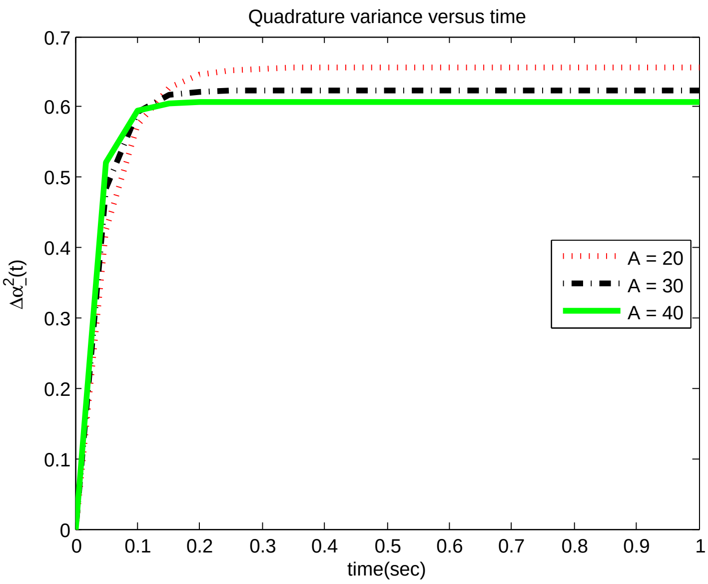

Fig. 4.1: Plots of a minus quadrature variance[Eq.(4.45)] versus time for $\bar{n}_a = 0,\ \eta = 0.2,\ \theta = 0,\ \kappa = 0.8,\ r = 2,\ e = 2.72$, and different values of the linear gain coefficient, for A = 20(Dotted curve), A = 30(dash-dot curve) and A = 40(Solid curve).

It is clearly shown in Fig.4.1 that the degree of a single mode squeezing increases with time and the rate at which the atoms are injected into the cavity. This entails that as time progresses more atoms traverse across the cavity where by the non-classical correction gets more strength. This type of reasoning gives sense if one compares the results obtained here with earlier studies at a steady state[18].

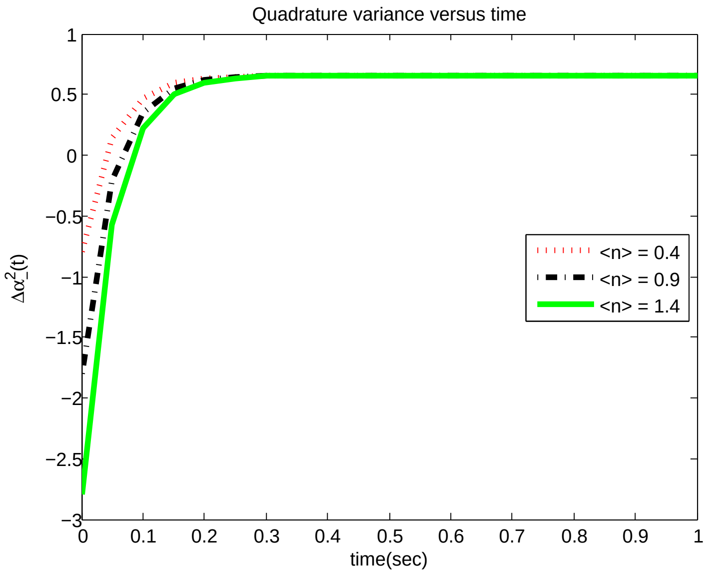

Fig. 4.2: Plots of a minus quadrature variance[Eq.(4.45)] versus time for $A = 20,\ \eta = 0.2,\ \theta = 0,\ \kappa = 0.8,\ r = 2,\ e = 2.72$, and different values of $\bar{n}_a$, for $\langle\hat{n}_a\rangle = 0.4$(Dotted curve), $\langle\hat{n}_a\rangle = 0.9$ (dash-dot curve) and $\langle\hat{n}_a\rangle = 1.4$(Solid curve).

It is clearly shown in Fig.4.2 that the degree of squeezing at large time scale is independent of the mean photon number of the chaotic light. This is mainly because as time progresses there are more atoms that pass the cavity in a way absorb the available radiation.
At steady state the quadrature variance of Eqs.(4.44) and (4.45) for $\theta = 0$, takes the form

$$(\Delta\alpha_+^2)_{ss} = 1 + \frac{1}{2(\kappa + A\eta)}[A((1-\eta) + (1-\eta^2)^{1/2}) - \kappa(1-e^{2r})], \tag{4.46}$$

$$(\Delta\alpha_-^2)_{ss} = 1 + \frac{1}{2(\kappa + A\eta)}[A((1-\eta) - (1-\eta^2)^{1/2}) + \kappa(1-e^{-2r})], \tag{4.47}$$

where ss stands for steady state. We easily notice that the squeezing occurs in the minus quadrature. It is also clear that the best squeezing in a degenerate three-level laser is achieved for all values of η between zero and one and the degree of squeezing increases with the linear gain coefficient.

