ADAMA SCIENCE AND TECHNOLOGY UNIVERSITY

SCHOOL OF GRADUATE STUDIES

APPLIED MATHEMATICS PROGRAM

A Comparison of Adomian Decomposition Method and Variational Iteration Method for Solving Burger's Fisher Equation

A thesis submitted to the school of graduate studies of Adama Science and Technology University, in partial fulfillment of the requirements for the degree of Master of science in Numerical analysis.

By: Desalegn Ayele

January, 2017

Adama, Ethiopia

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By: Desalegn Ayele

Major Advisor :- Lemi Guta(PhD)

Co - Advisor :- Mr. Yadeta Chimdessa(M.sc.)

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As thesis research advisors, we hereby certify that we have read and evaluated this thesis prepared, under our guidance, by Desalegn Ayele titled: " A comparison of Adomian Decomposition Method and Variational Iteration Method For Solving Burger's Fisher Equation" and we hereby also confirm that the thesis can be submitted for evaluation by examiners and eventual defense.

Dr. Lemi Guta	___________	___________
Major advisor	Signature	Date
Yadeta chimdessa (M.SC)	___________	___________
Co-advisor	Signature	Date

As members of the board of examiners of the M.SC thesis open defense examination, we certify that we have read, evaluated the thesis prepared by Desalegn Ayele and examined the candidate. We recommended that the thesis be accepted as fulfilling the thesis requirement for the degree of master of science in Mathematics (Numerical Analysis).

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Final approval and acceptance of the thesis is contingent upon the submission of the final copy of the thesis to the council of Graduate studies (CGS) through the Departmental Graduate Committee (DGC) of the candidate's major department.

Declaration

I declare that this thesis is the result of my own research, except for the materials referred to as cited in the reference section. The thesis had not been accepted for any M.SC degree and is concurrently submitted in candidature of any degree.

Name: Desalegn Ayele

Place : ASTU

Date:

Signature:____________

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Abbreviations

PDE	partial differential equation
BFE	Burger's Fisher equation
ADM	Adomian decomposition method
RDTM	Reduced differential transformation method
VIM	Variation iteration method
HPM	Homotopy perturbation method
FDM	Finite difference method
NLPDE	Nonlinear partial differential equation
BHE	Burger's Huxley equation

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ABSTRACT

This study provides a comparative study of the Adomian Decomposition Method and the He's Variational iteration method to obtain analytic approximations of nonlinear space time Burger's fisher equation that arises in a modeling of gas dynamics, financial mathematics, sound waves in a viscous medium etc. ADM and VIM are applied for solving Burger's fisher equation, which is a nonlinear second order partial differential equation involving the first derivative in time. Adomian decomposition method (ADM) needs Adomian polynomial formula. But He's variational iteration method (VIM) needs determining the general Lagrange Multiplier, which is optimal value.Moreover, two test examples were solved both by ADM and VIM based on the analysis obtained and their convergence and accuracy has been seen. Approximate analytic solutions calculated by ADM and Variational iteration method (VIM) are compared with the exact solutions for different values of x and t in order to obtain the absolute errors. The maximum absolute errors have been seen in ADM than VIM and it has been proved that VIM has a good convergence and accuracy than ADM in solving Burger's fisher equation. Finally, the results are presented graphically.

UNIT ONE

1. Introduction

1.1 Back ground of the study

In Mathematics, a partial differential equation or so called PDE is any equation involving a function of more than one independent variable and at least one partial derivative of that function. Although a PDE can be expressed in multiple dimensions, but the smallest number for the illustration is two, which one of them may be time. The order of a PDE is the order of the highest order derivative that appears in the PDE. It is said to be linear if the unknown function u and all its partial derivatives appear in an algebraically linear, that is, of the first degree.

The general second order linear partial differential equation is of the form:

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial x\,\partial y} + C\frac{\partial^2 u}{\partial y^2} + D = 0$$

Which can be written as $Au_{xx} + Bu_{xy} + Cu_{yy} + D = 0$ (1.1)

Where A, B, C, and D are all functions of x and y. The PDE in eq.(1.1) is nonlinear if A, B, or C include u, u_x or u_y or if D is nonlinear in u and /or its first derivatives. Equations of the form (1.1) can be classified into three distinct groups, which are shown as equation (1.2), (1.3), (1.4) as below [1]:

$$B^2 - 4AC < 0, \quad \text{Elliptic} \tag{1.2}$$

$$B^2 - 4AC = 0, \quad \text{Parabolic} \tag{1.3}$$

$$B^2 - 4AC > 0, \quad \text{Hyperbolic} \tag{1.4}$$

Most physical phenomena arising in various fields of engineering and science are modeled by nonlinear partial differential equations (NLPDEs). The investigation of solutions to NPDEs has attracted much attention due to their potential applications and many numerical schemes have been proposed. The generalized Burger's Fisher Equation is one of the important NLPDE which appears in varies applications, such as fluid mechanics, shock wave formation, Heat conduction, traffic flow, gas dynamics, sound waves in viscous medium, and some other fields of applied science[2].

Almost, all of models represented by PDEs cannot be solved analytically. So, to study the predictions of PDE models of such phenomena it is often necessary to approximate their solution numerically.

The Adomian Decomposition method (ADM) is a semi-analytical method for solving ordinary and partial nonlinear differential equations. The method, developed by American Mathematician George Adomian in the 1985's is one of the methods. Adomian proposed a new and ingenious method to obtain exact solution of linear and nonlinear equations of various kinds like algebraic, differential for both Ordinary and partial, integral etc problems.

The techniques uses a decomposition of the nonlinear operator as a series of Adomian functions. Each term of this series is a generalized polynomial called the Adomian polynomial. Some techniques which assume essentially that the nonlinear system is almost linear after equivalent linearization will not be able to retain the originality of the problem.

Adomian Decomposition Method consists of splitting (separating) the given equation into linear, remainder and nonlinear parts, inverting the highest order differential operator contained in the linear operator on both sides, identifying the initial or boundary conditions and the terms involving the independent variable alone as initial approximation, decomposing the unknown function into a series whose components are to be determined , decomposing the nonlinear function in terms of special polynomials, and finding the successive terms of the series solution[3].

Recently, a great deal of interest has been focused on the application of Adomian's decomposition method for the solution of different problems.

The ADM, which accurately computes the series solution, is of great interest to applied sciences. The method provides the solution in a rapidly convergent series with components that are elegantly computed.

The main advantage of the ADM is that it can be applied directly for all types of differential and integral equations, linear or nonlinear, homogeneous or in homogeneous, with constant coefficients or with variable coefficients. Another advantage is that the method is capable of greatly reducing the size of computation work while still maintaining high accuracy of numerical solution [4].

The variational iteration method (VIM) was first developed in 1999 by Chinese Mathematician Ji-Huan He, professor at Donghua University. The VIM was initially proposed towards the end of the most recent century and completely grew in 2006 and 2007.

The method VIM is used to solve effectively, easily and accurately a large class of nonlinear problems with approximations, which converge rapidly to the accurate solutions. For linear problems, its exact solution can be obtained by only one iteration step due to the fact that the Lagrange multiplier can be exactly identified.

This method VIM introduces a reliable and efficient process for a wide variety of scientific and engineering applications like linear or nonlinear, homogenous or inhomogeneous equations, and systems of equations etc. Also VIM is more powerful than other existing methods, such as RDTM(reduced differential transformation method), perturbation method, etc. The VIM gives rapidly convergent successive approximations of the exact solution if such a solution exists, otherwise a few approximations can be used for numerical purpose. The existing numerical techniques suffer from the restrictive assumptions that are used to handle nonlinear terms. The VIM has no specific requirements, such as linearization, small parameters, Adomian polynomials etc, for non linear operators. Another important advantage of VIM is capable of greatly reducing the size of calculation while still maintaining high accuracy of numerical solution. Moreover, the power of the method gives it a wider applicability in handling a huge number of analytical and numerical applications in real life problems.

The VIM is relatively new approaches to provide approximate solutions to linear and nonlinear problems. The VIM was successfully applied to find the solutions of several classes of variational problems.

The VIM is an iterative method based on the use of Lagrange multiplier, restricted variations and correction functional which has found a wide application for the solution of nonlinear ordinary and partial differential equations. The VIM does not require the presence of small parameters in the differential equation, and provides the solution (or an approximation to it) as a sequence of iterates. The method does not require that the nonlinearities be differentiable with respect to the dependent variable and its derivatives. Being different from the other nonlinear analytic methods, such as perturbation method, this method does not depend on small parameters and initial guess can easily chosen even also with unknown parameters so it gives a wide application in nonlinear problems without linearization or small perturbation[5].

In this paper, we focus on the comparison of Adomian Decomposition Method (ADM) and Variational iteration method (VIM) for the solution of space time Burger's Fisher Equation. The space time generalized Burger's Fisher Equation is of the form [6]:

$$U_t + \alpha U^{\delta} U_x - U_{xx} = \beta U\,(1 - U^{\delta}) \quad \text{for all } x \in [0,1], \quad t \geq 0, \quad U = U(x,t) \tag{1.5}$$

Subject to the initial condition given by

$$U(x,0) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right\}^{\frac{1}{\delta}} \tag{1.6}$$

And boundary conditions

$$U(0,t) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(\frac{\alpha}{2(\delta+1)}\left(\frac{\alpha}{\delta+1} + \beta\frac{(\delta+1)}{\alpha}\ t\right)\right)\right)\right\}^{\frac{1}{\delta}} \tag{1.7}$$

$$U(1,t) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(1 - \left(\frac{\alpha}{\delta+1} + \beta\frac{(\delta+1)}{\alpha}\right)t\right)\right)\right\}^{\frac{1}{\delta}}, \quad t \geq 0 \tag{1.8}$$

Where $\alpha, \beta\ and\ \delta$ are parameters.

The exact solution is given by [6].

$$U_{exact}(x,t) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(x - \left(\frac{\alpha}{\delta+1} + \beta\frac{(\delta+1)}{\alpha}\right)t\right)\right)\right\}^{\frac{1}{\delta}} \tag{1.9}$$

1.2 Statement of the problem

Burger's Fisher equation arise in different areas of applied science such as financial Mathematics, gas dynamics, traffic flow, sound waves in viscous medium, applied Mathematics and physics applications as already mentioned above in section 1.1. It is difficult to get solution of such type of equation analytically because of its variety of category and the behavior it usually depict. Research on numerical solution of Burger's Fisher equation (BFE) was done by long established and well known methods, that is, Finite difference method (FDM), Reduced Differential transformation method (RDTM), Variational iteration method (VIM), Homotopy perturbation method (HPM), etc are among the methods used to treat such equation. But in many of the reviews, there is not detail comparison on these numerical methods for the solution of Burger's fisher equation. Therefore, the researcher wants to focus, in this research to compare Adomian decomposition method (ADM) and Variational iteration method (VIM) for the solution of Burger's fisher equation. So, this work was conducted to answer the following research questions.

