

Adaptive Sliding Mode Controller for Attitude and Position of Quadrotor UAV



Kemeriya Major Husen

A Thesis Submitted to
The Department of Electrical Power and Control Engineering
School of Electrical Engineering and Computing

Presented in Partial Fulfillment of the Requirements for Degree of Master's
in Electrical Power and Control Engineering (Control Engineering)

Office of Graduate Studies
Adama Science and Technology University

July, 2023
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DECLARATION

I hereby declare that this Master Thesis entitled “**Adaptive sliding mode control for attitude and position of quadrotor UAV**” is my original work. That is, it has not been submitted for the award of any academic degree, diploma or certificate in any other university. All sources of materials that are used for this thesis have been duly acknowledged through citation.

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LIST OF ACRONYMS

3D	Three dimensional
ABSMC	Adaptive back-stepping sliding mode control
ASMC	Adaptive sliding mode control
BS	Back-stepping
DC	Direct current
DO	Disturbance observer
DOF	Degree of freedom
HIL	Hardware-in-loop
LQR	Linear quadratic regulator
PD	Proportional derivative
PID	Proportional integral derivative
PWM	Pulse width modulation
RBF	Radial basis function
SMC	Sliding mode control
SMC-SMDO	Sliding mode control-sliding mode disturbance observer
UAV	Unmanned aerial vehicle
VSS	Variable structure system
VTOL	Vertical take-off and landing
WWII	World war second

ABSTRACT

In recent years Unmanned Aerial Vehicles (UAVs) have become increasingly popular research topic due to their wide applications and advantages. The quadrotor is a highly non-linear and coupled system which requires a good controller. In this research work, an adaptive law-based Sliding Mode Control (SMC) is proposed for attitude and position of quadrotor UAVs under external disturbance and internal parameter uncertainty, considered mass in this study. Since quadrotor dynamics is highly non-linear and strongly coupled, the system dynamics is given first and open loop verification is done to show that the system functions according to the given input signal properly. The Adaptive Sliding Mode Control (ASMC) is then designed and the chattering effect on the proposed controller performance is decreased by adding a boundary layer around the sliding surface by tanh function. Lyapunov stability analysis is used to achieve asymptotic stability. The system has inner and outer loop which controls attitude and position respectively. The outer loop output responses, X and Y positions, are given as a desired trajectory for inner loop, Phi and Theta, while an arbitrary reference trajectory is given for Psi angle. The controller and the system dynamics is integrated and simulated using MATLAB/Simulink environment. The simulation result shows, the system performs very well while exhibiting robustness on the set point path with less than 2 second settlement and 0% overshoot. To confirm the robustness of the control method, uncertain parameter value of mass and external step wind disturbance is applied on the system. Finally, the proposed controller is compared with classical SMC to show the performance of both controllers. The proposed controller has good tracking and better disturbance rejection ability.

Key words: Quadrotor, Adaptive sliding mode control, Lyapunov stability, Chattering, Robustness

CHAPTER ONE

1 INTRODUCTION

1.1 Background

Quadrotor Unmanned Aerial Vehicles (UAVs) have become a popular research topic in recent years due to their low cost, maneuverability, structural simplicity, ability to hover, Vertical Take-Off and Landing (VTOL) capability and ability to perform a wide range of tasks in control and aerospace engineering.

This study is motivated by the numerous applications of these devices, which include inspection, surveillance, law enforcement, videography, rescue, search and many others. From World War I (WWI) to the middle of the nineteenth century UAVs were only used for aviation training, implying that they were designed and built for military purposes only. However, in the twenty-first century, technology has advanced to the point where UAVs are being considered for use in other areas of aviation (Norris, 2014). Since it has been widely used in the military department over the last decade and the success of these military applications is increasingly driving efforts in technologically advanced countries to establish UAVs in non-military roles. The trend in UAVs usage is not for a single purpose, but for multiple purposes by modifying some parts, such as security systems, aerial cameras, and videography. Through time, the use of UAVs for military purposes has decreased and the use of UAVs for other technology transfer has increased. Having a UAV system that can fly autonomously and perform an action simplifies human contribution while exponentially increasing its capability.

Quadrotors are small UAVs that get their lift force from four rotors or motors. It has 6 Degrees of Freedom (6-DOF), three of which are rotational and the rest are translational. This, in turn, improves the vehicle's maneuverability and controllability in space (Quan, 2017). The shape of the quadcopter's rotor arm is symmetrical, combined with a central plate where all the devices and useful things are placed. This symmetry in shape helps the quadcopter move vertically, operate the hovering function, and steer the aircraft in any direction. Quadrotor typically have a larger payload, excellent maneuverability, small size, efficiency, safety and are easier to manufacture than conventional helicopters. As a result,

quadrotor helicopters have received a lot of attention in UAV research and history (Clothier & Walker, 2015; Quan, 2017).

Quadrotor UAVs control system is very complex due to the high non-linearity, strong couplings, under actuation and complex structure of the dynamics. UAV flight can be controlled fully autonomously by on-board computers or with a certain degree of remote control from an operator on the ground or in another vehicle (Clothier & Walker, 2015; Norris, 2014). This aerial vehicle has four rotors whose thrust force and torque are generated by power transmission to the quadrotors blades or propellers and controlled and stabilized by motor rpm or thrust force and torque. By controlling the angular velocity of each rotor, including the motor and propeller assembly, the quadrotor changes its attitude and position (Mukherjee & Alemayehu, 2021).

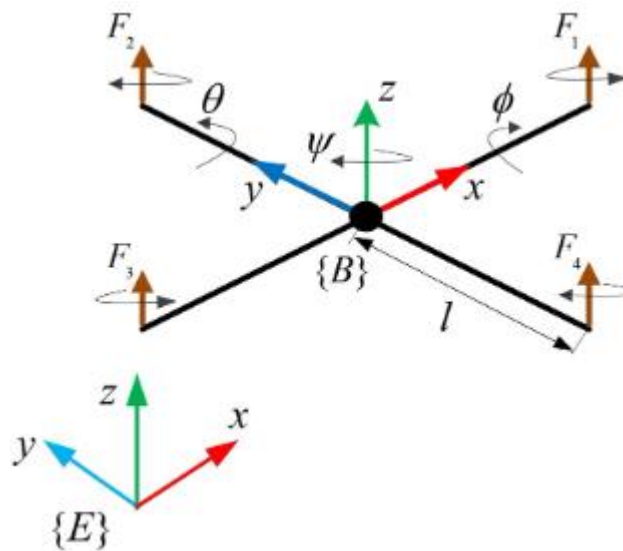


Figure 1.1: Quadrotor configuration (L1)

A quadrotor has been controlled using a variety of linear methods, including classical Proportional, Integral Derivative (PID) and Linear Quadratic (LQ) controllers (Bouabdallah, Noth, & Siegwart, 2004). Nonlinear control methods, such as adaptive control (Koksal, An, & Fidan, 2020) and SMC (Pang, Shang, & Yang, 2020) have been developed to control attitude and position of quadrotor because of its robustness to parameter uncertainty, fast dynamic response, and insensitivity to bounded disturbance. Compared to other controllers, SMC has received research attention a lot (Zeng et al., 2011). However, there are some drawbacks to SMC such as chattering and limited design freedom for designers using the sliding function. The chattering effect is caused by the inclusion of the sign function in the switching control as well as the system's non-ideal behavior. To reduce chattering and

improve performance and reliability when dealing with external disturbances and aerodynamic parameter uncertainties, the extended SMC method has been proposed (Jia et al., 2017). But, implementation of these methods necessitates understanding of the bounds on the uncertainties. SMC tend to cause large input gains to compensate for these uncertainties. The shortcomings of sliding mode controllers are mitigated by modifying the estimates of the uncertainty upper bound (Lee, Jin Kim, & Sastry, 2009). It can be difficult to estimate the upper bound accurately at times, and the assumption of uncertainty can be difficult to apply.

Some researchers present ASMC for robust control of non-linear systems with adaptive tuning. In this work, an adaptive law for robust control of quadrotors with uncertain parameters is given based on SMC. Without knowing the upper bound of the system uncertainty beforehand, the recommended control design method adaptively alters the control gains. The performance of the quadcopter's attitude and position tracking is ensured by the closed-loop system's stability and resilience, which are demonstrated using the direct Lyapunov stability theory. The uniqueness of this study approach lies in the tuning scheme that may be carefully selected to regulate the rate of adaptation.

1.2 Statement of the problem

Any UAVs modeling and control are difficult. One category of multirotor UAVs is quadcopters. It is a multivariable, under actuated structure that exhibits strong nonlinearity. Systems that are under actuated have fewer inputs for control than their total levels of flexibility. Because of the nonlinear coupling between the actuator's degrees of freedom, they are exceedingly challenging to regulate. Whereas PID flight controllers and other linear flight control algorithms are the most often used, these controllers are only effective when the drone is flying in a specified pattern. Every time the quadrotor departs from the nominal conditions or makes forceful maneuvers while operating in an abnormal situation, its efficiency is severely degraded. ASMC is suggested in this study, since it has a well performance ability to control attitude and position while rejecting disturbances and internal parameter uncertainty. ASMCs internal chattering issues pose a disadvantage and as a result of this the system drives may receive varied current and voltage dispersion causing the drone aircraft to vibrate due to the various forces across each rotor. ASMC is presented to regulate the quadcopter UAVs altitude, attitude, and heading in order to alleviate this issue. Utilizing MATLAB/Simulink environment analysis, the system performance is examined.

1.3 Objectives

1.3.1 General objective

The general objective of this research work is to design an adaptive sliding mode controller for attitude, altitude and heading of the quadrotor system.

1.3.2 Specific objectives

-  Deriving the quadrotor dynamics nonlinear mathematical model.
-  Designing a nonlinear ASMC controller with a tanh function switching law to reduce the chattering effect.
-  Performing a Lyapunov stability analysis to get the precise boundary of the proposed controller gain and check the overall stability of the system.
-  Analyzing the performance of the proposed controller by simulating on MATLAB/Simulink environment and comparing with SMC.

1.4 Scope of the study

This research work scope includes from developing a nonlinear mathematical model to designing ASMC for altitude and attitude dynamics of quadrotor UAV. A Lyapunov stability analysis and bounding the sliding surface to reduce a high oscillation effect is also included in the designing method of the proposed controller. Finally, simulating the suggested controller system with MATLAB/Simulink environment to verify the controller performance.

1.5 Motivation of the study

New sectors of application for the quadcopter drone have been created by the development and deployment of outstanding technology. This kind of vehicle can fit into tight locations and is employed for a variety of activities. Drones are currently used for a variety of applications across many different fields. Therefore, when creating these high-tech systems, a precise and perfect control algorithm design is crucial.

1.6 Significance of the study

A very fascinating topic of research in the control system fields is controlling the UAV's altitude and orientation. Due to the versatility of their applications and the benefits of multirotor UAVs like; delivery, monitoring, photography and mapping, these types of studies are currently being conducted in numerous nations.

1.7 Limitation of the study

Stability of the adaptive control system is not treated rigorously. The high gain observer is needed to avoid full state measurement. In addition, this research is restricted to just simulating the quadcopter system because creating actual prototype implementations would be complicated, expensive, and time-consuming.

CHAPTER TWO

2 LITERATURE REVIEW

2.1 Chapter overview

In this chapter a brief description, history, types and uses of the UAV quadrotor is presented. Then the concept and the structure of the quadrotor-type UAVs are discussed. Finally, closely related works along with their achievement and drawbacks are reviewed and discussed.

2.2 Unmanned aerial vehicles (UAVs)

UAVs are small aircraft that are designed or modified for operation without a human pilot and operated by electronic input activated by a flight controller. They can be remotely operated by humans or they can be autonomous. Self-driving vehicles are controlled by onboard computers, which can be pre-programmed to perform specific tasks or a wide range of tasks. Aircraft UAVs are mainly used for military applications, but recently they have begun to be used in civilian applications (Giordan et al., 2020).

2.2.1 History of UAVs

A precise model of the dynamic system is required for quadrotor control. The Lagrangian approach was used to create the first dynamic quadrotor model. It was a partial differential equation for the dynamics of a quadrotor system that was obtained from a quadrotor Lagrangian model (Das, Lewis, & Subbarao, 2009). It was practically taken off with a rotorcraft in the very first experiment, which was mostly done using a multicopter. According to Jacques and Louis Breguet, a pair of French brothers created and tested the Gyroplane No. 1 quadcopter model in 1907. They were able to launch it, despite the fact that the design controller system was discovered to be extremely unstable and so unsuitable (Norris, 2014).

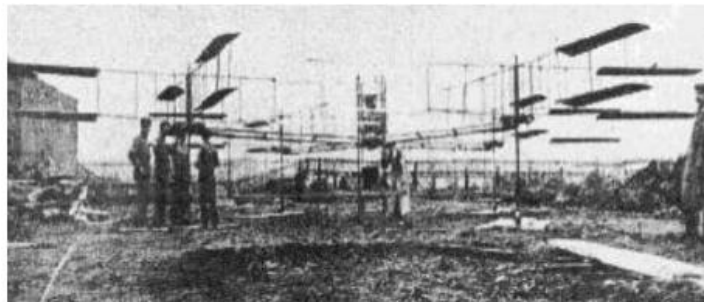


Figure 2.1: The Breguet-Richet gyroplane No.1-quadrotor of 1907 (Quan, 2017)

The first scientist and researcher to test rotorcraft ideas was Etienne Oehmichen in 1920. He showed off six alternative concepts, the second of which was a multicopter with four rotors and eight propellers, all driven by a single motor. It had a steel tube frame and four arms with two-bladed rotors at the ends. These propellers' angles could be changed by warping. Five of the propellers stabilized the machine laterally while rotating in the horizontal surface plane. At the nose, another six propellers were placed for steering. For forward propulsion, the remaining propellers were used (Zeng et al., 2011). It set the first-ever helicopter distance record of 360m for the Federation Aeronautique Internationale (FAI) in 1924. Eventually, it successfully accomplished the first 1-kilometer closed-circuit rotorcraft flight (Bouabdallah et al., 2004; Carnduff, 2015; Clothier & Walker, 2015).

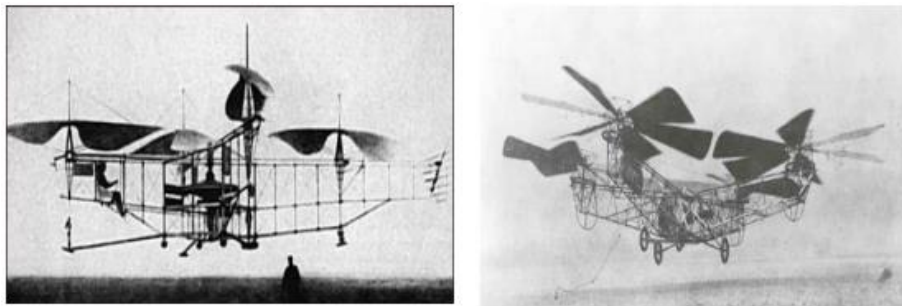


Figure 2.2: Oehmichen quadrotor No. 2, 1924 and Bothezat quadrotor (Quan, 2017)

Following Oehmichen, another aircraft was modified by G. de Bothezat and I. Jerome with six bladed rotors at the end of an X-shaped layout. For propulsion and yaw control, a pair of tiny propellers with variable pitch were utilized (Clothier & Walker, 2015).

It was suggested that the Convert wings quadrotor aircraft serve as the prototype for a series of considerably bigger military and civilian quadrotor helicopters. The design idea has wings added for more lift in forward flight and employs two engines to drive four rotors. No tail rotor was required; the control method was achieved by changing the thrust between the rotors. This helicopter, which was successfully flown a number of times in the middle of the 1950s, proved to be the ideal quadrotor type and the first quadrotor to demonstrate successful forward flight. The project, however, was abandoned because there were not enough materials for either the commercial or military versions (Lee et al., 2009).



Figure 2.3: Convert-wings quadrotor model 1956 (Quan, 2017)

2.2.2 Application of UAVs

Along with military applications, UAVs can also be utilized for various civilian or commercial applications that are too tedious, muted, or hazardous for manned aircraft. These practices comprise functions like (ElKholy, 2014):

- ✚ UAVs applications can be used in combination with satellite observations.
- ✚ UAVs equipped with infrared sensors, are directed flying over fire prone forests to spot it in time and lead the particular place of the fire to the ground station before its blowout.
- ✚ UAVs are used as a cost-effective substitute to traditional manned police helicopters.
- ✚ UAVs are used to patrol the border to prevent illegal immigration, drug and gun smuggling.
- ✚ UAVs are also used in research conducted by universities to test certain theories. In addition, environmental research institutions use drones equipped with appropriate sensors to monitor certain environmental phenomenon, such as pollution in large cities.
- ✚ UAVs are used in various industrial applications, such as pipeline inspection or surveillance and surveillance of nuclear plants.
- ✚ UAVs also have agricultural uses, such as crop spraying.

2.2.3 Classification of UAVs

There are different ways to classify UAVs, either according to their range of action, aerodynamic configuration, size and payload and, according to their levels of autonomy based on their (Angelov, 2012);

- ✚ Range of actions with maximum altitude and endurance.
- ✚ Size and payload configuration classifications.
- ✚ Aerodynamic configuration,
 - Fixed-wing UAVs.

- Blimp UAVs.
- Flapping-wing UAVs.
- Rotary-wing UAVs; single-rotor, coaxial, quadrotor and multi-rotor UAVs.

2.2.4 Quadrotors

Quadrotors are rotary-wing UAVs whose concept of aircraft was developed a long time ago. According to reports, the Breguet Richet quadrotor manufactured in 1907 has already flown. Since its configuration will be discussed later, the quadrotor is considered a rotary wing UAV(ElKholi, 2014) (Angelov, 2012).



Figure 2.4: Modern quadrotor example (L2)

The quadrotor can be configured in two ways as a plus and cross configuration and it consists of four rotors each of which is installed at one end of the cross shaped frame, as shown in Figure 2.5. Each rotor consists of a propeller mounted on a separately powered DC motor. Propellers 1 and 3 rotate in the same direction, while propellers 2 and 4 rotate in opposite directions, thus balancing the total torque of the system and counteracting the gyro and aerodynamic torque in hovering flight (Hou, Zhuang, Xia, Wang, & Yu, 2010).

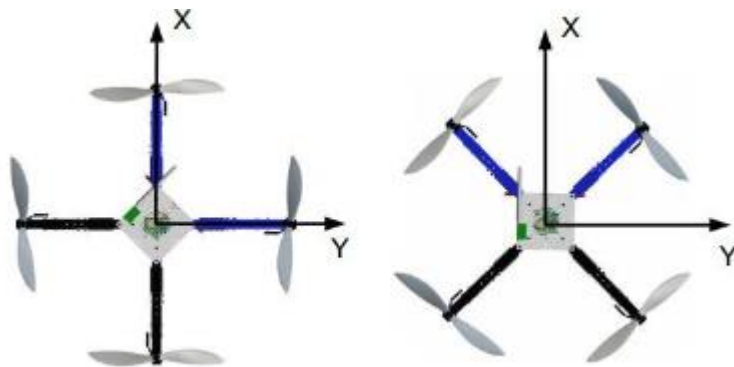


Figure 2.5: Plus and cross type configuration of quadrotor (Zhang, Li, Wang, & Lu, 2014)

The quadrotor is a 6-DOF object, so it uses 6 variables to represent its position in space $(X, Y, Z, \phi, \theta, \varphi)$. X , Y and Z represent the distance of the quadrotors center of mass from the earths fixed inertial system along the X , Y and Z axes, respectively. ϕ , θ and φ are the three Euler angles that represent the direction of the quadrotor. ϕ is called the roll angle, it is the angle concerning the X axis, θ is called the pitch angle concerning the Y axis, and φ is the yaw angle for the Z axis. Figure 2.6 clearly explains the Euler angles. The roll and pitch angles are usually called the quadrotor attitude, and the yaw angle is called the quadrotor heading.