4.2 Squeezing spectrum of the output mode

The squeezing spectrum for a single-mode light can then be defined in the normal order as

$$S^{out}_{\pm}(\omega) = 1 \pm 2Re\int_0^{\infty} \langle : \hat{a}^{out}_{\pm}(t), \hat{a}^{out}_{\pm}(t+\tau) :\rangle_{ss} e^{i(\omega)\tau} d\tau, \tag{4.48}$$

in which : refers to normal ordering. This can be expressed interims of c-number variables associated with the normal ordering as

$$S^{out}_{\pm}(\omega) = 1 \pm 2Re\int_0^{\infty} \langle : \alpha^{out}_{\pm}(t), \alpha^{out}_{\pm}(t+\tau) :\rangle_{ss} e^{i(\omega)\tau} d\tau. \tag{4.49}$$

For a cavity mode coupled to a squeezed vacuum reservoir, the output and cavity variables can be related by

$$\alpha^{out}_{\pm} = \sqrt{\kappa}\alpha_{\pm} + \alpha^{in}_{\pm}, \tag{4.50}$$

on account of Eq.(4.50), the squeezing spectrum takes the form

$$\begin{aligned} S^{out}_{\pm}(\omega) = 1 &\pm 2\kappa Re\int_0^{\infty} \langle \alpha_{\pm}(t)\alpha_{\pm}(t+\tau)\rangle_{ss} e^{i(\omega)} d\tau \mp 2\sqrt{\kappa} Re\int_0^{\infty} \langle \alpha_{\pm}(t)\alpha^{in}_{\pm}(t+\tau)\rangle_{ss} e^{i(\omega)} d\tau \\ &\mp 2\sqrt{\kappa} Re\int_0^{\infty} \langle \alpha^{in}_{\pm}(t)\alpha_{\pm}(t+\tau)\rangle_{ss} e^{i(\omega)} d\tau \pm 2Re\int_0^{\infty} \langle \alpha^{in}_{\pm}(t)\langle \alpha^{in}_{\pm}(t+\tau)\rangle_{ss} e^{i(\omega)} d\tau. \end{aligned} \tag{4.51}$$

The solution of Eq.(3.134), can be written as

$$\alpha_{\pm}(t+\tau) = \alpha_{\pm}(t)e^{-\mu_{\pm}\tau} + \int_0^{\tau} e^{-\mu_{\pm}(t-\tau')}[f^*(t+\tau') \pm f(t+\tau')]d\tau'. \tag{4.52}$$

Now applying this equation along with the quantum regression theorem, one can establish that

$$\langle \alpha_{\pm}(t)\alpha_{\pm}(t+\tau)\rangle_{ss} = \langle \alpha^2_{\pm}(t)\rangle_{ss} e^{-\mu\tau}, \tag{4.53}$$

$$\langle \alpha_{\pm}(t)\alpha^{in}_{\pm}(t+\tau)\rangle_{ss} = 0, \tag{4.54}$$

$$\langle \alpha^{in}_{\pm}(t)\alpha_{\pm}(t+\tau)\rangle_{ss} = 2\sqrt{\kappa}(M \pm N)e^{-\mu\tau}, \tag{4.55}$$

$$\langle \alpha^{in}_{\pm}(t+\tau)\rangle_{ss} = 2(M \pm N)\delta(\tau). \tag{4.56}$$

In view of these results, the squeezing spectrum takes the form

$$\begin{aligned} S^{out}_{\pm}(\omega) = 1 + 2\kappa\langle \alpha^2_{\pm}(t)\rangle_{ss} Re\int_0^{\infty} e^{-(\mu - i\omega\tau)} d\tau \mp 4\kappa(M \pm N) Re\int_0^{\infty} e^{-(\mu - i\omega\tau)} d\tau \pm 4(M \pm N) \\ Re\int_0^{\infty} e^{-(\mu - i\omega\tau)} d\tau. \end{aligned} \tag{4.57}$$

Thus upon performing the integration and using Eq.(3.107), we easily find

$$S_{\pm}^{out}(\omega) = 1 \pm \frac{2\kappa(A\rho_{ca}^{0} - \kappa M) - 4\kappa(M \pm N)\mu}{\mu^2 + \omega^2} \pm 2(M \pm N). \tag{4.58}$$

Employing Eqs.(3.52), (4.41), (4.42) and (4.43), squeezing spectrum can be written as

$$S_{+}^{out}(\omega) = 1 + \frac{\kappa e^{2r}[2A(1-\eta^2)^{\frac{1}{2}}e^{-i\theta} - \kappa(e^{-2r} - e^{2r}) + 4(1-e^{-2r})(\kappa + A\eta)]}{2((\kappa + A\eta)^2 + \omega^2)}, \tag{4.59}$$

$$S_{-}^{out}(\omega) = 1 - \frac{\kappa e^{-2r}[2A(1-\eta^2)^{\frac{1}{2}}e^{-i\theta} - \kappa(e^{-2r} - e^{2r}) + 4(1-e^{2r})(\kappa + A\eta)]}{2((\kappa + A\eta)^2 + \omega^2)}. \tag{4.60}$$

We observed that the squeezed vacuum reservoir has the effect of increasing the degree of squeezing. We also realized that for $\omega = 0$, the noise in the minus quadrature is completely suppressed, with noise in the plus quadrature going to infinity. Finally, for $\theta = 0$ and r = 0, Eqs.(4.59) and (4.60), reduced to

$$S_{+}^{out}(\omega) = 1 + \frac{\kappa A(1-\eta^2)^{\frac{1}{2}}}{(\kappa + A\eta)^2 + \omega^2}, \tag{4.61}$$

$$S_{-}^{out}(\omega) = 1 - \frac{\kappa A(1-\eta^2)^{\frac{1}{2}}}{(\kappa + A\eta)^2 + \omega^2}, \tag{4.62}$$