1. How can we apply the Adomian Decomposition Method (ADM) and the Variational iteration method (VIM) to solve Burger's Fisher Equation?
2. How can we solve the nonlinear and linear portion of the Burger's Fisher equation for ADM?
3. How can we determine the general Lagrange multiplier for the solution of BFE for VIM?
4. Which method gives more accurate result?

1.3 Objective of the study

1.3.1 General Objective

The main objective of this study is to introduce a comparative study to the solution of Burger's Fisher Equation by using Adomian Decomposition Method (ADM) and Variational iteration method (VIM).

1.3.2 Specific Objectives

The specific objectives of this study are:

- ✓ To describe the Adomian Decomposition Method (ADM) and Variational iteration method (VIM).
- ✓ To apply ADM and VIM for solving Burger's Fisher Equation.
- ✓ To find the approximate analytic solution of Burger's Fisher Equation by ADM and VIM.
- ✓ Comparing results of ADM, VIM and exact solution of Burger's Fisher equation for different values of x and t.
- ✓ To investigate which method facilitates the computational work and gives the solution rapidly.

1.4 Significance of the study

The main purposes of the study are:

- ✓ Applying ADM and VIM in the solution of Burger's fisher equation.
- ✓ Developing skills in using ADM and VIM to solve any nonlinear partial differential equations.
- ✓ Besides this, the study provides several guides to all natural science students, teachers and any one who works in this area.

1.5 Scope of the study

The study was carried out on the solution of space –time (one variable time) Burger's Fisher equation, which is a nonlinear partial differential equation with initial and boundary conditions using Adomian Decomposition Method and Variational iteration method (VIM).

1.6 Limitation

There are problems or shortages under this study Some of them are:

- ✓ As the internet is inaccessible in the area of the research, the would be information might not be included.
- ✓ Lack of related reference books may have determined the accuracy of the result.

UNIT TWO

2. REVIEW LITERATURE

The Adomian Decomposition Method was proposed by the American Mathematician, George Adomian. It is based on the search for a solution in the forms of a series and on decomposing the nonlinear operator into a series in which the terms are calculated recursively using Adomian polynomial formula [7].

Burger's Fisher equation is very important in fluid dynamics model and the study of this model has been considered by many authors both for conceptual understanding of physical flows and testing various numerical methods. Burger's Fisher equation has been found some applications in fields of gas dynamics, numerical theory, heat conduction, elasticity, sound waves in viscous medium etc [8].

Nonlinear phenomena that appears in many areas of scientific fields such as fluid dynamics, Mathematical Biology, Chemical kinetics solid state physics and plasma physics can be modeled by partial differential equations. A broad class of analytical solutions methods and numerical solutions method were used in handle these problems. Then a discussion on a new application of ADM for non linear physical equations has been made by EI-Waki, and Abdou[9]. They investigated the behavior of Adomian solutions and the effects of different values of t. The ADM has been proved to be effective and reliable for handling differential equations, linear or non linear. In addition to that, ADM has been used to solve effectively, easily, and accurately a large class of linear and non linear equations with approximates which converge rapidly.

According to Biazar and Ebrahim [10], they applied ADM to solve hyperbolic partial differential equations and make up a results comparison with the characteristic method. The results shows that ADM can provide a more approximate solution by a smaller size of computation compare to characteristics method. Since ADM has been applied to solve many functional equations and systems of functional equations, we can see that ADM is such a very effective method and results in considerable saving in computation time. Besides this, ADM is very sensitive to initial conditions.

The variational iteration method (VIM), was first proposed by He[11]. VIM has successfully been applied to many situations. For example, He solved the classical Blasiu's equation using Variational iteration method (VIM)[12]. He used Variational iteration method (VIM) to give approximate solutions for some well-known nonlinear problems. He[13] solved strongly nonlinear equations using VIM. Momani et al.[14] applied VIM to the Helmholtz equation. The Variational iteration Method has been applied for solving nonlinear differential equations of fractional order by Odibat et al. [15]. Bildik et al. [16] used VIM to solve different types of nonlinear partial differential equations. Abbasbandy [17] solved the quadratic Ricciti differential equation by VIM incorporating Adomian's polynomials. Junfeng [18] applied VIM to solve singular two point boundary value problems. Abdou and Soliman used Variational iteration method to obtain the solution of Burger's equation and coupled Burger's equations. Batiha et al.[19] applied variational iteration method to solve the generalized Huxley equations and they employed Variational iteration method to the generalized Burger's Huxley equation.

2.1 Numerical Treatments of the Burger's Fisher Equation

In order to treat Burger's Fisher Equation, plenty of numerical methods were investigated during the past years.

Rashid et.al [20] employed homotopy perturbation method (HPM), for obtaining approximate solutions of the generalized Burger's Fisher Equation.

Ismail et al. [21], Wazwaz and Gorguis [22] studied Adomian decomposition method for Burger's Huxley and Burger's Fisher equations.

In [23], Javid and Golbabi studied spectral collocation method and spectral domain decomposition method for the solution of the generalized BFE.

Numerically, many researchers used various numerical methods to solve the BFE. Mickens [24] gave a nonstandard finite difference scheme for the Burger's Fisher Equation. Kaya and Sayed [25] introduced a numerical simulation and explicit solutions of the generalized Burger's Fisher equation. Ismail and Rabolian [26], presented a restrictive pade approximation for the solution of the generalized BFEs.

Rabolian and Saeidian [27], Wazwaz [28], gave the analytical approaches for Burger's fisher Equation and BHE. Zhang et al.[29] obtained numerical solutions for the generalized Burger's fisher equation by using the local discontinuous Galerkin method. The obtained results are in good agreement with the exact solution.

Zhao et.al [30] obtained numerical solutions of the generalized Burger's Fisher equation using the pseudospectral method. The space discretization is based on the Legendre Galerkin formulation, while the chebyshev – Gauss-lobatto nodes have been used in practical computations.

Khattak [31] presented a computational mesh less method for the Burger's Fisher equation. Zhu and Kang [32] investigated the solution of the BFE by using the B – Spline quasi-interpolation method.

Convergence of the Adomian Decomposition Method when applied to some class of ordinary and partial differential equations is discussed by many researchers. For instance, K.Abbao and Y. cherrualt [33, 34] proved the convergence of the ADM for differential and operators equations. Lesnic [35] investigated convergence of the ADM when applied to time dependent heat, wave and beam equations for both forward and back ward time evoluation. He showed that the convergence was faster for forward problems than for back ward problems.

Al-khaled and Allah [36] implemented the Adomian method for variable depth shallow water equations with a source term and illustrated the convergence numerically.

UNIT THREE

3. Methods and Materials

This section consists of methods and materials used to undertake the study. These are study design, study procedure and materials.

3.1 Study Design

The study involves numerical implementation with MATLAB programming language. The researcher applies different operators such as L, R and N for linear and nonlinear portion respectively, of the Burger's Fisher Equation. The nonlinear portion of the equation is decomposed into a series Adomian polynomials for ADM. With the help of prescribed initial condition, we determine the first term of the series solution for ADM. Finally, the unknown function is solved with the help of the first term of the series using recurrence relation. For VIM, we construct a correction functional formula and we determine the general Lagrange multiplier. Using the obtained Lagrange multiplier we can find the successive approximations.

3.2 Study procedure

For the study, we have followed the procedures written below:

3.2.1 Study procedures for ADM

- ✓ Use of different operators for linear and nonlinear portion of the equation.
- ✓ Rewriting the equation in terms of the operators.
- ✓ With the help of initial condition, determining the first term of the series solution.
- ✓ Solving the nonlinear part (portion) of the equation using Adomian polynomial formula.
- ✓ Solving the remainder linear portion of the equation if it exists.
- ✓ With the help of first term of the series and using MATLAB programming language, determining the remaining terms of the unknown function using recurrence relation.

- ✓ Validating the method using test examples.
- ✓ Discussing the method from the graphs.

3.2.2 Study procedures for VIM

- ✓ Constructing a correction functional formula.
- ✓ Determining the general Lagrange multiplier.
- ✓ Using initial condition and the obtained Lagrange multiplier, determining the successive approximations of the unknown solution.
- ✓ Validating the method using test examples.
- ✓ Discussing the method from the graphs.

3.3 Materials

Sources in the web and libraries were used to collect all the pieces of information about the ADM and VIM for the solution of Burger's Fisher equation.

- ✓ Relevant journals and texts were addressed to gather information about the ADM and VIM for Burger's Fisher equation.
- ✓ Matlab programming language was applied suitably to reduce the difficulty of the computation by the methods and graphs were plotted using the program.

UNIT FOUR

4. Description of the methods, Results and Discussion.

4.1 Some Basic Definitions and Mathematical preliminaries

4.1.1 Operators

Definition 4.1 An operator is a function that takes a function as an argument instead of numbers as we used to deal with in functions.

We already know a couple of operators even if we did not know that they were operators.

Here are some examples of operators.

$$L(.) = \frac{d(.)}{dx}, \quad L(.) = \frac{\partial(.)}{\partial x}, \quad L^{-1}(.) = \int_0^x (.)dx$$

Or if a function is plugged in, say U(x, t), in each of the above, the following can be obtained.

$$LU(x,t) = \frac{dU(x,t)}{dx}, \quad LU(x,t) = \frac{\partial U(x,t)}{\partial x}, \quad L^{-1}U(x,t) = \int_o^t U(x,t)\,dt$$

The operator L is the operator of the highest ordered derivatives.

$L(.) = \frac{\partial^n(.)}{\partial x^n}$ for n - order differential equations and thus its inverse $L^{-1}(.)$ follows as the n – fold definite integration operator from 0 to x or t.

An operator that is not linear is known as nonlinear operator. In this study, non linear operators in which nonlinear functions are plugged are symbolized by some various representative like N(U) and P(U). These operators are used to determine the Adomian polynomial by the help of Adomian formula.

Definition 4.2 The order of a PDE is the order of the highest derivative in the partial differential equation.

I .A first order linear homogeneous partial differential equation.

Example: - $U_t + U_x = 0$ is a first order PDE in x and t, since the highest derivative is a first order partial derivative.

II. A first order linear inhomogeneous PDE

Example: - $U_t + U_x = \sin(x - t)$ is a first order PDE in x and t, since the highest derivative is a first order partial derivative.

III. A first order nonlinear homogeneous PDE

Example : - $U_t + UU_x = 0$ (the Hopf or inviscid Burgers equation) is a first order PDE in x and t, since the highest derivative is a first order partial derivative.

IV. A nonlinear second order homogeneous PDE

Example: - $U_t + UU_x - U_{xx} = 0$ is a second order PDE in x and a first order PDE in x and t. However, this PDE is classified as second order PDE. Since the highest derivative in the equation is a second order partial derivative.

4.2 General Description of the ADM and VIM.

4.2.1 The Adomian decomposition method (ADM).

In the 1985‘s, George Adomian introduced a new method to solve linear or nonlinear functional equations. This method has since been termed the Adomian Decomposition Method and has been the subject of many investigations. The method involves splitting or separating the equation under investigation into linear and nonlinear portions. The linear operator represents the linear portion or terms of the equation. The inverse operator is then applied to the equation.