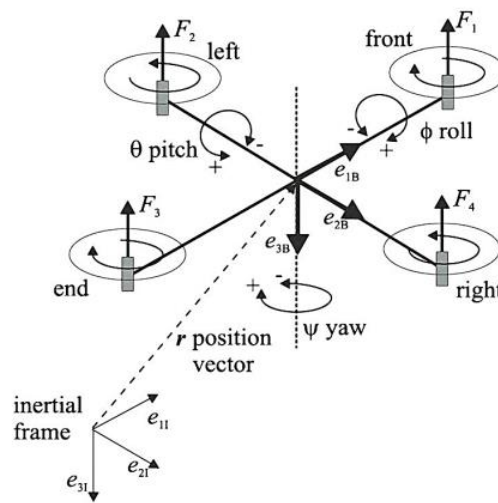


Figure 2.6: Mechanical structure and configuration of the quadrotor with body and inertial frame (L3)

All four propellers' speeds are simultaneously increased to create vertical upward movement, while they are simultaneously decreased to create vertical downward movement. The second and fourth propellers' speeds must be modified to produce a roll revolution along the x-axis, and the first and third propellers' speeds must be changed to produce a pitch rotation along the y-axis. One issue with the quadcopter design is that each pair of propellers' reverse torque must be different in order to achieve yaw rotation.

2.3 Controller history

A control system uses control loops to manage, command, direct, or regulate the behavior of other systems or devices. Automatic control, particularly the use of feedback, has played a significant role in the development of automation. Its predecessors included water clocks, level control, and pneumatic/hydraulic systems. Beginning in the 17th century, systems were

created for mechanically managing mills, steam engines, and temperature. During the 19th century, the tendency for instability of feedback systems became more and more apparent. An independent stability criterion was created around the turn of the century by Routh in England and Hurwitz in Switzerland. With the advent of flight, the issue reached a completely new (literal) level. The essence of three-term control was made clear by Minorsky's theoretical analysis of ship control in the 1920s, which was also applied to process applications by the 1930s. Cybernetics ideas and the coherent body of theory known as classical control emerged in the US, the UK, and other nations during and immediately after WWII. It was inspired by the demand for high-performance gun control systems and built on the servo and communications technical developments of the 1930s. Cold War demands for missile control systems swiftly led to the emergence of state-space or contemporary control techniques in both the East and the West. This method of information dissemination was gradual. The early post-war period was marked by significant concerns about automation, even while the development of the digital computer opened up new possibilities for automatic control.

2.4 Related works

Several researchers have been working on the recent UAV research report.

In (Bouadi, Aoudjif, & Guenifi, 2015) a direct adaptive sliding mode flight control for quadrotor attitude steadying and altitude trajectory tracking was developed where minimal phase and observability requirements were confirmed. A sliding mode adaptive flight control algorithm was created with ideal navigational conditions after the attainment of quadrotor flight dynamics under particular presumptions. The study also considered centered white gaussian noise and various parameter uncertainties with respect to the quadrotor mass and inertia matrix, respectively. Lyapunov theory was utilized to guarantee the overall stability of the synthesized controller with the adaptation mechanism. Finally, numerical simulation tests that demonstrated the efficacy and reliability of the suggested adaptive control algorithm are given, which showed a good convergence and tracking ability.

In (Martín Alay, 2013), it was shown that it is possible to linearize the dynamics around an operating point, typically selected to be the hover, in order to control the quadrotor UAV using linear control techniques. In that instance, several unmodeled dynamics were disregarded and excluded from the system during linearization, which negatively impacted controller performance. To solve the above-mentioned problem the study utilized nonlinear

control techniques which considered a more general form of the vehicle's dynamics across all flight zones, a larger flight envelope and excellent performance was achieved. Backstepping (Madani & Benallegue, 2006), sliding mode controller (Razmi & Afshinfar, 2019), and feedback linearization (Oriolo, De Luca, & Vendittelli, 2002) have all been shown to be efficient quadrotor control techniques.

In (Chingozha & Nyandoro, 2014), SMC formulation with adaptive components was used to create an adaptive sliding backstepping control (ASBSC) law for quadcopter attitude control. A priori knowledge of the upper boundaries on the uncertainty was avoided by the control law. The key accomplishment of this technique was its ability to handle extended matching; nevertheless, it had the drawback of being over parameterized, i.e., requiring several estimates of the same parameter. The over parameterization was eliminated by the use of tuning functions. A potent method for ensuring stable system performance, SMC has the downside that it necessitates awareness of the uncertainties beforehand. The sliding BS methodology developed by the research assured that the tracking error advances along the sliding hyperplane by combining the ABS technique with tuning functions and SMC. The ASBSC tracked constant and time-varying signals almost precisely, according to simulations of the device shown.

In (Bouadi et al., 2015), an ASM flight controller was designed for tracking heading and altitude trajectories, stabilizing quadrotor bank and pitch angles, checking observability and minimum phase requirements. A white gaussian noise with a center and some parameter uncertainties were considered in relation to the quadrotor mass and inertia matrix, respectively. The effectiveness of the adaptive control laws was evaluated by numerical simulations, showing good convergence results. Lyapunov theory was developed in order to ensure the overall stability of the synthesized controller with the adaptation mechanism. Indeed, parameters estimation in the presence of external disturbance needs to be improved from accuracy point of view to demonstrate the efficacy and reliability of the suggested adaptive control algorithm.

In (Nadda & Swarup, 2018) An adaptive scheme for SMC was developed and applied for the rotational/ altitude control of a quadrotor. The application of SMC provided robustness against parametric uncertainties, but it required knowledge of the upper bounds of uncertainties. An adaptation strategy was used to implement SMC, which did not require the upper bound of the uncertainties. The adaptive control law was derived on the basis of Lyapunov stability theory, which guaranteed the tracking performance. The controller

improved the performance of the quadrotor and also provided robustness against the parameter variations and external disturbances. The stabilization and robustness of the control method was demonstrated and compared with existing classical SMC. The control law had a flexibility to adjust the adaptation speed. It was demonstrated that the performance of quadrotor altitude tracking and convergence were considerably improved while maintaining stability, even in presence of external disturbances and parameter uncertainties. Simulation results showed that the controller had effectively stabilized the quadrotor without knowledge of the upper bound of the system uncertainty. Also, the altitude tracking performance of the proposed method was better than that of the existing control techniques.

In (T. Huang, Huang, Wang, & Shah, 2019) the asymptotic convergence of the tracking error in altitude and attitude was studied using a robust ASMC system designed for a quadrotor UAV. To estimate the unknowable SMC parameters, adaptive laws were developed. Lyapunov analysis was used to carry out a thorough demonstration of the suggested control algorithm's stability. Different from the most parameter-estimation-based controllers that always assume the availability of partial parameters of the UAV and estimate the remaining, the proposed controller showed stronger applicability for different types of quadrotor UAV. Numerical simulation, comparison with other controllers like LQR and real experiment were carried out to verify the effectiveness of the proposed controller. The tracking convergence of the ASMC was verified by rigorous Lyapunov-based analysis and the robustness of the proposed controller was guaranteed by the novel learning mechanism that takes a parallel structure.

In (S. Huang, Huang, Cai, & Cui, 2021), the trajectory tracking control issue of a quadrotor UAV based on actuator malfunction and outside disturbance was held using ABSMC approach. By employing differential iteration to effectively suppress the chattering effect of SMC, the method built the switching gain of ASMC throughout the BS design process. The system guaranteed to converge in a finite amount of time using the specified controller, and the time was demonstrated. In order to assure system convergence, the BS approach and the sliding mode method were merged. The ABSMC method was used because the traditional SMC method was insensitive to nonlinear problems such as disturbance and failure in the flight of the quadrotor UAV system, and because the setting of its switching gain can cause enormous chattering of input. Since the upper bound of the disturbance is unknown, the switching gains adaptive approach presented in the study updated the switching gain adaptively to ensure the robustness of SMC as well as the accuracy of the control tracking.

Prior to designing the controllers using the ABSMC method, a dynamic model of the quadrotor UAV with an actuator defect and external disturbance was provided. The comparative experiments between the ABSMC method and the SMC method demonstrated that the ABSMC approach not only performed a flawless control effect but also successfully suppressed the chattering issue for the SMC method.

In (Nguyen et al., 2021), for the quadcopter's attitude and altitude system in the presence of outside disturbances, a neural network-based SMC was provided. The gains of the sliding surface employed in the control strategy were modified using the backpropagation rule. The neural network, whose weight was updated by the backpropagation algorithm, was used to adjust the gain of a sliding surface. To manage external disturbances, the control system integrated a disturbance observer (DO). Additionally, the control system had the ability to predict the size of an unidentified boundary disturbance. The stability of the control mechanism was demonstrated using Lyapunov theory. The efficiency of the recommended method was shown by contrasting it with the adaptive SMC approach. The results demonstrated that the strategy outperformed only adaptive sliding mode control in terms of tracking performance and disturbance rejection capability. Yet, the quadcopter model's persistent disruption was the only factor considered by the study.

In (B. Zheng, Wu, Li, & Chen, 2022), Using the delta operator framework, a novel adaptive sliding-mode control technique was done for the attitude control of quadrotor UAVs. The primary contribution of the study was the discretization of the quadrotor UAV's attitude system using the delta operator technique. The ideas for a linear sliding surface and an adaptive sliding mode that reached a control law were then displayed in the delta domain. The asymptotic stability of the quadrotor UAVs during sliding motion was guaranteed by the asymptotic structure of the linear sliding surface, which was based on the linear matrix inequality technique. The adaptive sliding mode controller was created utilizing radial basis function (RBF) neural network to estimate the external disturbance and ensure that all quadrotor UAV attitudes were driven to the sliding surface, enabling attitude management. Finally, evaluations of the simulation outcomes showed how efficient and better the attitude control technique was.

Table 2.1: Summary of literature review

Title	Author and year	Methodology	Gap
Adaptive flight control for quadrotor UAV in the presence of external disturbances	Bouadi, H., Aoudjif, A., & Guenifi, M. (2015)	By designing quadrotor system dynamics first, an adaptive flight controller is designed to apply numerical simulations while considering white gaussian external disturbance and some parameter uncertainties	Parameter estimation is considered as arbitrary within the model
On adaptive sliding mode control for improved quadrotor tracking	Nadda, S., & Swarup, A. (2018)	ASMC is designed for quadrotor UAV with Lyapunov stability analysis while considering white gaussian external disturbance	The parameter estimation and internal parameter uncertainty is not considered
Robust tracking control of a quadrotor UAV based on adaptive sliding mode control	Huang, T., Huang, D., Wang, Z., & Shah, A. (2019)	ASMC is designed for quadrotor UAV while considering internal parameter uncertainties with parallel structure learning algorithm	Does not briefly considers the external disturbance
Adaptive sliding mode control for attitude and altitude system of a quadcopter UAV via neural network	Nguyen, N. P., Mung, N. X., Thanh, H. L. N. N., Huynh, T. T., Lam, N. T., & Hong, S. K. (2021)	A novel adaptive method via neural network is designed with SMC for quadrotor altitude and attitude dynamics through online disturbance observer estimation.	Consideration of gain reduction is very poor
Adaptive sliding mode attitude control of quadrotor UAV's based on the delta operator framework	Zheng, B., Wu, Y., Li, H., & Chen, Z. (2022)	Using the delta operator framework, a novel ASMC technique was designed for the attitude control of quadrotor	Reliability problem, overlapping issues and approximation of the parameters are not addressed clearly

CHAPTER THREE

3 MATERIALS AND METHODS

3.1 Chapter overview

This chapter deals with the methods used to complete this research work, materials, the derivation of dynamic mathematical modeling of the quadrotor system based on Euler-Newton formalism and design of proposed ASMC controller.

3.2 Materials

In this research work, software such as MATLAB/2017a, Microsoft Office 2019, Draw.io and Math Type 6 equation is used. MATLAB is the computing software that consists of technical tool boxes and Simulink. Microsoft Office is used for editing the thesis documentation. Draw.io is drawing software which is used to draw different kinds of diagrams and flow charts. Math Type 6 version is used for writing mathematical formulas and equation in Microsoft office word and PowerPoint presentation tools.

3.3 Methods

The method used for this research study to accomplish the required task is shown in Figure 3.1. The first step in achieving the goals of this thesis was to do a study of the literature, which involved looking at previous attempts to model and regulate the altitude, attitude, and heading of quadrotor or quadcopter. The collected data is analyzed mathematically and graphically for developing system modeling utilizing both the Newton-Euler formalization and the rotor dynamic effects. The dynamics modeling of quadrotor is analyzed. Then the ASMC controller is designed with a normal Proportional Derivative (PD) sliding surface. The system is eventually interfaced to simulate on MATLAB/Simulink by constructing an adaptive law-based sliding mode controller with the tanh function reaching law. Finally, the proposed modeling and simulation result of the system is analyzed and discussed

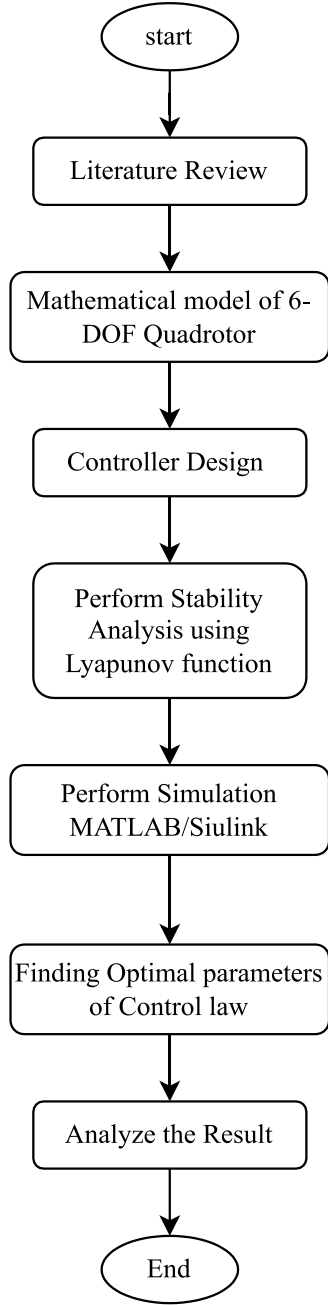


Figure 3.1: Flowchart of the proposed methodology

3.4 Workflow of the proposed ASMC controller for quadrotor

The full control scheme of the system is proposed as a multi-loop consisting of an inner loop that controls attitude and heading and an outer loop to control the position of the quadrotor system. The objective of the research study is to design a robust ASMC for position, attitude and heading dynamics, that guarantees a fast response and rapid adaptation with parameter uncertainty and external disturbance. The block diagram of the proposed ASMC for the quadrotor system is shown in Figure 3.2. The desired reference altitude is given to the four

states of the quadrotor and then to the ASMC. And finally, outputs of the controller are given to the quadrotor system and the actual values of the states are obtained.

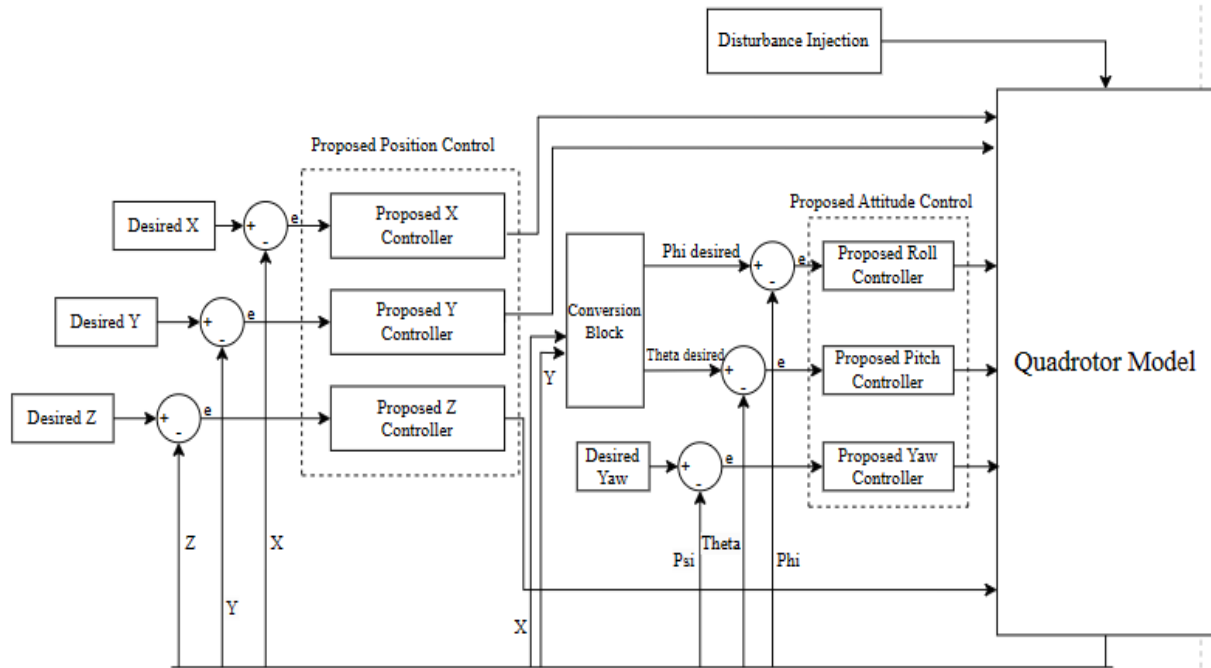


Figure 3.2: Overall application of quadrotor control strategy

3.5 Mathematical modeling and control system design

This section discusses the quadrotor's geometrical construction and reference frame while obtaining its nonlinear dynamic model.

3.6 Considerations for mathematical modeling of quadrotor UAV

3.6.1 Six degrees of freedom (6-DOF)

To express the motion of the quadcopter in terms of equations, it is important to have the idea of 6-DOF. The 6-DOF concept gives the position and the orientation of the quadcopter in 3-Dimensions (3D). The quadcopter dynamic equations represent the change in position and orientation overtime. To achieve the control of the quadcopter system in space, six coordinates are required three for describing the position and three for describing the orientation in space (Usman, 2020).

3.6.2 Quadrotor system assumptions

The following assumptions are considered for the derivation of kinematics and dynamics models of the quadcopter systems (Tüzel, 2019).

- ✚ The quadrotor structure is rigid and symmetrical.
- ✚ The quadrotor's center of gravity coincides with the origin of the body-fixed frame.

- ✚ The propellers are rigid.
- ✚ Thrust and drag forces are proportional to the square of the propeller's speed.

3.6.3 Control inputs

There are four identical propeller and DC motor pairs that can generate lift forces to generate the motion of the quadrotor. Two opposite side motors rotate clockwise, while the other pair turns counter-clockwise. There are four rotors of the quadcopter that means four control inputs can be defined.

- ✚ U1: throttle input, which is the total force produced by the four propellers.
- ✚ U2: roll input, which is the moment difference around x-axis due to left-side and right-side propellers.
- ✚ U3: pitch input, which the moment difference around y-axis due to front and rear propellers.
- ✚ U4: yaw input, which is the net torque exerted on the system around z-axis by the four propellers.

3.6.4 Reference frames

A reference frame is a collection of spatial points where the separation between any two locations is always constant (Frazzoli, 2011). Reference frames are employed here to investigate the motion of a reference frame with regard to an inertial reference frame after first identifying a reference frame with a rigid body. Hence, the terms rigid bodies and reference frames are used synonymously. So, a two-coordinate system is necessary. There are two frame systems: the earth frame system, which is fixed to the earth and sometimes referred to as an inertial coordinate system, and the body frame system, which is coupled to the quadrotor at its center of gravity.

In this research work, the body and inertial frames of the investigated quadrotor rotorcraft are described. The division of the model into translational and rotational coordinates occurs naturally (Frazzoli, 2011). Figure 3.3 shows quadrotor configuration and how it works.

