Enhancing bipartite entanglement through OPA coupled to squeezed vacuum reservoir in an optomechanical system

Sisay Belachew Yimenu

A Thesis Submitted to Department of Applied Physics
School of Applied Natural Science

Presented in Partial Fulfillment of the Requirement for the Degree of Master's in Applied Physics (Quantum Optics)

Office of Graduate Studies

Adama Science and Technology University

June, 2022

Adama, Ethiopia

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Advisor: Tamirat Abebe (Ph.D)

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DECLARATION

I hereby declare that this Master Thesis entitled " **Enhancing bipartite entanglement through OPA coupled to squeezed vacuum reservoir in an optomechanical system**."is my original work. That is, it has not been submitted for the award of any academic degree, diploma or certificate in any other university. All sources of materials that are used for this thesis have been duly acknowledged through citation .

Sisay Belachew Yimenu	________________	________________
Student Name	Signature	Date

Recommendation

I, the advisor(s) of this thesis, hereby certify that I have read the revised version of the thesis entitled " **Enhancing bipartite entanglement through OPA coupled to squeezed vacuum reservoir in an optomechanical system**" prepared under my guidance by Sisay Belachew submitted in partial fulfillment of the requirements for the degree of Mater's of Science in Applied Physics (Quantum Optics). Therefore, I recommend the submission of revised version of the thesis to the department following the applicable procedures.

Major Advisor	Signature	Date
Co-advisor	Signature	Date

Approval Page

I, the advisors of the thesis entitled " **Enhancing bipartite entanglement through OPA coupled to squeezed vacuum reservoir in an optomechanical system**"and and developed by Sisay Belachew, hereby certify that the recommendation and suggestions made by the board of examiners are appropriately incorporated into the final version of the thesis.

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Major Advisor	Signature	Date
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Co-advisor	Signature	Date

We, the undersigned, members of the Board of Examiners of the thesis by Sisay Belachew have read and evaluated the thesis entitled " **Enhancing bipartite entanglement through OPA coupled to squeezed vacuum reservoir in an optomechanical system**"and examined the candidate during open defense. This is, therefore, to certify that the thesis is accepted for partial fulfillment of the requirement of the degree of Master of Science in Applied Physics (Quantum Optics).

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List of Abbreviations

OMC	Optomechanical cavitis.
OPA	Optical parametric amplifier.
QLEs	Quantum langevin equations
CM	Covariance matrix.

Abstract

The aim of this thesis is to Enhancing bipartite entanglement through OPA coupled to squeezed vacuum reservoir in an optomechanical system. The Hamiltonian of system derived as entanglement between a cavity field and a movable mirror filled with an optical parametric amplifier coupled to squeezed vacuum resersoir interacts with a nonlinear crystal inside a cavity. By using Heisenberg equations of the system, the dynamics of optomechanical system incorporating new features, obtained based on the linearized quantum Langevin equations. On the other hand the dissipation, quantum noise, fluctuation terms and covariance matrix are obtained by using the quantum Langevin equations. Moreover, the entanglement of optical parametric amplifier coupled to squeezed vacuum reservoir are quantifying by using logarithmic negativity. It is found that the presence of optical parametric amplifier coupled to squeezed vacuum reservoir enhancing entanglement of the cavity radiation. Thus, when an squeezed vacuum reservoir is added inside a cavity, the optomechanical coupling and the entanglement are improved.

Keywords:
Optomechanical system, Entanglement, OPA, Sqeezed vacuum reservoir, Logarithmic negativity

CHAPTER 1

1 INTRODUCTION

1.1 Background of the Study

In recent years, optomechanical system is considered as an ideal device for studying basic quantum mechanics Fiaschi et al. (2021). Much of the fundamental interest in quantum optics is connected with its implications for the conceptual foundations of quantum mechanics. One of recent progress in the field of quantum optics is optomechanics. Optomechanics is an emerging field exploring the interaction between light and mechanical motion(a moving object). Recently, quantum optomechanics is a rapidly growing field in modern quantum physics and is considered as an excellent candidate to couple an optical mode with mechanical mode via radiation pressure.

Cavity optomechanics studies the radiation pressure interaction between a cavity field and the motion of a mechanical oscillator Aspelmeyer et al. (2014a). The simplest form of radiation pressure coupling is the momentum transfer due to reflection that occurs in a Fabry Perot cavity Aspelmeyer et al. (2014b). The momentum transfer from light to a mechanical object exerts a pressure force on the object. One of the goals of cavity optomechanics is to coherently control the quantum motion of mechanical resonators. Even for the stable optomechanical system, provided that the radiation pressure coupling is strong enough, many details of the optomechanical dynamics due to the nonlinear optomechanical interactions are not resolved by the linearized optomechanical model, which provides another reason for us to study the nonlinear effects in the optomechanical system. Cavity optomechanical systems display a parametric coupling between the mechanical displacement ($\hat{x}$) of a mechanical vibration mode and the energy stored inside a radiation mode. With a strong optomechanical coupling strength, the influence of the nonlinear optomechanical interaction becomes even more significant than it is in case of weak coupling regime.

One way to enhance the optomechanical interaction is to apply a strong driving field to the optical cavity which leads to an effective linear optomechanical interaction

Lemonde et al. (2016). Such a field-enhanced linear optomechanical interaction has been proved to be responsible for the cooling of the mechanical motion, as well as, the creation of various quantum correlations between the cavity light field and mechanical oscillator Clerk et al. (2010). One of the simplest non-trivial quantum states of a harmonic oscillator is a squeezed state, where the position and momentum uncertainty still achieve the Heisenberg bound, but are not equal to the ground state uncertainties. Squeezed states are non classical states characterized by a reduction of quantum fluctuations (noise)in one quadrature component below the vacuum level (quantum standard limit), or below that achievable in a coherent state at the expense of increased fluctuations in the other component such that the product of these fluctuations still obeys the uncertainty relation.

Entanglement is one of the most attractive characteristic of quantum mechanics, which is believed as important resource for quantum information Penrose (1998). Quantum communication is the art of transferring a quantum state from one location to another. Entangled or inseparable states are fundamental to the field of quantum information, specifically to quantum teleportation of discrete and continuous variables and to quantum dense coding of discrete and continuous variables, to name a few examples. The majority of experiments on entanglement to date deal with entangled states of light

In recent years, OPA plays more important role in generating entangled and squeezed states of light Z. Yan & Jia (2017); Gräfe & Szameit (n.d.). Zhang et al. Z. Yan & Jia (2017) showed that a squeezed vacuum field can be amplified and quantum fluctuations can be manipulated in the optical parametric amplifiers (OPAs) system. In this paper, we propose placing OPAs inside two spatially separated coupled optomechanical systems to increase the coupling between the movable mirror and the cavity mode. In order to increase the gain, the parametric medium may be placed inside an optical cavity where it is coherently pumped. Moreover, the entanglement increases with the increasing parametric gain and larger input laser power C.-y. Zhang et al. (2016).

Currently, in many of the previous analysis, the strong driving field to the optical cavity coupled to mechanical oscillator influenced by a thermal noise on optomechanical system is neglected. Therefore, the novelty of this work relies on enhancing entanglement produced by OPA coupled to squeezed vacuum reservoir, which used to minimize the influence of the squeezing-induced noise. With this notion, to produce enhancing entanglement between a cavity field and a movable mirror filled with an optical parametric amplifier coupled to squeezed vacuum resersoir in an optomechanical system interacts with a nonlinear crystal inside a cavity. Entanglement among macroscopic systems with the existence of optical parametric amplifiers including degenerate OPA and squeezed vacuum reservoir can be pronouncedly increased compared to that without of optical parametric amplifiers coupled squeezed vacuum reservoir.

1.2 Statement of the Problem

Recently, quantum optomechanics is a rapidly growing field in modern quantum physics and is considered as an excellent candidate to couple an optical mode with mechanical mode via radiation pressure Aspelmeyer et al. (2014b); Meystre (2013). It offer us several promising application in both theoretical and experimental such as cooling a mechanical system to its quantum ground state Marquardt et al. (2008); Kingni et al. (2020), entangling microscopic and macroscopic system Kuzyk et al. (2013); Wang et al. (2015), generation of bipartite entanglementLi et al. (2016), Gaussian states correlations Marian et al. (2015); Ghiu et al. (2018), quantum synchronization Li et al. (2016), optomechanical weak force-sensing Kuzyk et al. (2013); Møller et al. (2017), photon transmission Zeng et al. (2018) , performing high-precision measurements Ma et al. (2014); K. Zhang et al. (2015) and processing quantum information Zeng et al. (2019); Li et al. (2015). Entanglement among macroscopic systems with the improvement of the qualities of the nonlinear crystals, optical parametric amplifiers including degenerate OPA and nondegenerate OPA are the larger entanglement of the optomechanical system corresponds to the larger input laser power.

All these are the new events becomes an interesting research topic and potential platforms to study enhancing entanglement by OPA coupled to squeezed vacuum reservoir with Optomechanical system by using a method of logarithmic negativitym. Specifically, the thesis attempts to answer the following questions:

- How the Hamiltonian model describing the system of we will develop?

- How can linearized quantum langevin equation deduced to matrix form?
- How does the entanglement generated by optomechanical system with optical parametric amplifier coupled to squeezed vacuum reservoir is quantifying by using logarithmic negativity?

1.3 Objectives of the Study

1.3.1 General objective

The general objective of thesis is to study Enhancing Entanglement by Optical parametric amplifier coupled to squeezed vacuum reservoir with Optomechanical system

1.3.2 Specic objectives

The specific objectives of the study are;

- Designing the model and Hamiltonitian of system
- Determining nonlinear quantum Langevin equations of the system
- Obtaining quadrature fluctuation for catity field
- Determine the determinant of drift matrix and explain nature of steady state solution
- Quantifying entanglement by OPA using a method of logarithmic negativity
- Explain Entanglement improved by optical parametric amplifier coupled to squeezed vacuum reservoir

1.4 Significance of the Study

This study has great importance to the quantum entanglement in macroscopic physical system and entanglement between the two basic components (i.e. the cavity and mechanical modes) interacts with a nonlinear crystal inside a cavity filled with an optical parametric amplifier. Such entanglements results may have spectacular the crucial resources for quantum information processing and quantum communication network and contribution for constructing technologies. In this paper, we propose a scheme to produce the entanglement between a cavity field and a movable mirror filled with an optical parametric amplifier coupled to squeezed vacuum resersoir in an optomechanical system interacts with a nonlinear crystal inside a cavity .