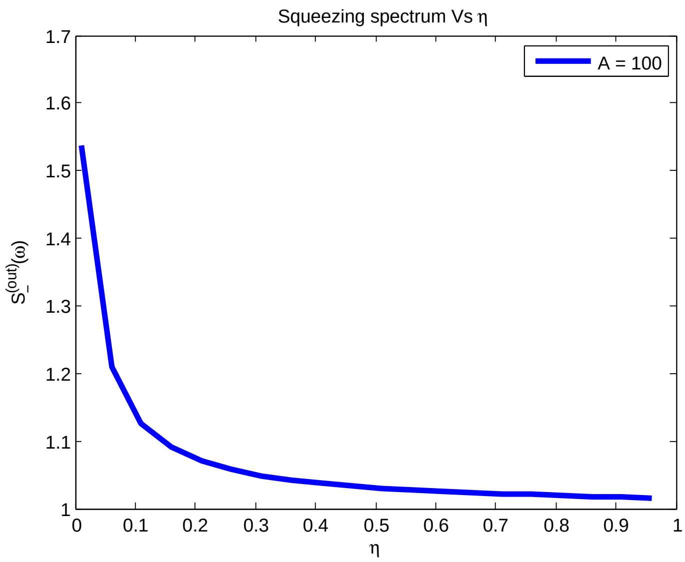

Fig. 4.3: Plots of a minus squeezing spectrum [Eq.(4.60)] versus η for $\omega = 0,\ \theta = 0,\ \kappa = 0.8,\ r = 0.5, e = 2.72,$ and value of A = 100(Solid curve).

Fig.4.3 Show that there appears to be perfect squeezing around $A = 100$ and for values of η very closed to zero.

4.3 Photon statistics

4.3.1 The mean photon number

For the system under consideration the number of photons in cavity can be represented by the number operator

$$\hat{n}_a = \langle \hat{a}^\dagger \hat{a} \rangle, \tag{4.63}$$

where $\hat{n}_a = \hat{a}^\dagger \hat{a}$ is the photon number operator for mode $\hat{a}$. The mean photon number for the cavity mode can also be expressed in terms of c-number variable associated with the normal ordering as

$$\hat{n}_a = \langle \alpha^*(t)\alpha(t) \rangle. \tag{4.64}$$

Thus on accounting Eq.(3.136) and assuming the cavity mode to be initially in a single mode thermal light, one can readily verify that

$$\langle \alpha(t) \rangle = 0. \tag{4.65}$$

Upon multiplying Eq.(3.136) on the left by $\alpha^*(t)$ and taking expectation value of the resulting expression, we have

$$\langle \alpha^*(t)\alpha(t) \rangle = A_+(t)\langle \alpha^*(t)\alpha(0) \rangle + \langle \alpha^*(t)B_+(t) \rangle - \langle \alpha^*(t)B_-(t) \rangle. \tag{4.66}$$

It can be shown using Eq.(3.136) and its complex conjugate and taking expectation value of the resulting expression, we have

$$\langle \alpha^*(t) \rangle = A_+(t)\langle \alpha_*(0) \rangle + \langle B_+^*(t) \rangle - \langle B_-^*(t) \rangle, \tag{4.67}$$

upon multiplying Eq.(4.67) on the right by $\alpha^*(0)$, we obtain

$$\langle \alpha^*(t)\alpha(0) \rangle = A_+(t)\langle \alpha_*(0)\alpha(0) \rangle + \langle B_+^*(t)\alpha(0) \rangle - \langle B_-^*(t)\alpha(0) \rangle. \tag{4.68}$$

On the basis of the fact that the noise force at time t does not affect the cavity mode variable at earlier time and taking the cavity mode to be initially in a chaotic state, we get

$$\langle \alpha^*(t)\alpha(0) \rangle = A_+(t)\bar{n}_a, \tag{4.69}$$

$$\langle \alpha^*(t)B_+(t) \rangle = \langle B_+^*(t)B_+(t) \rangle + \langle B_-^*(t)B_+(t) \rangle, \tag{4.70}$$

$$\langle \alpha^*(t)B_-(t) \rangle = \langle B_-^*(t)B_-(t) \rangle + \langle B_+^*(t)B_-(t) \rangle. \tag{4.71}$$

Thus substitution of Eqs.(4.69), (4.70) and (4.71) into Eq.(4.66), result in

$$\langle\alpha^*(t)\alpha(t)\rangle = A_+^2\bar{n}_a + \langle B_+^*(t)B_+(t)\rangle + \langle B_-^*(t)B_+(t)\rangle - \langle B_-^*(t)B_-(t)\rangle - \langle B_+^*(t)B_-(t)\rangle, \quad (4.72)$$

employing Eq.(3.138), one can write

$$B_+(t) = \frac{1}{2}\int_0^t e^{-\mu(t-t'')/2}(f^*(t'') + f(t''))dt'', \quad (4.73)$$

$$B_+^*(t) = \frac{1}{2}\int_0^t e^{-\mu(t-t')/2}(f^*(t') + f(t'))dt', \quad (4.74)$$

$$B_-(t) = \frac{1}{2}\int_0^t e^{-\mu(t-t'')/2}(f^*(t'') - f(t''))dt'', \quad (4.75)$$

$$B_-^*(t) = \frac{1}{2}\int_0^t e^{-\mu(t-t')/2}(f^*(t') - f(t'))dt'. \quad (4.76)$$

Next we evaluate the correlation functions in Eq.(4.72), to this end, on the basis of Eqs.(4.73) and (4.74), we have

$$\begin{aligned}\langle B_+^*(t)B_+(t)\rangle = \frac{1}{4}\int_0^t\int_0^t e^{-\mu(2t-t'-t'')/2}[\langle f^*(t')f^*(t'')\rangle + \langle f(t')f(t'')\rangle \\ + \langle f(t')f^*(t'')\rangle + \langle f^*(t')f(t'')\rangle]dt'dt''.\end{aligned} \quad (4.77)$$