The nonlinear portion(term) is decomposed into a series Adomian polynomials. This method generates a solution in the form of a series whose terms are determined by a recursive relationship using the polynomials.

Consider a general differential equation in an operator form:

$$LU + RU + NU = g \qquad (4.1)$$

Where L is an operator of the highest ordered derivatives representing the linear portion which is easily invertible, N is the nonlinear operator representing the nonlinear term and R is a linear operator for the remainder of the linear operator.

The inverse, $L^{-1} = \int_0^t (\,.\,) dt$ (4.2)

Applying the inverse operator L^{-1} on both sides of eq. (4.1) the equation becomes

$$L^{-1}(LU + RU + NU) = L^{-1} g\,(t\,) \qquad (4.3)$$

Since L is a linear, L^{-1} would represent integration and with any given boundary conditions or initial conditions, will give an equation for U including these conditions. This gives

$$U(x, t) = f(x) - L^{-1}RU - L^{-1}NU \qquad (4.4)$$

Where f(x) represents the function generated by integrating 'g' and using initial conditions.

The Adomian Decomposition method assume that the unknown function U(x, t) can be expressed by an infinite series of the form :

$$U\,(x, t) = \sum_{n=0}^{\infty} U_n\,(\,x\,, t\,) \qquad (4.5)$$

And

The nonlinear operator N (U) can be decomposed by an infinite series of polynomials given by

$$N\,(U_0, U_1, U_2, \ . \ . \ . \ , U_n) = \sum_{n=0}^{\infty} A_n(\,U_0,\ U_1\ , \ldots\ , U_n) \qquad (4.6)$$

Where A_n, $n \geq 0$ are the Adomian polynomials that can be determined by Adomian formula.

$$A_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[N\left(\sum_{n=0}^{\infty} \lambda^i U_i\right)\right]_{\lambda=0}, \qquad n = 0, 1, 2, \ . \ . \ . \qquad (4.7)$$

So, for n = 0 eq.(4.7) reduces to

$$A_0 = N(U_0)$$

For n = 1, it reduces to

$$A_1 = \frac{d}{d\lambda}[N(U_0 + \lambda U_1)]_{\lambda=0}$$

$$\Rightarrow A_1 = U_1 N'(U_0)$$

For n = 2, it reduces to

$$A_2 = \frac{1}{2!}\frac{d^2}{d\lambda^2}[N(U_0 + \lambda U_1 + \lambda^2 U_2)]_{\lambda=0}$$

$$\Rightarrow A_2 = \frac{1}{2!}U_1^2 N''(U_0) + U_2 N'(U_0)$$

The desired Adomian polynomials can be determined using similar way .

$$\Rightarrow A_3 = U_3 N'(U_0) + U_1 U_2 N''(U_0) + \frac{1}{3!}U_1^3 N'''(U_0)$$

It can be observed that A_0 depends only on U_0, A_1 depends on U_0 and U_1, A_2 depends on U_0, U_1, and U_2 and so forth .

Now, substituting (4.6) and (4.5) into (4.4) we get

$$\sum_{n=0}^{\infty} U_n(x,t) = f(x) - L^{-1} R(U_n) - L^{-1} N(U_n)$$

$$\sum_{n=0}^{\infty} U_n(x,t) = f(x) - L^{-1} \sum_{n=0}^{\infty} R(U_n) - L^{-1} \sum_{n=0}^{\infty} (A_n) \tag{4.8}$$

The recursive relationship is found to be

$$U_0(x,t) = f(x)$$

$$U_{n+1}(x,t) = -L^{-1} R(U_n) - L^{-1}(A_n), \quad n \geq 0 \tag{4.9}$$

And hence, $U_0(x,t) = f(x)$

$$U_1(x,t) = -L^{-1} R(U_0) - L^{-1}(A_0)$$

$U_2(x, t) = -L^{-1} R(U_1) - L^{-1}(A_1)$

.   .

.   .

$U_i(x, t) = -L^{-1} R(U_{i-1}) - L^{-1}(A_{i-1})$ for $i = 1, 2, 3, \ldots$

So, having determined the components $U_n(x, t)$, $n \geq 0$ the solution $U(x, t)$ in a series form immediately by (4.5). The series may be summed to provide the solution in a closed form. However, for concrete problems, the n-term partial sum may be used to give the approximate solutions.

$$\Phi_n = \sum_{i=0}^{n-1} U_i(x, t) \tag{4.10}$$

4.2.2 The Variational iteration method (VIM)

VIM is based on the general Lagrange's multiplier method. The main feature of the method is that the solution of a mathematical problem with linearization assumption is used as initial approximation or trial function. Then a more highly precise approximation at some special point can be obtained. This approximation converges rapidly to an accurate solution.

To illustrate the basic concepts of VIM, we consider the following nonlinear differential equation.

$$Lu + Nu = g(x, t) \tag{4.11}$$

Where L is a linear operator, N is a nonlinear operator, and g(x, t) is an inhomogeneous (source) term. According to VIM, we can construct a correction functional for eq.(4.11) as follows:

$$U_{n+1}(x, t) = U_n(x, t) + \int_0^t \lambda(\tau)\{LU_n(x, \tau) + N\widetilde{U}_n(x, \tau) - g(x, \tau)\}d\tau, \; n \geq 0 \tag{4.12}$$

where λ is a general Lagrange multiplier which can be identified optimally via the variational theory, the subscript n denotes the n^{th} order approximation, $\widetilde{U}_n(x, \tau)$ is considered as a restricted variation, that is, $\delta\widetilde{U}_n(x, \tau) = 0$. The solution of the linear problem can be achieved in a single iteration step due to the exact identification of the Lagrange multiplier. This method requires that the Lagrange multiplier $\lambda(\tau)$ is first determined optimally. The successive approximation $U_n(x,t)$, $n \geq 0$, of the solution U(x,t) can be readily obtained by using this determined Lagrange multiplier and the zeroth approximation, that is, initial condition $U_0(x,t)$. Consequently, the solution is given by U(x,t) = $\lim_{n\to\infty} U_n\,(x,t)$.

4.3 ADM and VIM for nonlinear space time Burger's Fisher equation.

Consider the generalized Burger's Fisher Equation

$$U_t + \alpha U^{\delta} U_x - U_{xx} = \beta U\,(1 - U^{\delta}) \qquad \text{for all } x \in [0, 1],\quad t \geq 0,\ U = U(x,t)$$

$$U_t = U_{xx} - \alpha U^{\delta} U_x + \beta U(1 - U^{\delta}) \tag{4.13}$$

Subject to the initial condition given by

$$U(x, 0) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right\}^{\frac{1}{\delta}} \tag{4.14}$$

And boundary conditions

$$U(0,t) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(\frac{\alpha}{2(\delta+1)}\left(\frac{\alpha}{\delta+1} + \beta\,\frac{(\delta+1)}{\alpha}\,t\right)\right)\right)\right\}^{\frac{1}{\delta}} \tag{4.15}$$

$$U(1,t) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(1 - \left(\frac{\alpha}{\delta+1} + \beta\,\frac{(\delta+1)}{\alpha}\right)t\right)\right)\right\}^{\frac{1}{\delta}}, \quad t \geq 0 \tag{4.16}$$

Where α, β *and* δ are parameters.

The exact solution is given by

$$U_{exact}(x, t) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}\left(x - \left(\frac{\alpha}{\delta+1} + \beta\,\frac{(\delta+1)}{\alpha}\right)t\right)\right)\right\}^{\frac{1}{\delta}} \tag{4.17}$$

When $\alpha = 0$ *and* $\delta = 1$ equation (4.13) is reduced to $U_t = U_{xx} + \beta U(1 - U)$ which is called the Huxley equation which describes nerve pulse propagation in nerve fibre and wall motion in liquid crystals.

When $\beta = 0$ equation (4.13) gives the generalized Burger equation will be obtained. That is, $U_t + \alpha U^{\delta} U_x - U_{xx} = 0$. This equation has been used to investigate sound waves in a viscous medium. However, it was originally introduced by Burgers to model one dimensional turbulence and can also be applied to waves in fluid filled viscous elastic tubes and magneto hydrodynamic waves in a medium with finite electrical conductivity.

4.3.1 ADM for the solution of generalized Burger's fisher equation.

To obtain the approximate solution of equation (4.13) using ADM, which is a Burger's Fisher equation apply the inverse operator ${L_t}^{-1}$on both sides of equation (4.13) and using initial condition we obtain,

U (x, t) = f(x) $-{L_t}^{-1}(\alpha U^{\delta} U_x - U_{xx} - \beta U(1 - U^{\delta}))$

U(x, t) = f(x) $+ \beta {L_t}^{-1} P(U) - \alpha {L_t}^{-1} N(U) + {L_t}^{-1} R(U)$ (4.18)

Where P (U) = U(1 – $(U)^{\delta}$), N (U) = $U^{\delta} U_x$, R (U) = U_{xx}

The operators R(U), P(U) and N(U) are usually represented by the infinite series of the Adomian polynomials as follows. P (U) = $\sum_{n=0}^{\infty} A_n$, N(U) = $\sum_{n=0}^{\infty} B_n$, R(U) = $\sum_{n=0}^{\infty} K_n$ Where A_n, B_n, and K_n, n ≥ 0 are the Adomian Polynomials. By using (4.2) and (4.4) into the functional equation (4.5) gives

U(x, t) = $\sum_{n=0}^{\infty} U_n(x,t)$ = f (x) $+ \beta\, {L_t}^{-1}(\sum_{n=0}^{\infty} A_n) - \alpha {L_t}^{-1}(\sum_{n=0}^{\infty} B_n) + {L_t}^{-1}(\sum_{n=0}^{\infty} K_n)$

(4.19)

Identifying the zeros component U_0(x , t) by f(x), the remaining components n ≥ 0 can be determined by using the recurrence relation.

U_0(x ,t) = f (x)

U_{n+1}(x, t) = $\beta {L_t}^{-1}(A_n) - \alpha {L_t}^{-1}(B_n) + {L_t}^{-1}(K_n)$, n ≥ 0 (4.20)

Where A_n, B_n, and K_n (n ≥ 0) are Adomian polynomials and they are defined by

$G_n = \frac{1}{n!} \frac{d^n}{d\lambda^n} \left[F(\sum_{n=0}^{\infty} \lambda^i U_i) \right]_{\lambda=0}$

Then

$$\left.\begin{aligned} A_0 &= U_0(1 - {U_0}^{\delta}) \\ A_1 &= U_1 - (1+\delta){U_0}^{\delta}U_1 \\ A_2 &= U_2 - (1+\delta){U_0}^{\delta}U_2 - (\delta + \tfrac{1}{2}\delta(\delta-1)){U_0}^{\delta-1}{U_1}^2 \end{aligned}\right\} \tag{4.21}$$

.