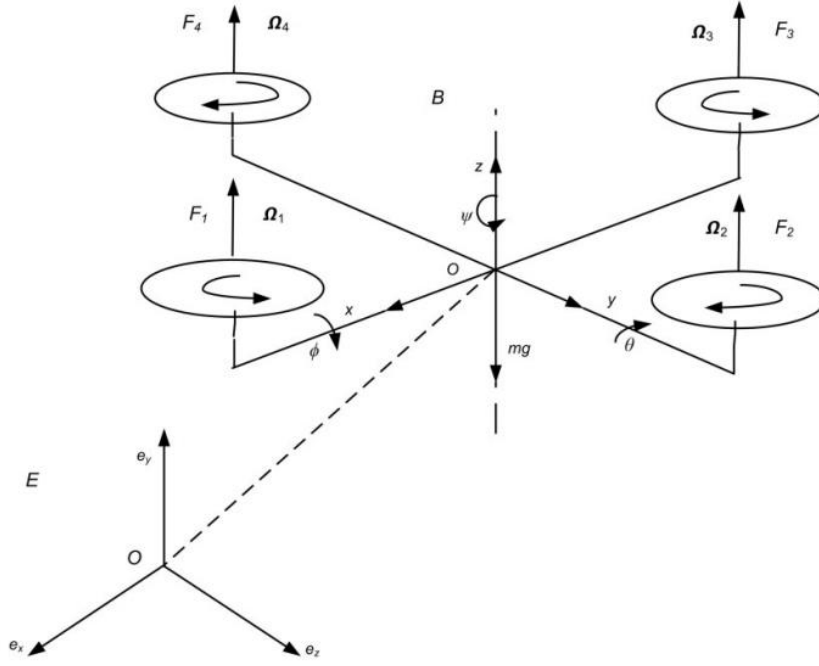


Figure 3.3: Quadrotor UAV representation (Nadda & Swarup, 2018)

3.6.5 Rotational matrix

State variables for location are in the global frame, whereas those for velocity are in the body frame. As a result, defining a rotation matrix is necessary to convert variables between coordinate systems. We can apply the rotation matrix to discover the relationship between the angular rates and the time derivatives of the Euler angles because the angular rates are defined in the body frame and the Euler angles are defined in intermediate coordinate frames. The angular velocities do not equal the time derivative of the Euler angles since they are vectors pointing along each rotational axis.

3.7 Dynamic model of quadrotor

To precisely describe the flight condition of a quadrotor, we can define flat earth frame E and rigid body frame B according to the right-hand rule as shown in Figure 3.3. The aerodynamic forces and moments of a propeller are affected by a number of factors, including the angle of attack of the aero foil and the inflow ratio. The aerodynamic forces F_i and moments T_i of four propellers were assumed to be proportional to the square of the angular velocity w_i .

$$F_i = k_f w_i^2 \quad (3.1)$$

$$T_i = k_t w_i^2 \quad (3.2)$$

Where $i=1, 2, 3, 4, \dots$, K_t is aerodynamic force, K_f is anti-torque coefficient

Since the forces and moments are derived by modifying the speed of each rotor, then the lifting force and three moments can be given:

Lifting force (the sum of four rotor forces)

$$F = F_1 + F_2 + F_3 + F_4 = k_f (w_1^2 + w_2^2 + w_3^2 + w_4^2) \quad (3.3)$$

Roll torque (right hand rule)

$$T_p = L(F_1 - F_2 + F_3 - F_4) = Lk_f (w_1^2 - w_2^2 + w_3^2 - w_4^2) \quad (3.4)$$

Pitch torque (right hand rules)

$$T_q = L(-F_1 + F_2 - F_3 + F_4) = Lk_f (-w_1^2 + w_2^2 - w_3^2 + w_4^2) \quad (3.5)$$

Yaw torque (right hand rule)

$$T_r = L(F_1 + F_2 - F_3 - F_4) = Lk_t (w_1^2 + w_2^2 - w_3^2 - w_4^2) \quad (3.6)$$

where L is the distance from motor position to the center of quadrotor mass.

$$\begin{bmatrix} T_p \\ T_q \\ T_r \\ F_z \end{bmatrix} = \begin{bmatrix} Lk_f & -Lk_f & Lk_f & -Lk_f \\ -Lk_f & Lk_f & -Lk_f & Lk_f \\ k_t & k_t & -k_t & -k_t \\ k_f & k_f & k_f & k_f \end{bmatrix} \begin{bmatrix} w_1^2 \\ w_2^2 \\ w_3^2 \\ w_4^2 \end{bmatrix} \quad (3.7)$$

The Euler angles which denote the rotation of quadrotor in the flat earth frame coordinate E is the variables ϕ, θ and φ which denotes roll, pitch and yaw respectively.

3.7.1 6-DOF model of rigid body

The dynamics of a rigid body under external forces applied to the center of mass expressed in the body fixed frame are in Newton-Euler formalism.

$$m\dot{v}_b + \Omega_b \times mv_b = F_b + F_g + F_e \quad (3.8)$$

$$I\dot{\Omega}_b + \Omega_b \times I\Omega_b = T_b \quad (3.9)$$

Where the $v_b = [u \ v \ w]^T \in R^3$ is the velocity in the body frame, $\Omega_b = [p \ q \ r]^T$ is the angular velocity in the body frame, $F_b, F_g, F_e \in R^3$ denotes the external propeller forces acted on the rotors, the weight forces of mass center, the elastic force of landing gear only on the ground respectively, $T_b = [T_p \ T_q \ T_r]^T \in R^3$ is external torques in the body frame, $I^{3 \times 3} \in R^3$ are the inertia matrix of quadrotor, m is the mass of quadrotor. The 3×3 symmetrical inertia

$$I = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix} \quad (3.10)$$

And the cross-product operator $\times$ is defined as:

$$\Omega_b \times = \begin{bmatrix} 0 & -r & q \\ r & 0 & -p \\ -q & p & 0 \end{bmatrix} \quad (3.11)$$

The external propeller forces F_b acted on the rotors is defined as:

$$F_b = [0 \quad 0 \quad F_z]^T \quad (3.12)$$

The weight forces of mass center are as follows:

$$F_g = [mg \sin \theta \quad -mg \cos \theta \sin \varphi \quad -mg \cos \theta \cos \varphi]^T \quad (3.13)$$

The elastic force of landing gear on the ground is modeling the characteristics of taking off situation for quadrotor, so it disappears while flying or leaving the ground. Its modeling is as follows:

$$F_e = [0 \quad 0 \quad k_l(z_l - z)\delta(z_l - z)]^T \quad (3.14)$$

Where k_l and z_l is elastic coefficient and elastic deformation on ground, and the function $\delta(z_l - z)$ denotes:

$$\delta(z_l - z) = \begin{cases} 1 & z_l - z > 0 \\ 0 & z_l - z \leq 0 \end{cases} \quad (3.15)$$

Then equation (3.16) can be expanded as follows:

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix} = \begin{bmatrix} -rv + qw \\ ru - pw \\ -qu + pv \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ F_z / m \end{bmatrix} + \begin{bmatrix} g \sin \theta \\ -g \cos \theta \sin \varphi \\ -g \cos \theta \cos \varphi \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ k_l(z_l - z)\delta(z_l - z) / m \end{bmatrix} \quad (3.17)$$

$$\begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = - \begin{bmatrix} q_r(I_{zz} - I_{yy}) / I_{xx} \\ p_r(I_{xx} - I_{zz}) / I_{yy} \\ p_q(I_{yy} - I_{xx}) / I_{zz} \end{bmatrix} + \begin{bmatrix} T_p / I_{xx} \\ T_q / I_{yy} \\ T_r / I_{zz} \end{bmatrix} \quad (3.18)$$

3.7.2 Navigation equations

To obtain the translational and rotational motions from the body frame to the flat earth coordinate system, the well-known navigation equations are described as follows:

$$\dot{E}_e = S\Omega_b \quad (3.19)$$

$$\dot{N}_e = RV_b \quad (3.20)$$

Where the vectors $N_e = [x \quad y \quad z]^T$ Stands for the position of quadrotor in flat earth coordinate, $E_e = [\phi \quad \theta \quad \varphi]^T$ Stands for the attitude angles of body frame in the flat earth

coordinate. The matrix R and S is translational and rotational transform matrix. The equation (3.21) can be extended as follows:

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\varphi} \end{bmatrix} = \begin{bmatrix} 1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\ 0 & \cos \phi & -\sin \phi \\ 0 & \sin \phi / \cos \theta & \cos \phi / \cos \theta \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (3.22)$$

$$\begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \cos \theta \cos \varphi & -\cos \phi \sin \varphi + \sin \phi \sin \theta \cos \varphi & \sin \phi \sin \varphi + \cos \phi \sin \theta \cos \varphi \\ \cos \theta \sin \varphi & \cos \phi \cos \varphi + \sin \phi \sin \theta \sin \varphi & -\sin \phi \cos \varphi + \cos \phi \sin \theta \sin \varphi \\ -\sin \theta & \sin \phi \cos \theta & \cos \phi \cos \theta \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix} \quad (3.23)$$

The motor dynamics:

$$\dot{w}_i = -k_1 w_i - k_2 w_i^2 + k_3 w_i \quad (3.24)$$

Where k_1, k_2 and k_3 are constants in dynamic model.

$$\phi, \theta, \varphi \in R \text{ and } (-\pi/2 < \phi < \pi/2, -\pi/2 < \theta < \pi/2, -\pi < \varphi < \pi) \quad (3.25)$$

The nonlinear dynamic equations of the quadcopter in presence of disturbances and uncertainties can be expressed as.

$$\ddot{\phi} = \frac{I_{YY} - I_{ZZ}}{I_{XX}} \dot{\theta} \dot{\varphi} + \dot{\theta} \frac{I_m}{I_{XX}} \Omega_m + \frac{U_2}{I_{XX}} + \delta_\phi(t) \quad (3.26)$$

$$\ddot{\theta} = \frac{I_{ZZ} - I_{XX}}{I_{YY}} \dot{\phi} \dot{\varphi} - \dot{\phi} \frac{I_m}{I_{YY}} \Omega_m + \frac{U_3}{I_{YY}} + \delta_\theta(t) \quad (3.27)$$

$$\ddot{\varphi} = \frac{I_{XX} - I_{YY}}{I_{ZZ}} \dot{\phi} \dot{\theta} + \frac{U_4}{I_{ZZ}} + \delta_\varphi(t) \quad (3.28)$$

$$\ddot{z} = g - \frac{(\cos \phi \cos \theta) U_1}{m} + \delta_z(t) \quad (3.29)$$

$$\ddot{x} = U_1 (\cos \phi \sin \theta \cos \varphi + \sin \phi \sin \varphi) / m \quad (3.30)$$

$$\ddot{y} = U_1 (\cos \phi \sin \theta \sin \varphi - \sin \phi \cos \varphi) / m \quad (3.31)$$

where control inputs U_i ($i=1,2,3,4$) is defined as

$$\begin{bmatrix} U_1 \\ U_2 \\ U_3 \\ U_4 \\ \Omega_m \end{bmatrix} = \begin{bmatrix} b(w_1^2 + w_2^2 + w_3^2 + w_4^2) \\ bL(w_4^2 - w_2^2) \\ bL(w_1^2 - w_3^2) \\ d(-w_1^2 + w_2^2 - w_3^2 + w_4^2) \\ -w_1 + w_2 - w_3 + w_4 \end{bmatrix} \quad (3.32)$$

Where w_i is the angular speed of i th motor ($i=1, 2, 3, 4$); $g=9.81\text{m/s}^2$ is the gravitational acceleration; $\delta_\phi, \delta_\theta, \delta_\varphi, \delta_z$ are the disturbances of the roll, pitch, yaw and altitude motion respectively.

The equation (3.24) to (3.29) can be represented in state space form as:

$$\dot{x} = f(x, U) + \delta(x, U) \quad (3.33)$$

Where x is the state vector; U is the control input vector; $\delta(x, U)$ is the uncertainty vector, denoted for quadrotor as:

$$x = [x_1, x_2, x_3, \dots, x_{12}]^T = [\phi, \dot{\phi}, \theta, \dot{\theta}, \varphi, \dot{\varphi}, z, \dot{z}, x, \dot{x}, y, \dot{y}]^T \quad (3.34)$$

$$U = [U_1, U_2, U_3, U_4] \quad (3.35)$$

Where $x_1 = \phi$, $x_2 = \dot{x}_1 = \dot{\phi}$, $x_3 = \theta$, $x_4 = \dot{x}_3 = \dot{\theta}$, $x_5 = \varphi$, $x_6 = \dot{x}_5 = \dot{\varphi}$, $x_7 = z$, $x_8 = \dot{x}_7 = \dot{z}$,

$x_9 = x$, $x_{10} = \dot{x}_9 = \dot{x}$, $x_{11} = y$, $x_{12} = \dot{x}_{11} = \dot{y}$

From the equations the dynamic equation of the quadcopter is formulated as:

$$f(x, U) = \begin{bmatrix} x_2 \\ x_4 x_6 a_1 + x_4 a_2 \Omega_m + b_1 U_2 \\ Y_4 \\ x_2 x_6 a_3 + x_2 a_4 \Omega_m + b_2 U_3 \\ x_6 \\ x_2 x_4 a_5 + b_3 U_4 \\ x_8 \\ -g + \frac{1}{m} (\cos x_1 \cos x_3) U_1 \\ x_{10} \\ \frac{1}{m} T_x U_1 \\ x_{12} \\ \frac{1}{m} T_y U_1 \end{bmatrix} \quad (3.36)$$

$$\delta(x, U) = [0, \delta_\phi, 0, \delta_\theta, 0, \delta_\varphi, 0, \delta_z, 0, 0, 0, 0]^T \quad (3.37)$$

where $a_1 = (I_{YY} - I_{ZZ}) / I_{XX}$, $a_2 = I_m / I_{XX}$,

$a_3 = (I_{ZZ} - I_{XX}) / I_{YY}$, $a_4 = I_m / I_{YY}$,

$a_5 = (I_{XX} - I_{YY}) / I_{ZZ}$, $b_1 = 1 / I_{XX}$, $b_2 = 1 / I_{ZZ}$,

$$T_x = \cos x_1 \sin x_3 \cos x_5 + \sin x_1 \sin x_5 ,$$

$$T_y = \cos x_1 \sin x_3 \sin x_5 - \sin x_1 \cos x_5$$

Remark 1: the disturbances are assumed to be bounded with

$$|\delta_\phi| \leq D_1, |\delta_\theta| \leq D_2, |\delta_\varphi| \leq D_3, |\delta_z| \leq D_4 \text{ where } D_i = \text{constant} > 0, i=1,2,3,4.$$

3.8 Open-loop model verification

The open loop simulation analysis includes a loop opening, which breaks the signal flow and removes the effects of the feedback loop to verify the validity of the dynamics to apply a controller on. It explains the consequence of input on the output state by using rotor speed for calculating input U1, U2, U3 and U4 or throttle input, roll input, pitch input and yaw input respectively.

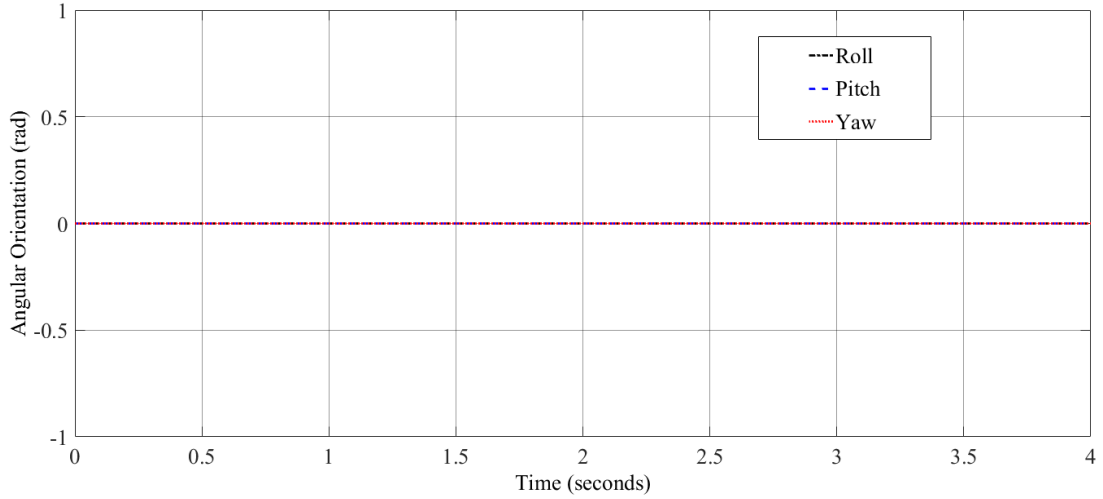


Figure 3.4: Response of nonlinear orientation to throttle input

$$\Omega_1 = \Omega_2 = \Omega_3 = \Omega_4 = 103 \text{ (rad/sec)}$$

The simulation result of Figure 3.4 shows that when all the four angular speeds of the quadrotor motors are the same, there will not be any variation in all the three dynamics (phi, theta, and psi) but only in altitude which is vertical or Z-dynamics since the quadrotor must take off from the ground to specified altitude to stabilize its orientation within the desired angle. This effect is occurred by controller U1.

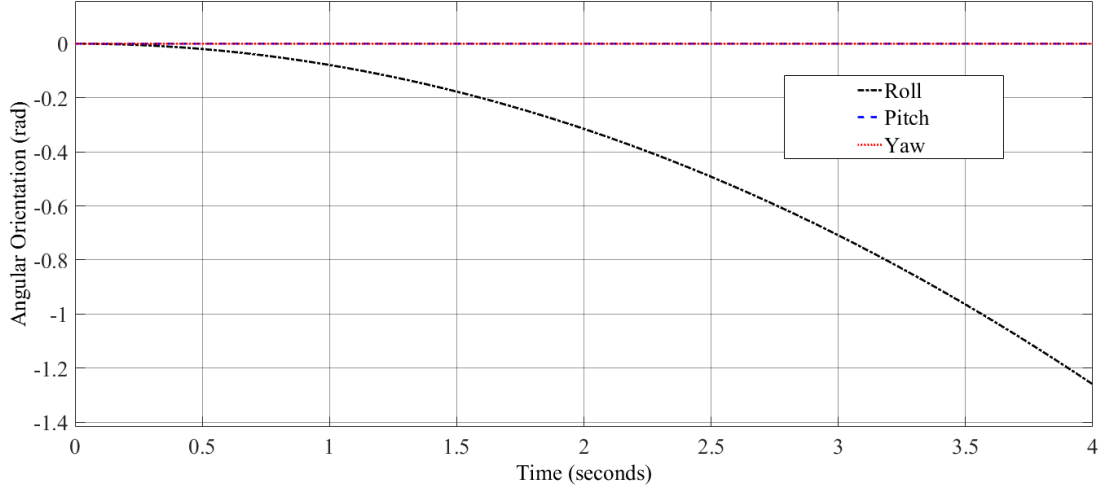


Figure 3.5: Response of nonlinear orientation to roll input at
 $\Omega_1 = \Omega_3 = 103, \Omega_2 = 102.9, \Omega_4 = 103.1$ (rad/sec)

Figure 3.5 illustrates, when two of the angular speeds of the quadrotor varies (w_2 and w_4) only the phi angle or roll movement is changed or affected since the other two rotor speeds are kept on the same value. This effect occurs due to U2 controller which is pendent on w_2 and w_4 .