Application of Eq.(3.123) and (3.131) in Eq.(4.77), results in

$$\langle B_+^*(t)B_+(t)\rangle = \frac{1}{2}[A(\rho_{ac}^{(0)} + \rho_{aa}^{(0)}) - (\kappa M - \kappa N)]\int_0^t\int_0^t e^{-\mu(2t-t'-t'')/2}\delta(t'-t'')dt'dt'', \quad (4.78)$$

upon carrying out the integration Eq.(4.78), we see that

$$\langle B_+^*(t)B_+(t)\rangle = \frac{1}{2\mu}[A(\rho_{ac}^{(0)} + \rho_{aa}^{(0)}) - (\kappa M - \kappa N)](1 - e^{-\mu t}). \quad (4.79)$$

With the aid of Eq.(4.73) and (4.76), we get

$$\begin{aligned}\langle B_-^*(t)B_+(t)\rangle = \frac{1}{4}\int_0^t\int_0^t e^{-\mu(2t-t'-t'')/2}[\langle f^*(t')f^*(t'')\rangle - \langle f(t')f(t'')\rangle \\ + \langle f^*(t')f(t'')\rangle - \langle f(t')f^*(t'')\rangle]dt'dt'',\end{aligned} \quad (4.80)$$

with the aid of Eqs.(3.123), (3.131) and (4.80), we see that

$$\langle B_-^*(t)B_+(t)\rangle = 0. \quad (4.81)$$

Further more, employing Eq.(4.75) and its complex conjugate, we have

$$\begin{aligned}\langle B_-^*(t)B_-(t)\rangle = \frac{1}{4}\int_0^t\int_0^t e^{-\mu(2t-t'-t'')/2}[\langle f^*(t')f^*(t'')\rangle + \langle f(t')f(t'')\rangle \\ - \langle f^*(t')f(t'')\rangle - \langle f(t')f^*(t'')\rangle]dt'dt'',\end{aligned} \quad (4.82)$$

taking Eqs.(3.123) and (3.131) into account Eq.(4.82), we get

$$\langle B_-^*(t)B_-(t)\rangle = \frac{1}{2}[A(\rho_{ac}^{(0)} - \rho_{aa}^{(0)}) - (\kappa M + \kappa N)]\int_0^t\int_0^t e^{-\mu(2t-t'-t'')/2}\delta(t'-t'')dt'dt'', \tag{4.83}$$

so that performing the integration, there follows

$$\langle B_-^*(t)B_-(t)\rangle = \frac{1}{2\mu}[A(\rho_{ac}^{(0)} - \rho_{aa}^{(0)}) - (\kappa M + \kappa N)](1 - e^{-\mu t}). \tag{4.84}$$

Moreover, using Eqs.(4.74) and (4.75), we get

$$\langle B_+^*(t)B_-(t)\rangle = 0. \tag{4.85}$$

Hence substitution of Eqs.(4.79), (4.80), (4.84) and (4.85) into Eq.(4.72), the results in

$$\begin{aligned}\langle \alpha^*(t)\alpha(t)\rangle = A_\pm^2\bar{n}_a + [\frac{A(\rho_{ac}^{(0)} + \rho_{aa}^{(0)}) - (\kappa M - \kappa N)}{2\mu}](1 - e^{-\mu t}) \\ + [\frac{A(\rho_{ac}^{(0)} - \rho_{aa}^{(0)}) - (\kappa M + \kappa N)}{2\mu}](1 - e^{-\mu t}).\end{aligned} \tag{4.86}$$

Finally, on account of Eqs.(3.47), (3.52), (3.137), (4.41), (4.42) and (4.43) the mean photon number Eq.(4.86), takes the form

$$\langle \hat{n}\rangle = e^{-(\kappa + A\eta)t}\bar{n}_a + [\frac{2A((1-\eta^2)^{\frac{1}{2}}e^{i\theta}) + \kappa(e^{-2r} + e^{2r})}{4(\kappa + A\eta)}](1 - e^{-(\kappa + A\eta)t}). \tag{4.87}$$

At steady state the mean photon number Eq.(4.87) for $\theta = 0$ and $r = 0$, takes the form

$$\langle \hat{n}\rangle_{ss} = \frac{A(1-\eta^2)^{\frac{1}{2}}}{2(\kappa + A\eta)}. \tag{4.88}$$

This is represent the mean of the photon number for the cavity mode a for a degenerate three-level laser. Which is identical with the expression obtained by Fesseha[9] upon direct use of steady state solutions of c-number Langevin equations.

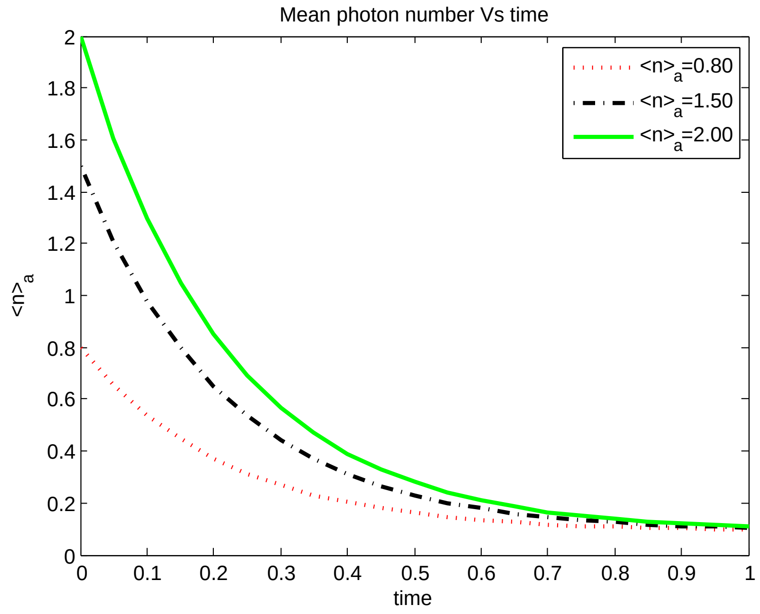

Fig. 4.4: Plots of the mean photon number of the cavity a single-mode [Eq.(4.87)] versus time for $A = 10$, $\eta = 0.2$, $\theta = 0$, $\kappa = 0.8$, $r = 2$, $e = 2.72$, $e^{2r} \approx 0$, $r = 2$, and different values of $\bar{n}_a$, for $\langle \hat{n} \rangle_a = 0.80$(Dotted curve), $\langle \hat{n} \rangle_a = 1.50$(dash-dot curve) and $\langle \hat{n} \rangle_a = 2.00$(Solid curve).