.

$$\left.\begin{aligned} B_0 &= {U_0}^{\delta}U_{0x} \\ B_1 &= \delta {U_o}^{\delta-1}U_1U_{0x} + {U_0}^{\delta}U_{1x} \\ B_2 &= \tfrac{1}{2}\delta(\delta-1){U_0}^{\delta-2}U_{0x}{U_1}^2 + \delta {U_0}^{\delta-1}U_{0x}U_2 + \delta {U_0}^{\delta-1}U_1U_{1x} + {U_0}^{\delta}U_{2x} \end{aligned}\right\} \tag{4.22}$$

.

.

$$\left.\begin{aligned} K_0 &= U_{0xx} \\ K_1 &= U_{1xx} \\ K_2 &= U_{2xx} \end{aligned}\right\} \tag{4.23}$$

.

.

The polynomials A_n, B_n, K_n, $n \geq 3$ can be computed in a similar manner. The first few components of $U_n(x,t)$ follows immediately upon setting.

$$
\left.\begin{aligned}
U_0(x,t) &= f(x)\\
U_1(x,t) &= \beta L_t^{-1} A_0 - \alpha L_t^{-1} B_0 + L_t^{-1} K_0\\
U_2(x,t) &= \beta L_t^{-1} A_1 - \alpha L_t^{-1} B_1 + L_t^{-1} K_1\\
U_3(x,t) &= \beta L_t^{-1} A_2 - \alpha L_t^{-1} B_2 + L_t^{-1} K_2
\end{aligned}\right\} \qquad (4.24)
$$

$$
\begin{array}{cc}
. & .\\
. & .\\
. & .
\end{array}
$$

The scheme in (4.24) can easily determine the components $U_n(x,t)$, $n \geq 0$. It is in principle, possible to calculate more components in the decomposition series. consequently, can recursively determine every term of the series $\sum_{n=0}^{\infty} U_n(x,t)$, and hence the solution $U(x,t)$ is readily obtained in a series form. Here, it is interesting to note that we obtain the solution by the initial condition only: This leads to the complete determination of the components U_n of U (x, t). The series solution defined by eq. (4.5) follows immediately. For numerical purposes, the n term approximant defined by (4.10) can be used to approximate the exact solution.

4.3.2 The VIM for the solution of generalized Burger's fisher equation.

To solve equation (4.13) by means of the VIM, we substitute in equation (4.12) by

$$
\mathrm{L}U_n = \frac{\partial U_n}{\partial t}, \quad \mathrm{R}U_n = -\frac{\partial^2 U_n}{\partial x^2}, \quad \mathrm{N}U_n = \alpha {U^\delta}_n \frac{\partial U_n}{\partial x} - \beta U_n(1 - {U^\delta}_n)
$$

To get

$$
U_{n+1}(x,t) = U_n(x,t) + \int_0^t \lambda(\tau)\{\frac{\partial U_n(x,\tau)}{\partial \tau} - \frac{\partial^2 \tilde{U}_n(x,\tau)}{\partial x^2} + \alpha {\tilde{U}^\delta}_n(x,\tau)\frac{\partial \tilde{U}_n(x,\tau)}{\partial x} - \beta \tilde{U}_n(x,\tau)(1 - {\tilde{U}^\delta}_n(x,\tau))\}d\tau, \ n \geq 0 \qquad (4.25)
$$

$\delta \tilde{U}_n(x,\tau)$ is considered as a restricted variation, that is, $\delta \tilde{U}_n(x,\tau) = 0$. Making the correction functional equation(4.25), stationary, noticing that $\delta \tilde{U}_n(x,\tau) = 0$.

The variation of equation (4.25) is :

$$\delta U_{n+1}(x,t) = \delta U_n(x,t) + \delta \int_0^t \lambda(\tau)\{\frac{\partial U_n(x,\tau)}{\partial \tau} - \frac{\partial^2 \tilde{U}_n(x,\tau)}{\partial x^2} + \alpha \tilde{U}^{\delta}{}_n(x,\tau)\frac{\partial \tilde{U}_n(x,\tau)}{\partial x} - \beta \tilde{U}_n(x,\tau)(1 - \tilde{U}^{\delta}{}_n(x,\tau))\}d\tau, n \geq 0$$

(4.26)

Where $N(\tilde{U}_n(x,\tau)) = \alpha \tilde{U}^{\delta}{}_n(x,\tau)\frac{\partial \tilde{U}_n(x,\tau)}{\partial x} - \beta \tilde{U}_n(x,\tau)(1 - \tilde{U}^{\delta}{}_n(x,\tau))$.

$$\delta U_{n+1}(x,t) = \delta U_n(x,t) + \delta \int_0^t \lambda(\tau)\{\frac{\partial U_n(x,\tau)}{\partial \tau}\}d\tau \tag{4.27}$$

Using integration by parts equation (4.27) becomes

$$\delta U_{n+1}(x,t) = \delta U_n(x,t) + \lambda(\tau)\delta U_n(x,\tau)|_{\tau=t} - \int_0^t \delta U_n(x,\tau)\lambda'(\tau)d\tau = 0 \tag{4.28}$$

Yields the following stationary conditions:

$$\left.\begin{aligned} &\delta U_n:\ \lambda'(\tau) = 0 \\ &\delta U_n:\ 1 + \lambda(\tau)|_{\tau=t} = 0 \end{aligned}\right\} \tag{4.29}$$

From equation (4.29) we obtain $\lambda(\tau) = -1$ (4.30)

Which is called the Lagrange multiplier.

As a result, substituting eqn.(4.30) in eqn.(4.25), we obtain the following iteration formula.

$$U_{n+1}(x,t) = U_n(x,t) - \int_0^t \{\frac{\partial U_n(x,\tau))}{\partial \tau} - \frac{\partial^2 U_n(x,\tau)}{\partial x^2} + \alpha U_n{}^{\delta}(x,\tau)\frac{\partial U_n(x,\tau)}{\partial x} - \beta U_n(x,\tau)\big(1 - U_n{}^{\delta}(x,\tau)\big\}d, n \geq 0$$

(4.31)

By considering the initial condition given by equation(4.14) as the zeroth approximation, we can obtain the following results:

$$U_0(x,t) = \left\{\frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-\alpha\delta}{2(\delta+1)}x\right)\right\}^{\frac{1}{\delta}} \tag{4.32}$$

$$U_1(x,t) = U_0(x,t) - \int_0^t \{\frac{\partial U_0(x,\tau))}{\partial \tau} - \frac{\partial^2 U_0(x,\tau)}{\partial x^2} + \alpha {U_0}^{\delta}(x,\tau)\frac{\partial U_0(x,\tau)}{\partial x} - \beta U_0(x,\tau)\left(1 - {U_0}^{\delta}(x,\tau)\right\} d\tau \qquad \textbf{(4.33)}$$

$$U_2(x,t) = U_1(x,t) - \int_0^t \{\frac{\partial U_1(x,\tau))}{\partial \tau} - \frac{\partial^2 U_1(x,\tau)}{\partial x^2} + \alpha {U_1}^{\delta}(x,\tau)\frac{\partial U_1(x,\tau)}{\partial x} - \beta U_1(x,\tau)\left(1 - {U_1}^{\delta}(x,\tau)\right\} d\tau \tag{4.34}$$

-

-

$U_n(x,t) =$ - - -

Continue for n-iterative.

In equation (4.18) we assume f(x) is bounded for all x, in J = [0,L]. The term R(U), P(U) and N(U) are Lipschitz continuous with $|R(U) - R(U^*)| \leq L_3|U - U^*|$,

$|N(U) - N(U^*)| \leq L_2|U - U^*|$, $|P(U) - P(U^*)| \leq L_1|U - U^*|$ and

$\alpha_1 := T(|\beta|L_1 + |\alpha|L_2 + L_3)$

$\beta_1 := 1 - T(1 - \alpha_1)$

$\beta_2 := 1 - TL\alpha_1$

Theorem: Let $0 < \alpha_1 < 1$, then Burger's Fisher Equation (eq.4.18), has a unique solution [37].

Proof : Let U and U^* be two different solutions of (4.18)

$|U - U^*| = |\beta L_t^{-1}P(U) - \alpha L_t^{-1}N(U) + L_t^{-1}R(U) - (\beta L_t^{-1}P(U^*) - \alpha L_t^{-1}N(U^*) + L_t^{-1}R(U^*))|$

$= |\beta \int_0^t P(U)dt - \alpha \int_0^t N(U)dt + \int_0^t R(U)dt - \beta \int_0^t P(U^*)dt + \alpha \int_0^t N(U^*)dt - \int_0^t R(U^*)dt|$

$\leq |\beta|\int_0^t |P(U) - P(U^*)|\, dt + |\alpha| \int_0^t |N(U) - N(U^*)|dt + \int_0^t |R(U) - R(U^*)|\, dt$

$\leq T(|\beta|L_1 + |\alpha|L_2 + L_3)|U - U^*|$

$= \alpha_1|U - U^*|$

$\Rightarrow |U - U^*| = \alpha_1|U - U^*|$

$\Rightarrow |U - U^*| - \alpha_1|U - U^*| = 0$

$\Rightarrow (1 - \alpha_1)|U - U^*| = 0$

Since $0 < \alpha_1 < 1$, then $|U - U^*| = 0$

This implies $U = U^*$. The proof is complete

4.4 Numerical examples and results

In order to examine which method is efficient, accurate and convergent rapidly to the exact solution for the solution of Burger's Fisher Equation, we work on the following two examples.

Example 1. Let us consider the following Burger's Fisher Equation,

assuming α = -1 $\beta = \delta = 1$

Eq. (4.13) becomes

$$U_t - UU_x - U_{xx} + U(U-1) = 0, \quad x \in [0,1],\ t \geq 0$$

$$U_t = U(1-U) + UU_x + U_{xx} \tag{4.35}$$

With initial condition

$$U(x,0) = \frac{1}{2} + \frac{1}{2} tanh\left(\frac{x}{4}\right) \tag{4.36}$$

And boundary conditions

$$U(0,t) = \frac{1}{2} + \frac{1}{2} \tanh\left(\frac{1}{32} + \frac{t}{8}\right) \tag{4.37}$$

$$U(1,t) = \frac{1}{2} + \frac{1}{2} \tanh\left(\frac{3}{8} + \frac{t}{4}\right) \tag{4.38}$$

And exact solution of eq. (4.35) is

$$U(x,t) = \frac{1}{2} + \frac{1}{2} tanh\left(\frac{x}{4} + \frac{5}{8}t\right) \tag{4.39}$$

Method 1: Using ADM

Recall that the ADM involves separating the equation under investigation into linear and non linear portions. The linear operator representing the linear portion of the equation is inverted and the inverse operator is then applied to the equation. Any given conditions are taken into consideration. The nonlinear portion is decomposed into a series of

Adomian polynomials. This method generates a solution in the form of a series whose terms are determined by a recursive relationship using these Adomian polynomials.