CHAPTER 2

2 LITERATURE REVIEW

2.1 Overview of the Study

In this chapter, we describe a cavity optomechanical system, a degenerate parametric amplifier which is used to generate squeezed light, Hamiltonian formulation of optomechanical system, Heisenberg equation and quantum langevin equation to determine the system dynamics and logarithmic negativity which is used to quantifying entanglement of an optomechanical system with optical parametric amplifier coupled to squeezed vacuum reservoir.

2.2 Optical and Optomechanical cavities

Optomechanical cavities are optical cavities with at least one mechanical mode coupled to at least one intracavity optical mode Aspelmeyer et al. (2012). An optical cavity (also referred to as an optical resonator or a resonating cavity) is an optical device that temporarily traps incoming light. The most common form of the optical cavity is the Fabry-Perot cavity Paternostro et al. (2006). These devices utilize two at mirrors to trap and amplify input light.

It seems clear that a new field of cavity optomechanics has emerged, and will soon evolve into cavity quantum optomechanics cavity (QOM) whose goal is the obser-

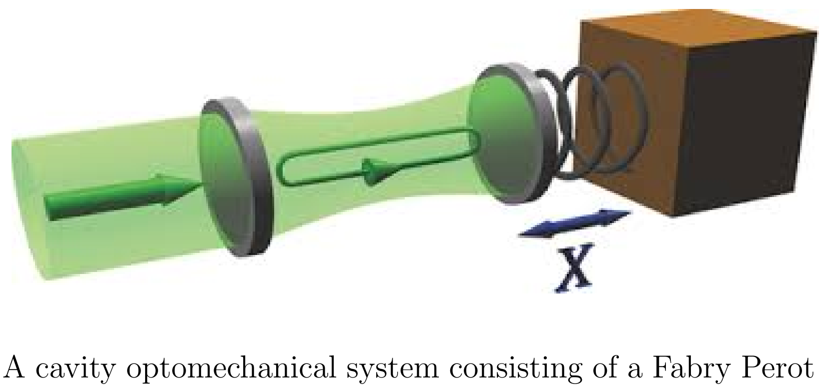

Figure 1: A cavity optomechanical system consisting of a Fabry Perot cavity with a harmonically bound end mirror

vation and exploration of quantum phenomena of mechanical systems Schwab &

Roukes (2005) as well as quantum phenomena involving both photons and mechanical systems. An optomechanical cavity (OMC) is an optical cavity that includes a mechanical oscillator that is coupled to the intracavity electromagnetic field. These types of OMC's couple the optical to mechanical fields via the momentum transfer that comes from the intracavity radiation pressure. Cavity optomechanics, which is based on the interaction between light and mechanical resonators by radiation pressure, can provide a good platform for studying quantum entanglement at macroscopic scales R.-G. Yang et al. (2017). Moreover, many theoretical and experimental progresses have been achieved in the field of optomechanics, such as ground-state cooling and strong mechanical squeezing Tan & Zhan (2019)

2.3 Radiation Pressure in Quantum Optomechanical System

When a laser pump light incidents and hits on the surface of the mirror in optomechanical system, it exert a force at its surface area and the pressure formed. The fundamental mechanism that couples the properties of the cavity radiation field to the mechanical motion is the momentum transfer of photons Aspelmeyer et al. (2014b). Moreover, radiation pressure is the mechanical pressure exerted upon any surface due to the exchange of momentum between the object and the electromagnetic field. These includes the momentum of light or electromagnetic radiation of any wavelength which is absorbed, reflected or otherwise emitted by matter on any scale. The radiation pressure force originates from the fact that light carries momentum. The momentum transfer from light to a mechanical object exerts a pressure force on the object.The typical experimental setup in optomechanics consists of an optical cavity where one of the endmirrors can move (Fig. 2)Marquardt & Girvin (2009) . The photons inside the cavity with wavenumber κ, each transfer momentum of with $2\hbar\kappa$ (with $\hbar$:Planck's constant) onto the movable mirror, displace it and bring a phase shift depending on its position. A single photon transfers the momentum $\Delta p = 2\hbar\kappa$ onto the mirror due to the conservation of momentum. The radiation pressure force is simply the derivative of interaction Hamiltonian

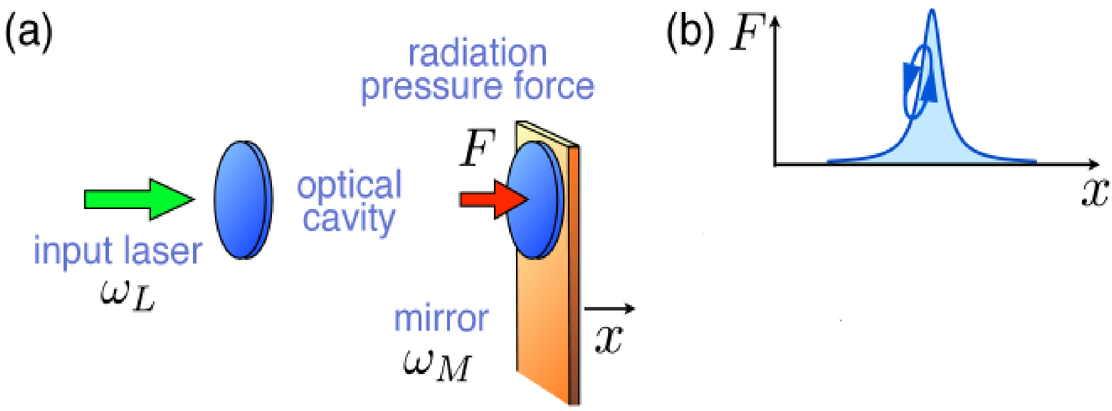

Figure 2: (a) Schematic optomechanical setup. (b) Radiation pressure force vs. position.

with respect to displacement Westervelt (1957)

$$\langle \hat{F} \rangle = -\frac{dH_{in}}{dx} = \hbar \frac{\omega_c}{\ell} \langle a^{\hat{\dagger}} \hat{a} \rangle, \tag{2.1}$$

where hence, F is radiation pressure force and H_{in} is interaction Hamiltonian of optomechanical system. Therefore, $\hbar\frac{\omega_c}{\ell}$ describes the radiation-pressure force caused by one intra-cavity photon. The number of photons inside the cavity, causes the radiation pressure force and consequently the displacement of the mirror, the loop is closed; the radiation pressure force effectively depends on the mirror position Marquardt & Girvin (2009). Another advantage of optical cavities is that the modulation of the cavity length through an oscillating mirror can directly be seen in the spectrum of the cavity.

2.4 Entanglement of Optomechanical systems

Entanglement is one of the most attractive topics of quantum mechanics, which is basic and practical significance that we can generate entanglement among even macroscopic systems Bell & Bell (2004). Entanglement, especially the entanglement of macroscopic objects, is of great interest both for fundamental physics and for possible applications in quantum information processing Kotler et al. (2020). Quantum entanglement is a fundamental phenomenon providing the crucial resources for quantum information processing and quantum communication Nielson & Chuang (2000). Recent experimental progress makes it possible to manipulate quantum states of macroscopic mechanical objects by means of optical or microwave radiation pressure.

A cavity optomechanical system is an available candidate system to study the entanglement of macroscopic states. With the improvement of the qualities of the nonlinear crystals, optical parametric amplifiers (OPAs) including degenerate OPA and non degenerate OPA, have shown tremendous applications in generating squeezed and entangled states C.-y. Zhang et al. (2016).

2.5 Optical Parametric Amplifiers

The Optical Parametric Amplifiers gives rise to single-mode squeezing of the cavity fields, which results in significant improvement of the two-mode entanglement Streshinsky et al. (2013). An optical parametric amplifier is generally used to induce a squeezed cavity mode Wu et al. (1986). In the context of optomechanical systems, an OPA can improve optomechanical cooling Huang & Agarwal (2009a), and modify the normal-mode splitting behavior of the coupled movable mirror and the cavity field Huang & Agarwal (2009c), as well as enhance the optomechanical interaction strength into the single-photon strong-coupling regime Lü et al. (2015). It is one of the most interesting and well characterized optical devices in quantum optics. This simple dissipative quantum system plays an important role in the study of squeezed states. In Optical Parametric Amplifiers a strong pump photon interacts with a nonlinear-medium (crystal) inside a cavity and is down-converted into two photons of smaller frequencies Abebe et al. (2020). Moreover, the mechanical entanglement is more robust to thermal noise and cavity decay with the assistant of OPAs. In addition, we find the optimal frequency for the laser pumping the OPAs, that maximize the mechanical entanglement Optical parametric amplifier (OPA) including degenerate OPA and nondegenertae OPA, is a nonlinear laser frequency conversion device to amplify intensity of signal light by nonlinear crystal Pan et al. (2016). In degenerate OPA, we assume that the strong pump photon is down converted into a pair of signal photons each of frequency ω and these modes are referred to as signal modes. In recent years, OPA plays more important role in generating entangled and squeezed states of lightZhu et al. (2012); J. Zhang et al. (2008) Zhang et al. J. Zhang et al. (2008) showed that a squeezed vac-

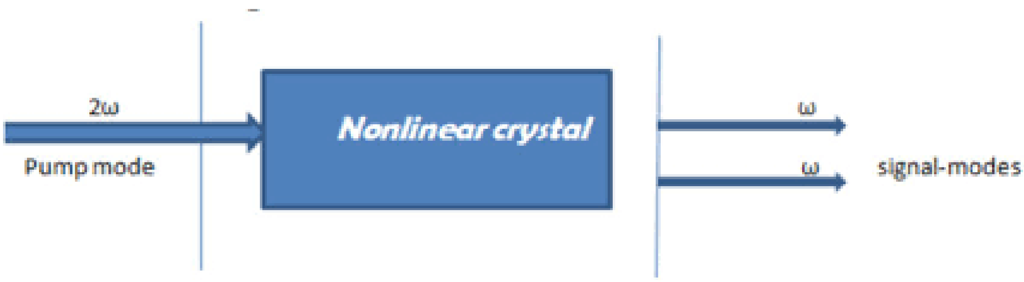

Figure 3: A degenerate parametric amplifier with nonlinear crystal pumped by coherent light.

uum field can be amplified and quantum fluctuations can be manipulated in the optical parametric amplifiers (OPAs) system. Its essential to generate squeezed states with non-classical properties which have potential applications in quantum optical communications and computation, gravitational wave detection, interferometry, spectroscopical measurements and for the study of fundamental concepts Xavier et al. (2021).