The plots in Fig.4.5, indicate that the mean photon number increases with time at the beginning and then it has no effect at time progresses. This is mainly due to the fact that at initial time the atoms emits until the radiation in the cavity is sufficient enough to frustrate the transition from energy level $|a\rangle$ to $|b\rangle$.

4.3.2 The variance of the photon number

The variance of the photon number can be expressed as

$$\Delta n^2 = \langle \hat{n}^2 \rangle - \langle \hat{n} \rangle^2, \tag{4.89}$$

applying Eq.(4.63), the variance can be put in the form

$$\Delta n^2 = \Delta n_a^2. \tag{4.90}$$

On the other hand, the photon number variance for mode a can be written in the normal order as

$$\Delta n_a^2 = \langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle + \langle \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2. \tag{4.91}$$

This can be expressed interims of c-number variables associated with the normal ordering as

$$\Delta n_a^2 = \langle \alpha^{*2}\alpha^2\rangle + \langle \alpha^*\alpha\rangle - \langle \alpha^*\alpha\rangle^2. \tag{4.92}$$

In addition, one can also write

$$\langle \hat{n}_a\rangle = \langle \alpha^*\alpha\rangle, \tag{4.93}$$

hence substitution of Eq.(4.93) into Eq.(4.92), we obtain

$$\Delta n_a^2 = \langle \hat{n}_a\rangle - \langle \hat{n}_a\rangle^2 + \langle \alpha^{*2}\alpha^2\rangle. \tag{4.94}$$

Further more Eq.(3.136), can be written as

$$\alpha(t) = A_+\alpha(0) + B_+(t) - B_-(t) + \eta_\alpha(t), \tag{4.95}$$

where $A_+(t)$ is defined by Eq.(3.137)

$$\eta_\alpha(t) = \frac{1}{2}[\int_0^t e^{-\mu(t-t')/2}(f^*(t') + f(t'))dt' + \int_0^t e^{-\mu(t-t')/2}(f^*(t') - f(t'))dt']. \tag{4.96}$$

Employing Eq.(4.95) and its complex conjugate along with the assumption that the cavity radiation is initially in a single-mode chaotic state, we get

$$\alpha^2(t) = 2\alpha(t)\eta_\alpha(t) + \eta_\alpha^2(t). \tag{4.97}$$

The complex conjugate of Eq.(4.100), we have

$$\alpha^{*2}(t) = 2\alpha^*(t)\eta_\alpha^*(t) + \eta_\alpha^{*2}(t). \tag{4.98}$$

Taking the product of Eqs.(4.97) and (4.98), then the expectation value, we have

$$\begin{aligned}\langle \alpha^{*2}\alpha^2\rangle &= 4\langle \bar{n}_a\rangle\langle \eta_\alpha^*(t)\eta_\alpha(t)\rangle + 2\langle \alpha^*(t)\rangle\langle \eta_\alpha^*(t)\eta_\alpha^2(t)\rangle \\ &+ 2\langle \alpha(t)\rangle\langle \eta_\alpha^{*2}(t)\eta_\alpha(t)\rangle + \langle \eta_\alpha^{*2}(t)\eta_\alpha^2(t)\rangle.\end{aligned} \tag{4.99}$$

We recall that Gaussian variables with vanishing means satisfy the relation[1]

$$\langle ABCD\rangle = \langle AB\rangle\langle CD\rangle + \langle AC\rangle\langle BD\rangle + \langle AD\rangle\langle BC\rangle. \tag{4.100}$$

Next we need to show that $\eta_\alpha(t)$ is Gaussian variable. One can write Eq.(4.96) as

$$\eta_\alpha = \eta_+ + \eta_-, \tag{4.101}$$

in which

$$\eta_+ = \frac{1}{2}\int_0^t e^{-\mu(t-t')/2}(f^*(t') + f(t'))dt', \tag{4.102}$$

$$\eta_- = \frac{1}{2}\int_0^t e^{-\mu(t-t')/2}(f^*(t') - f(t'))dt']. \tag{4.103}$$

Employing Eqs.(4.102) and (4.103), the time evolution for the expectation values of η_+ and η_- can be expressed as

$$\frac{d}{dt}\langle\eta_+\rangle = -\frac{1}{4}\langle\eta_+\rangle, \tag{4.104}$$

$$\frac{d}{dt}\langle\eta_-\rangle = -\frac{1}{4}\langle\eta_-\rangle. \tag{4.105}$$