Following the method described above, the desired linear operator is defined as

$$L_t(U) = \frac{\partial U}{\partial t}$$

The inverse operator is then

$$L_t^{-1}(.) = \int_0^t (.)\ dt$$

Rewriting the differential equation (4.35) in an operator form, we have

$$L_t(U) = P(U) + \text{N (U)} + \text{R (U)} \tag{4.40}$$

Where P (U) = U (1 – U)

N (U) = UU_x and R (U) = U_{xx}

Applying the inverse operator L_t^{-1} on both sides of eq. (4.40)

$$L_t^{-1}L_t(U) = L_t^{-1}P(U)\ \ L_t^{-1}N(U) + L_t^{-1}R(U) \tag{4.41}$$

Using the initial condition, the left hand side becomes

$$L_t^{-1}L_tU = \text{U(x, t)} - \text{U(x, 0)} \tag{4.42}$$

Returning (4.42) to the eq. (4.41) it becomes

$$\text{U(x, t)} = \text{U (x, 0)} + L_t^{-1}P(U) + L_t^{-1}N(U) + L_t^{-1}R(U) \tag{4.43}$$

Next we need to generate the Adomian polynomials $A_n,\ B_n,\ and\ K_n$

Let U(x, t) be expanded as infinite series

$$\text{U(x, t)} = \sum_{n=0}^{\infty} U_n(x,t) \tag{4.44}$$

and define P(U) = $\sum_{n=0}^{\infty} A_n$, N(U) = $\sum_{n=0}^{\infty} B_n$, R(U) = $\sum_{n=0}^{\infty} K_n$

$$\text{Then } \sum_{n=0}^{\infty} U_n\,(x,t) = \text{U(x, 0)} + L_t^{-1}P(U) + L_t^{-1}N(U) + L_t^{-1}R(U) \tag{4.45}$$

To find A_n, B_n and K_n we Use Adomian polynomials formula. That is,

$$G_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{n=0}^{\infty}\lambda^i U_i\right)\right]_{\lambda=0}, \text{ n} = 0, 1, 2, ..$$

Then we find Adomian polynomials

$$\left.\begin{aligned} A_0 &= N(U_0) = U_0(1-U_0) \\ A_1 &= U_1 - 2U_1U_0 \\ A_2 &= U_2 - 2U_2U_0 - {U_1}^2 \\ A_3 &= U_3 - 2U_0U_3 - 2U_1U_2 \end{aligned}\right\} \tag{4.46}$$

.

.

$$\left.\begin{aligned} B_0 &= U_0U_{0x} \\ B_1 &= U_1U_{0x} + U_{1x}U_0 \\ B_2 &= U_2U_{0x} + U_{1x}U_1 + U_{2x}U_0 \\ B_3 &= U_3U_{0x} + U_{1x}U_2 + U_1U_{2x} + U_0U_{3x} \end{aligned}\right\} \tag{4.47}$$

.

.

$$\left.\begin{aligned} K_0 &= U_{0xx} \\ K_1 &= U_{1xx} \\ K_2 &= U_{2xx} \end{aligned}\right\} \tag{4.48}$$

$$K_3 = U_{3xx}$$

.

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The polynomials A_n , B_n, K_n, n ≥ 4 can be computed in a similar manner.

Substituting the Adomian polynomials to equation (4.45) we can determine the recursive relationship that will be used to generate the solution.

$U_0(x,t)$ = U(x, 0) = $\frac{1}{2}+\frac{1}{2}\tanh\left(\frac{x}{4}\right)$

$$U_{n+1}(x,t) \quad = \; L_t{}^{-1}(A_n) + \; L_t{}^{-1}(B_n) + L_t{}^{-1}(K_n),\; n\geq 0 \qquad (4.49)$$

MATLAB programming language are used to determine the required decomposition of this recurrence relation. For the sake of demonstration, the first four components of the solution computed by the Matlab programming language are given as follows.

$$U_0(x,t) = U(x,0) \;\; = \;\; \frac{1}{2}+\frac{1}{2}\tanh\left(\frac{x}{4}\right)$$

$$U_1(x,t) \quad = \quad L_t{}^{-1}(A_0) + L_t{}^{-1}(B_0) + L_t{}^{-1}(K_0)$$

$$= \; L_t{}^{-1}U_0(1-U_0) + L_t{}^{-1}(U_0U_{0x}) + L_t{}^{-1}(U_{0xx})$$

$$= \; \frac{5\,t}{16} - \frac{5\,t}{16}\tanh\left(\frac{x}{4}\right)^2$$

$$U_2(x,t) \quad = \quad L_t{}^{-1}(A_1) + L_t{}^{-1}(B_1) + L_t{}^{-1}(K_1)$$

$$= \; L_t{}^{-1}(U_1 - 2U_1U_0) + L_t{}^{-1}(U_1U_{0x} + U_{1x}U_0) + L_t{}^{-1}(U_{1xx})$$

$$= \frac{25\,t^2}{128}\tanh\left(\frac{x}{4}\right)^3 - \frac{25\,t^2}{128}\tanh\left(\frac{x}{4}\right)$$

So, the approximate solution, say using the first three components (iterations) can be determined as:

$$U_{appx.3}(x,t) = \; \sum_{n=0}^{2} U_n\,(x,t)$$

$$U_{appx.3}(x,t) = \; U_0(x,t) + U_1(x,t) + U_2(x,t)$$

$$U_{appx3.}(x,t) \;\; = \;\; \frac{1}{2}+\frac{5\,t}{16}+\frac{1}{2}\tanh\left(\frac{x}{4}\right) - \frac{5\,t}{16}\tanh\left(\frac{x}{4}\right)^2 - \frac{25\,t^2}{128}\tanh\left(\frac{x}{4}\right) + \frac{25\,t^2}{128}\tanh\left(\frac{x}{4}\right)^3 .$$

Therefore, the numerical solutions obtained by ADM are as follows:

All the computations and the plot are done by Matlab.

Table 1: Examle 1: Numerical Results for U(x , t) by 3 terms ADM.

x	t	Exact	ADM(by 3-terms)	Absolute error $\left\lvert U(x,t) - U_{appx.3}(x,t)\right\rvert$
0.1	0.2	0.5744	0.5748	0.0004
	0.4	0.6341	0.6366	0.0025
	0.6	0.6900	0.6981	0.0081
	0.8	0.7408	0.7592	0.0184
	1	0.7858	0.8199	0.0341
0.5	0.2	0.6225	0.6228	0.0003
	0.4	0.6792	0.6814	0.0022
	0.6	0.7311	0.7382	0.0071
	0.8	0.7773	0.7930	0.0157
	1	0.8176	0.8459	0.0283
0.9	0.2	0.6682	0.6684	0.0002
	0.4	0.7211	0.7229	0.0018
	0.6	0.7685	0.7742	0.0057
	0.8	0.8100	0.8221	0.0121
	1	0.8455	0.8667	0.0212

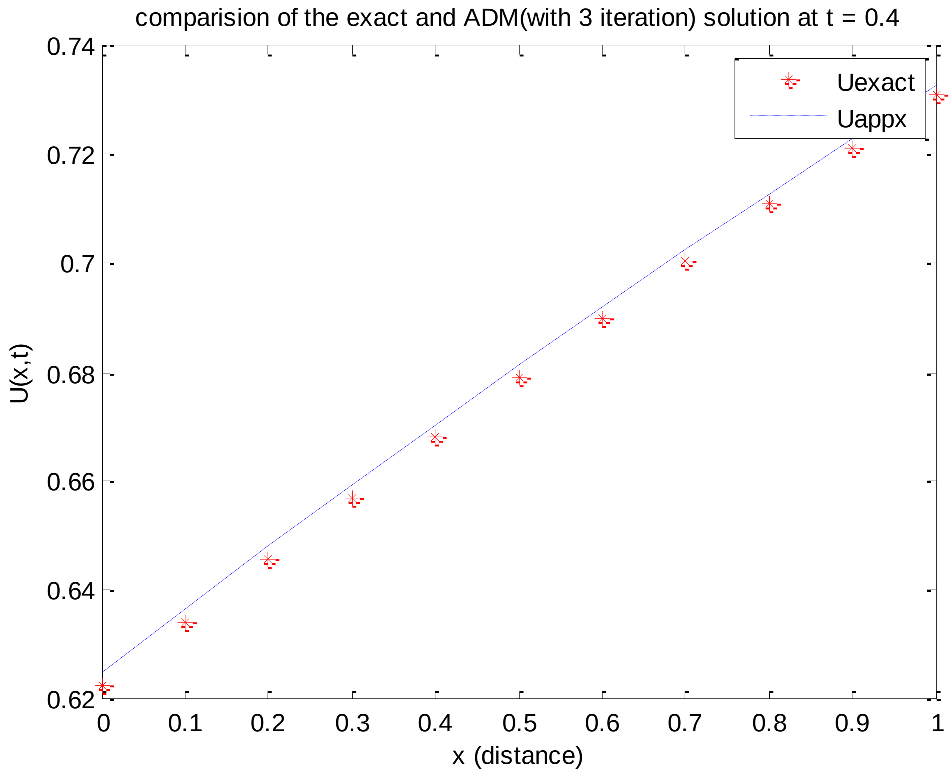

Figure 1: Example1: Comparision of the exact solution and ADM (by 3 iterations) solution at t = 0.4.

The exact solution is compared with the approximate solution $U_{appx.3}(x,t)$ obtained by ADM both in table 1 and figure 1.

Method 2 : using VIM

To solve equation (4.35) using VIM, we consider a correction functional as:

$$U_{n+1}(x,t) = U_n(x,t) + \int_0^t \lambda(\tau)\{U_{n\tau}(x,\tau) - \widetilde{U}_n(x,\tau)\widetilde{U}_{nx}(x,\tau) - \widetilde{U}_{nxx}(x,\tau) - \widetilde{U}_n(x,\tau)\big(1-\widetilde{U}_n(x,\tau)\big\}d\tau, n \geq 0 \tag{4.50}$$

where $\widetilde{U}_n(x,\tau)$ is the restricted variation, i.e, $\delta\widetilde{U}_n(x,\tau) = 0$, $\lambda(\tau)$ is a general Lagrangian multiplier,

$$\delta U_{n+1}(x,t) = \delta U_n(x,t) + \delta \int_0^t \lambda(\tau)\{U_{n\tau}(x,\tau)\}d\tau \, , \tag{4.51}$$

Using integration by parts

$$\int_0^t \lambda(\tau)\{U_{n\tau}(x,\tau)\}d\tau \;\; = \lambda(\tau)U_n(x,\tau)|_{\tau=t} - \int_0^t \lambda'(\tau)U_n(x,\tau)\,d\tau \tag{4.52}$$

Thus, substituting eq.(4.52) into eq.(4.51) we obtain

$$\delta U_{n+1}(x,t) = \delta U_n(x,t) + \lambda(\tau)\delta U_n(x,\tau)|_{\tau=t} - \int_0^t \lambda'(\tau)\delta U_n(x,\tau)\,d\tau = 0 \tag{4.53}$$