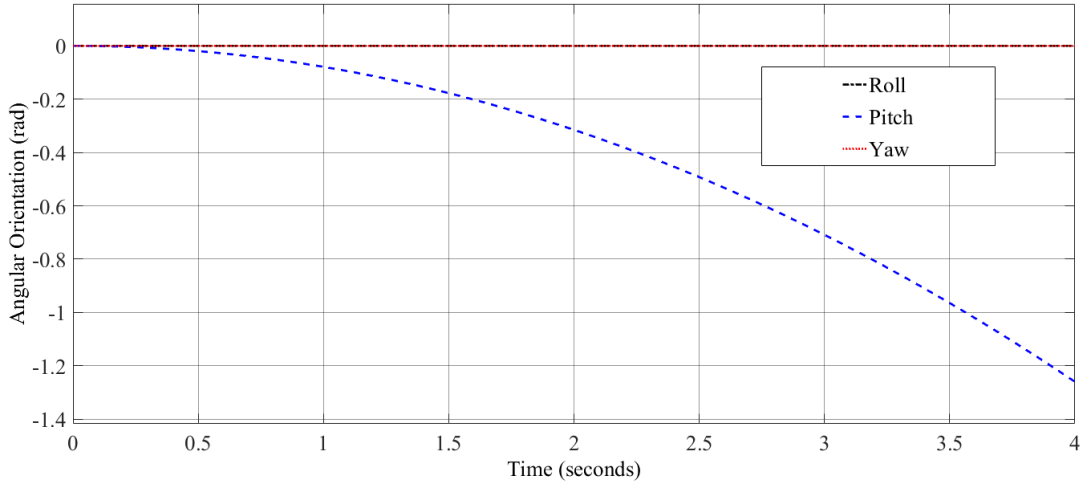


Figure 3.6: Response of nonlinear angular orientation to pitch input at
 $\Omega_2 = \Omega_4 = 103, \Omega_1 = 102.9, \Omega_3 = 103.1$ (rad/sec)

The simulation result of Figure 3.6 tells us when the angular speeds of w_1 and w_3 are varied and the other two (w_2 and w_4) are kept the same, theta orientation is affected while phi and psi are not for the matter of time which results in pitch angle variation. The input U3 is the one which affects the theta dynamics or pitch movement variation.

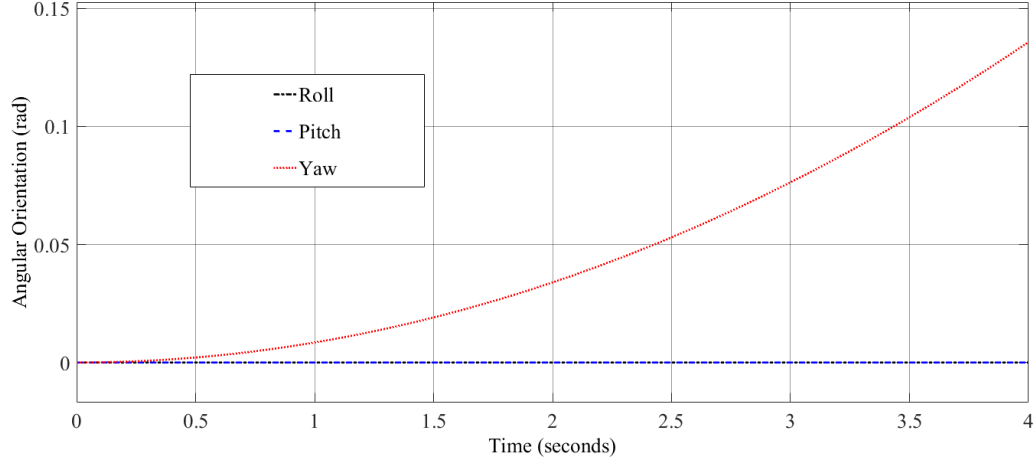


Figure 3.7: Angular orientation to yaw input at $\Omega_1 = \Omega_3 = 103$, $\Omega_2 = \Omega_4 = 104$ (rad/sec)

The simulation result of Figure 3.7 tells us when the angular speeds of w_1 and w_3 are kept the same and the other two (w_2 and w_4) are kept the same also, but in a different value, psi orientation is affected while phi and theta are not for the matter of time which results in yaw dynamics variation. U4 is the responsible input which affects the psi angle or yaw movement variation.

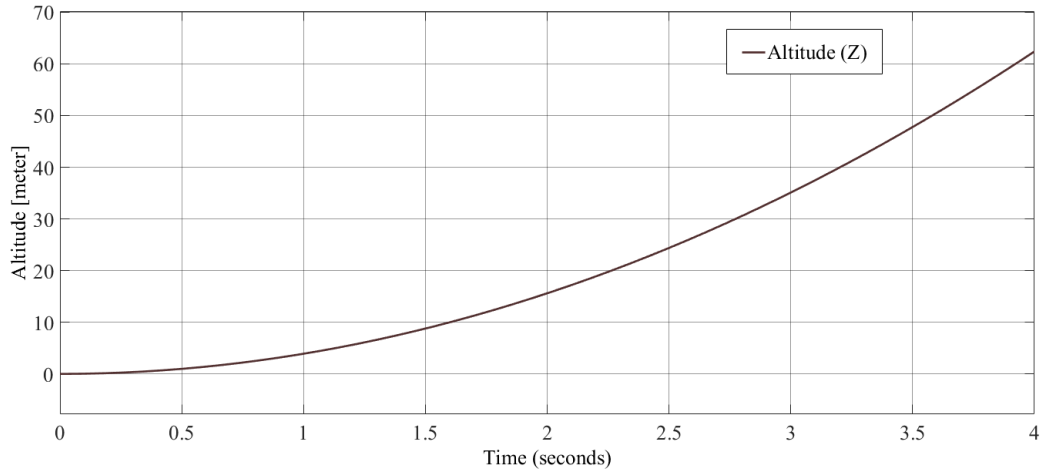


Figure 3.8: Response of nonlinear translational position to throttle input at

$$\Omega_1 = \Omega_2 = \Omega_3 = \Omega_4 = 103 \text{ (rad/sec)}$$

The open loop response of Figure 3.8 illustrates that, when all the angular speeds of the rotor are the same only input U1 exist and changes the altitude Z-dynamics which means the motion or the movement of the quadrotor system is along Z-axis only that is vertical.

CHAPTER FOUR

4 CONTROLLER DESIGN

4.1 Adaptive sliding mode control system design for quadrotor UAV

In this work, ASMC controller is designed for controlling the nonlinear model of quadrotor UAV. The objective of using ASMC controller is to control rotational or Euler dynamics (attitude and heading) and altitude of the quadrotor.

4.1.1 Sliding mode controller

For addressing abrupt and significant changes in system dynamics, SMC is an effective controller. As a technique for creating controllers for uncertain systems, it has becoming increasingly valuable. The system's state is forced to a predetermined sliding surface, also known as a switching surface, using the sliding mode's discontinuous feedback control rule. The system design must be capable of overcoming the associated uncertainties in order to fulfill the objective of SMC.

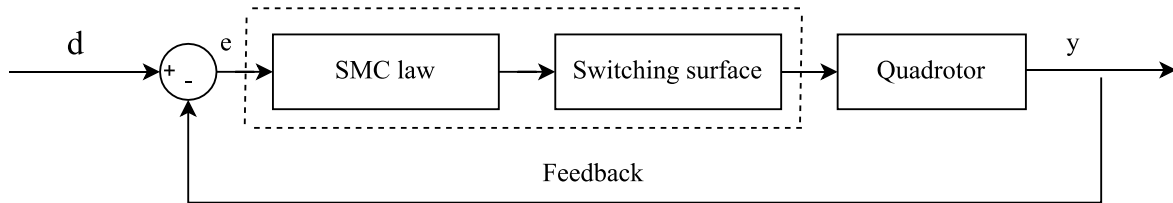


Figure 4.1: Block diagram of sliding mode controller for quadrotor

4.1.2 Adaptive controller

The Adaptive Control Approach's central tenet is the design of systems that exhibit the same dynamic characteristics under conditions of uncertainty based on the use of the most recent data. It entails changing the control law which a controller uses to deal for the fact that the parameters of the system being controlled are slowly changing over time or unclear. Even more, adaptive control entails enhancing dynamic qualities when the characteristics of a controlled plant or environment are changing.

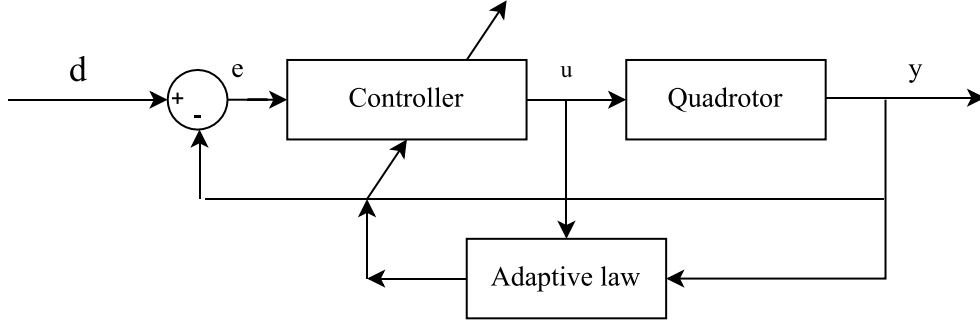


Figure 4.2: Adaptive control

4.1.3 Rotational control

Consider the fact that the control signal has two components

$$U = U_{eq} + U_{sw} \quad (4.1)$$

U_{eq} is the equivalent control equation without disturbances and U_{sw} is the switching control, guarantee the discontinuity of the control law across the sliding surface, giving farther control to manage the disturbance or uncertainties. The essential control action is given by equivalent control with uncertain internal disturbance of zero, that is, $g(x, U) = 0$, and switching control guarantees the discontinuity of the control law to make the sliding function converge to the surface of the sliding mode, giving farther control account for the presence of un-modeled dynamics and matched disturbances.

The traditional first-order sliding surface equation is (Eker, 2010)

$$s_i = \left(\lambda_i + \frac{d}{dt} \right)^{n-1} e_i \quad (i = 1, 2, \dots, 4) \quad (4.2)$$

Where n denotes the order of the uncontrolled system, λ_{ci} is a positive constant and $\lambda_i \in R^+$.

It depends on the tracking error e_i and the derivative of the tracking error.

From the dynamical model equations (3.26) to (3.31), the order of each degree of freedom, that is, $n = 2$, is two. Therefore, the sliding surface of (4.2) is expressed as

$$s_i = \lambda_i e_i + \dot{e}_i \quad (i = 1, 2, \dots, 4) \quad (4.3)$$

The tracking error will become closer to convergence to zero if the tracking error e_i is maintained on the sliding surface s_i . As a result, the polynomial $(\lambda_i e_i + \dot{e}_i)$ is strictly Hurwitz, meaning that its roots are located in the complex plane's left-hand half. This indicates that the system is globally asymptotically stable and the error is equal to zero for $t \rightarrow \infty$. The rotational subsystem, which contains control inputs U_2 , U_3 , and U_4 , and the

translational subsystem, which contains control input U_1 , are both derived from the dynamical model equations (3.26) to (3.31).

4.1.3.1 Roll controller

Let the required states for roll, pitch, yaw, and altitude be x_{1d} , x_{3d} , x_{5d} and x_{7d} respectively. The first time derivative of the sliding surface is taken into consideration to generate the control law for the roll subsystem as follows

$$\dot{s}_1 = \lambda_1 \dot{e}_1 + \ddot{e}_1 \quad (4.4)$$

Where $e_1 = x_{1d} - x_1$, $\dot{e}_1 = \dot{x}_{1d} - \dot{x}_2$, $\ddot{e}_1 = \ddot{x}_{1d} - \ddot{x}_2$ and the second time derivative of error $\ddot{e}_1$ can be written as:

$$\ddot{e}_1 = \ddot{x}_{1d} - x_4 x_6 a_1 - x_4 a_2 \Omega - b_1 U_2 + g_1 \quad (4.5)$$

Where, g_1 is the lumped uncertainty related to roll motion. Therefore, $\dot{s}_1$ is defined as:

$$\dot{s}_1 = \lambda_1 \dot{e}_1 + \ddot{x}_{1d} - x_4 x_6 a_1 - x_4 a_2 \Omega - b_1 U_2 + g_1 \quad (4.6)$$

Overall control is obtained when $\dot{s}_1 = 0$, which is a necessary condition for the tracking to remain on the sliding surface, and external uncertain disturbances are not taken in to account (U_{eq1}) for the roll angle is obtained from (4.1) when $\dot{s}_1 = 0$ as

$$U_{eq1} = U_2 = \frac{1}{b_1} (\ddot{x}_{1d} - x_4 x_6 a_1 - x_4 a_2 \Omega + \lambda_1 \dot{e}_1) \quad (4.7)$$

The switching function is introduced as:

$$U_{sw1} = \frac{K_{s1}}{b_1} \text{sign}(s_1) \quad (4.8)$$

Where, K_{s1} is a positive constant, $K_{s1} \in R^+$ and $K_{s1} > g_1$ and where the sign (signum function) (Slotine & Li, 1991) is defined as:

$$\text{sgn}(s_i) = \begin{cases} -1 & \text{if } s_i < 0 \\ 0 & \text{if } s_i = 0 \\ 1 & \text{if } s_i > 0 \end{cases} \quad i = 1, 2, \dots, 4 \quad (4.9)$$

On substituting equations (4.7) and (4.8) into equation (4.6), the expression is obtained as

$$\dot{s}_1 = -K_{s1}(\text{sign}(s_1)) - g_1 \quad (4.10)$$

The derived sliding mode control equations (4.7) and (4.8) do not manage the external disturbances and are constrained by the unknown upper bound on uncertainty. The switching mode control described above is ineffective due to these drawbacks. To adjust the controller,

gain adaptively without knowing the upper bound on the system uncertainty, a modified control rule is therefore proposed.

Adaptive law: Without being aware of the top bound on the quadrotor uncertainty, the suggested control rule adaptively adjusts the controller gain. The proposed adaptive control law is

$$U = U_{eq1} + U_{as1} \quad (4.11)$$

Where U_{eq1} is equivalent control and U_{as1} is the adaptive switching controls term. Corresponding to the uncertainties expressed in (4.10), the adaptive term U_{as} is expressed as

$$U_{as1} = -\frac{1}{b_1} \hat{\Gamma}_1 \text{sign}(s_1) \quad (4.12)$$

Where, $\hat{\Gamma}_1$ is an adjustable gain constant. Consider that there exists a positive number $\hat{\Gamma}_{d1}$ such that $U_{as1} = -\frac{1}{b_1} \hat{\Gamma}_1 \text{sign}(s_1)$ is a terminal solution for U_{as} , where Γ_1 must satisfy $\Gamma_1 > |g_1|$.

Let the adaptive law be defined as

$$\dot{\hat{\Gamma}}_1 = \frac{1}{\beta_1} |s_1| \quad (4.13)$$

Where β_1 is the adaptation gain and $\beta_1 > 0$. The adaptation rate of $\hat{\Gamma}_1$ can be tuned by β_1 . The result of the adaptation gain β_1 is taken properly to neglect high control activity in the reaching mode.

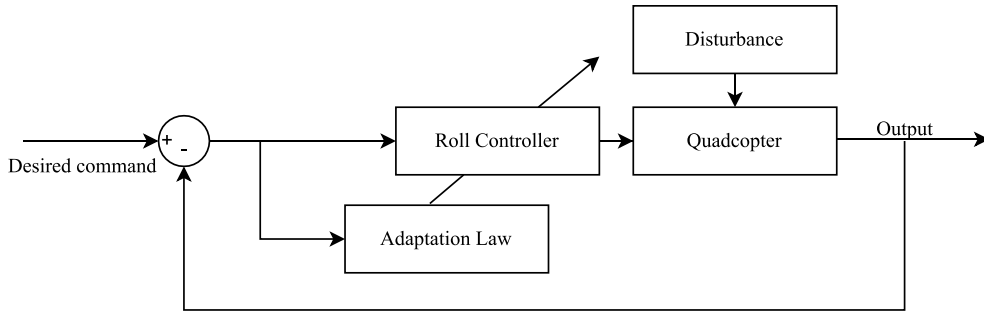


Figure 4.3: Roll control structure of quadcopter

4.1.3.2 Pitch and yaw controller

The adaptive sliding mode control for the pitch and yaw subsystems have been designed to obtain U_3 and U_4 , following the steps above identical to those for the roll subsystem. The equivalent control input U_{eq2} or U_3 is given as

$$U_{eq2} = U_3 = \frac{1}{b_2} (\ddot{x}_{3d} - x_2 x_6 a_3 - x_2 a_4 \Omega + \lambda_2 \dot{e}_2) \quad (4.14)$$

The adaptive law is given as

$$U_{as2} = \frac{1}{b_2} \hat{\Gamma}_2 \text{sign}(s_2) \quad (4.15)$$

Where $\hat{\Gamma}_2$ is also an adjustable gain constant for the pitch angle. Repeating the above procedure for the roll angle, consider that there is a positive number Γ_2 such that

$U_{as2} = \frac{1}{b_2} \hat{\Gamma}_2 \text{sign}(s_2)$ is a final solution for U_{as2} , where Γ_2 must fulfill $\Gamma_2 > |g_2|$. Then the

adaptive law is expressed as the following:

$$\hat{\Gamma}_2 = \frac{1}{\beta_2} |s_2| \quad (4.16)$$

Where β_2 is the adaptation gain and $\beta_2 > 0$. The adaptation rate of $\hat{\Gamma}_2$ can be tuned by β_2 .

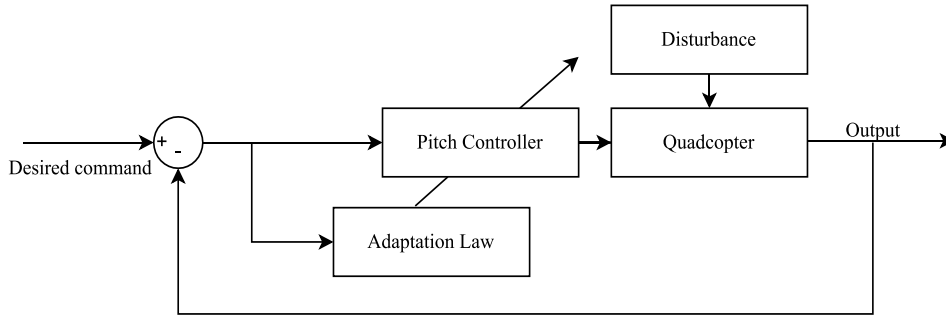


Figure 4.4: Pitch control structure of quadcopter

Identically, the control input for yaw motion is:

$$U_{eq3} = U_4 = \frac{1}{b_3} (\ddot{x}_{5d} - x_4 x_6 a_5 + \lambda_3 \dot{e}_3) \quad (4.17)$$

Repeatedly, the adaptive term is taken as:

$$U_{as3} = \frac{1}{b_3} \hat{\Gamma}_3 \text{sign}(s_3) \quad (4.18)$$

Where $\hat{\Gamma}_3$ is again an adjustable gain constant for the yaw angle. Repeating the above procedure for the yaw angle, consider that there is a positive number Γ_3 such that

$U_{as3} = \frac{1}{b_3} \hat{\Gamma}_3 \text{sign}(s_3)$ is a final solution for U_{as3} , where Γ_3 must fulfill $\Gamma_3 > |g_3|$. Then the

adaptive law is expressed as

$$\hat{\Gamma}_3 = \frac{1}{\beta_3} |s_3| \quad (4.19)$$

Where β_3 is the adaptation gain and $\beta_3 > 0$. The adaptation rate of $\hat{\Gamma}_3$ can be tuned by β_3 .