Inspection of Eqs.(4.104) and (4.105) indicates that η_+ and η_- are Gaussian variables. Thus in view of Eq.(4.101), we see that η_α is also Gaussian variable. In addition, using(4.96), one easily gets

$$\langle\eta_\alpha(t)\rangle = 0. \tag{4.106}$$

Hence on the basis of the relation Eq.(4.100), we have

$$\langle\eta_\alpha\rangle = \langle\eta_\alpha^*\rangle = \langle\eta_\alpha\eta_\alpha^{*2}\rangle = \langle\eta_\alpha^*\eta_\alpha^2\rangle = 0, \tag{4.107}$$

$$\langle\eta_\alpha^{*2}\eta_\alpha^2\rangle = \langle\eta_\alpha^{*2}\rangle\langle\eta_\alpha^2\rangle + 2\langle\eta_\alpha^*\eta_\alpha\rangle^2. \tag{4.108}$$

So that application of these results in Eq.(4.99), leads to

$$\langle\alpha^{*2}\alpha^2\rangle = 4\langle\bar{n}_a\rangle\langle\eta_\alpha^*(t)\eta_\alpha(t)\rangle + \langle\eta_\alpha^{*2}\rangle\langle\eta_\alpha^2\rangle + 2\langle\eta_\alpha^*\eta_\alpha\rangle^2. \tag{4.109}$$

Now with the help of Eq.(4.96), we can write

$$\begin{aligned}\langle\eta_\alpha^2\rangle = \frac{1}{4}[&\int_0^t e^{-\mu(2t-t'-t'')/2}\langle(f^*(t') + f(t'))(f^*(t'') + f(t''))dt'dt''\rangle \\ &+ \int_0^t e^{-\mu(2t-t'-t'')/2}\langle(f^*(t') - f(t'))(f^*(t'') - f(t''))dt'dt''\rangle \\ &+ \int_0^t e^{[-(\mu+\mu)t-\mu t'-\mu t'']/2}\langle(f^*(t') + f(t'))(f^*(t'') - f(t''))dt'dt''\rangle + \\ &\int_0^t e^{[-(\mu+\mu)t-\mu t'-\mu t'']/2}\langle(f^*(t') - f(t'))(f^*(t'') - f(t''))dt'dt''\rangle],\end{aligned} \tag{4.110}$$

$$\langle f(t')f^*(t'')\rangle = \langle f^*(t')f(t'')\rangle = 0. \tag{4.111}$$

Using Eq.(4.111), we get

$$\langle\eta_\alpha^2\rangle = 0, \tag{4.112}$$

we also note that

$$\langle\eta_\alpha^{*2}\rangle = 0. \tag{4.113}$$

Furthermore, applying Eq.(4.96) and its complex conjugate, we find

$$\langle \eta_\alpha^* \eta_\alpha \rangle = \frac{1}{4}[\langle \beta_1 \rangle + \langle \beta_2 \rangle + \langle \beta_3 \rangle + \langle \beta_4 \rangle], \tag{4.114}$$

in which

$$\beta_1 = \int_0^t e^{-\mu(2t-t'-t'')/2}(f^*(t') + f(t'))(f^*(t'') + f(t''))dt'dt'', \tag{4.115}$$

$$\beta_2 = \int_0^t e^{-\mu(2t-t'-t'')/2}(f^*(t') - f(t'))(f^*(t'') - f(t''))dt'dt'', \tag{4.116}$$

$$\beta_3 = \int_0^t e^{[-(\mu+\mu)t-\mu t'-\mu t'']/2}(f^*(t') + f(t'))(f^*(t'') - f(t''))dt'dt'', \tag{4.117}$$

and

$$\beta_4 = \int_0^t e^{[-(\mu+\mu)t-\mu_- t'-\mu t'']/2}(f^*(t') - f(t'))(f^*(t'') - f(t''))dt'dt''. \tag{4.118}$$

With the aid of Eqs.(3.123) and (3.131), we obtain

$$\langle \beta_1 \rangle = 2[A(\rho_{ac}^0 + \rho_{aa}^0) - (\kappa M - \kappa N)] \int_0^t e^{-\mu(2t-t'-t'')/2}\delta(t'-t'')dt'dt'', \tag{4.119}$$

$$\langle \beta_2 \rangle = 2[A(\rho_{ac}^0 - \rho_{aa}^0) - (\kappa M + \kappa N)] \int_0^t e^{-\mu(2t-t'-t'')/2}\delta(t'-t'')dt'dt'', \tag{4.120}$$

and

$$\langle \beta_3 \rangle = \langle \beta_4 \rangle = 0. \tag{4.121}$$

Upon carrying out the integration Eq.(4.119) and (4.120), is expressible as

$$\langle \beta_1 \rangle = 2\frac{[A(\rho_{ac}^0 + \rho_{aa}^0) - (\kappa M - \kappa N)]}{\mu}(1 - e^{-\mu t}), \tag{4.122}$$

$$\langle \beta_2 \rangle = 2\frac{[A(\rho_{ac}^0 - \rho_{aa}^0) - (\kappa M + \kappa N)]}{\mu}(1 - e^{-\mu t}). \tag{4.123}$$

Hence substitution of Eqs.(4.122) and (4.123) into Eq.(4.114), results in

$$\langle \eta_\alpha^* \eta_\alpha \rangle = \frac{1}{2}[S + T], \tag{4.124}$$

in which

$$S = \frac{[A(\rho_{ac}^0 + \rho_{aa}^0) - (\kappa M - \kappa N)]}{\mu}(1 - e^{-\mu t}), \tag{4.125}$$

$$T = \frac{[A(\rho_{ac}^0 - \rho_{aa}^0) - (\kappa M + \kappa N)]}{\mu}(1 - e^{-\mu t}). \tag{4.126}$$

Therefore, on account of Eqs.(4.112), (4.113) and (4.124), Eq.(4.109), takes the form

$$\langle\alpha^{*2}\alpha^2\rangle = 2\langle\bar{n}_a\rangle[S+T] + \frac{1}{2}[S+T]^2. \tag{4.127}$$