2.6 Hamiltonian Formulation of Harmonic Oscillator

The fundamental mechanism that couples the properties of the cavity radiation field to the mechanical motion is the momentum transfer of photons, i.e. radiation pressure. We assume it is a single harmonic oscillator with position $\hat{x}$ and momentum $\hat{p}$ satisfying $[\hat{x}, \hat{p}] = i\hbar$ Chang et al. (2002). For coupled oscillators,

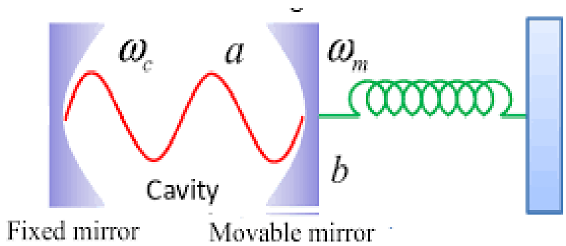

Figure 4: Optomechanical coupling between a optical cavity and a mechanical resonator .

the Hamiltonian can be written a (the sum of free hamiltonian plus the interaction hamiltonian of oscillators) will be;

$$\hat{H} = \hat{H}_{free} + \hat{H}_{int} \tag{2.2}$$

Then, the free Hamiltonian of Eq. (2.2) becomes

$$\hat{H}_{free} = \frac{p^2}{2m} + \frac{1}{2}m\omega^2 x^2, \tag{2.3}$$

where x and p correspond to the mean position and momentum, respectively

$$\hat{H}_{free} = \frac{1}{2}(P^2 + m\omega^2 x^2) \tag{2.4}$$

Now, $P^2 + m\omega^2 x^2 = (m\omega\hat{x} - ip)(m\omega\hat{x} + i\hat{p}) + m\omega\hbar$ this yielding as

$$\begin{aligned}
\hat{H}_{cav} &= \frac{1}{2m}[(m\omega\hat{x} - ip)(m\omega\hat{x} + i\hat{p}) + m\omega\hbar)] \\
&= \frac{\hbar\omega}{2m\omega\hbar}[m\omega\hat{x} - ip)(m\omega\hat{x} + i\hat{p}] + \frac{\omega\hbar}{2} \\
&= \hbar\omega[\frac{(m\omega\hat{x} - ip}{\sqrt{2\hbar m\omega}})\frac{(m\omega\hat{x} - ip}{\sqrt{2\hbar m\omega}})] + \frac{\omega\hbar}{2}
\end{aligned} \tag{2.5}$$

In quantum physics, it is conventional to introduce the lowering and raising operators of the harmonic oscillator as

$$\hat{a} = \sqrt{\frac{m\omega_m}{2\hbar}}\hat{x} + i\sqrt{\frac{1}{2\hbar\omega_m}}\hat{p}, \hat{a}^\dagger = \sqrt{\frac{m\omega_m}{2\hbar}}\hat{x} - i\sqrt{\frac{1}{2\hbar\omega_m}}\hat{p} \tag{2.6}$$

The corresponding commutation relations are $[\hat{a}, \hat{a}^\dagger] = 1$ and $[\hat{a}, \hat{a}] = [\hat{a}^\dagger, \hat{a}^\dagger] = 0$, and the harmonic oscillator of Eq. (2.5) Hamiltonian becomes

$$\hat{H}_{cav} = \hbar\omega(\hat{a}^\dagger\hat{a} + \frac{1}{2}) \tag{2.7}$$

The offset term, corresponding to the zero-point energy, makes no contribution to the Hamiltonian dynamics, and hence is often omitted. We have neglected the $\frac{1}{2}\hbar\omega$ since this will not affect our results.

$$\hat{H}_{cav} = \hbar\omega_c(x)\hat{a}^\dagger\hat{a}, \tag{2.8}$$

where ω_c is the frequency of the optical mode and x is the position of the mechanical resonator. In the same manner we can determine the Hamiltonian of mechanical resonator as

$$\hat{H}_{mech} = \hbar\omega_m\hat{b}^\dagger\hat{b}, \tag{2.9}$$

where $\hat{b}$ and $\hat{b}^\dagger$ are annihilation and creation operator of mechanical resonators respectively and ω_m is mechanical mode frequency.

$$\hat{b} = \sqrt{\frac{m\omega_m}{2\hbar}}\hat{x} + i\sqrt{\frac{1}{2\hbar\omega_m}}\hat{p}, \hat{b}^\dagger = \sqrt{\frac{m\omega_m}{2\hbar}}\hat{x} - i\sqrt{\frac{1}{2\hbar\omega_m}}\hat{p} \tag{2.10}$$

For a simple cavity of length ℓ, because the optical cavity and mechanical oscillator are coupled by radiation pressure and by the mechanical oscillator physically changing the length of the optical cavity. when the mirror moves by a distance x, the length of the optical cavity increases by $\ell + x$. Then,

$$\omega_c = \frac{A}{\ell + x}, \tag{2.11}$$

where $n = 1, 2, 3, ...$ and ℓ is the size of the cavity and $A = 2n\pi c$ is a constant and c is the speed of light in vacuum

In this case the coupling comes from the fact that the cavity frequency ω_c actually depends on the position of the mirror. When the mirror is allowed to vibrate we should then replace ℓ by $\ell + x$. Assuming that x is small compared to ℓ we can then get

$$\omega_c(\ell + x) \simeq \frac{A}{\ell}(1 - \frac{x}{\ell}) = \omega_c - \frac{\omega_C}{\ell}x \tag{2.12}$$

After solving Eq. (2.10) simultaneously we get

$$\hat{x} = \sqrt{\frac{\hbar}{2m\omega_m}}(\hat{b} + \hat{b}^\dagger), \tag{2.13}$$

where $\omega_c = \omega_c(\ell)$ is the equilibrium frequency of the cavity. Replace x by $\hat{x}$ consequently, we see that the Hamiltonian $\omega_c \hat{a}^\dagger \hat{a}$ is to be transformed into

$$\omega_c \hat{a}^\dagger \hat{a} \rightarrow \omega_c \hat{a}^\dagger \hat{a} - \frac{\omega_c}{\ell}\hat{a}^\dagger \hat{a}\hat{x} \tag{2.14}$$

Therefore we have a coupling between $\hat{a}^\dagger \hat{a}$ and $\hat{x}$. This is called the radiation pressure coupling. We replace $\hat{x}$ by $\sqrt{\frac{\hbar}{2m\omega_m}}(\hat{b} + \hat{b}^\dagger)$, then we get the following equation;

$$\hat{H}_{cav-int} = \omega_c \hat{a}^\dagger \hat{a} - \frac{\omega_c}{\ell}\hat{a}^\dagger \hat{a}\sqrt{\frac{\hbar}{2m\omega_m}}(\hat{b} + \hat{b}^\dagger), \tag{2.15}$$

where $g_o = \frac{\omega_c}{\ell} x_{zpt}$. This is the so-called radiation pressure optomechanical coupling. Where $x_{zpt} = \sqrt{\frac{\hbar}{2m\omega_m}}$ is the zero-point fluctuation amplitude of the mechanical oscillator. The larger the zero point uncertainty is, the easier it is to detect the quantum fluctuations of the resonator. For this reason, in order to reach the quantum limit, it would appear desirable to have a resonator with a small mass and a small frequency

$$\hat{H}_{cav-int} = \hbar\omega_c \hat{a}^\dagger \hat{a} - \hbar g_o \hat{a}^\dagger \hat{a}(\hat{b} + \hat{b}^\dagger) \tag{2.16}$$

suppose $\hat{b}+\hat{b}^{\dagger}=\hat{q}$,then collecting Eq. (2.9) and Eq. (2.16),we obtain

$$\hat{H}=\hbar\omega_c\hat{a}^{\dagger}\hat{a}+\hbar\omega_m\hat{b}^{\dagger}\hat{b}-\hbar g_o\hat{a}^{\dagger}\hat{a}\hat{q} \tag{2.17}$$

Eq. (2.17) serve as standard optomechanical Hamiltonian of the optical and mechanical mode with the optomechanical interaction between the optical mode and the mechanical mode with out driving field for any optomechanical systems.

2.7 Squeezed vacuum reservoir

The vacuum reservoir contains a vacuum mode radiations contributed by vacuum fields. This interaction is responsible for many quantum features of light which are not observed in the classical phenomena. Squeezed vacuum reservoir increases the degree of non classical features of light produced by the system. Furthermore, using the criteria developed by logarithm negativity the quantum entanglement of the cavity mode is quantified Gashu et al. (2020). The coupling between the cavity modes and the reservoirs can be described by the expression Y. Yan et al. (2015).

$$\hat{H}_{C-R}=i\sqrt{\frac{\kappa}{\pi}}\int d\omega(\hat{a}_{\omega}^{\dagger}\hat{a}-\hat{a}^{\dagger}\hat{a}_{\omega}), \tag{2.18}$$

where $\hat{a}_{\omega}(\hat{a}_{\omega}^{\dagger})$ are the annihilation(creation) operators of optical reservoir. The OPA is used to generate a squeezed-vacuum reservoir, which couples to the cavity mode to minimize the influence of the squeezing-induced noise.

2.8 Open Quantum System

2.8.1 Quantum Langevin equations

In a real system, dissipation of both photons and phonons are present. Due to fluctuation-dissipation processes Hu et al. (2020), the dynamics of such system subject to thermal environment should be described by the quantum Langevin equations (QLEs) i.e.

$$i\hbar\dot{\hat{Z}}=[\hat{Z},\hat{H}]+N-H_{dis}, \tag{2.19}$$

where N is quantum noise and H_{dis} describe the dissipation of each mode and $\hat{Z}$ is an arbitrary operator for $\hat{p},\hat{q}$ and $\hat{a}$.

2.8.2 Linearization of Quantum Langevin Equations

The linearization of the system operators around the steady-state under strong intense laser driving intracavity field amplitude and one can adopt the standard linearization technique to the QLEs. Since the problem is nonlinear, we write each operator of the system as the sum of its steady state mean value and a small fluctuation with zero mean value C.-y. Zhang et al. (2016)

$$\hat{Z} = Z_s + \delta Z, \tag{2.20}$$

where $\hat{Z} = \hat{a}, \hat{q}, \hat{p}$ and Z_s are amplitude of system and δZ are fluctuation with zero mean value. As shown in later sections, these linearized equations are able to fully describe several phenomena, including amplification, and parametric normal-mode splitting (i.e., strong, coherent coupling) Aspelmeyer et al. (2014b).