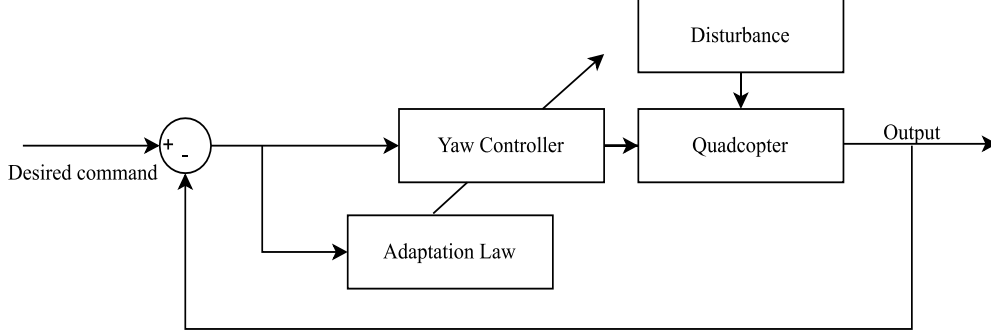


Figure 4.5: Yaw control structure of quadcopter

4.1.3.3 Altitude control

From the dynamical model of quadrotor equations (3.29), the altitude subsystem containing vertical force input U_1 is given by:

$$\ddot{Z} = -g + (\cos \phi \cos \theta) \frac{1}{m} U_1 \quad (4.20)$$

This can be obtained the same procedure as above. Then, the equivalent control law U_{eq4} is given by:

$$U_{eq4} = U_1 = \frac{m}{\cos x_1 \cos x_3} (g + \lambda_4 \dot{e}_4) \quad (4.21)$$

The adaptive function is taken as:

$$U_{as4} = \hat{\Gamma}_4 \text{sign}(s_4) \quad (4.22)$$

Where $\hat{\Gamma}_4$ is an adjustable gain constant and Γ_4 is a positive number, so that $U_{as4} = \hat{\Gamma}_4 \text{sign}(s_4)$ is a final solution for U_{as4} , where Γ_4 must satisfy $\Gamma_4 > |g_3|$. Let the adaptive law be:

$$\hat{\Gamma}_4 = \frac{1}{\beta_4} |s_4| \quad (4.23)$$

Where β_4 is the adaptation gain and $\beta_4 > 0$. The adaptation rate of $\hat{\Gamma}_4$ can be tuned by β_4 .

The controller inputs are saturated in order to increase the performance of the system. There are always adverse effects for the systems if ignoring the existence of input saturation non-linearity which reduces the performance indicators of the systems such as causing lag,

increasing overshoot, lengthening settling time, increasing steady-state error and aggravating oscillation and even affects the stability of the system.

$$0 \leq U1 \leq 4bw_{\max}^2 \quad (4.24)$$

$$-bw_{\max}^2 \leq U2 \leq bw_{\max}^2 \quad (4.25)$$

$$-bw_{\max}^2 \leq U3 \leq bw_{\max}^2 \quad (4.26)$$

$$-2dw_{\max}^2 \leq U4 \leq 2dw_{\max}^2 \quad (4.27)$$

Finally, the angular speeds for the four rotors of the quadrotor are expressed in terms of the controller inputs.

4.2 Chattering reduction

The switching term signum function in reaching law can cause chattering in sliding mode controllers, especially for a significant disturbance, which will harm system components like actuators. Additionally, it stimulates the dynamics of unmodeled systems. These could cause the system to become unstable and consume a lot of power. Sliding mode control is therefore more difficult to implement on actual systems since the effect cannot be neglected.

Numerous studies have been conducted in an effort to lessen chattering, testing various methods including neural networks, boundary conditions, surrounding surfaces, and quasi and terminal SMC (Thomas, 2017). By adding a boundary layer around the sliding surface, this issue is typically reduced. In this study, the specified method is used.

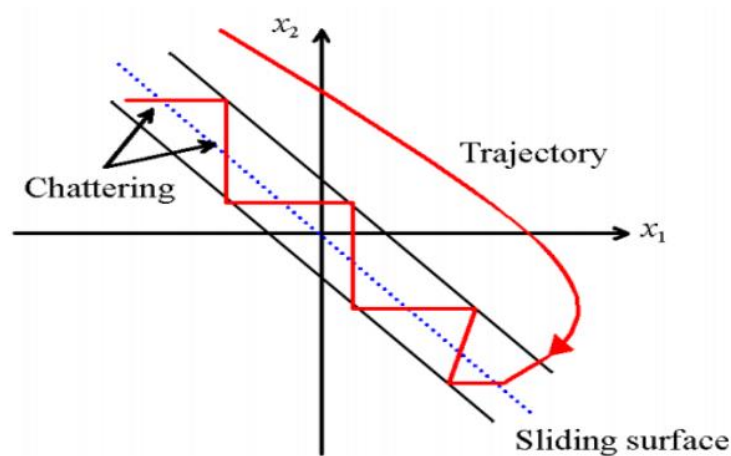


Figure 4.6: Chattering effect of SMC (Ahmed, 2004)

To restrain the effect of chattering, hyperbolic tangent function is adopted instead of $\text{sign}(s)$. the $\tanh$ function operates element-wise on arrays.

4.3 Lyapunov stability analysis of the rotational and altitude controllers

Stability theory deals with the effect of disturbances on time process in real systems. Differential equation or difference equation solutions characterizing dynamical systems may exhibit many sorts of stability. The type that concerns the stability of solutions close to an equilibrium point is the most significant. This may be discussed by the theory of Aleksander Lyapunov. For choosing Lyapunov function there is no general rule. In this work the positive Lyapunov candidate function for each single dynamic systems a suitable function is selected.

4.3.1.1 Roll controller stability analysis

Select the Lyapunov function as;

$$v = \frac{1}{2} s_1^2 \quad (4.28)$$

The first derivative of equation (4.28) is

$$\dot{v} = s_1 \dot{s}_1 \quad (4.29)$$

Inserting equation (4.4) and (4.6) in equation (4.29)

$$\dot{v} = s_1 [\ddot{\phi}_d - x_4 x_6 a_1 - x_4 a_2 \Omega - \delta_\phi + \lambda_1 \dot{e}_1 - b_1 U_2] \quad (4.30)$$

$$\dot{v} = s_1 [\ddot{\phi}_d - x_4 x_6 a_1 - x_4 a_2 \Omega - \delta_\phi + \lambda_1 \dot{e}_1 - (\ddot{\phi}_d - x_4 x_6 a_1 - x_4 a_2 \Omega + \lambda_1 \dot{e}_1 + \hat{T}_1 \text{sign}(s_1))] \quad (4.31)$$

$$\dot{v} = s_1 (-\hat{T}_1 \text{sign}(s_1) - \delta_\phi) \leq (-\hat{T}_1 + |\delta_\phi|) |s_1| \quad (4.32)$$

By selecting $\hat{T}_1 > |\delta_\phi|$ then $\dot{v} < 0$. This ensures that the control law U_2 satisfies the stability of system (3.25) by using Lyapunov theory.

4.3.1.2 Pitch controller stability analysis

By following the above same procedure, we can ensure the stability of system (3.27).

$$v = \frac{1}{2} s_2^2 \quad (4.33)$$

The first derivative of equation (4.33) is

$$\dot{v} = s_2 \dot{s}_2 \quad (4.34)$$

$$\dot{v} = s_2 [\ddot{\theta}_d - x_2 x_6 a_3 - x_2 a_4 \Omega - \delta_\theta + \lambda_2 \dot{e}_2 - b_2 U_3] \quad (4.35)$$

$$\dot{v} = s_2 [\ddot{\theta}_d - x_2 x_6 a_3 - x_2 a_4 \Omega - \delta_\theta + \lambda_2 \dot{e}_2 - (\ddot{\theta}_d - x_2 x_6 a_3 - x_2 a_4 \Omega + \lambda_2 \dot{e}_2 + \hat{T}_2 \text{sign}(s_2))] \quad (4.36)$$

$$\dot{v} = s_2 (-\hat{T}_2 \text{sign}(s_2) - \delta_\theta) \leq (-\hat{T}_2 + |\delta_\theta|) |s_2| \quad (4.37)$$

By selecting $\hat{T}_2 > |\delta_\theta|$ then $\dot{v} < 0$. This ensures that the control law U_3 satisfies the stability of system (3.27) by using Lyapunov theory.

4.3.1.3 Yaw (heading) controller stability analysis

Again, by proceeding the same step as above for yaw stability,

$$v = \frac{1}{2} s_3^2 \quad (4.38)$$

The first derivative of equation (4.38) is

$$\dot{v} = s_3 \dot{s}_3 \quad (4.39)$$

$$\dot{v} = s_3 [\ddot{\varphi}_d - x_2 x_4 a_5 - \delta_\varphi + \lambda_3 \dot{e}_3 - b_3 U_4] \quad (4.40)$$

$$\dot{v} = s_3 [\ddot{\varphi}_d - x_2 x_4 a_5 - \delta_\varphi + \lambda_3 \dot{e}_3 - (\ddot{\varphi}_d - x_2 x_4 a_5 + \lambda_3 \dot{e}_3 - \hat{T}_3 \text{sign}(s_3) - \delta_\varphi)] \quad (4.41)$$

$$\dot{v} = s_3 (-\hat{T}_3 \text{sign}(s_3) - \delta_\varphi) \leq (-\hat{T}_3 + |\delta_\varphi|) |s_3| \quad (4.42)$$

By selecting $\hat{T}_3 > |\delta_\varphi|$ then $\dot{v} < 0$. This ensures that the control law U_4 satisfies the stability of system (3.28) by using Lyapunov theory.

4.3.1.4 Altitude controller stability analysis

$$v = \frac{1}{2} s_4^2 \quad (4.43)$$

The first derivative of equation (4.43) is

$$\dot{v} = s_4 \dot{s}_4 \quad (4.44)$$

$$\dot{v} = s_4 [\ddot{Z}_d - (-g + (\cos \phi \cos \theta) \frac{1}{m} U_1 + \lambda_4 \dot{e}_4 - \delta_z)] \quad (4.45)$$

$$\dot{v} = s_4 [\ddot{Z}_d - (-g + (\cos \phi \cos \theta) \frac{1}{m} (\frac{m}{\cos x_1 \cos x_3} (g + \lambda_4 \dot{e}_4) + \hat{T}_4 \text{sign}(s_4)) + \lambda_4 \dot{e}_4 - \delta_z)] \quad (4.46)$$

$$\dot{v} = s_4 (-\hat{T}_4 \text{sign}(s_4) - \delta_z) \leq (-\hat{T}_4 + |\delta_z|) |s_4| \quad (4.47)$$

By selecting $\hat{T}_4 > |\delta_z|$ then $\dot{v} < 0$. This ensures that the control law U_3 satisfies the stability of system (3.29) by using Lyapunov theory.

CHAPTER FIVE

5 RESULT AND ANALYSIS

In this chapter, the performance and stability of the proposed ASMC for the quadrotor dynamics is simulated using MATLAB/Simulink environment without disturbance, with disturbance and with parameter uncertainty considered different values of mass in this study. The attitude and position stabilization are applied on the quadrotor for performing good tracking and enhancing the position accuracy while keeping the altitude on the desired point. To highlight the performance of the proposed control method, SMC method is chosen for comparison. The sliding surface and adaptive term parameters are tune using GA optimization technique to get the optimal values. The initial position and angles are chosen as $X_0 = Y_0 = Z_0 = 0$, $\phi_0 = \theta_0 = \varphi_0 = 0$. The trajectory given as a reference for X, Y and Z position is $\sin(t)$, $\cos(t)$ and 1 (meter) respectively. For the simulation of the proposed system, the following quadrotor system and controller parameters shown in Table 4.1 and Table 4.2 are used respectively.

Table 5.1: Quadcopter system's parameter (Nadda & Swarup, 2018)

Parameters	Definitions	Value
m	Mass	0.5 kg
l	Arm length	0.23 m
b	Thrust coefficients	$3.13 \times 10^{-5} \text{ N s}^2$
d	Drag coefficients	$7.5 \times 10^{-7} \text{ N m s}^2$
I_m	Rotor inertia	$6 \times 10^{-5} \text{ kg m}^2$
I_x	Inertia on x-axis	$7.5 \times 10^{-3} \text{ kg m}^2$
I_y	Inertia on y-axis	$7.5 \times 10^{-3} \text{ kg m}^2$
I_z	Inertia on z-axis	$1.3 \times 10^{-2} \text{ kg m}^2$
g	Acceleration of gravity	9.8 m/s^2

Table 5.2: Controller's parameter

Parameters	Value
$\lambda_1 = \lambda_2$	37
λ_3	10
λ_4	30
$\lambda_5 = \lambda_6$	6
β_1	0.15
β_2	0.45
$\beta_3 = \beta_4 = \beta_5 = \beta_6$	0.015

5.1 Attitude and position controller stability analysis without parameter uncertainty and external disturbance

5.1.1 Control inputs result analysis

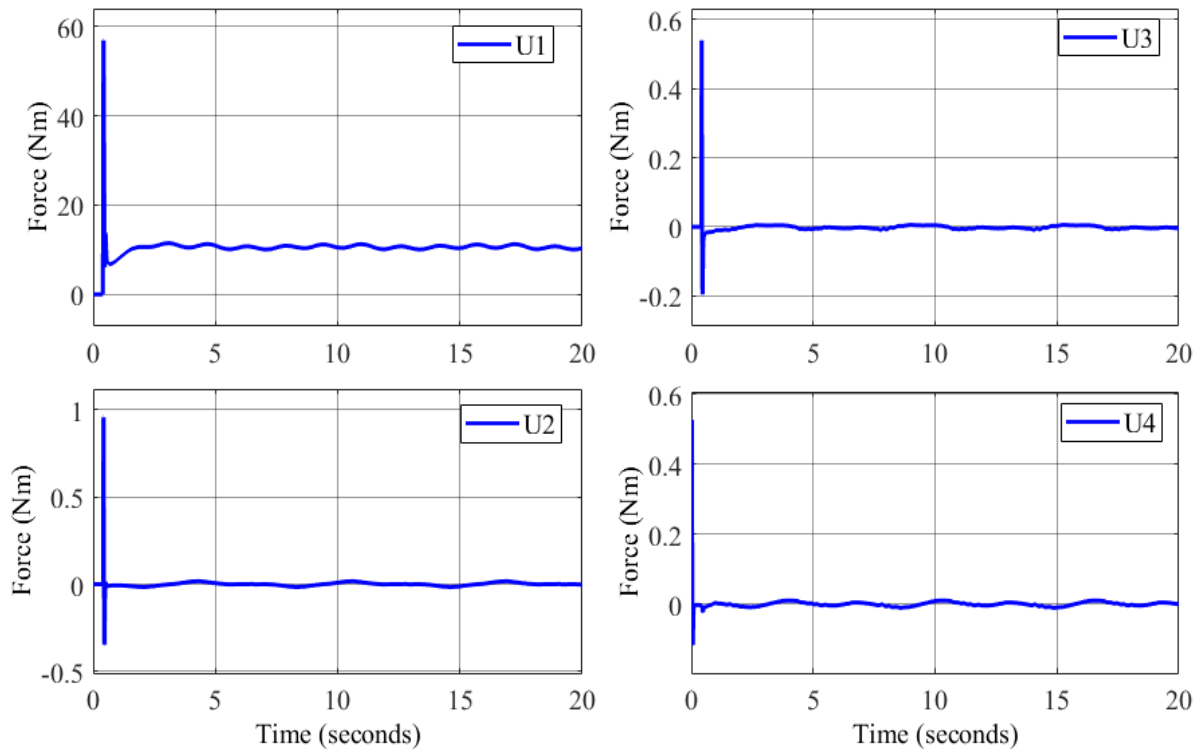


Figure 5.1: Control input signals for attitude and altitude dynamics of quadrotor without external disturbance

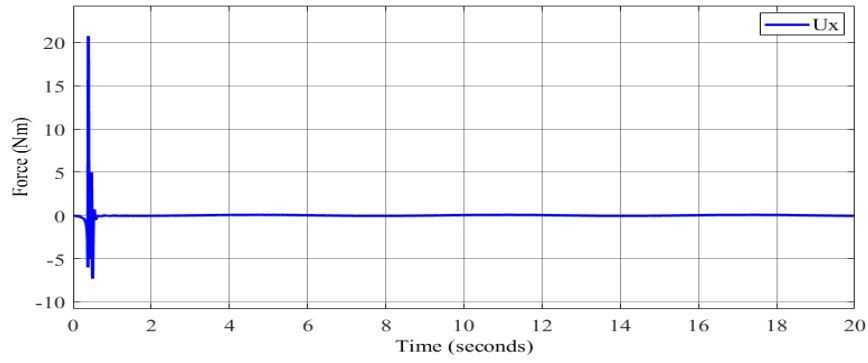


Figure 5.2: Control input signal for X position of quadrotor without external disturbance

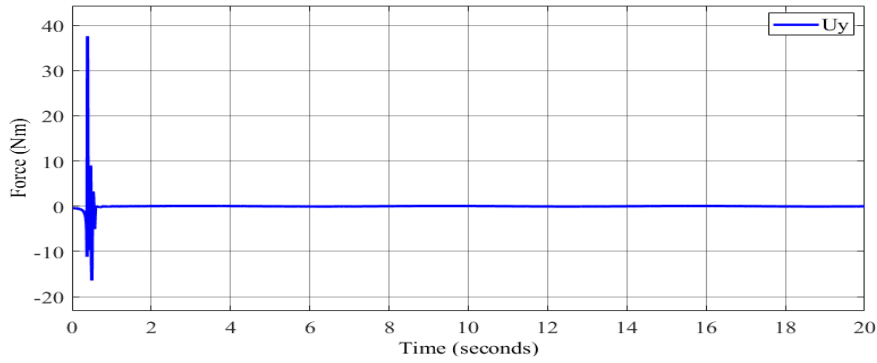


Figure 5.3: Control input signal for Y position of quadrotor without external disturbance

The control input U_1 shown in Figure 5.1 clarifies that when quadrotor system takes off from ground to hover; the thrust force or torque is increased to its maximum level to lift up and decreased from 58 to 10 N to set the quadrotor on its desired altitude level. The control inputs U_2 , U_3 and U_4 performs the same to stabilize the roll, pitch and yaw angles respectively at state zero.

The controller U_x and U_y shown in Figure 5.2 and 5.3 acts for X position and Y position respectively. These two controllers coupled with U_1 to further stabilize the position of the quadrotor when U_1 is not equals to zero.

5.1.2 Position controller analysis

The position control system is the external loop control in the quadrotor control system. It is used to extract the desired attitude roll and pitch orientations.

Figure 5.4 and Figure 5.6 shows position tracking performance of quadrotor system. For X position tracking sine wave is given as a desired path while cosine wave is given for Y position tracking. The tracking performance is good and has an error bounded within $[-0.01, 0.01]$. In Figure 5.8, the altitude controller experienced a good tracking performance on altitude of quadrotor system. Known that altitude is always positive and the quadrotor starts from ground level, $Z=0$, during take-off, the quadrotor reaches its desired given reference

altitude and stabilized within 0.5 seconds. Since an altitude is the first condition to be met in quadrotor control system it has to be controlled within few seconds because without some altitude rise up we cannot maintain orientation stability.