Moreover, with the aid of Eqs.(4.87), (4.124), (4.125) and (4.126), we find

$$\langle\hat{n}_a\rangle^2 = e^{-\mu t}\bar{n}_a^2 + [S+T]e^{-\mu t}\bar{n}_a + \frac{1}{4}[S+T]^2. \tag{4.128}$$

Finally, on account of Eqs.(4.127) and (4.128) into Eq.(4.94), can be put in the form

$$\Delta n_a^2 = \langle\hat{n}_a\rangle[1+2(S+T)] - (\bar{n}_a^2 - (S+T))e^{-\mu t} + \frac{1}{2}[S+T]^2. \tag{4.129}$$

With the help of Eqs.(4.90), (4.125), (4.126) and (4.129), the variance takes at steady state the form

$$\begin{aligned}\Delta n_{ss}^2 = \langle\hat{n}\rangle_{ss}[1+2(&\frac{[A(\rho_{ac}^0+\rho_{aa}^0)-(\kappa M-\kappa N)]}{\mu} + \frac{[A(\rho_{ac}^0-\rho_{aa}^0)-(\kappa M+\kappa N)]}{\mu})] \\ &+ \frac{1}{2}[\frac{(A(\rho_{ac}^0+\rho_{aa}^0)-(\kappa M-\kappa N))}{\mu} + \frac{(A(\rho_{ac}^0-\rho_{aa}^0)-(\kappa M+\kappa N))}{\mu}]^2.\end{aligned} \tag{4.130}$$

With the help of Eqs.(3.47), (3.52), (4.41), (4.42) and (4.43) into Eq.(4.130), the variance takes at steady state the form

$$\begin{aligned}\Delta n_{ss}^2 = \langle\hat{n}\rangle_{ss}[1 &+ \frac{2A(1-\eta^2)^{\frac{1}{2}}e^{i\theta} + \kappa(e^{2r}-e^{-2r})}{\kappa + A\eta}] \\ &+ \frac{1}{2}[\frac{2A(1-\eta^2)^{\frac{1}{2}}e^{i\theta} + \kappa(e^{2r}-e^{-2r})}{2(\kappa+A\eta)}]^2.\end{aligned} \tag{4.131}$$

Finally, the variance of the photon number of Eq.(4.131) for $\theta = 0$ and $r = 0$, takes the form

$$\Delta n_{ss}^2 = \langle\hat{n}\rangle_{ss}[1+\frac{2A(1-\eta^2)^{\frac{1}{2}}}{\kappa+A\eta}] + \frac{1}{2}[\frac{A(1-\eta^2)^{\frac{1}{2}}}{\kappa+A\eta}]^2. \tag{4.132}$$

With the help of Eq.(4.88) into Eq.(4.132), the relation between the variance and the mean of the photon number at steady state is expressible as

$$\Delta n_{ss}^2 = \langle\hat{n}\rangle_{ss} + 6\langle\hat{n}\rangle_{ss}^2. \tag{4.133}$$

This represents the variance of the photon number of the a single-mode produced by a degenerate three-lever laser. We note that $\Delta n_{ss}^2 > \langle\hat{n}\rangle_{ss}$ and hence the cavity radiation has super-Poissonian statistics.

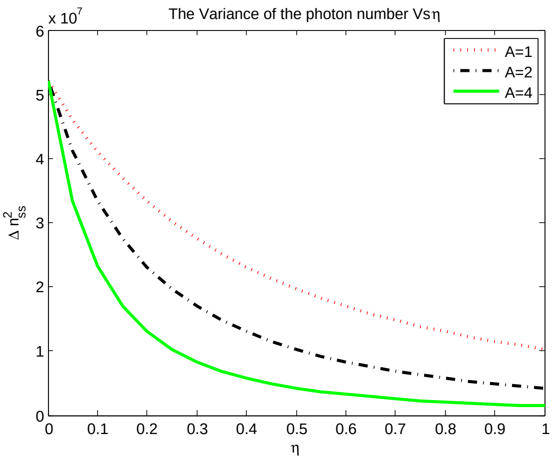

Fig. 4.5: Plots of the variance of the photon number cavity a single-mode [Eq.(4.131)] versus η for $\bar{n}_{ss} = 0,\ \theta = 0,\ \kappa = 0.8,\ r = 2,\ e = 2.72$ and different values of the linear gain coefficient, for A = 1(Dotted curve), A = 2(dash-dot curve) and A = 4(Solid curve).

The plots indicated in Fig.4.5 the variance of the photon number is decreases as linear gain coefficient increases.

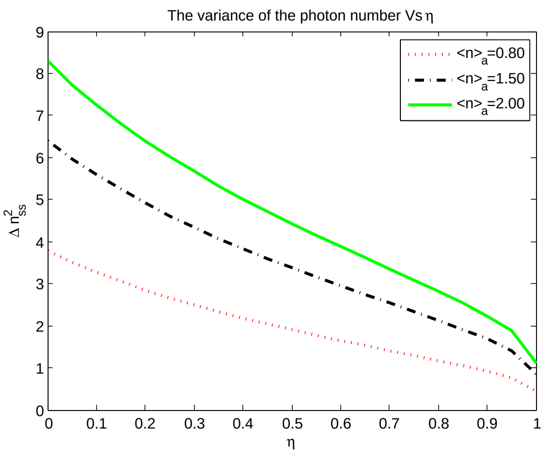

Fig. 4.6: Plots of the variance of the photon number of the cavity a single-mode [Eq.(4.131)] versus η for $A = 1,\ \theta = 0,\ \kappa = 0.8,\ r = 2,\ e^{2r} \approx 0,\ e = 2.72$ and different values of $\bar{n}_a$, for $\langle\hat{n}\rangle_a = 0.80$(Dotted curve), $\langle\hat{n}\rangle_a = 1.50$(dash-dot curve) and $\langle\hat{n}\rangle_a = 2.00$(Solid curve).