2.8.3 Covariance Matrix of the System

The quantum fluctuations of the system between appropriate quadrature is the interaction of cavity fields and mechanical resonator can be generated. Hence, we can introduce the quadrature operator of optical field for position and momentum for the optical cavity field quadratures and input noise quadratures Mancini et al. (2003). For this purpose, we define the quadrature operators for Eq. 2.20 in terms of the dimensionless quadratures, respectively, for the optical and mechanical modes as The cavity field quadratures;

$$\begin{aligned} \delta\hat{x} &= \frac{\delta\hat{a}^{\dagger} + \delta\hat{a}}{\sqrt{2}} \\ \delta\hat{y} &= i\frac{\delta\hat{a}^{\dagger} - \delta\hat{a}}{\sqrt{2}} \end{aligned} \tag{2.21}$$

the input noise quadratures

$$\begin{aligned} \delta\hat{x}_{in} &= \frac{\delta\hat{a}_{in}^{\dagger} + \delta\hat{a}_{in}}{\sqrt{2}} \\ \delta\hat{y}_{in} &= i\frac{\delta\hat{a}_{in}^{\dagger} - \delta\hat{a}_{in}}{\sqrt{2}} \end{aligned} \tag{2.22}$$

Consequently, the linearized quantum Langevin equations allow us to deduce the covariance matrix

2.9 Entanglement Quantification Criteria

A pair of particles is taken to be entangled in quantum theory, if its states cannot be expressed as a product of the states of its separate constituents. The preparation and manipulation of these entangled states that have non-classical and non-local properties lead to a better understanding of the basic quantum principles. Here, we study the single mode entanglement in the cavity. Hence, a lot of criteria have been developed to measure, detect and manipulate the entanglement generated by various quantum optical devices. Some of the criteria are includes Logarithmic negativity, Duan *et al.*, Hillary Zubairy criteria and violation of Cauchy-Schwarz inequality inseparability criteria to measure the degree of entanglement Gashu et al. (2020).

2.9.1 Logarithmic Negativity

Quantum entanglement can be observable in bipartite or tripartite forms that as key tool for information processing such as super-dense coding, quantum teleportation and protocols of quantum cryptography. The negativity of correlation matrix as well as the logarithmic E_N can be formulated in a practical or experimentally and easy to calculate way using the sympletic egien value $\bar{\eta}$ of the partial transpose of the correlation matrix defined i.e,

$$\bar{\eta} = \sqrt{\frac{\Sigma V - \sqrt{V^2 - 4\det V}}{2}}, \tag{2.23}$$

where $\bar{\eta}$ is the minimum symplectic eigenvalue of partial transposed cavity mode corresponding to the bipartite system C.-y. Zhang et al. (2016). The logarithmic negativity is an entanglement measure which is easily computable and an upper bound to the distillable entanglement. It is defined as

$$E_N = max\{0, -\ln 2\bar{\eta}\}, \tag{2.24}$$

where V can be expressed in a form of block matrix as

$$V = \begin{pmatrix} F_A & F_C \\ F_C^T & F_B \\ & \end{pmatrix} \tag{2.25}$$

The negative partial transpose must be parallel with respect to the entanglement monotone in order to obtain the degree of entanglement. And

$$\Sigma V = \det F_A + \det F_B - 2 \det F_C, \tag{2.26}$$

where F_A, F_B and F_C are 2 x 2 are matrices extracted from the correlation matrix. The entanglement is achieved when E_N is positive within the region of the lowest eigenvalue of covariance matrix $\bar{\eta}$<1

CHAPTER 3

3 METHODOLOGY

3.1 Overview of the Study

In this chapter, we study how to enhancing entanglement produced by optomechanical system with optical parametric amplifier coupled to squeezed vacuum reservoir. By using Heisenberg equations of the system, the dynamics of optomechanical system incorporating new features, obtained based on the linearized quantum Langevin equations. On the other hand the dissipation, quantum noise, fluctuation terms and covariance matrix are obtained by using the quantum Langevin equations.

3.2 Methods

In our work, the Heisenberg equations for the single-mode entanglement by the OPA coupled to a squeezed vacuum reservoirs incorporating new features, a degenerate optical parametric amplifier can be derived based on the linearized quantum Langevin equations. The solutions of the steady-state condition resulting from the differential equations can be determined. To determine entanglement, we need to choose a convenient measure. Some of the criteria are includes Logarithmic negativity, Duan *et al.*, Hillary Zubairy criteria and violation of Cauchy-Schwarz inequality inseparability criteria to measure the degree of entanglement.Here, our results can be quantify by using the logarithmic negativity entanglement criteria

3.3 Model and Hamiltonian Formulation

We consider a setup consisting of two mirrors with Fabry Perot cavity, where an optical parametric amplifier is inserted in a Fabry Perot cavity with a fixed mirror and a perfectly reflective movable mirror driven by a laser. For a given parametric gain, the larger entanglement between a cavity modes and movable mirrors and the interaction between the driving field and the cavity mode and parametric interaction by larger input laser power as shown in figure 5. In this way, enhancing entanglement produced by optomechanical system with optical parametric amplifer coupled to squeezed vacuum reservoir pumped by classical laser field and a cavity comprises by an optical parametric amplifier is inserted in a Fabry–Perot cavity with a fixed mirror and a movable mirror as observed from fig.5. By this principle,

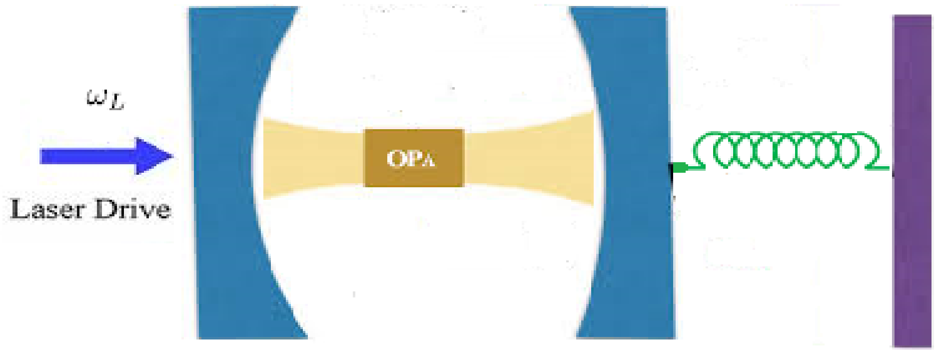

Figure 5: Entanglement of OPA coupled to squeezed vacuum reservoir in an optomechanical system.

our system consists of a partially reflected mirror (fixed mirror) and a fully reflected mirror (movable mirror), which is in harmonic oscillator and a degenerate parametric amplifier placed between them and the cavity mode is driven by a laser field and the degenerate parametric amplifier is pumped by another laser at a frequency of ω_L.

The Hamiltonian of driven by a laser field C.-y. Zhang et al. (2016), can be

$$\hat{H}_d = i\hbar\varepsilon(\hat{a}^\dagger e^{-i\omega_L t} - \hat{a}e^{i\omega_L t}) \tag{3.1}$$

And the coupling between the OPA and the cavity field also,

$$\hat{H}_{PA} = i\hbar G(e^{i\theta}\hat{a}^{\dagger 2}e^{-i\omega_L t} - e^{-i\theta}\hat{a}^2 e^{i\omega_L t}) \tag{3.2}$$

The interaction between cavity modes and optical reservoirs

$$\hat{H}_{int}(t) = i\hbar A(t)[\hat{a}^\dagger_{in}(t')\hat{a} - \hat{a}^\dagger\hat{a}_{in}(t)], \tag{3.3}$$

where

$$A = \sqrt{\frac{\kappa}{\pi}}\int_{-\infty}^{\infty} e^{i\omega(t-t')}d\omega \tag{3.4}$$

The total Hamiltonian of coupled system is given as

$$\begin{aligned}\hat{H}_{eff} &= \hbar\omega_c\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega_m(\hat{p}^2 + \hat{q}^2) - \hbar g_o\hat{a}^\dagger\hat{a}\hat{q} + i\hbar\varepsilon(\hat{a}^\dagger e^{-i\omega_L t} - \hat{a}e^{i\omega_L t})\\ &+ i\hbar A(t)[\hat{a}^\dagger_{in}(t')\hat{a} - \hat{a}^\dagger\hat{a}_{in}(t)] + i\hbar G(e^{i\theta}\hat{a}^{\dagger 2}e^{-\omega_L t} - e^{-i\theta}\hat{a}^2 e^{\omega_L t}),\end{aligned} \tag{3.5}$$

where ω_c resonance frequency, ω_L is driving and pump mode frequency, ω_m is mechanical frequency, $\hat{p}$ and $\hat{q}$ are dimensionless position and momentum of a mechanical resonator respectively, $\hat{a}$ and $\hat{a}^\dagger$ are annihilation and creation operator of cavity mode respectively, g_o is the single optomechanical coupling strength between the cavity field and mechanical mirror, ε is amplitude of the field driving laser and the cavity field (input field intensity related to the input power $\varepsilon = \sqrt{\frac{2\kappa p}{\hbar\omega_L}}$, where $\kappa = \frac{\pi c}{2f\ell}$ is optical decay rate with f is the cavity finesse, $P = \hbar\omega_L\langle\hat{a}^\dagger\hat{a}\rangle$ is the input laser power, i.e $\langle\hat{a}^\dagger\hat{a}\rangle$ is the rate of photons arriving at the cavity and ℓ is cavity length, G is a nonlinear gain of the OPA, which is proportional to the power of the pump mode and θ is the phase of the optical field driving the OPA Huang & Agarwal (2009b).

The movable mirrors are considered as quantum mechanical harmonic oscillators, with mass m, frequency ω_m and γ_m damping rate.