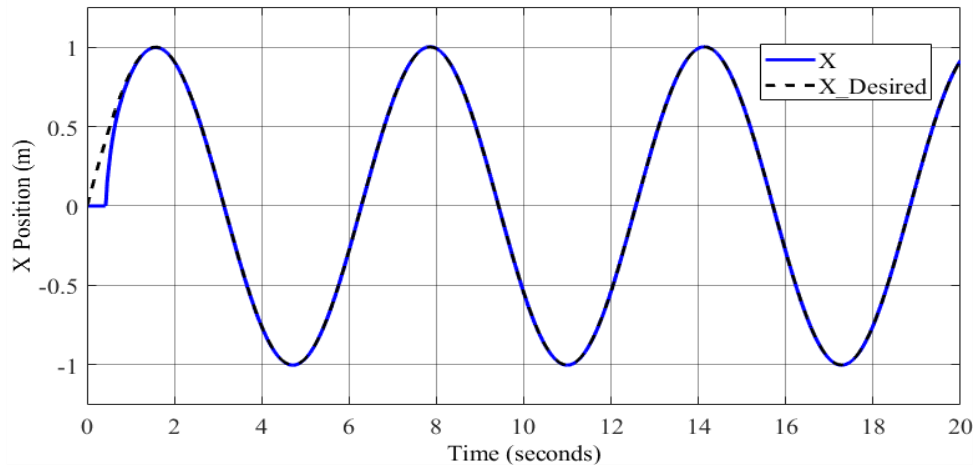


Figure 5.4: X position tracking performance

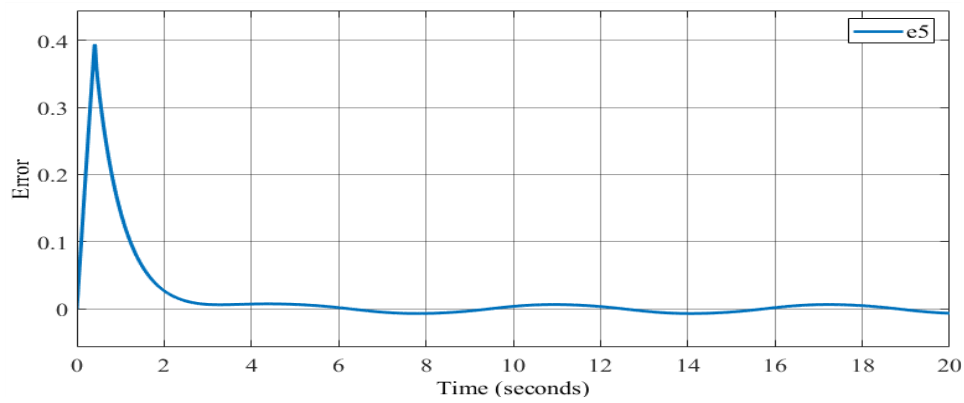


Figure 5.5: X position tracking performance error

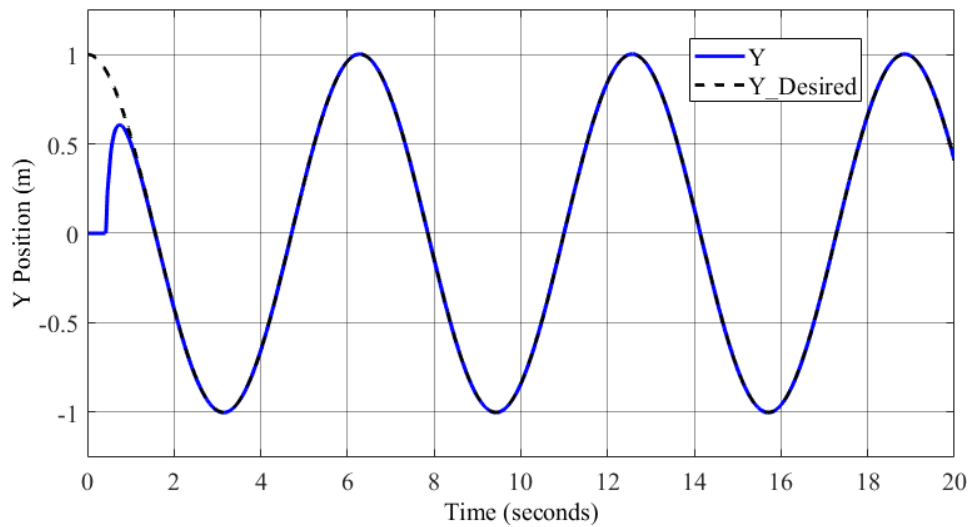


Figure 5.6: Y position tracking performance

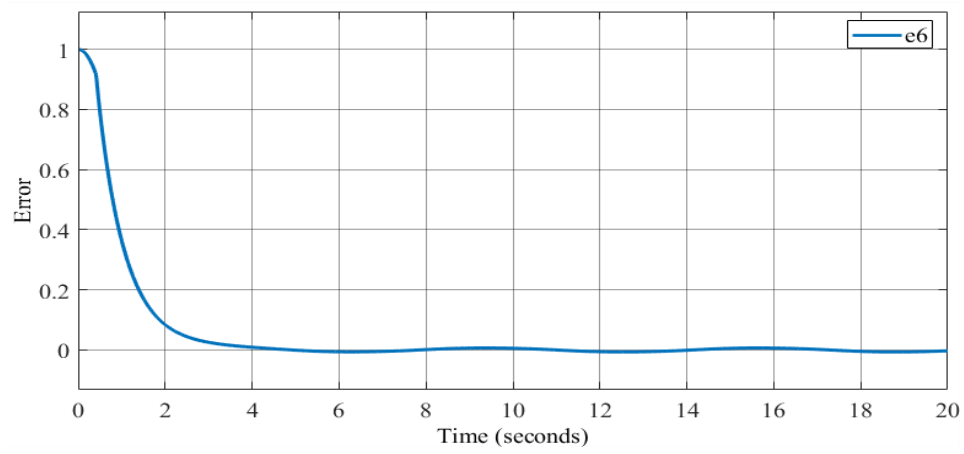


Figure 5.7: Y position tracking performance error

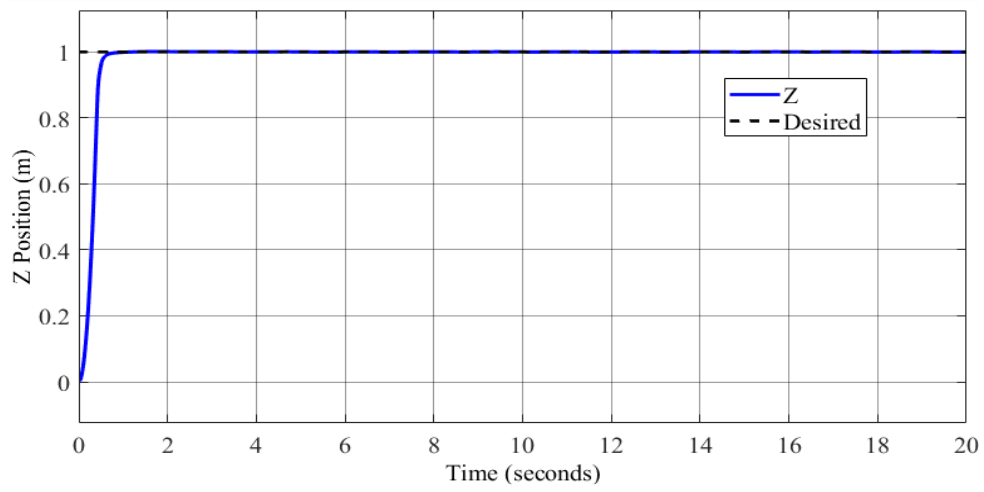


Figure 5.8: Altitude (Z) tracking performance

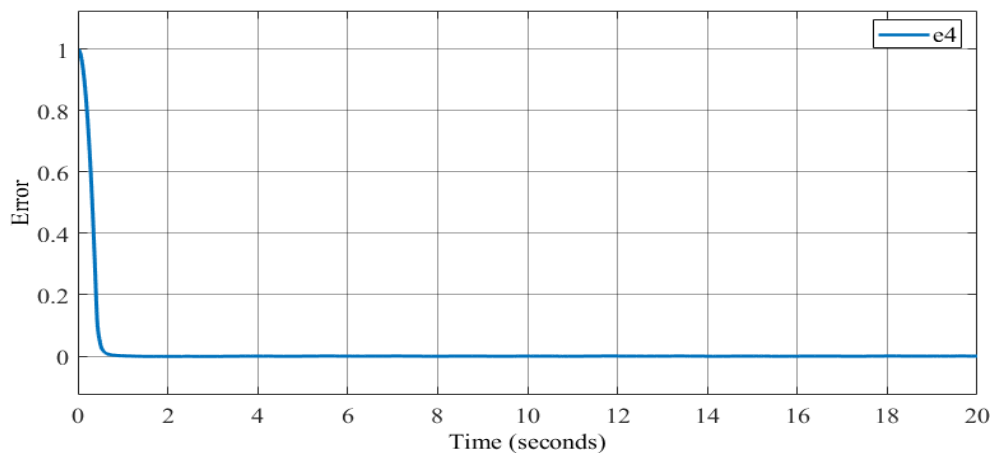


Figure 5.9: Altitude (Z) tracking performance error

5.1.3 Attitude controller analysis

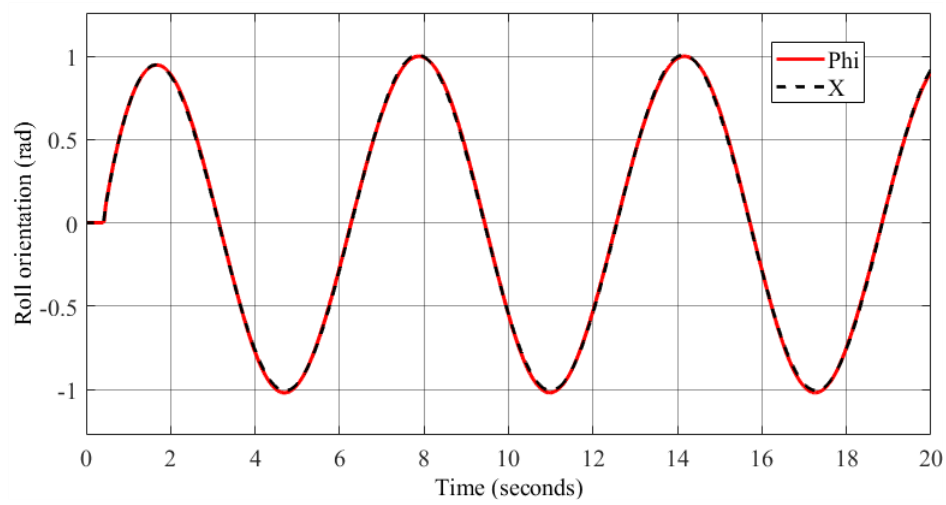


Figure 5.10: Roll orientation tracking performance

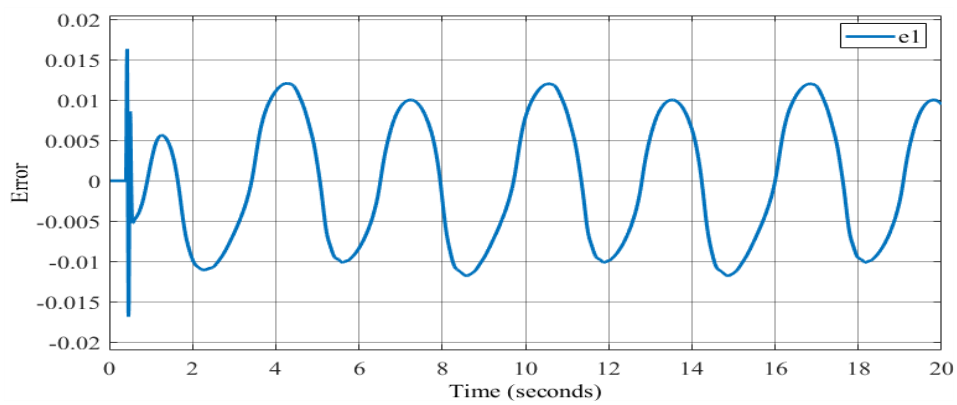


Figure 5.11: Roll orientation tracking performance error

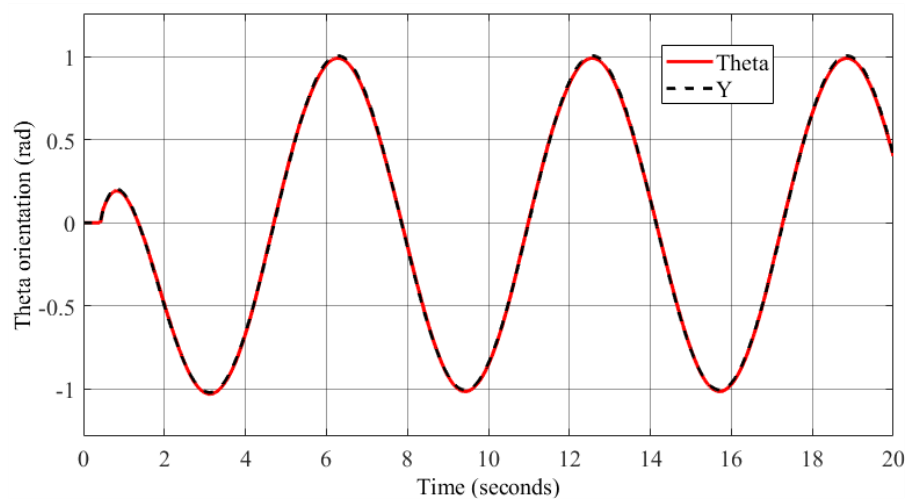


Figure 5.12: Pitch orientation tracking performance

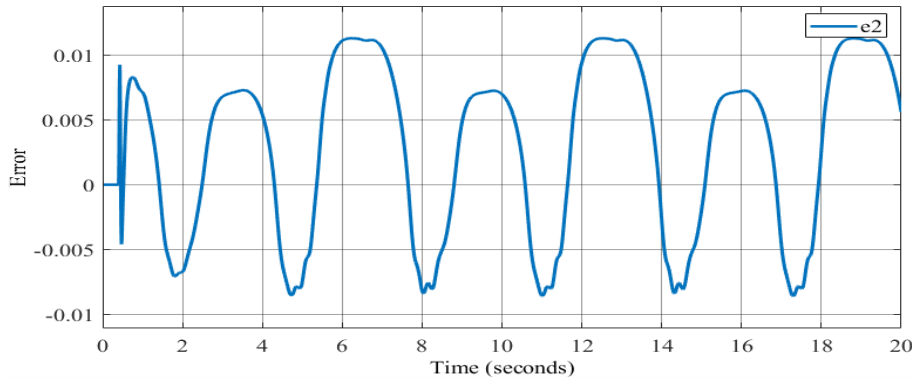


Figure 5.13: Pitch orientation tracking performance error

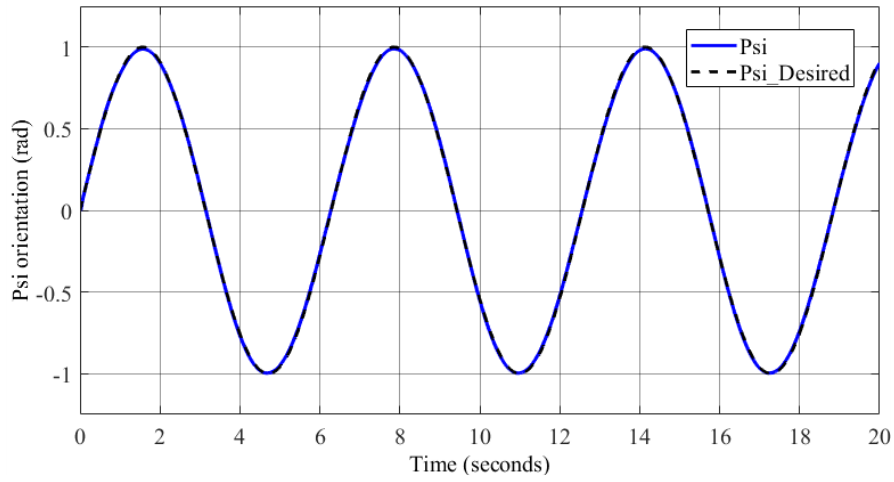


Figure 5.14: Yaw orientation tracking performance

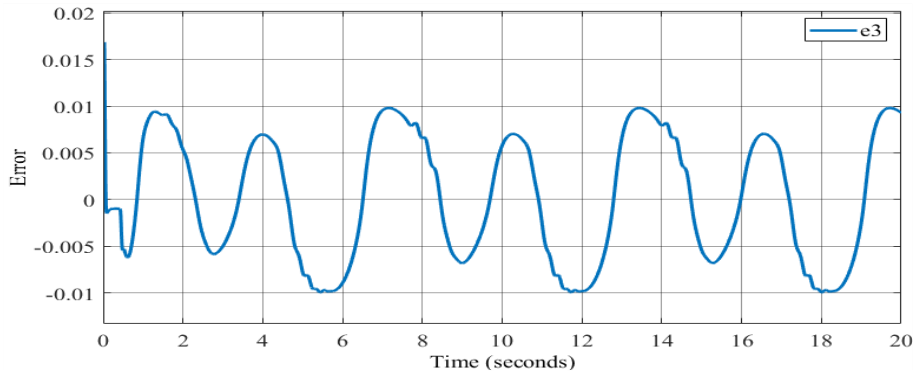


Figure 5.15: Yaw orientation tracking performance error

Figure 5.10 to 5.15, shows that the controller tracking performance of roll, pitch and yaw with their respective tracking error signal respectively. The roll movement is rotation around X-axis and the pitch movement on the other hand is rotation around Y-axis. Since, this coupling is highly non-linear in the quadrotor system, the orientation takes its desired path from the output of the position loop and tracks. Both roll and pitch start from initial angle 0

rad and stabilized at desired angle from the start keeping a lower convergence to set point. The yaw or heading controller is used as a direction controller and guider for the quadrotor. Therefore, a controller must have a better and fast control action as seen in Figure 5.14. The reference angle for yaw is given arbitrarily a sine wave in this study.

Figure 5.11, Figure 5.13 and Figure 5.15 shows the error between set point value and actual state of the quadrotor system dynamics without disturbance injection. Where e_1 , e_2 and e_3 are roll, pitch and yaw errors respectively. Tracking error is a subtraction of points of actual orientation and reference throughout the performance of the system and the value is bounded within $[-0.01 \ 0.01]$, which is an acceptable result. As $t \rightarrow \infty$ the actual attitude tracks the desired position and the tracking error becomes 0.

5.2 Attitude and position controller stability analysis with external disturbance

In this section, the robustness and ability of the proposed controller to track a set point reference in the presence of parameter uncertainty and external disturbance is given. The external disturbance is considered as step input wind disturbance, which has a constant magnitude ranges from -1.8 to 1.8 and the internal uncertain parameter is quadrotor mass variation from 0.1 to 7 kg.

5.2.1 Control inputs result analysis

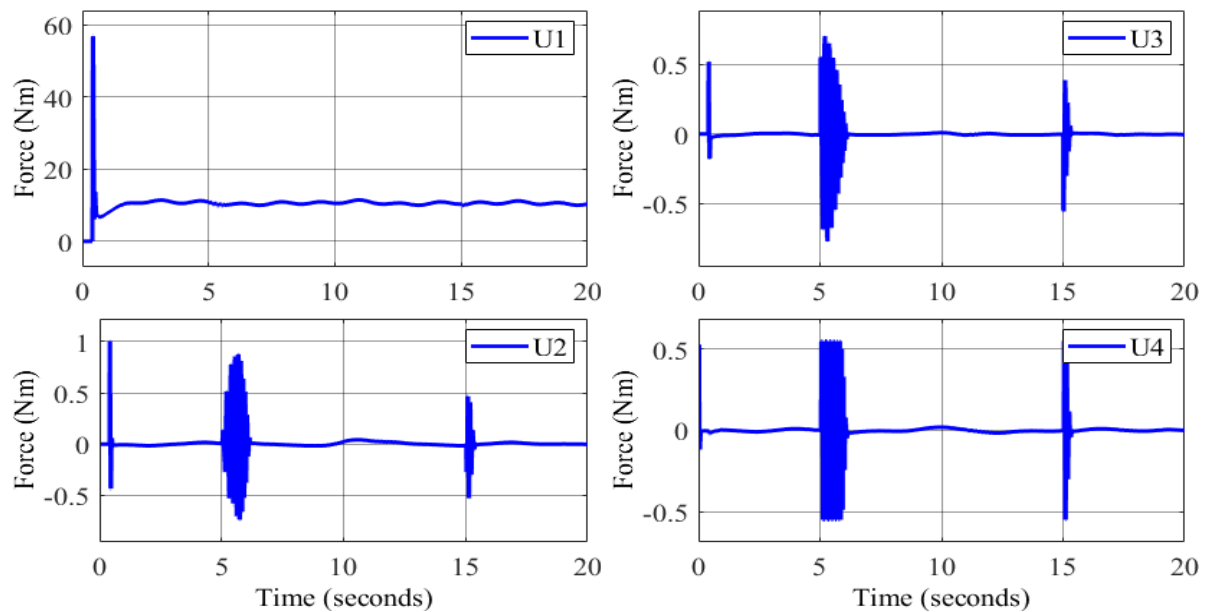


Figure 5.16: Control input signals for quadrotor system dynamics with external disturbance

The control inputs U2, U3 and U4 shows variation of torque when the external disturbance is introduced and rejected from the roll, pitch and yaw system respectively, at 5 and 15 seconds, to compensate the effect as shown in Figure 5.16. This is due to the torque that a controller needed to reject the external disturbance and become robust again on the set point varies with respect to the disturbance amount.