Yields the following stationary conditions:

$$1+\lambda(\tau) = 0 \quad \text{and} \quad \lambda'(\tau)|_{\tau=t} = 0 \tag{4.54}$$

From eq.(4.54), the Lagrange multiplier can therefore be identified as:

$$\lambda(\tau) = -1 \tag{4.55}$$

Substituting eq.(4.55) into the correction functional eq.(4.50) results in the following iteration formula.

$$U_{n+1}(x,t) = U_n(x,t) - \int_0^t \{U_{n\tau}(x,\tau) - U_n(x,\tau)U_{nx}(x,\tau) - U_{nxx}(x,\tau) - U_n(x,\tau)(1-U_n(x,\tau)\}d\tau, \;\; n \geq 0 \tag{4.56}$$

We start with the initial approximation given the following equation.

$$U_0(x,t) \;\; = \;\; \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{x}{4}\right) \tag{4.57}$$

Using the above iteration formula eq.(4.56) and the initial approximation eq.(4.57), we can obtain the following iterations: All the computations and the plot are done by Matlab.

$$U_1(x,t) = U_0(x,t) - \int_0^t \{U_{0\tau}(x,\tau) - U_0(x,\tau)U_{0x}(x,\tau) - U_{0xx}(x,\tau) - U_0(x,\tau)(1 - U_0(x,\tau)\}d\tau,$$

$$= U_0(x,t) - \int_0^t \left\{-\frac{5}{16} + \frac{5}{16}\tanh\left(\frac{x}{4}\right)^2\right\} d\tau$$

$$= U_0(x,t) + \frac{5}{16}t - \frac{5}{16}t\,(\tanh\,(\frac{x}{4})^2.$$

$$U_1(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\,(\frac{x}{4}) + \frac{5}{16}t - \frac{5}{16}t\,(\tanh\,(\frac{x}{4})^2).$$

Similarly

$$U_2(x,t) = U_1(x,t) - \int_0^t \{U_{1\tau}(x,\tau) - U_1(x,\tau)U_{1x}(x,\tau) - U_{1xx}(x,\tau) - U_1(x,\tau)(1 - U_1(x,\tau)\}d\tau,$$

$$= U_1(x,t) + \frac{25}{1536}t^3\tanh\,(\frac{x}{4})^5 - \frac{t^2}{64}\tanh\,(\frac{x}{4})^4 + \frac{50}{1536}t^3\tanh\,(\frac{x}{4})^3 + \frac{15}{128}t^2\tanh\,(\frac{x}{4})^3$$

$$-\frac{165}{1024}t^2\tanh\left(\frac{x}{4}\right) - \frac{25}{1536}t^3\tanh(\frac{x}{4}) - \frac{25}{768}t^3 + \frac{50}{768}t^3\tanh\,(\frac{x}{4})^2 - \frac{25}{768}t^3\tanh\,(\frac{x}{4})^4.$$

$$U_2(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\,(\frac{x}{4}) + \frac{5}{16}t - \frac{5}{16}t\,(\tanh\,(\frac{x}{4})^2)$$

$$+\frac{25}{1536}t^3\tanh\,(\frac{x}{4})^5 - \frac{t^2}{64}\tanh\,(\frac{x}{4})^4 + \frac{50}{1536}t^3\tanh\,(\frac{x}{4})^3 + \frac{15}{128}t^2\tanh\,(\frac{x}{4})^3$$

$$-\frac{165}{1024}t^2\tanh\left(\frac{x}{4}\right) - \frac{25}{1536}t^3\tanh(\frac{x}{4}) - \frac{25}{768}t^3 + \frac{50}{768}t^3\tanh\,(\frac{x}{4})^2 - \frac{25}{768}t^3\tanh\,(\frac{x}{4})^4.$$

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Other components can be easily obtained.

Thus, the numerical solutions obtained by VIM (by 2 terms) are as follows:

Table 2: Example 1: shows the comparison of exact solution with VIM (by 2 terms) solution.

x	**t**	exact	VIM (by 2 terms)	Absolute error
0.1	0.2	0.5744	0.5745	0.0001
	0.4	0.6341	0.6347	0.0006
	0.6	0.6900	0.6913	0.0013
	0.8	0.7408	0.7429	0.0021
	1	0.7858	0.7879	0.0021
0.5	0.2	0.6225	0.6226	0.0001
	0.4	0.6792	0.6799	0.0007
	0.6	0.7311	0.7324	0.0013
	0.8	0.7773	0.7785	0.0012
	1	0.8176	0.8165	0.0011
0.9	0.2	0.6682	0.6684	0.0002
	0.4	0.7211	0.7219	0.0008
	0.6	0.7685	0.7695	0.0010
	0.8	0.8100	0.8096	0.0004
	1	0.8455	0.8407	0.0048

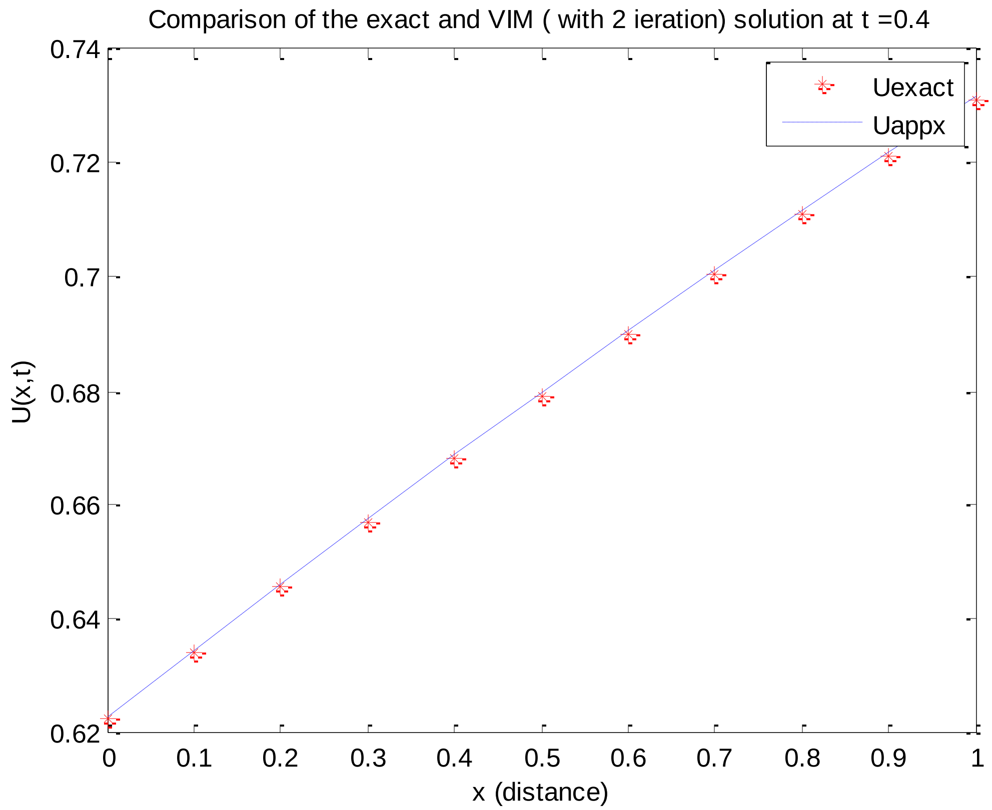

Figure 2: Example 1: comparison of the exact solution and VIM(by 2 terms) solution at t = 0.4.

The exact solution is compared with the approximate solution $U_{appx.2}(x,t)$ obtained by VIM both in table 2 and figure 2.

Example 2. consider the following Burger's Fisher Equation,

Assuming, $\alpha = 1, \beta = 0 \; and \; \delta = 1$

$U_t + UU_x - U_{xx} = 0$ x ∈ [0,1], t ≥ 0

$$U_t = U_{xx} - UU_x \tag{4.58}$$

With initial condition

$$U(x,0) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) \tag{4.59}$$

And boundary conditions

$$U(0,t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{1}{32}\right) \tag{4.60}$$

$$U(1,t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{-1}{4} + \frac{t}{8}\right) \tag{4.61}$$

And exact solution

$$U(x,t) = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{t}{8} - \frac{x}{4}\right) \tag{4.62}$$

Method 1: Using ADM

We begin by introducing a differential operator

$$L_t(.) = \frac{\partial(.)}{\partial t}$$

Then the inverse operator is

$$L_t^{-1}(.) = \int_0^t (.)dt$$

Rewriting the differential equation (4.58) in an operator form, we have

$$L_t U_t = R(U) - N(U) \tag{4.63}$$

Where N (U) = U U_x and R (U) = U_{xx}

Applying the inverse Operator L_t^{-1} on both sides of equation eq. (4.58) yields

$$L_t^{-1}L_tU_t = L_t^{-1}R(U) - L_t^{-1}N(U) \qquad (4.64)$$

Using initial condition the left hand side expression of eq.(4.64) becomes

$$L_t^{-1}L_tU_t = \mathrm{U(x, t) - U(x, 0)} \qquad (4.65)$$

Returning eq. (4.65) into eq.(4.64) it becomes

$$\mathrm{U(x, t)} = \mathrm{U(x,0)} + L_t^{-1}R(U) - L_t^{-1}N(U) \qquad (4.66)$$

Next we need to generate the Adomian polynomials B_n and K_n.

Let U(x, t) be expanded as infinite series

$$\mathrm{U(x, t)} = \sum_{n=0}^{\infty} U_n(x,t) \qquad (4.67)$$

Define R (U) = $\sum_{n=0}^{\infty} K_n$ and N (U) = $\sum_{n=0}^{\infty} B_n$

Then $\sum_{n=0}^{\infty} U_n(x,t) = \mathrm{U(x, 0)} + L_t^{-1}R(U) - L_t^{-1}N(U)$ (4.68)

Next we need to determine the Adomian polynomials to find K_n and B_n, we use Adomian formula. That is

$$G_n = \frac{1}{n!}\frac{d^n}{d\lambda^n}\left[F\left(\sum_{n=0}^{\infty} \lambda^i U_i\right)\right]_{\lambda=0}, \quad n = 0, 1, 2, 3, \ldots$$

Then we find the Adomian polynomials

$$\left.\begin{aligned} B_0 &= \mathrm{N}(U_0) = U_0U_{0x} \\ B_1 &= U_1U_{0x} + U_{1x}U_0 \\ B_2 &= U_0U_{2x} + U_{0x}U_2 + U_1U_{1x} \\ B_3 &= U_0U_{3x} + U_{1x}U_2 + U_{0x}U_3 + \frac{1}{3}U_1U_{2x} \end{aligned}\right\} \qquad (4.69)$$

.

.

$$\left.\begin{aligned} K_0 &= U_{0xx} \\ K_1 &= U_{1xx} \\ K_2 &= U_{2xx} \end{aligned}\right\} \qquad (4.70)$$

.

The polynomials K_n *and* B_n, n ≥ 3 can be computed in a similar way.