Applying the unitary transformation $\hat{U} = exp^{i\omega_\ell\hat{a}^\dagger\hat{a}t}$ makes the driving terms time-independent, and generates a new Hamiltonian $\hat{H}_{eff} = \hat{U}H_{old}\hat{U} + i\hbar\frac{\partial\hat{U}}{\partial t}\hat{U}^\dagger$. In particular, raising and lowering operators are related as $\hat{a} = \hat{a}e^{-i\omega_L t}$ and $\hat{a}^\dagger = \hat{a}^\dagger e^{i\omega_L t}$ Eq.(3.6) becomes

$$\begin{aligned}
\hat{H}_d &= \hat{U}H_d\hat{U} + i\hbar\frac{\partial\hat{U}}{\partial t}\hat{U}^\dagger \\
&= e^{i\omega_L\hat{a}\hat{a}^\dagger t} i\hbar\varepsilon(\hat{a}^\dagger e^{i\omega_L t}e^{-i\omega_L t} - \hat{a}e^{-i\omega_L t}e^{i\omega_L t})e^{-i\omega_L\hat{a}\hat{a}_\dagger t} \\
&+ i\hbar\frac{\partial}{\partial t}(e^{i\omega_L\hat{a}\hat{a}^\dagger t})e^{-i\omega_L\hat{a}\hat{a}^\dagger t} \\
&= e^{i\omega_L\hat{a}\hat{a}^\dagger t} i\hbar\varepsilon(\hat{a}^\dagger - \hat{a})e^{-i\omega_L\hat{a}\hat{a}^\dagger t} + i\hbar i\omega_L\hat{a}\hat{a}^\dagger \\
&= i\hbar\varepsilon(\hat{a}^\dagger - \hat{a}) - \hbar\omega_L\hat{a}\hat{a}^\dagger
\end{aligned} \tag{3.6}$$

In the same calculus, the coupling between the OPA and the cavity field becomes,

$$\hat{H}_{PA} = i\hbar G(e^{i\theta}\hat{a}^{\dagger 2} - e^{-i\theta}\hat{a}^2) \tag{3.7}$$

The last term of Eq.(3.7) vanishes, it does not affect our result and then the equation becomes

$$\hat{H}_{PA} = i\hbar G(e^{i\theta}\hat{a}^{\dagger 2} - e^{-i\theta}\hat{a}^2) \tag{3.8}$$

Then, the total Hamiltonian of the system yielding as the following expressions

$$\begin{aligned}\hat{H}_{eff} =&\hbar\omega_c\hat{a}^\dagger\hat{a} - \hbar g_o\hat{a}^\dagger\hat{a}\hat{q} + \frac{1}{2}\hbar\omega_m(\hat{p}^2 + \hat{q}^2) + i\hbar\varepsilon(\hat{a}^\dagger - \hat{a}) - \hbar\omega_L\hat{a}\hat{a}^\dagger \\ &+ \hbar A(t)[\hat{a}_{in}^\dagger(t')\hat{a} - \hat{a}^\dagger\hat{a}_{in}(t)] + i\hbar G(e^{i\theta}\hat{a}^{\dagger 2} - e^{-i\theta}\hat{a}^2)\end{aligned} \tag{3.9}$$

The first term of Eq.(3.9) is cavity mode energy. The second term arises from the coupling of the movable mirror to the cavity field via radiation pressure (the interaction energy between the cavity field and mechanical mirror), the third term gives the energy of the movable mirror, the fourth term describes the coupling between the input laser field and the cavity field, the fifth term indicate coupling between the cavity modes and the reservoirs and the last term is the coupling between the OPA and the cavity field.

3.4 Dissipation and quantum noise of open quantum system

In a real system, dissipation of both photons and phonons is presentUnder this condition, the full dynamics of the system is quantized using the standard approach of Heisenberg-Langevin equation of motion, and taking the corresponding damping and noise effects of the reservior, optical cavity as well as the mechanical modes. In addition, there is necessarily vacuum noise (and possibly thermal noise) entering the system via these dissipation channels. If we assume the photon loss rate to be κ (is the cavity decay rate) Brennecke et al. (2008), then the equation of motion contains additional terms.

Due to fluctuation-dissipation processes, the dynamics of such system subject to thermal environment should be described by the quantum Langevin equations (QLEs) Aspelmeyer et al. (2014a)i.e

$$i\hbar\dot{\hat{Z}} = [\hat{Z}, \hat{H}_{eff}] + N - H_{dis}, \tag{3.10}$$

where H_{dis}(describe the dissipation of each mode as well as the driving the cavity mode) in the Langevin approach and $\hat{Z}$ is an arbitrary operator for $\hat{p}, \hat{q}$ and $\hat{a}$. The nonlinear quantum Langevin equations can be calculated as following;

for $\hat{q}$,

$$
\begin{aligned}
i\hbar\frac{dq}{dt} &= [\hat{q}, \hat{H}_{eff}] \\
&= [\hat{q}, \hbar\omega_c\hat{a}^\dagger\hat{a} + \frac{1}{2}\hbar\omega_m(\hat{p}^2 + \hat{q}^2) - \hbar g_o\hat{a}^\dagger\hat{a}\hat{q} + i\hbar\varepsilon(\hat{a}^\dagger + \hat{a}) \\
&- \hbar\omega_L\hat{a}^\dagger\hat{a} + i\hbar A(t)(\hat{a}_{in}^\dagger(t')\hat{a} - \hat{a}^\dagger\hat{a}_{in}(t)) + i\hbar G(e^{i\theta}\hat{a}^{\dagger 2} - e^{-i\theta}\hat{a}^2)]
\end{aligned} \tag{3.11}
$$

From the commutation relation we have

$$
[\hat{A}, \hat{B}\hat{C}] = \hat{B}[\hat{A}, \hat{C}] + [\hat{A}, \hat{B}]\hat{C} \tag{3.12}
$$

and

$$
[\hat{A}\hat{B}, \hat{C}] = \hat{A}[\hat{B}, \hat{C}] + [\hat{A}, \hat{C}]\hat{B} \tag{3.13}
$$

Using the above Eq. 3.12 and Eq. 3.12 into Eq. 3.11 we get

$$
\begin{aligned}
i\hbar\frac{dq}{dt} &= [\hat{q}, \hat{H}_{eff}] \\
&= \hbar\omega_c[\hat{q}, \hat{a}^\dagger\hat{a}] + \frac{1}{2}\hbar\omega_m[\hat{q}, (\hat{p}^2 + \hat{q}^2)] - \hbar g_o[\hat{q}, \hat{a}^\dagger\hat{a}\hat{q}] \\
&+ i\hbar\varepsilon[\hat{q}, (\hat{a}^\dagger - \hat{a})] - \hbar\omega_L[\hat{q}, \hat{a}^\dagger\hat{a}] + i\hbar A(t)[\hat{q}, (\hat{a}_{in}^\dagger(t')\hat{a} - \hat{a}^\dagger\hat{a}_{in}(t))] \\
&+ i\hbar G[\hat{q}, (e^{i\theta}\hat{a}^{\dagger 2} - e^{-i\theta}\hat{a}^2)]
\end{aligned} \tag{3.14}
$$

In above equation, there are three important annihilation operators such as $\hat{p}, \hat{q}$, and $\hat{a}$. Out of each three operator two of them can commute with operator $\hat{q}$,

$$
[\hat{q}, \hat{q}] = [\hat{q}, \hat{a}] = 0 \tag{3.15}
$$

and does not commute with operator $\hat{p}$

$$
[\hat{q}, \hat{p}] = i\hbar \tag{3.16}
$$

Using Eq.(3.15) and Eq.(3.16), Eq. (3.14) becomes,

$$
i\hbar\dot{\hat{q}} = i\hbar^2\omega_m\hat{p} \tag{3.17}
$$

$$
\dot{\hat{q}} = \hbar\omega_m\hat{p} \tag{3.18}
$$

For $\hat{p}$

$$
\begin{aligned}
i\hbar\frac{dp}{dt} &= [\hat{p}, \hat{H}_{eff}] - \gamma_m\hat{p} + \xi(t) \\
&= \hbar\omega_c[\hat{p}, \hat{a}^\dagger\hat{a}] + \frac{1}{2}\hbar\omega_m[\hat{p}, (\hat{p}^2 + \hat{q}^2)] - \hbar g_o[\hat{p}, \hat{a}^\dagger\hat{a}\hat{q}] + i\hbar\varepsilon[\hat{p}, (\hat{a}^\dagger \\
&- \hat{a})] - \hbar\omega_L[\hat{p}, \hat{a}^\dagger\hat{a}] + i\hbar A(t)[\hat{p}, (\hat{a}_{in}^\dagger(t')\hat{a} - \hat{a}^\dagger\hat{a}_{in}(t))] \\
&+ i\hbar G[\hat{p}, (e^{i\theta}\hat{a}^{\dagger 2} - e^{-i\theta}\hat{a}^2)] - \gamma_m\hat{p} + \xi(t)
\end{aligned} \tag{3.19}
$$

Similarly operator $\hat{p}$ commute with $\hat{p}, \hat{a}$ and $\hat{a}_{in}$ i.e

$$[\hat{p}, \hat{p}] = [\hat{p}, \hat{a}] = [\hat{p}, \hat{a}_{in}] = 0 \tag{3.20}$$

and

$$[\hat{p}, \hat{q}] = -i\hbar \tag{3.21}$$

Using above Eq. (3.20) and Eq. (3.21), Eq. (3.19)becomes

$$\begin{aligned} i\hbar\dot{\hat{p}} &= -i\hbar^2\omega_m\hat{p} + i\hbar^2 g_o\hat{a}^\dagger\hat{a} - \gamma_m\hat{p} + \xi(t) \\ \dot{\hat{p}} &= -\hbar\omega_m\hat{q} + \hbar g_o\hat{a}^\dagger\hat{a} - \gamma_m\hat{p} + \xi(t), \end{aligned} \tag{3.22}$$

where γ_m is the mechanical damping rate and the force ξ is the Brownian noise operator resulting from the coupling of the movable mirror to the thermal bath, whose mean value is zero.

For $\hat{a}$

$$\begin{aligned} i\hbar\frac{da}{dt} &= [\hat{a}, \hat{H}_{eff}] + \sqrt{2\kappa}\hat{a}_{in} \\ &= \hbar\omega_c[\hat{a}, \hat{a}^\dagger\hat{a}] + \frac{1}{2}\hbar\omega_m[\hat{a}, (\hat{p}^2 + \hat{q}^2)] - \hbar g_o[\hat{a}, \hat{a}^\dagger\hat{a}\hat{q}] \\ &+ i\hbar\varepsilon[\hat{a}, (\hat{a}^\dagger - \hat{a})] - \hbar\omega_L[\hat{a}, \hat{a}^\dagger\hat{a}] + i\hbar G[\hat{a}, (e^{i\theta}\hat{a}^{\dagger 2} \\ &+ e^{-i\theta}\hat{a}^2)] + i\hbar A(t)[\hat{a}, (\hat{a}_{in}^\dagger(t')\hat{a} - \hat{a}^\dagger\hat{a}_{in}(t))] + \sqrt{2\kappa}\hat{a}_{in} \end{aligned} \tag{3.23}$$

For squeezed vacuum reservoir

$$\langle\hat{a}_{in}\rangle = 0 = \langle\hat{a}_{in}^\dagger\rangle \tag{3.24}$$

$$\sqrt{\kappa}\langle\hat{a}^\dagger(t)\hat{a}_{in}(t)\rangle = \frac{\kappa}{2}N = \sqrt{\kappa}\langle\hat{a}_{in}^\dagger(t)\hat{a}(t)\rangle \tag{3.25}$$

$$\langle\hat{a}_{in}^\dagger(t)\hat{a}_{in}(t)\rangle = N \tag{3.26}$$

By using

$$[\hat{a}, \hat{a}^\dagger] = 1 \tag{3.27}$$

$$[\hat{a}, \hat{a}] = 0 \tag{3.28}$$

and

$$[\hat{a}, \hat{H}_{int}] = -\sqrt{\pi}\kappa N\hat{a} \tag{3.29}$$

$$[\hat{a}^\dagger, \hat{H}_{int}] = -\sqrt{\pi}\kappa N\hat{a}^\dagger \tag{3.30}$$