5.2.2 Position controller analysis

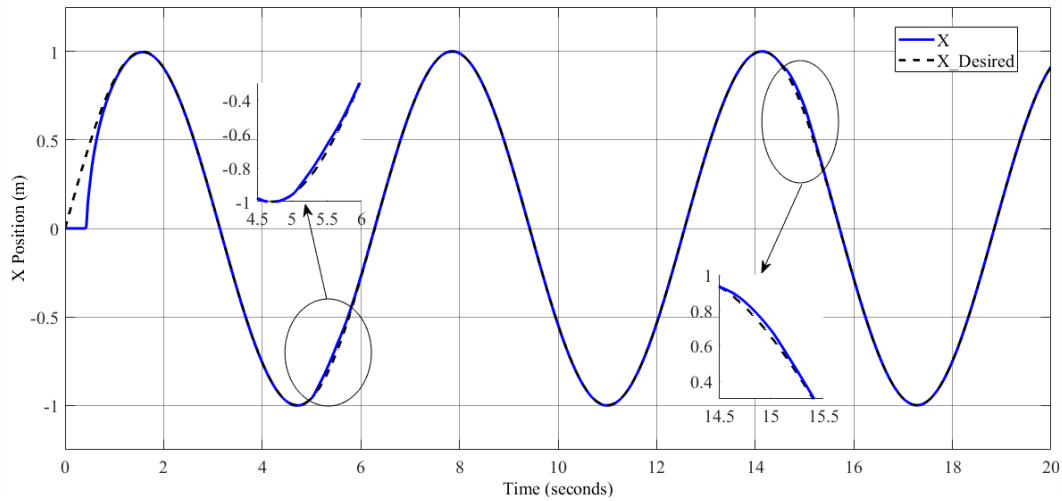


Figure 5.17: X position tracking performance with external disturbance

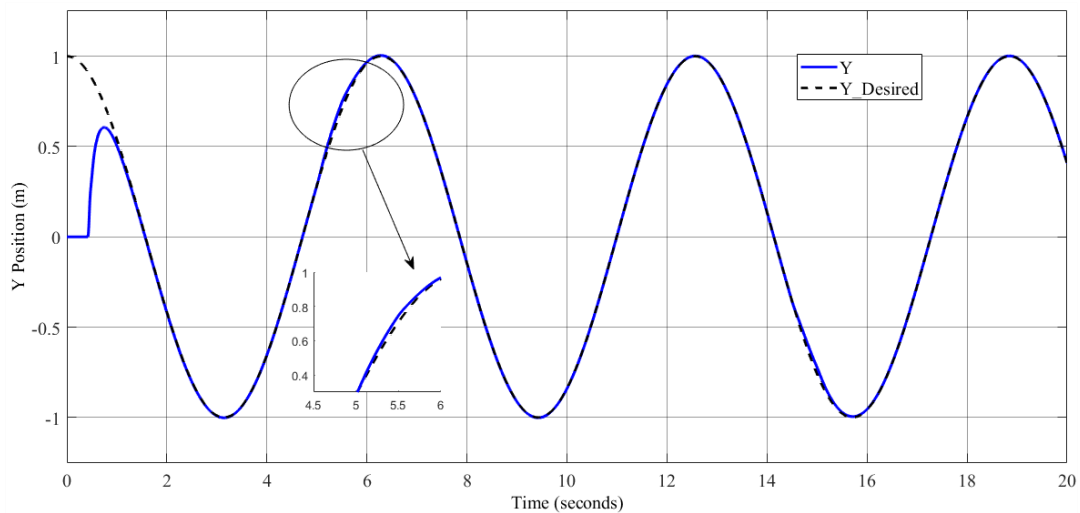


Figure 5.18: Y position tracking performance with external disturbance

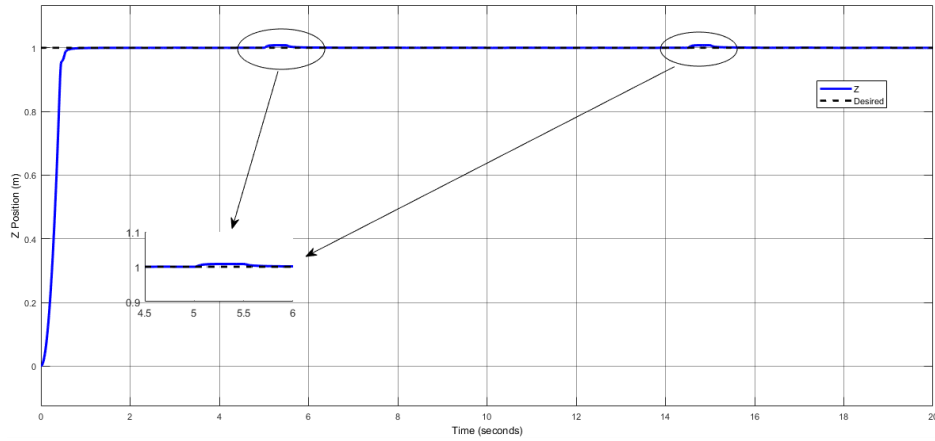


Figure 5.19: Altitude (Z) tracking performance with external disturbance

Figure 5.17 to Figure 5.19 shows, X, Y and altitude (Z) tracking ability of the proposed controller in presence of external disturbance. It settles in a desired reference position within 0.5 second and when the disturbance is introduced at 5 second and leaves from the system at 15 second it shows variation for 0.5 second and becomes stable in between them as expected to be like robust. The comparative SMC controller in fact, performs exactly like the proposed controller ASMC in case of position stabilization.

5.2.3 Attitude controller analysis

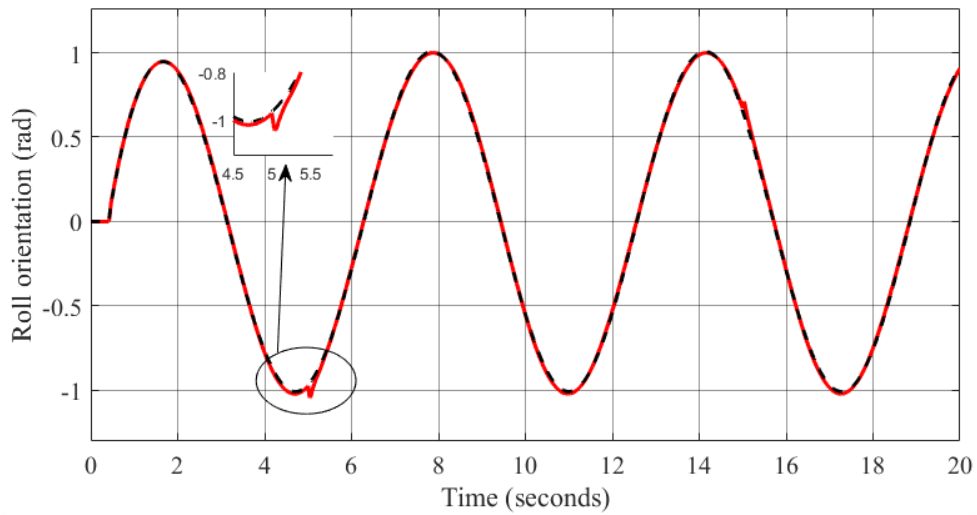


Figure 5.20: Roll orientation tracking performance with external disturbance

As shown in Figure 5.20, when the disturbance is implanted to roll dynamics from 5 second to 15 second the proposed controller handles the disturbance well although there is a small change at the moment of injection and leaving of a disturbance from the dynamics. This effect is expected because when a sudden change in the dynamics occurs, it cannot

completely ignore and stabilize itself perfectly in that specific time. After the small change the feedback value converges exactly to the set point orientation.

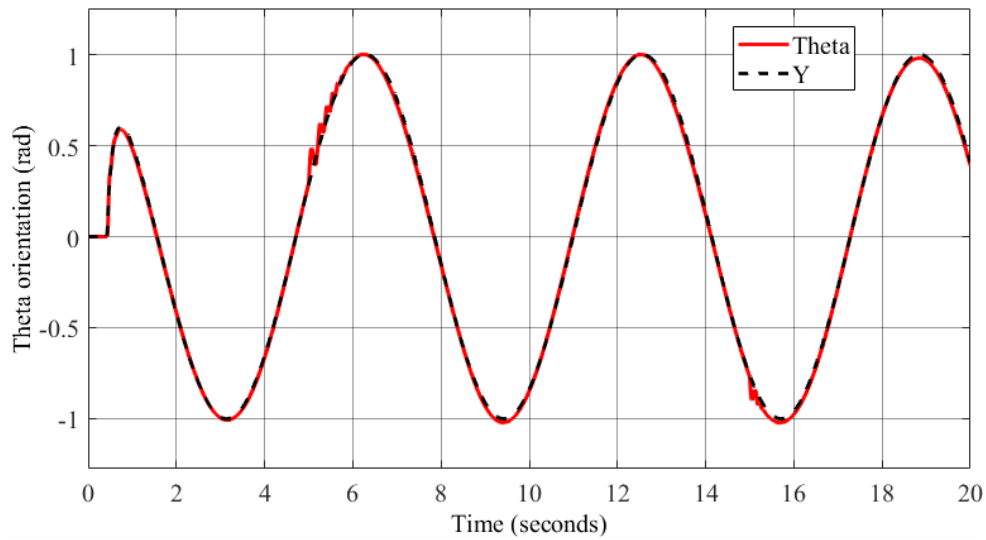


Figure 5.21: Pitch orientation tracking performance with external disturbance

Figure 5.21 shows that, the theta orientation within time t in presence of disturbance from 5 second to 15 second and experienced a good tracking and compensation due to the proposed controller. As explained in roll controller performance the same effect happens when the disturbance is injected and leaves from the dynamics. The effect is compromised very well again by the proposed controller.

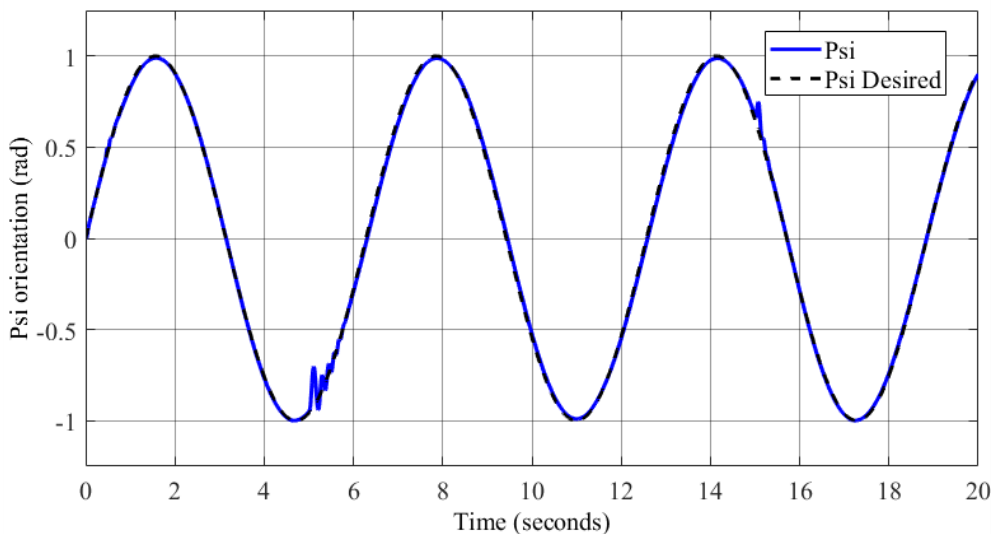


Figure 5.22: Yaw orientation tracking performance with external disturbance

In Figure 5.22, it is shown that the yaw orientation or yaw movement is tracked very well considering the injected disturbance by the proposed controller. Starting from initial orientation the controller causes the yaw movement to stabilize itself on the desired angle within 0.5 second. At 5 and 15 seconds the orientation experiences some spike due to the moment of injection and breaking of disturbance from the dynamics while becoming robust in between them.

5.3 Attitude and position controller stability analysis with parameter uncertainty

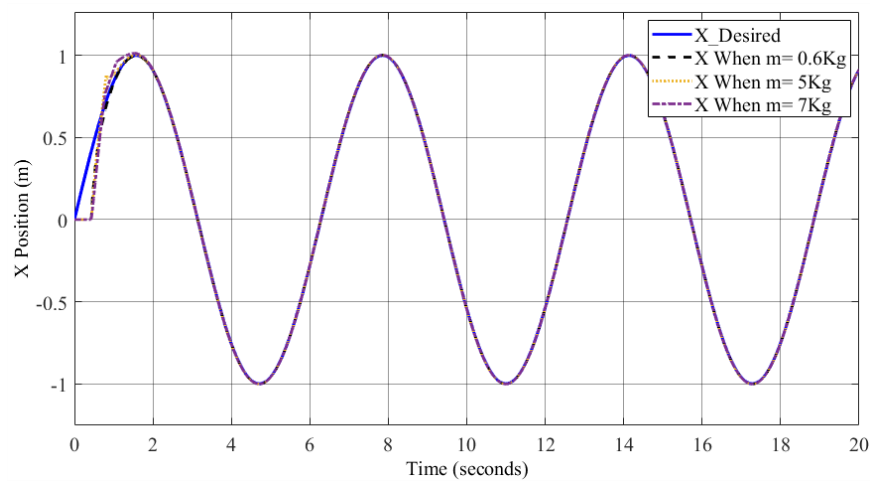


Figure 5.23: X position performance when different mass values applied on

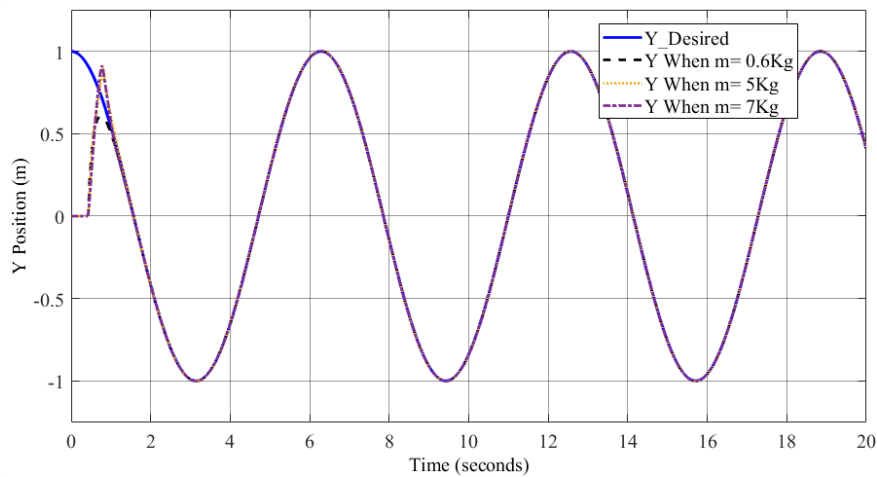


Figure 5.24: Y position performance when different mass values applied on

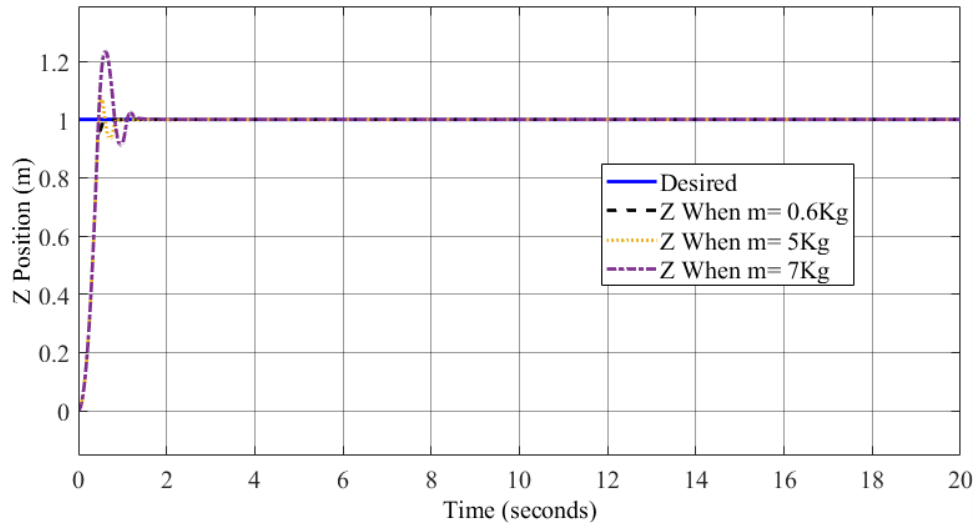


Figure 5.25: Altitude performance when different mass values applied on

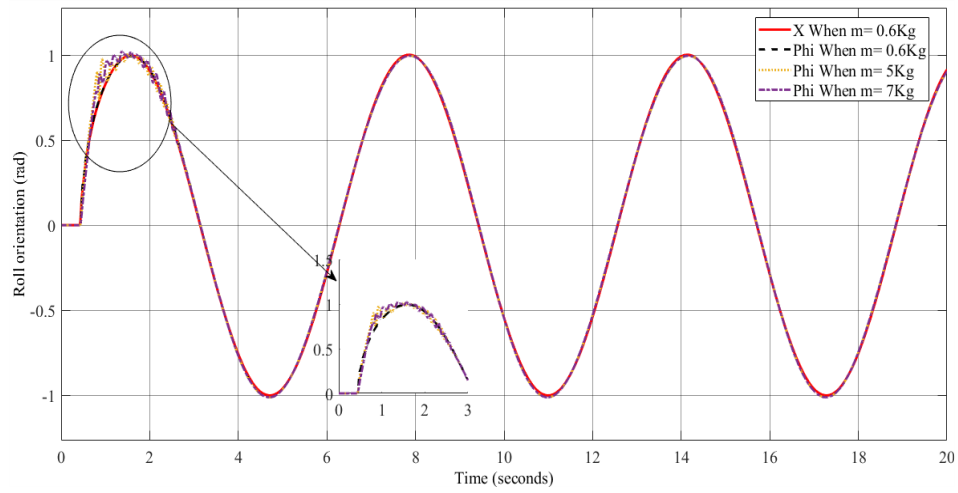


Figure 5.26: Roll angle performance when different mass values applied on

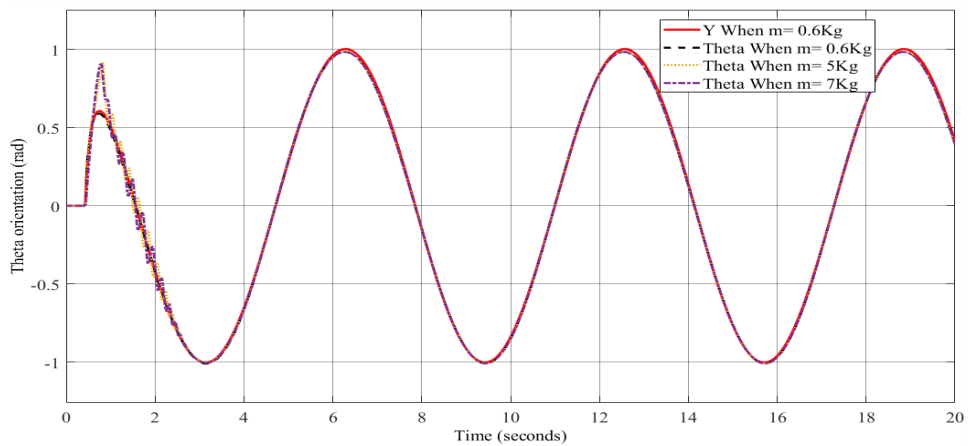


Figure 5.27: Pitch angle performance when different mass values applied on

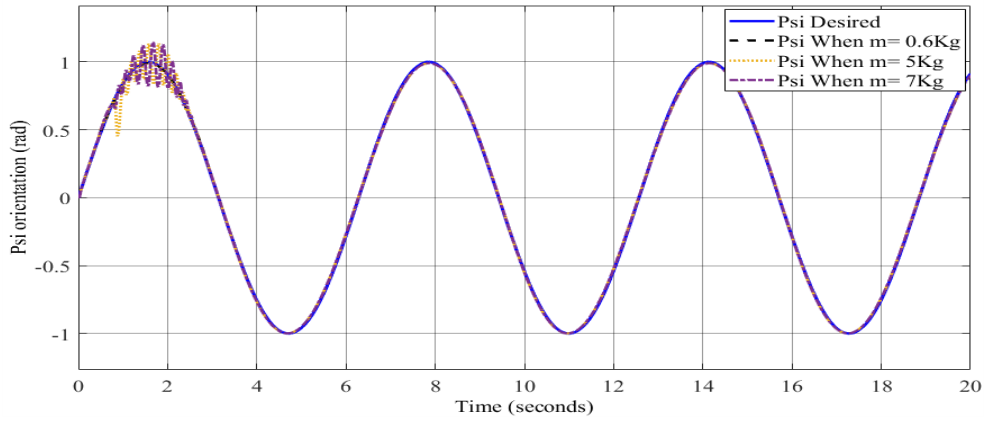


Figure 5.28: Yaw angle performance when different mass values applied on

In Figure 5.23 to Figure 5.28 it is shown that, when different mass values, as internal model parameter uncertainty, are applied on the quadrotor system dynamics the proposed controller performs just fine to handle the variation. Although it exhibits some overshoot and vibration up to 3 second as mass value increases to 7 Kg, the controller stabilizes the state of the quadrotor on its reference path.