Substituting the Adomian polynomials eq. (4.69) and eq. (4.70) to the equation (4.68)

We can determine the recursive relationship that will be used to generate the solution.

$$U_0(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right)$$

$$U_{n+1}(x,t) = L_t^{-1}(K_n) - L_t^{-1}(B_n), \qquad n = 0, 1, 2, \ldots \qquad (4.71)$$

Using the same procedure applied under example 1, the three terms of the components can be obtained by the help of MATLAB programming language. The terms U_1 *and* U_2 are as follows.

$$U_0(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right)$$

$$U_1(x,t) = L_t^{-1}(K_0) - L_t^{-1}(B_0)$$

$$= L_t^{-1}(U_{0xx}) - L_t^{-1}(U_0 U_{0x})$$

$$= \frac{t}{16} - \frac{t}{16}\tanh\left(\frac{x}{4}\right)^2$$

$$U_2(x,t) = L_t^{-1}(K_1) - L_t^{-1}(B_1)$$

$$= L_t^{-1}(U_{xx}) - L_t^{-1}(U_1 U_{0x} + U_{1x} U_0)$$

$$= \frac{1}{128} t^2 \tanh\left(\frac{x}{4}\right) - \frac{1}{128} t^2 \tanh\left(\frac{x}{4}\right)^3$$

Hence by (eq.4.10)

$$U_{appx\,3}(x,t) = \sum_{n=0}^{2} U_n(x,t) = U_0(x,t) + U_1(x,t) + U_2(x,t).$$

$$= \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) + \frac{t}{16} - \frac{t}{16}\tanh\left(\frac{x}{4}\right)^2 + \frac{1}{128}t^2\tanh\left(\frac{x}{4}\right) - \frac{1}{128}t^2\tanh\left(\frac{x}{4}\right)^3.$$

$$U_{appx\,3}(x,t) = \frac{1}{2} + \frac{t}{16} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) - \frac{t}{16}\tanh\left(\frac{x}{4}\right)^2 + \frac{1}{128}t^2\tanh\left(\frac{x}{4}\right) - \frac{1}{128}t^2\tanh\left(\frac{x}{4}\right)^3.$$

Which is the approximate solution using the first three iterations only. MATLAB programming language is used to plot the graphs of ADM and exact solution together in figure 3 below.

Table 3: Numerical results for U(x,t) of example 2 by 3 terms ADM.

X	T	Exact	ADM(3 terms)	Absolute error
0.1	0.5	0.5187	0.5188	0.0001
	1	0.5498	0.5502	0.0004
	2	0.6106	0.6132	0.0026
0.5	0.5	0.4688	0.4688	0.0000
	1	0.5000	0.5003	0.0003
	2	0.5622	0.5647	0.0025
0.9	0.5	0.4195	0.4195	0.0000
	1	0.4502	0.4504	0.0002
	2	0.5125	0.5148	0.0023

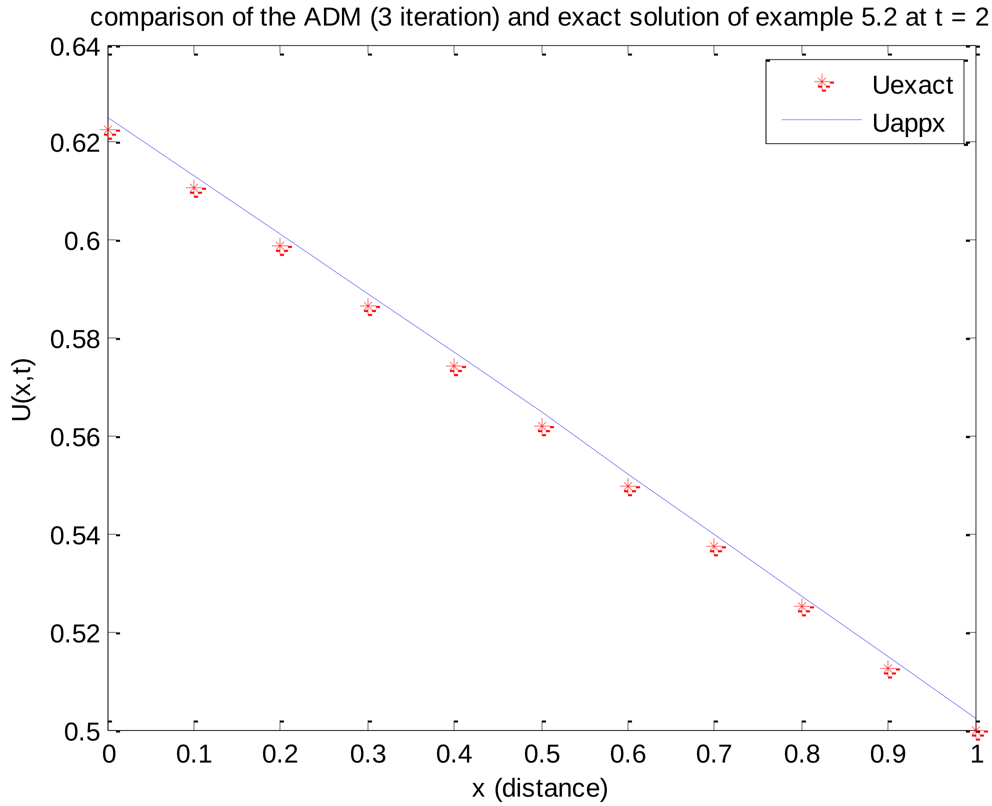

Figure 3: Example 2: Comparison of the exact solution and ADM (by 3 terms) solution at t = 2.

Method 2 : Using VIM

To solve equation (4.58) using VIM, we consider a correction functional as:

$$U_{n+1}(x,t) = U_n(x,t) + \int_0^t \lambda(\tau)\{U_{n\tau}(x,\tau) + \widetilde{U}_n(x,\tau)\widetilde{U}_{nx}(x,\tau) - \widetilde{U}_{nxx}(x,\tau)\}d\tau \quad (4.72)$$

where, $\widetilde{U}_n$ is the restricted variation, that is, $\delta\widetilde{U}_n$= 0, and $\lambda(\tau)$ is a general Lagrange multiplier.

$$\delta U_{n+1}(x,t) = \delta U_n(x,t) + \delta \int_0^t \lambda(\tau)\{U_{n\tau}(x,\tau)\}\mathrm{d}\tau \quad (4.73)$$

Using integration by parts

$$\int_0^t \lambda(\tau)\{U_{n\tau}(x,\tau)\}\mathrm{d}\tau = \lambda(\tau)U_n(x,\tau)|_{\tau=t} - \int_0^t \lambda'(\tau)U_n(x,\tau)d\tau \quad (4.74)$$

Thus, substituting eq.(4.74) into eq.(4.73) we obtain

$$\delta U_{n+1}(x,t) = \delta U_n(x,t) + \lambda(\tau)\delta U_n(x,\tau)|_{\tau=t} - \int_0^t \lambda'(\tau)\delta U_n(x,\tau)d\tau = 0 \quad (4.75)$$

Yields the following stationary conditions:

$$1+\lambda(\tau) = 0$$

$$\lambda'(\tau)|_{\tau=t} = 0 \quad (4.76)$$

From eq.(4.76), the Lagrange multiplier can therefore be identified as

$$\lambda(\tau) = -1 \quad (4.77)$$

Substituting eq.(4.77) into the correctional equation (4.72) results in the following iteration formula.

$$U_{n+1}(x,t) = U_n(x,t) - \int_0^t \{U_{n\tau}(x,\tau) + U_n(x,\tau)U_{nx}(x,\tau) - U_{nxx}(x,\tau)\}d\tau \quad (4.78)$$

We start with the initial approximation given the following equation

$$U_0(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) \quad (4.79)$$

Now, using the above iteration formula eq.(4.78) and the initial approximation eq.(4.79), we can obtain the following iterations: All the computations and the plot are done by matlab.

$$U_1(x,t) = U_0(x,t) - \int_0^t \{U_{0\tau}(x,\tau) + U_0(x,\tau)U_{0x}(x,\tau) - U_{0xx}(x,\tau)\}d\tau$$

$$= U_0(x,t) - \int_0^t \{0 + \left(\frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right)\right)\left(\frac{1}{8}\tanh\left(\frac{x}{4}\right)^2 - \frac{1}{8}\right) - \left(\frac{1}{16}\tanh\left(\frac{x}{4}\right) - \frac{1}{16}\tanh\left(\frac{x}{4}\right)^3\right)\}d\tau$$

$$U_1(x,t) = \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) + \frac{t}{16} - \frac{t}{16}\tanh\left(\frac{x}{4}\right)^2.$$

Similarly,

$$U_2(x,t) = U_1(x,t) - \int_0^t \{U_{1\tau}(x,\tau) + U_1(x,\tau)U_{1x}(x,\tau) - U_{1xx}(x,\tau)\}d\tau$$

$$= \frac{1}{2} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) + \frac{t}{16} - \frac{t}{16}\tanh\left(\frac{x}{4}\right)^2 - \frac{t^2}{128}\tanh\left(\frac{x}{4}\right)^3 + \frac{t^2}{128}\tanh\left(\frac{x}{4}\right) - \frac{t^3}{768}\tanh\left(\frac{x}{4}\right)^3$$
$$+ \frac{t^3}{1536}\tanh\left(\frac{x}{4}\right) + \frac{t^3}{1536}\tanh\left(\frac{x}{4}\right)^5.$$

$$U_3(x,t) = U_2(x,t) - \int_0^t \{U_{2\tau}(x,\tau) + U_2(x,\tau)U_{2x}(x,\tau) - U_{2xx}(x,\tau)\}d\tau$$

$$= \frac{5}{66060288}t^7\tanh\left(\frac{x}{4}\right)^{11} - \frac{t^7}{3145728}\tanh\left(\frac{x}{4}\right)^9 + \frac{17}{33030144}t^7\tanh\left(\frac{x}{4}\right)^7 - \frac{13}{33030144}t^7\tanh\left(\frac{x}{4}\right)^5$$
$$+ \frac{t^7}{7340032}\tanh\left(\frac{x}{4}\right)^3 - \frac{t^7}{66060228}\tanh\left(\frac{x}{4}\right) - \frac{t^6}{589824}\tanh\left(\frac{x}{4}\right)^9 + \frac{13}{2359296}t^6\tanh\left(\frac{x}{4}\right)^7$$
$$- \frac{5}{786432}t^6\tanh\left(\frac{x}{4}\right)^5 + \frac{7}{2359296}t^6\tanh\left(\frac{x}{4}\right)^3 - \frac{t^6}{2359296}\tanh\left(\frac{x}{4}\right) - \frac{7}{491520}t^5\tanh\left(\frac{x}{4}\right)^8$$
$$+ \frac{3}{327680}t^5\tanh\left(\frac{x}{4}\right)^7 + \frac{11}{245760}t^5\tanh\left(\frac{x}{4}\right)^6 - \frac{7}{327680}t^5\tanh\left(\frac{x}{4}\right)^5 - \frac{1}{20480}t^5\tanh\left(\frac{x}{4}\right)^4$$
$$+ \frac{1}{65536}t^5\tanh\left(\frac{x}{4}\right)^3 + \frac{1}{49152}t^5\tanh\left(\frac{x}{4}\right)^2 - \frac{1}{327680}t^5\tanh\left(\frac{x}{4}\right)$$
$$- \frac{t^5}{491520} + \frac{3}{16384}t^4\tanh\left(\frac{x}{4}\right)^7 + \frac{25}{98304}t^4\tanh\left(\frac{x}{4}\right)^6$$
$$- \frac{23}{49152}t^4\tanh\left(\frac{x}{4}\right)^5 - \frac{55}{98304}t^4\tanh\left(\frac{x}{4}\right)^4 + \frac{19}{49152}t^4\tanh\left(\frac{x}{4}\right)^3 + \frac{35}{98304}t^4\tanh\left(\frac{x}{4}\right)^2$$
$$- \frac{5}{49152}t^4\tanh\left(\frac{x}{4}\right) - \frac{5}{98304}t^4 - \frac{1}{1024}t^3\tanh\left(\frac{x}{4}\right)^4 + \frac{1}{768}t^3\tanh\left(\frac{x}{4}\right)$$
$$- \frac{t^3}{3072} - \frac{t^2}{128}\tanh\left(\frac{x}{4}\right)^3 + \frac{t^2}{128}\tanh\left(\frac{x}{4}\right) - \frac{t}{16}\tanh\left(\frac{x}{4}\right)^2 + \frac{t}{16} - \frac{1}{2}\tanh\left(\frac{x}{4}\right) + \frac{1}{2}.$$

Other components can be easily obtained using the same manner.