From the commutation relation of Eq. (3.12)

$$[\hat{a}, \hat{a}^\dagger \hat{a}] = \hat{a}^\dagger [\hat{a}, \hat{a}] + [\hat{a}, \hat{a}^\dagger]\hat{a} = \hat{a} \tag{3.31}$$

Using Eq. (3.27), Eq. (3.28), Eq. (3.31) and Eq. (3.23) becomes

$$i\hbar\dot{\hat{a}} = \hbar\omega_c\hat{a} - \hbar g_o\hat{a}\hat{q} + i\hbar\varepsilon - \hbar\omega_L\hat{a} + 2Ge^{i\theta}\hat{a}^\dagger + -\sqrt{\pi}\kappa N\hat{a} - \kappa\hat{a} + \sqrt{2\kappa}\hat{a}_{in} \tag{3.32}$$

This implies that,

$$\begin{aligned}\dot{\hat{a}} &= (i(\omega_c - \sqrt{\pi}\kappa N - \omega_L - g_o\hat{q}) - \kappa)\hat{a} + \varepsilon - \omega_L\hat{a} + 2i\hbar Ge^{i\theta}\hat{a}^\dagger + \sqrt{2\kappa}\hat{a}_{in} \\ &= -(\kappa + \sqrt{\pi}\kappa N - i\Delta)\hat{a} + \varepsilon + 2Ge^{i\theta}\hat{a}^\dagger + \sqrt{2\kappa}\hat{a}_{in},\end{aligned} \tag{3.33}$$

where $\Delta = \omega_c - \omega_L - g_o\hat{q}$ is effective cavity detuning, including the radiation pressure effects as. The motion of the system can be described by the Heisenberg equations of motion and adding the corresponding damping and input vacuum noise operator, which leads to the following quantum Langevin equations:

$$\dot{\hat{q}} = \omega_m\hat{p} \tag{3.34}$$

$$\dot{\hat{p}} = \omega_m\hat{q} + g_o\hat{a}^\dagger\hat{a} - \gamma_m\hat{p} + \xi \tag{3.35}$$

$$\dot{\hat{a}} = -(\kappa + \sqrt{\pi}\kappa N\hat{a} - i\Delta)\hat{a} + \varepsilon + 2Ge^{i\theta}\hat{a}^\dagger + \sqrt{2\kappa}\hat{a}_{in} \tag{3.36}$$

where $\hat{a}_{in}$ is the zero-mean vacuum cavity input noise operator R. Yang et al. (2017), satisfying the auto-correlation function. For cavity mode coupled to squeeze vacuum reservoir, input noise operator $\hat{a}_{in}(t)$, satisfies the following correlation

$$\langle\hat{a}_{in}(t)\rangle = 0 \tag{3.37}$$

$$\langle\hat{a}^\dagger_{in}(t)\hat{a}_{in}(t')\rangle = N\delta(t - t') \tag{3.38}$$

$$\langle\hat{a}_{in}(t)\hat{a}^\dagger_{in}(t')\rangle = (N + 1)\delta(t - t') \tag{3.39}$$

$$\langle\hat{a}_{in}(t)\hat{a}_{in}(t')\rangle = \langle\hat{a}^\dagger_{in}(t)\hat{a}^\dagger_{in}(t')\rangle = M\delta(t - t') \tag{3.40}$$

Since for squeezed vacuum reservoirs

$$N = \sinh^2(r) \tag{3.41}$$

$$M = \sinh(r)\cosh(r), \tag{3.42}$$

where N and M are mean number of cavity modes and degree of two photon correlations respectively and r is the squeezing parameter taken to be real and positive for convenience.

3.4.1 Linearization of Quantum Langevin Equations

When the system is strongly driven by the external driving laser, one can adopt the standard linearization technique to the QLEs in above equation and rewrite each Heisenberg operator as $\hat{Z} = Z_s + \delta Z(\hat{Z} = \hat{a}, \hat{q}, \hat{p})$ Bai et al. (2019); Chen et al. (2014). So we write each operator of the system as the sum of its steady state mean value and a small fluctuation with zero mean value. This can be expand as a sum of its coherent amplitudes semi-classical steady-state value plus an additional small fluctuation operators.

$$\hat{q} = q_s + \delta\hat{q} \tag{3.43}$$

$$\hat{p} = p_s + \delta\hat{p} \tag{3.44}$$

$$\hat{a} = \alpha_s + \delta\hat{a} \tag{3.45}$$

where q_s, p_s, α_s and α_s^* are amplitude of system and $\delta\hat{q}$, $\delta\hat{p}$, $\delta\hat{a}$ and $\delta\hat{a}^\dagger$ are fluctuation with zero mean value. The corresponding average values are obtained by setting the fluctuations with zero-mean values and applying derivatives. Eq.3.44 becomes and it's mean value of momentum is zero.

$$\delta\dot{\hat{q}} = \omega_m(p_s + \delta\hat{p}) \tag{3.46}$$

The momentum p_s, it's mean value (steady-state)is equal to zero and the mechanical damping does not affect the steady-state of the system. Then, $p_s = 0$

$$\delta\dot{\hat{q}} = \omega_m\delta\hat{p} \tag{3.47}$$

To find q_s, lets insert Eq.(3.40), Eq. (3.44) and Eq.(**??**) into Eq. (3.35)

$$\begin{aligned} \delta\dot{\hat{p}} &= -\omega_m(q_s + \delta\hat{q}) + g_o(\alpha_s^* + \delta\hat{a}^\dagger)(\alpha_s + \delta\hat{a}) - \gamma_m(p_s + \delta\hat{p}) + \xi(t) \\ &\omega_m q_s - \omega_m\delta\hat{q} + g_o(\alpha_s^*\alpha_s + \alpha_s^*\delta\hat{a} + \alpha_s\delta\hat{a}^\dagger + \delta\hat{a}\hat{a}^\dagger) \\ &- \gamma_m p_s - \gamma_m\delta\hat{p} = 0 \end{aligned} \tag{3.48}$$

Since the mean values of fluctuation terms $\delta\hat{q}, \delta\hat{p}, \delta\hat{a}$ are zero and due to the momentum in the steady-state, p_s being equal to zero. So q_s can be written as

$$q_s = -\frac{g|\alpha_s|^2}{\omega_m} \tag{3.49}$$

Then, Eq. (3.48) becomes

$$\delta\dot{\hat{p}} = -g_o|\alpha_s|^2 + g_o|\alpha_s|^2 - \omega_m\delta\hat{q} + g_o\alpha_s(\delta\hat{a} + \delta\hat{a}^\dagger) - \gamma_m\delta\hat{p} + \xi(t) \tag{3.50}$$

$$\delta\dot{\hat{p}} = -\omega_m\delta\hat{q} + g_o(\alpha_s\delta\hat{a}^\dagger + \alpha_s^*\delta\hat{a}) - \gamma_m\delta\hat{p} + \xi(t) \tag{3.51}$$

In similar manner, substitute Eq. (3.40), Eq. (3.44) and Eq.(**??**) into Eq.(3.36) we get

$$\delta\dot{\hat{a}} = -(\kappa + \sqrt{\pi}\kappa N + i\Delta)(\alpha_s + \delta\hat{a}) + \varepsilon + 2Ge^{i\theta}(\alpha_s^* + \delta\hat{a}^\dagger) + \sqrt{2\kappa}\hat{a}_{in} \tag{3.52}$$

The corresponding to Eq. 3.52 average values of α_s are obtained by setting the derivatives to zero

$$\alpha_s = \frac{\kappa + \sqrt{\pi}\kappa N - i\Delta + 2Ge^{i\theta}}{\kappa^2 + \pi\kappa^2 N^2 + \Delta^2 - 4G^2}\varepsilon \tag{3.53}$$

Substituting Eq.(3.53) into Eq. 3.52, we can easily obtain the following equation

$$\delta\dot{\hat{a}} = -(\kappa + \sqrt{\pi}\kappa N + i\Delta)\delta\hat{a} + ig_o\alpha_s\delta\hat{q} + 2Ge^{i\theta}\delta\hat{a}^\dagger + \sqrt{2\kappa}\hat{a}_{in} \tag{3.54}$$

We get the linearized quantum Langevin equations for the fluctuation operators

$$\delta\dot{\hat{q}} = \omega_m\delta\hat{p} \tag{3.55}$$

$$\delta\dot{\hat{p}} = -\omega_m\delta\hat{q} + g_o(\alpha_s\delta\hat{a}^\dagger + \alpha_s^*\delta\hat{a}) - \gamma_m\delta\hat{p} + \xi(t) \tag{3.56}$$

$$\delta\dot{\hat{a}} = -(\kappa + \sqrt{\pi}\kappa N + i\Delta)\delta\hat{a} + ig_o\alpha_s\delta\hat{q} + 2Ge^{i\theta}\delta\hat{a}^\dagger + \sqrt{2\kappa}\hat{a}_{in} \tag{3.57}$$

3.5 Quantification of Bipartite Entanglement

The mechanical entanglement measured by logarithmic negativity will be numerically investigated based on the linearized QLEs Eq. (3.55 -3.57). We first introduce the amplitude and phase fluctuations for a cavity modes Mancini et al. (2003). The cavity field quadratures;

$$\delta\hat{x} = \frac{\delta\hat{a}^\dagger + \delta\hat{a}}{\sqrt{2}} \tag{3.58}$$

$$\delta\hat{y} = i\frac{\delta\hat{a}^\dagger - \delta\hat{a}}{\sqrt{2}} \tag{3.59}$$

the input noise quadratures

$$\delta\hat{x}_{in} = \frac{\delta\hat{a}_{in}^\dagger + \delta\hat{a}_{in}}{\sqrt{2}} \tag{3.60}$$

$$\delta\hat{y}_{in} = i\frac{\delta\hat{a}_{in}^\dagger - \delta\hat{a}_{in}}{\sqrt{2}} \tag{3.61}$$