5.4 Comparison of proposed controller (ASMC) with SMC

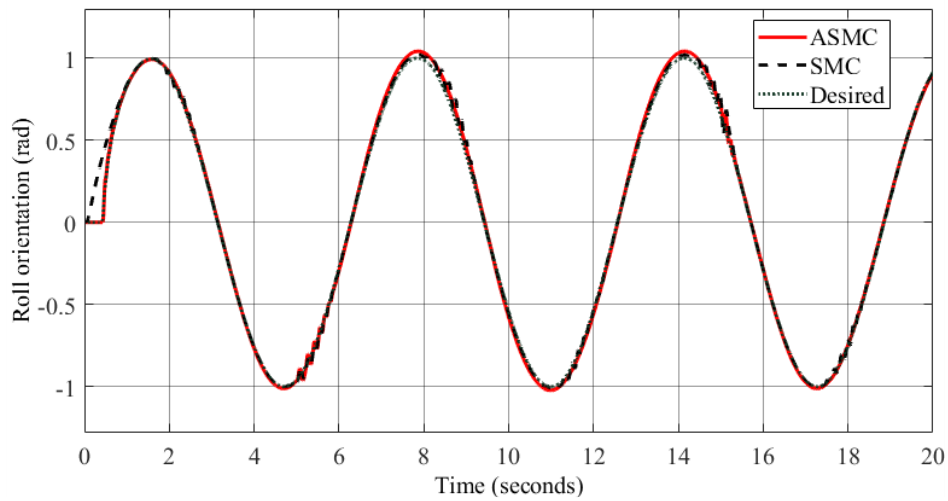


Figure 5.29: Roll angle performance when external disturbance is applied on

Figure 5.29 shows, when external disturbance is applied on the roll angle the proposed controller shows a good performance to stabilize the angle on its desired point while SMC shows some vibrations throughout the path. The robustness of the proposed controller is better since it has no vibrations other than at the moment of injection and leaving or has a good disturbance rejection ability while both controllers has no overshoot. The proposed controller has very fast settling time (0.2 second) and SMC has a settling time of 1 second.

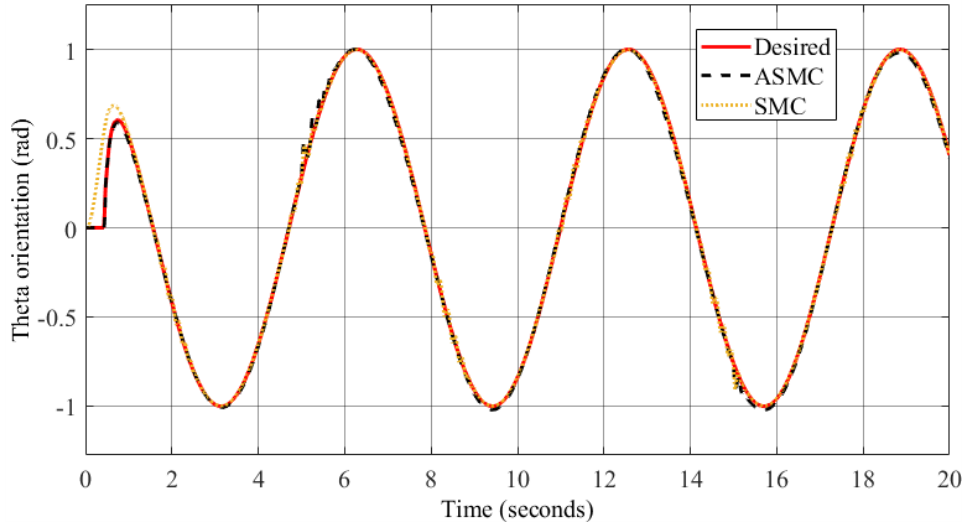


Figure 5.30: Pitch angle performance when external disturbance is applied on

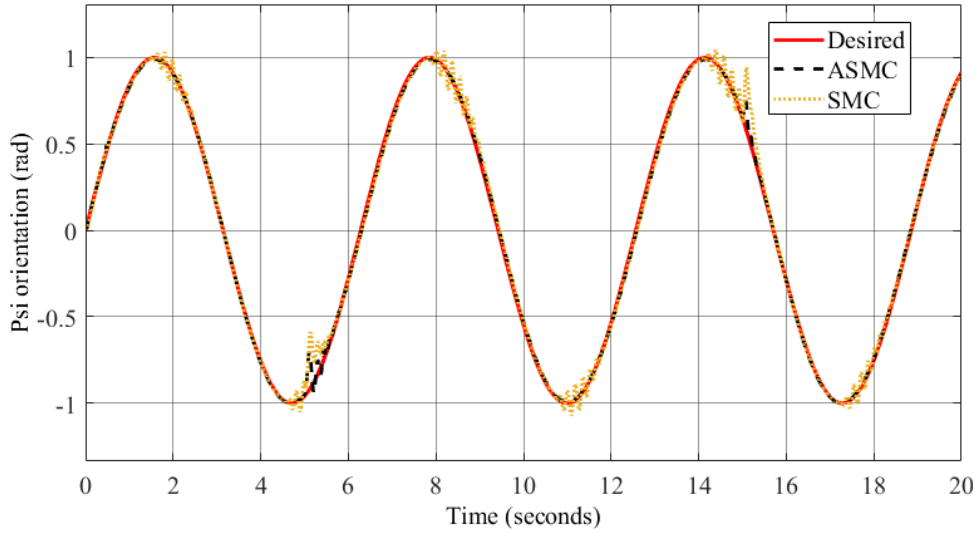


Figure 5.31: Roll angle performance when external disturbance is applied on

In Figure 5.30, both controllers perform almost the same to stabilize the theta angle to its reference path. In case of overshoot SMC has some overshoot while the proposed controller does not. Settling time of the proposed controller is very fast (0.2 second) and that of SMC is 1 second. In Figure 5.31, the proposed controller tracks the desired trajectory better than SMC. When disturbance is injected in to the system the proposed controller shows some spike and become robust while SMC vibrates more and continues to vibrate until the disturbance leaves the system. The disturbance rejection ability of the proposed controller is way better than SMC due to SMC shows unstable tracking performance particularly on the turning point of the trajectory. Both controllers settle on to the reference at equal time and has no overshoot.

6 CONCLUSION AND RECOMMENDATION

6.1 Conclusion

In this work, the design of a quadrotor UAV attitude and position stabilization controller using adaptive law-based SMC is presented. The highly non-linear and coupled quadrotor dynamics is designed through Newton-Euler formalism to apply the controller on and verify its ability to stabilize the system. In the quadrotor dynamics, the rotational motion is independent of the translational motion and the translational motion is dependent on the rotational motion. Due to this fact, designing an attitude or rotational motion controller is possible by feeding the translational output from the outer loop to stabilize the orientation of the quadrotor system.

The proposed control scheme improves the performance of the quadrotor by providing robustness against the parameter variations and external disturbance. The proposed controller uses a nonlinear PD sliding surface where the gains are tuned through adaptive tuning law which has a flexibility to adjust the adaptation speed. Since only one component of the control law is being made to adapt the changes, the implementation becomes simpler. The adaptation mechanism is a direct type, which takes the estimated parameters directly to use them in the controller. Numerical simulations have been carried out to validate the effectiveness of the proposed method and shows that the proposed controller has effectively stabilized the quadrotor without knowledge of the upper bound of the system uncertainty. The error signal also shows that the state of the quadrotor precisely tracks the reference orientation as required.

6.2 Recommendation

This research work recommends for future researchers to consider:

-  Designing a controller that is capable of estimating and handling a time varying disturbance.
-  Modelling a quadrotor control system that has multiple control techniques which can identify and choose the suitable one according to the performed environment.
-  Implementing the controller on hardware to examine the performance.

Acknowledgements

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- (L3) <https://www.rs-online.com/designspark/tuning-a-drone-flight-control-systemwithout-flight>

APPENDIXES

Appendixes A

Simulink block diagram

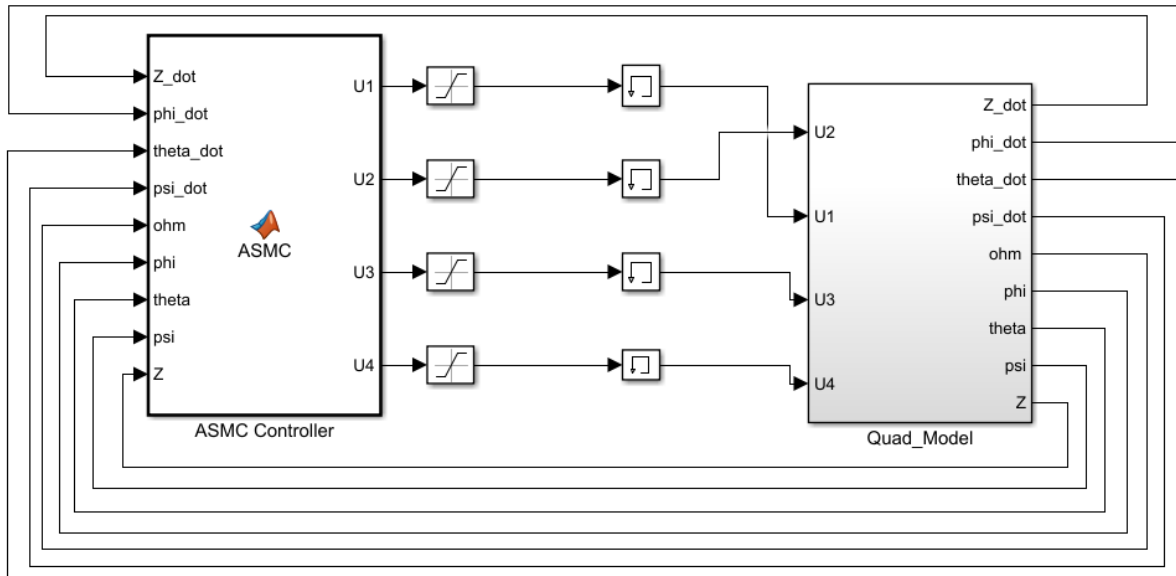


Figure A.1: Simulink block for overall system

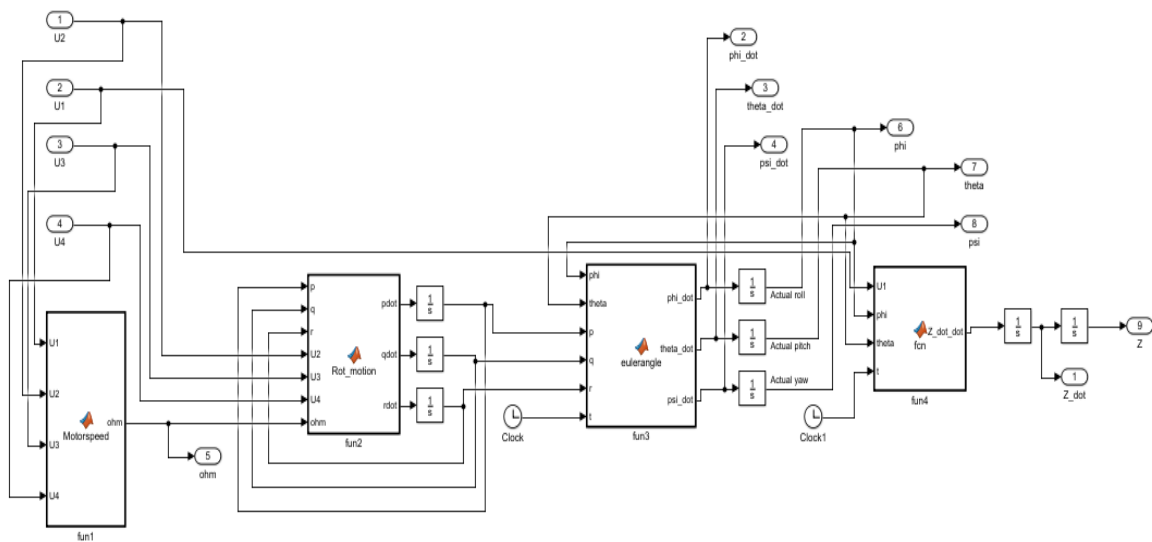


Figure A.2: Simulink block for quadrotor system

Appendixes B

```
function [U1,U2,U3,U4,Ux,Uy,j,h] =
ASMC(t,Z_dot,X_dot,X_dot_dot,Y_dot,Y_dot_dot, phi_dot, theta_dot, psi_dot,
ohm, phi, theta, psi, Z,X,Y)
phi_max= pi/2;
phi_min= -pi/2;
if (phi> phi_max)
    phi= pi/2;
end
if (phi< phi_min)
    phi= -pi/2;
end
if (theta> phi_max)
    theta= pi/2;
end
if (theta< phi_min)
    theta= -pi/2;
end
g = 9.81;
m = 0.6;
d = 3.2320e-7;
l = 0.5;
Ixx = 3.8278e-3;
Iyy = 3.8278e-3;
Izz = 7.6566e-3;
Im = 2.8385e-5;
x1 = phi;
x2 = phi_dot;
x3 = theta;
x4 = theta_dot;
x5 = psi;
x6 = psi_dot;
x7 = Z;
x8 = Z_dot;
x9 = X;
x10 = X_dot;
x11 = Y;
x12 = Y_dot;
a1 = (Iyy - Izz)/Ixx;
a2 = Im/Ixx;
a3 = (Izz - Ixx)/Iyy;
a4 = - Im/Iyy;
a5 = (Ixx - Iyy)/Izz;
b1 = 1/Ixx; b2 = 1/Iyy; b3 = 1/Izz;
alpha1 = 0.15; alpha2 = 0.45; alpha3 = 0.015; alpha4 = 0.015; alpha5 =
0.015; alpha6 = 0.015;
lamda1 = 37;
e1 = X - x1;
de1 = X_dot- x2;
S1 = lamda1*e1 + de1;
T1 = (1/alpha1)*abs(S1);
Uas1 = (1/b1)*T1*tanh(S1);
U2 = (1/b1)*(X_dot_dot - (x4*x6*a1) - (x4*a2*ohm) + (lamda1*de1)) + Uas1;
lamda2 = 37;
e2 = Y - x3;
de2 = Y_dot- x4;
S2 = lamda2*e2 + de2;
T2 = (1/alpha2)*abs(S2);
Uas2 = (1/b2)*T2*tanh(S2);
U3 = (1/b2)*(Y_dot_dot - (x2*x6*a3) - (x2*a4*ohm) + (lamda2*de2)) + Uas2;
```

```

lamda3 = 10;
e3 = sin(t) - x5;
de3 = cos(t) - x6;
S3 = lamda3*e3 + de3;
T3 = (1/alpha3)*abs(S3);
Uas3 = (1/b3)*T3*tanh(S3);
U4 = (1/b3)*(-sin(t) - (x4*x2*a5) + (lamda3*de3)) + Uas3;
lamda4 = 30;
e4 = 1 - x7;
de4 = - x8;
S4 = lamda4*e4 + de4;
T4 = (1/alpha4)*abs(S4);
Uas4 = T4*tanh(S4);
U1 = (m/(cos(x1)*cos(x3))) * ( g - (lamda4*de4) - Uas4);
j=1;
lamda5 = 6;
e5 = sin(t) - x9;
de5 = cos(t) - x10;
S5 = lamda5*e5 + de5;
T5 = (1/alpha5)*abs(S5);
Uas5 = T5*tanh(S5);
lamda6 = 6;
e6 = cos(t) - x11;
de6 = -sin(t) - x12;
S6 = lamda6*e6 + de6;
T6 = (1/alpha6)*abs(S6);
Uas6 = T6*tanh(S6);
if U1==0
    Ux=0;Uy=0;
else
    Ux = (m*(-sin(t) + lamda5*de5 + Uas5))/U1;
    Uy = (m*(-cos(t) + lamda6*de6 + Uas6))/U1;
end
h=cos(t);
function ohm = Motorspeed(U1, U2, U3, U4)
b = 2.9842e-5;
d = 3.2320e-7;
om1= U1/(4*b) - U4/(4*d) + U3/(2*b);
om2= U1/(4*b) + U4/(4*d) - U2/(2*b);
om3= U1/(4*b) - U4/(4*d) - U3/(2*b);
om4= U1/(4*b) + U4/(4*d) + U2/(2*b);
om1= max(min(om1,920),0);
om2= max(min(om2,920),0);
om3= max(min(om3,920),0);
om4= max(min(om4,920),0);
w1 = sqrt(om1);
w2 = sqrt(om2);
w3 = sqrt(om3);
w4 = sqrt(om4);
ohm = - w1 + w2 - w3 + w4;
function [pdot,qdot,rdot] = Rot_motion(p,q,r,U2,U3,U4,ohm)
Ixx = 3.8278e-3;
Iyy = 3.8278e-3;
Izz = 7.6566e-3;
Im = 2.8385e-5;
l = 0.5;
pdot = ((Iyy-Izz)*q*r + l*U2 + Im*q*ohm)/Ixx;
qdot = ((Izz-Ixx)*p*r + l*U3 - Im*p*ohm)/Iyy;
rdot = ((Ixx-Iyy)*p*q + U4)/Izz;

```

```

function [phi_dot, theta_dot, psi_dot] = eulerangle(phi, theta, p, q, r, t)
phi_max= pi/2;
phi_min= -pi/2;
D=0;
if (phi> phi_max)
    phi= pi/2;
end
if (phi< phi_min)
    phi= -pi/2;
end
if (theta> phi_max)
    theta= pi/2;
end
if (theta< phi_min)
    theta= -pi/2;
end
if (t <= 5)
    D=0;
end
if (t > 5&&t<15)
    D=3.8;
end
if (t > 15)
    D=0;
end
phi_dot = p + sin(phi)*tan(theta)*q + cos(phi)*tan(theta)*r;
theta_dot = cos(phi)*q - sin(phi)*r;
psi_dot = (sin(phi)*q + cos(phi)*r)/cos(theta);
function [Z_dot_dot, X_dot_dot, Y_dot_dot] = fcn(U1, Ux, Uy, phi, theta, t)
g = 9.81;
m = 0.6;
phi_max= pi/2;
phi_min= -pi/2;
if (phi> phi_max)
    phi= pi/2;
end
if (phi< phi_min)
    phi= -pi/2;
end
if (theta> phi_max)
    theta= pi/2;
end
if (theta< phi_min)
    theta= -pi/2;
end
Z_dot_dot = g - ((cos(phi)*cos(theta)*U1)/m);
X_dot_dot = (U1*Ux)/m;
Y_dot_dot = (U1*Uy)/m;

```