Thus, the numerical solutions obtained by VIM (by 3 terms) are as follows:

Table 4: Example 2: Shows the comparison of exact solution with VIM (by 3 terms).

x	t	exact	VIM (by 3 terms)	Absolute error
0.1	0.5	0.5187	0.5187	0.0000
	1	0.5498	0.5498	0.0000
	2	0.6106	0.6097	0.0009
0.5	0.5	0.4688	0.4688	0.0000
	1	0.5000	0.4999	0.0001
	2	0.5622	0.5613	0.0009
0.9	0.5	0.4195	0.4195	0.0000
	1	0.4502	0.4501	0.0001
	2	0.5125	0.5118	0.0007

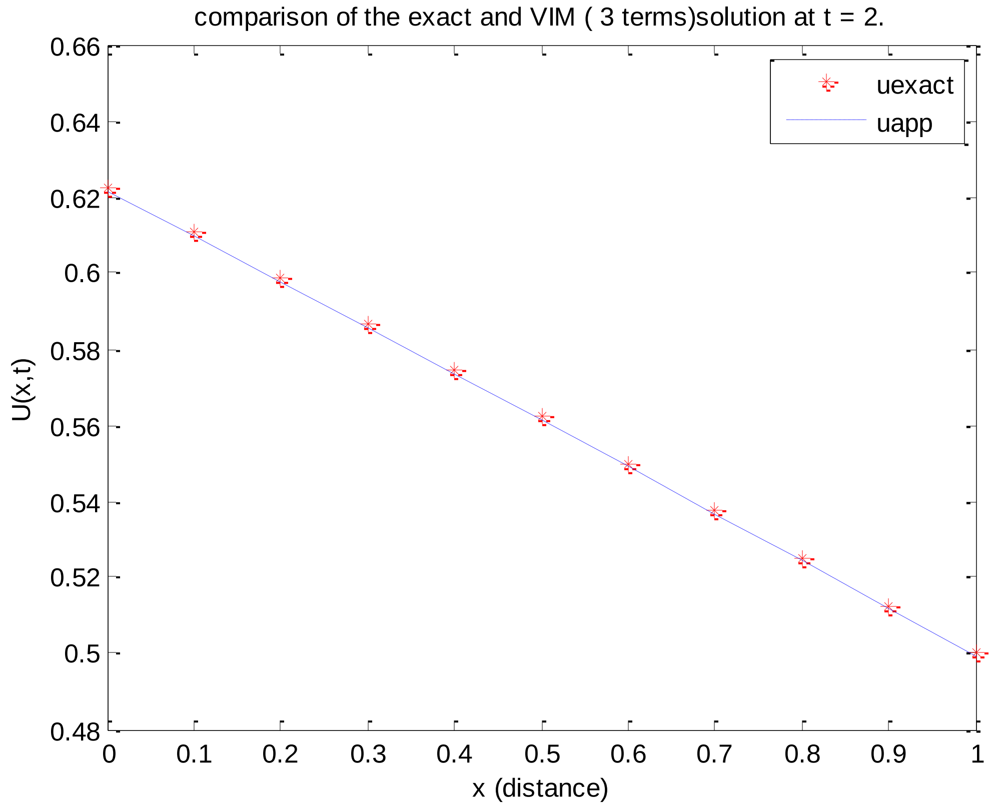

Figure 4 : Example 2: Comparison of the exact solution and VIM (with 3 terms) solution at t = 2.

The exact solution is compared with the approximate solution $U_{appx.3}(x,t)$ obtained by VIM both in table 4 and figure 4.

Table 5: Example 1: Comparison of absolute errors of the solutions of Burger's fisher equation, by ADM and by He's VIM for different values of x and t, assuming $\alpha = -1 \; and \; \beta = \delta = 1$.

x	t	Exact	ADM (by 3 terms)	VIM (by 2 terms)	ADM (Absolute error)	VIM (Absolute error)
0.1	0.2	0.5744	0.5748	0.5745	0.0004	0.0001
	0.4	0.6341	0.6366	0.6347	0.0025	0.0006
	0.6	0.6900	0.6981	0.6913	0.0081	0.0013
	0.8	0.7408	0.7592	0.7429	0.0184	0.0021
	1	0.7858	0.8199	0.7879	0.0341	0.0021
0.5	0.2	0.6225	0.6228	0.6226	0.0003	0.0001
	0.4	0.6792	0.6814	0.6799	0.0022	0.0007
	0.6	0.7311	0.7382	0.7324	0.0071	0.0013
	0.8	0.7773	0.7930	0.7785	0.0157	0.0012
	1	0.8176	0.8459	0.8165	0.0283	0.0011
0.9	0.2	0.6682	0.6684	0.6684	0.0002	0.0002
	0.4	0.7211	0.7229	0.7219	0.0018	0.0008
	0.6	0.7685	0.7742	0.7695	0.0057	0.0010
	0.8	0.8100	0.8221	0.8096	0.0121	0.0004
	1	0.8455	0.8667	0.8407	0.0212	0.0048

In the above table 5, it consider the errors of both the ADM and VIM of example 1, it can be seen that VIM is better in accuracy in comparison with ADM.

Table6: Example 2: Comparison of Absolute errors of the solutions of Burger's fisher equation, by ADM, and He's VIM for different values of x and t,

assuming $\alpha = 1 = \delta \; and \; \beta = 0$.

x	t	Exact	ADM (3 terms)	VIM (3 terms)	ADM (Absolute error)	VIM (Absolute error)
0.1	0.5	0.5187	0.5188	0.5187	0.0001	0
	1	0.5498	0.5502	0.5498	0.0004	0
	2	0.6106	0.6132	0.6097	0.0026	0.0009
0.5	0.5	0.4688	0.4688	0.4688	0	0
	1	0.5000	0.5003	0.4999	0.0003	0.0001
	2	0.5622	0.5647	0.5613	0.0025	0.0009
0.9	0.5	0.4195	0.4195	0.4195	0	0
	1	0.4502	0.4504	0.4501	0.0002	0.0001
	2	0.5125	0.5148	0.5118	0.0023	0.0007

In the above table 6, it consider the errors of both the ADM and VIM of example 2, it can be seen that VIM is better in accuracy in comparison with ADM.

4.5 Discussion

In this study, we applied Adomian Decomposition Method(ADM) and He's Variational iteration method (VIM) for solving nonlinear space time Burger's Fisher equation, being a nonlinear partial differential equation that arises in different areas of science such as in a modeling of gas dynamics, financial mathematics, sound wave in a viscous medium, etc. In the above worked example one and example two, we considered Adomian decomposition method(ADM) and Variational iteration method (VIM). For the first tested example in case one, three components(iterations) were used for ADM and compared with the exact solution and in case two, we determined two components(iterations) for VIM and compared with the exact solution for different values of x and t. And also, for the second example in case one three terms (iterations) were used for ADM and compared with the exact solution and in case two, 3 terms were used for VIM and compared with the exact solution for different values of x and t.

From the above table 1, and table 2 and from figure 1 and figure 2 of example one, we can observe that both methods are very powerful and efficient for solving such a partial differential equation and converges faster to the exact solution with in a few iterations. But when we compare them, the VIM converges faster than ADM. Even though, the comparison between the 3 terms of ADM and 2 terms of VIM performed, VIM converges faster to the exact solution. And also from the above table 3 and table 4 and from figure 3 and figure 4 of example two, we can conclude that the VIM is a powerful method than ADM. The numerical experiments show that the absolute error of VIM is very small as compared to the absolute error of ADM. Thus, the VIM is highly accurate. Generally, comparisons of the two methods of example one were shown in table 5 whereas comparisons of the two methods of example two were shown in table 6. Finally, we recognized that, from the above analyzed errors of the two test examples, VIM has good rate of convergence and better accuracy within a few iterations than ADM. Therefore, it is better to apply VIM for solving non linear space time Burger's Fisher equation.

UNIT FIVE

5.Conclusion

This work presents the basic understanding of Adomian decomposition method(ADM) and He's Variational iteration method (VIM) to solve Burger's Fisher equation, which is a nonlinear partial differential equation. The main idea of Adomian decomposition method is using different operators for linear and nonlinear terms and then determined the Adomian polynomials using Adomian polynomial formula and finally finding the unknown function in a series form. VIM uses the general Lagrange multiplier. In order to state which method is efficient, accurate and convergent, we observe the results which were obtained in example one and example two by using table 1, table 2, table 3 and table 4. Especially table 4 of example two shows that a very good approximation is achieved with a very few iteration of the Variational iteration method as compare to ADM.

From the tasted example one, figure 1 and figure 2 shows the results obtained by ADM at t = 0.4 and exact solution at t = 0.4 and the results obtained by VIM at t = 0.4 and exact solution at t = 0.4 respectively, on the same coordinate plane. From the tasted example two, figure 3 and figure 4 shows the results obtained by ADM at t = 2, exact at t = 2 and VIM at t = 2, exact at t = 2 respectively. This shows that the VIM has good convergence and accuracy than ADM.And also, VIM facilitates the computational work and gives the solution rapidly if compared with ADM. Thus, VIM is convenient and efficiency for solving partial differential equation problems that arising in various fields of science and engineering than ADM.

Based on the study, the researcher observes the following disadvantages of the ADM and VIM. Although both methods are very powerful, the methods gives the solution in a series form, which must be truncated for practical application. And also, the Adomian decomposition method requires the use of Adomian polynomials for nonlinear portions of the equation, and this need more work.

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