After inserting, Eq. (3.57-3.59) into Eq. (3.55) we get

$$\begin{aligned}
\delta\dot{\hat{p}} &= -\omega_m\delta\hat{q} + g_o(\alpha_s(\sqrt{2}\delta\hat{x} - \delta\hat{a}) + \alpha_s^*(-i\sqrt{2}\delta\hat{y} + \delta\hat{a}^\dagger)) - \gamma_m\delta\hat{p} + \xi \\
&= -\omega_m\delta\hat{q} + g_o(\alpha_s\sqrt{2}\delta\hat{x} - \alpha_s\delta\hat{a} - i\alpha_s^*\sqrt{2}\delta\hat{y} + \alpha_s^*\delta\hat{a}^\dagger) - \gamma_m\delta\hat{p} + \xi \\
&= -\omega_m\delta\hat{q} + g_o(\alpha_s\sqrt{2}\delta\hat{x} - i\alpha_s^*\sqrt{s}\delta\hat{y}) - \gamma_m\delta\hat{p} + \xi \\
\delta\dot{\hat{p}} &= -\omega_m\delta\hat{q} + g_o\alpha_s\sqrt{2}\delta\hat{x} - \gamma_m\delta\hat{p} + \xi
\end{aligned} \tag{3.62}$$

Now let's derivate Eq. (3.58) on both sides with respect to time

$$\delta\dot{\hat{x}} = \frac{\delta\dot{\hat{a}}^\dagger + \delta\dot{\hat{a}}}{\sqrt{2}} \tag{3.63}$$

After inserting Eq. (3.56) and Eq. (3.57) into Eq. (3.63) we get

$$\begin{aligned}
\delta\dot{\hat{x}} = &\frac{(-\kappa - \sqrt{\pi}\kappa N + i\Delta)\delta\hat{a}^\dagger - ig_o\alpha_s^*\delta\hat{q} - 2Ge^{-i\theta}\delta\hat{a} + \sqrt{2\kappa}\hat{a}_{in}^\dagger}{\sqrt{2}} \\
&+ \frac{-(\kappa + \sqrt{\pi}\kappa N + i\Delta)\delta\hat{a}^\dagger + ig_o\alpha_s\delta\hat{q} + 2Ge^{i\theta}\delta\hat{a}^\dagger + \sqrt{2\kappa}\hat{a}_{in}}{\sqrt{2}}
\end{aligned} \tag{3.64}$$

But, from Euler equations we have

$$e^{i\theta} = \cos\theta + i\sin\theta \tag{3.65}$$

and

$$e^{-i\theta} = \cos\theta - i\sin\theta \tag{3.66}$$

Now after inserting Eq. (3.65) and Eq. (3.66) into Eq. (3.64) and rearranging we get

$$\begin{aligned}
\delta\dot{\hat{x}} = &\frac{-\kappa(\delta\hat{a}^\dagger + \delta\hat{a}) - \sqrt{\pi}\kappa N(\delta\hat{a}^\dagger + \delta\hat{a}) + i\Delta)(\delta\hat{a}^\dagger - \delta\hat{a}) - 2G\cos\theta\delta\hat{a}^\dagger +)}{\sqrt{2}} \\
&+ \frac{2iG\sin\theta\delta\hat{a} + \sqrt{2\kappa}(\delta\hat{a_{in}}^\dagger + \delta\hat{a}_{in}) + ig_o(\alpha_s - \alpha_s^*)\delta\hat{q} + 2G\cos\delta\hat{a} + 2iG\sin\theta\delta\hat{a}}{\sqrt{2}} \\
= &\frac{-\kappa(\delta\hat{a}^\dagger + \delta\hat{a}) - \sqrt{\pi}\kappa N(\delta\hat{a}^\dagger + \delta\hat{a}) + i\Delta(\delta\hat{a}^\dagger - \delta\hat{a}) - 2G\cos\theta(\delta\hat{a}^\dagger + \delta\hat{a}) +}{\sqrt{2}} \\
&\frac{\sqrt{2\kappa}(\delta\hat{a}_{in}^\dagger + \delta\hat{a}_{in}) + ig_o(\alpha_s - \alpha_s^*)\delta\hat{q} + 2iG\sin(\delta\hat{a}^\dagger - \delta\hat{a})}{\sqrt{2}} \\
= &\frac{-\kappa\delta\hat{x} + \sqrt{\pi}\kappa N\delta\hat{x} + i\Delta\delta\hat{x} - 2G\cos\theta\delta\hat{x} + \sqrt{2\kappa}\delta\hat{x}_{in} + ig_o(\alpha_s - \alpha_s^*)}{\sqrt{2}} \\
&+ \frac{\delta\hat{q} + 2iG\sin(\delta\hat{a}^\dagger - \delta\hat{a})}{\sqrt{2}} \\
\delta\dot{\hat{x}} = &g_o\alpha_s\sqrt{2}\delta\hat{p} + (-\kappa + \sqrt{\pi}\kappa N + 2G\cos\theta)\delta\hat{x} + (\Delta + 2G\sin\theta)\delta\hat{y} \\
&+ \sqrt{2\kappa}\delta\hat{x}_{in}
\end{aligned} \tag{3.67}$$

Again let's derivate Eq. (3.59) on both sides with respect to time

$$\delta\dot{\hat{y}} = i(\frac{\delta\dot{\hat{a}}^\dagger - \delta\dot{\hat{a}}}{\sqrt{2}}) \tag{3.68}$$

After inserting Eq. (3.55) and Eq. (3.56) into Eq. (3.68) we get

$$\begin{aligned}
\delta\dot{\hat{y}} &= \frac{i(-\kappa - \sqrt{\pi}\kappa N + i\Delta)\delta\hat{a}^\dagger - g_o\alpha_s^*\delta\hat{q} - 2iGe^{-i\theta}\delta\hat{a} + i\sqrt{2\kappa}\hat{a}_{in}^\dagger}{\sqrt{2}} \\
&- \frac{i(\kappa + \sqrt{\pi}\kappa N + i\Delta)\delta\hat{a}^\dagger - g_o\alpha_s\delta\hat{q} - 2iGe^{i\theta}\delta\hat{a}^\dagger - i\sqrt{2\kappa}\hat{a}_{in}}{\sqrt{2}} \\
&= \frac{-i\kappa(\delta\hat{a}^\dagger + \delta\hat{a}) - \sqrt{\pi}\kappa N(\delta\hat{a}^\dagger + \delta\hat{a}) - \Delta(\delta\hat{a}^\dagger + \delta\hat{a}) - 2iG\cos\theta\delta\hat{a}^\dagger +}{\sqrt{2}} \\
&\frac{2G\sin\theta\delta\hat{a} + i\sqrt{2\kappa}(\delta\hat{a_{in}}^\dagger - \delta\hat{a}_{in}) - ig_o(\alpha_s + \alpha_s^*)\delta\hat{q} + 2G\cos\delta\hat{a} + 2iG\sin\theta\delta\hat{a}}{\sqrt{2}} \\
&= \frac{-i\kappa(\delta\hat{a}^\dagger - \delta\hat{a}) - \Delta(\delta\hat{a}^\dagger - \delta\hat{a}) - 2iG\cos\theta(\delta\hat{a}^\dagger - \delta\hat{a}) + i\sqrt{2\kappa}(\delta\hat{a}_{in}^\dagger - \delta\hat{a}_{in})}{\sqrt{2}} \\
&\frac{-ig_o(\alpha_s + \alpha_s^*)\delta\hat{q} + 2G\sin(\delta\hat{a}^\dagger + \delta\hat{a})}{\sqrt{2}} \\
&= \frac{-\kappa\delta\hat{y} - \sqrt{\pi}\kappa N\delta\hat{y} - \Delta\delta\hat{x} - 2iG\cos\theta\delta\hat{x} + \sqrt{2\kappa}\delta\hat{x}_{in} - ig_o(\alpha_s + \alpha_s^*)\delta\hat{q}}{\sqrt{2}} \\
&\frac{+2G\sin(\delta\hat{a}^\dagger + \delta\hat{a})}{\sqrt{2}}
\end{aligned}$$

$$\delta\dot{\hat{y}} = (-\Delta + 2G\sin\theta)\delta\hat{x} + (-\kappa - \sqrt{\pi}\kappa N - 2G\cos\theta)\delta\hat{y} + \sqrt{2\kappa}\delta\hat{y}_{in} \tag{3.69}$$

The linearized QLEs for the quantum fluctuation can be rewritten as

$$\delta\dot{\hat{q}} = \omega_m\delta\hat{p} \tag{3.70}$$

$$\delta\dot{\hat{p}} = -\omega_m\delta\hat{q} + g_o\alpha_s\sqrt{2}\delta\hat{x} - \gamma_m\delta\hat{p} + \xi \tag{3.71}$$

$$\begin{aligned}\delta\dot{\hat{x}} &= g_o\alpha_s\sqrt{2}\delta\hat{p} + (-\kappa - \sqrt{\pi}\kappa N + 2G\cos\theta)\delta\hat{x} + (\Delta + 2G\sin\theta)\delta\hat{y} \\ &+ \sqrt{2\kappa}\delta\hat{x}_{in}\end{aligned} \tag{3.72}$$

$$\delta\dot{\hat{y}} = (-\Delta + 2G\sin\theta)\delta\hat{x} + (-\kappa - \sqrt{\pi}\kappa N - 2G\cos\theta)\delta\hat{y} + \sqrt{2\kappa}\delta\hat{y}_{in} \tag{3.73}$$

Eq. (3.70) upto Eq. (3.73) can be written in the matrix form

$$\dot{f}(t) = Af(t) + \eta(t), \tag{3.74}$$

where $f(t)$ is the column vector of the fluctuations, and $\eta(t)$ is the column vector of the noise sources. For the sake of simplicity, their transposes are

$$f(t)^T = (\delta\hat{q}, \delta\hat{p}, \delta\hat{x}, \delta\hat{y}) \tag{3.75}$$

And with corresponding vector noise correlations operators;

$$\eta(t)^T = (0, \xi, \sqrt{2\kappa}\delta\hat{x}_{in}, \sqrt{2\kappa}\delta\hat{y}_{in}) \tag{3.76}$$

$$\begin{pmatrix} \delta\dot{\hat{q}} \\ \delta\dot{\hat{p}} \\ \delta\dot{\hat{x}} \\ \delta\dot{\hat{y}} \end{pmatrix} = \begin{pmatrix} 0 & \omega_m & 0 & 0 \\ -\omega_m & \gamma_m & g_o\alpha_s\sqrt{2} & 0 \\ 0 & g_o\alpha_s\sqrt{2} & \nu_1 & \nu_2 \\ 0 & 0 & \nu_3 & \nu_4 \end{pmatrix} \begin{pmatrix} \delta\hat{q} \\ \delta\hat{p} \\ \delta\hat{x} \\ \delta\hat{y} \end{pmatrix} + \begin{pmatrix} 0 \\ \xi \\ \sqrt{2\kappa}\delta\hat{x}_{in} \\ \sqrt{2\kappa}\delta\hat{y}_{in}, \end{pmatrix} \tag{3.77}$$

where the abbreviation notations are defined as $\nu_1 = -\kappa - \sqrt{\pi}\kappa N + 2G\cos\theta$, $\nu_2 = \Delta + 2G\sin\theta$, $\nu_3 = -\Delta + 2G\sin\theta$ and $\nu_4 = -\kappa - \sqrt{\pi}\kappa N - 2G\cos\theta$. The coefficient of drift matrix A can be simplified as;

$$A = \begin{pmatrix} 0 & \omega_m & 0 & 0 \\ -\omega_m & \gamma_m & g_o\alpha_s\sqrt{2} & 0 \\ 0 & g_o\alpha_s\sqrt{2} & \nu_1 & \nu_2 \\ 0 & 0 & \nu_3 & \nu_4 \\ \end{pmatrix} \tag{3.78}$$