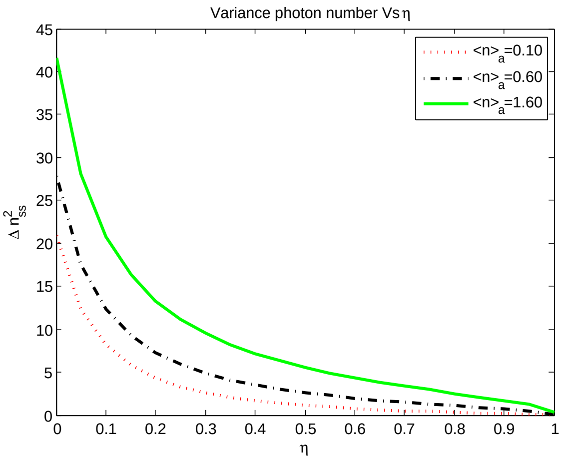

Fig. 4.7: Plots of the variance of the photon number of the cavity a single-mode at steady state [Eq.(4.132)] versus η for $A = 5,\ \theta = 0,\ \kappa = 0.8,\ r = 0,\ e = 2.72$ and different values of $\bar{n}_a$, for $\langle\hat{n}\rangle_a = 0.10$(Dotted curve), $\langle\hat{n}\rangle_a = 0.60$(dash-dot curve) and $\langle\hat{n}\rangle_a = 1.60$(Solid curve).

We see from Fig.4.6 and Fig.4.7 that the variance of the photon number at a steady state are positive. This indicates that the photon number statistics is super-Poissonian. Furthermore, we note that one effect of the coupling of the top and bottom levels is to decrease the variance of the photon number. In addition, the same plots show that the variance of the photon number decreases with η.

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CONCLUSION

In this thesis, we have studied the squeezing and statistical properties of light produced by a degenerate three-level laser whose cavity initially in thermal light and the cavity mode is coupled to squeezed vacuum reservoir. We first obtained the master equation for the system under consideration. Using this master equation, we obtained c-number Langevin equation associated with the normal ordering and the correlation properties of the noise forces, we have calculated the quadrature and squeezing spectrum. We have seen that the quadrature squeezing at large time scale is independent of the mean photon number of the chaotic light. This implies that the thermal light in the cavity is converted to a correlated light. One effect of the injecting thermal light in cavity is to decreases the degree of squeezing of the radiation. The more intense the injecting thermal light the more squeezing is damaged. It also observed that the minimum value of the quadrature variance described by Eq.(4.47) for A = 100, $\kappa = 0.8$, r = 2, and is found to be $\Delta\alpha_{ss}^2 = 0.0568$ and occurs at $\eta = 0.01$. This result implies that the maximum intracavity for the given value is 94.32% below the coherent state level. In addition, we have seen that the degree of squeezing increases with the parameter of squeezed vacuum reservoir.

On the other hand, employing the solution of c-number Langevin equations. We have determined the mean and the variance of the photon number. From the results obtained, we realized that the injecting thermal light in cavity to increases the mean and the variance of the photon number at the beginning and then it has no effect at time progresses.We have also observed that the photon number statistics is super-Poissonian.

Reference

[1] E.Alebachew and K.Fesseha opt, Common, 265, **314** (2006).

[2] M. Bekele, M.sc Thesis, Adama Science and Technology University(2014)

[3] W. Mekonnen, M.sc Thesis, Addis Ababa University(2007)

[4] M. Getahun, Ph.D.dissertation, Addis Ababa University (2009)

[5] T.Yirgashewa, M.sc Thesis, Addis Ababa University (2006)

[6] T.Yirgashewa, Ph.D.dissertation, Addis Ababa University(April, 2010)

[7] S.Getachew, M.sc Thesis, Addis Ababa University (2011)

[8] N.Lu.F.X Zhao, and J.Bergou, phys.Rev.A**39**, 5189 (1988).

[9] K.Fessesa, Fundamentals of Quantum Optics,(Lulu,North Carolina, 2008).

[10] K.Fesseha, phys.Rev.A**63**, 033811 (2001).

[11] B. Abiti, M.sc Thesis, Addis Ababa University (2011)

[12] N.A.Ansari, Phys.Rev.A**48**, 4686(1993).

[13] M.O.scully, K.Wodkiewicz, M.s.Zubairy, J.Bergou, N, Lu, and J.Meyerte Vehn, phys.Rev.Latt.60,1832 (1988).

[14] S.Tesfa, Phys.Rev A**74**, 043816 (2006).

[15] F. Gebremedhin, M.sc Thesis, Dilla University (2011).

[16] A. Abebe, M.sc Thesis, Adama1 Science and Technology University(2015)

[17] F.kassahun, Three-Level Laser Dynamic with the Atoms Pumped by Electron Bombardment, Addis Ababa University(2012).

[18] S.Tesfa, J. Phys.B: At.Mol.Opt.Phys.**40**, 2373 (2007).

[19] A.Samuel, M.sc Thesis, Addis Ababa University (2003).

[20] X.Fen, China, phys, Susie, **15**, 0235 (2006).

[21] Duan LM, Giedke G, Cirac J I and Zoller P, 2000, Phys. Rev.Lett **84**, 2722.

[22] M.Melaku, M.sc Thesis, Addis Ababa University (April, 2014)

[23] Y.Terefe, M.sc Thesis, Addis Ababa University (June, 2013)