When the real parts of all the eigenvalues of matrix A are negative, the system is stable and reaches its steady state Paternostro et al. (2012); Han et al. (2013). We define the element of the covariance matrix of the quantum fluctuations

$$\Omega_{i,j}(\infty) = \langle f_i(\infty)f_j(\infty) + f_j(\infty)f_i(\infty)\rangle/2 \tag{3.79}$$

and investigate the nature of the steady state of the cavity optomechanical system. The steady state covariance matrix Ω can be obtained by solving the Lyapunov equation and which contains all information about the steady-state Abdi et al. (2011)

$$A\Omega + \Omega A^T = -Q \tag{3.80}$$

Q is the diffusion matrix from noise correlations, which is defined as

$$Q_{i,j}\delta(t-t') = \frac{1}{2}\langle \varsigma_i(t)\varsigma_j(t') + \varsigma_j(t')\varsigma_i(t)\rangle \tag{3.81}$$

In vacuum, $\hat{a}_{in}$ is the input vacuum noise operator for the cavity, of which the only nonzero correlation function is

$$\langle \hat{a}_{in}(t)\hat{a}^\dagger_{in}(t')\rangle = \delta(t-t') \tag{3.82}$$

The forces ξ is the Brownian noise operator resulting from the coupling of the movable mirror to the thermal bath, whose mean value is zero, and it has the following correlation

$$\frac{1}{2}\langle \xi_i(t)\xi_j(t') + \xi_j(t')\xi_i(t)\rangle = \gamma_m(2\bar{n}+1), \tag{3.83}$$

r, which is assumed to stay at the same environmental temperature, this means that the evolution of the mechanical resonator is a Markovian process and

$$Q = diag[0, \gamma_m(2\bar{n}+1), \kappa, \kappa] \tag{3.84}$$

is the noise correlation matrix. We aim to investigate the properties of entanglement of the cavity optomechanical system containing OPA coupled to squeezed vacuum reservoir.

3.6 Logarithmic negativity criteria

Logarithmic negativity criteria is another widely used entanglement measure, which is used for a two-mode continues variables based on the negativity of the partial transpose. The logarithmic negativity for a two-mode cavity is defined as

$$E_N = max\{0, -ln2\eta\} \tag{3.85}$$

Where the smallest possible eigenvalue of the covariance matrix is defined as

$$\bar{\eta} = \sqrt{\Gamma - \sqrt{\Gamma^2 - 4\det V}/2} \tag{3.86}$$

with covariant matrix

$$\Gamma = \det V_A + \det V_B - 2\det V_C \tag{3.87}$$

and invariant matrix

$$V = \begin{pmatrix} V_A & V_C \\ V_C^T & V_B \end{pmatrix} \tag{3.88}$$

The elements of the covariance matrix V_A, V_B and V_C are sub-block matrices of 2×2 from the drift matrix A equation 3.78. According to the definition of equation 3.85 the system is entangled if E_N>0 within the region of the smallest eigenvalue of the covariance matrix Vs $<$ 1

CHAPTER 4

4 RESULT AND DISCUSSION

In this chapter by using the previous results, we present measure the entanglement property of optical parametric amplifier coupled to squeezed vacuum reservoir. The entanglement properties can be verified by numerical results of the quantification, measuring the corresponding CM through logarithmic negativity. Accordingly, we study effects of OPA parametric gain, cavity decay rate, input laser power, temperatures and squeezed parameter on the formation. The enhancement of entanglement are investigated by plotting graphs using numerical value in table below are used based on current of our statements of parameters as follows.

Table 4.1: The experimental parameters for the coupled optomechanicaics systems used in our numerical simulation. Marquardt et al. (2007); Gigan et al. (2006)

No	Quantity	symbol	Expression
1	frequency of mechanical oscillator	ω_m	$20\pi\times10^6 Hz$
2	damping rate	γ	200π Hz
3	finesse	f	1.07×10^4
4	Speed of light	c	$3\times10^8 m/s$
5	Cavity–field wavelength	λ	$810\times10^{-9} m$
6	mass of mechanical oscillator	m	$5\times10^{-12} Hz$
7	Input power	p	$20\times10^{-3} p$

4.1 Numerical results and discussions

In this section, we present the numerical results of the entanglement property of optical parametric amplifier coupled to squeezed vacuum reservoir. When OPA is introduced, the entanglement increases with the increasing G as see from fig.6. The reason is that increasing the parametric gain causes a stronger coupling between the movable mirror and the cavity field due to the increase of the photon number inside the cavities. More importantly, it can strongly improve the optomechanical entanglement, leading to as large parametric gain, which appears at the parameter amplifier with large cavity driving strength.

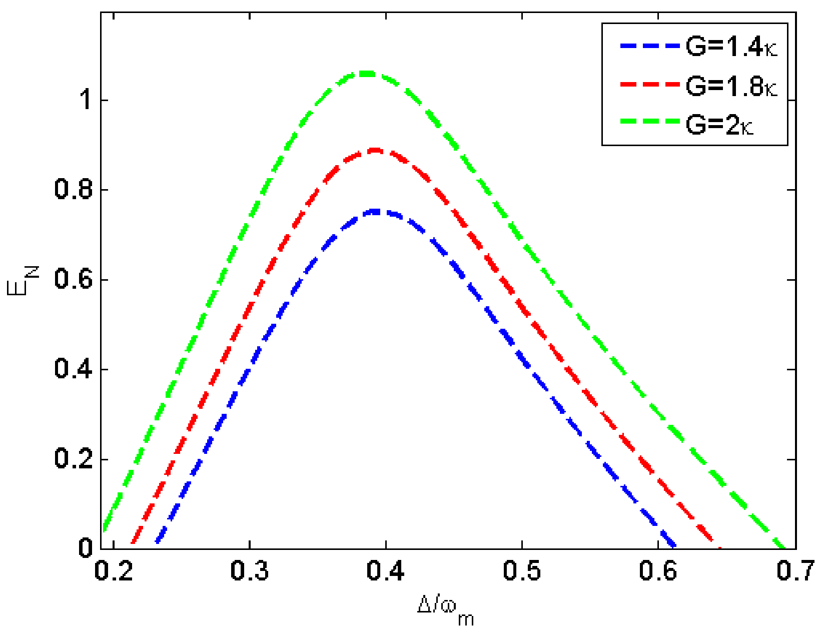

Figure 6: plot of mechanical entanglement E_N versus detuning of OPA Δ/ω_m for different value of parametric gain G and r=0.5

In fig.6, one can be observed that the entanglement with the OPA can be notably enhanced compared to increasing the value of parametric gain between the movable mirror and the cavity field. This shows that the bipartite entanglement between two cavity can be sharing and transfer information between each other. This suggests that the possibility of entanglement can be conveniently observed in the optomechanical system in the presence OPAs between two mode. The height difference between any two lines becomes larger with the increase of the normalized detuning. In the context of optomechanical systems, an OPA can modify the normal-mode splitting behavior of the coupled movable mirror and the cavity field Huang & Agarwal (2009c), as well as enhance the optomechanical interaction strength into the single-photon strong-coupling Lü et al. (2015). Therefore, in the presence of the optical parametric amplifier coupled to squeezed vacuum reservoir, the mechanical entanglement first reaches the maximum and then goes down as see fig.6 and fig.7. We observe that the entanglement have a maximum peak value. This implies that the peaks of positively of the bipartite see fig.7 show entanglement sharing and exchange information between each other. Furthermore, from fig.(6,7) can see that the logarithmic negativity behave the a peaks entanglement sharing is evident due to the presence of a mechanical resonator

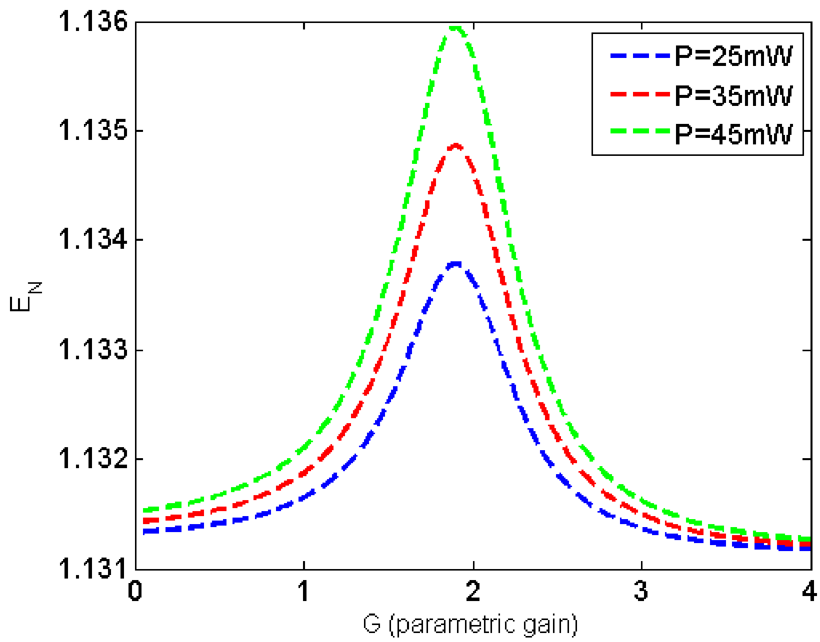

Figure 7: plot of mechanical entanglement E_N versus parametric gain for different value of input laser power for L= 1mm, G=2κ, T= 100$\times 10^{-3}$ and r=0.5

On the other hand, from fig.7 and fig. .6 it can be seen that the entanglement of the coupled cavity optomechanical systems increase when the laser power and parametric gain G are increased. It is because that the coupling between the mirror and the cavity field is increased with the increase of the input laser power due to the increase in photon number.

However, while κ becomes larger, then the effective squeezing parameter becomes too small to further increase the mechanical entanglement, in order to revive the mechanical entanglement, the OPA gain should be enlarged by increasing the pumping strength, as shown in fig.8. This shows increasing the ratio of para-

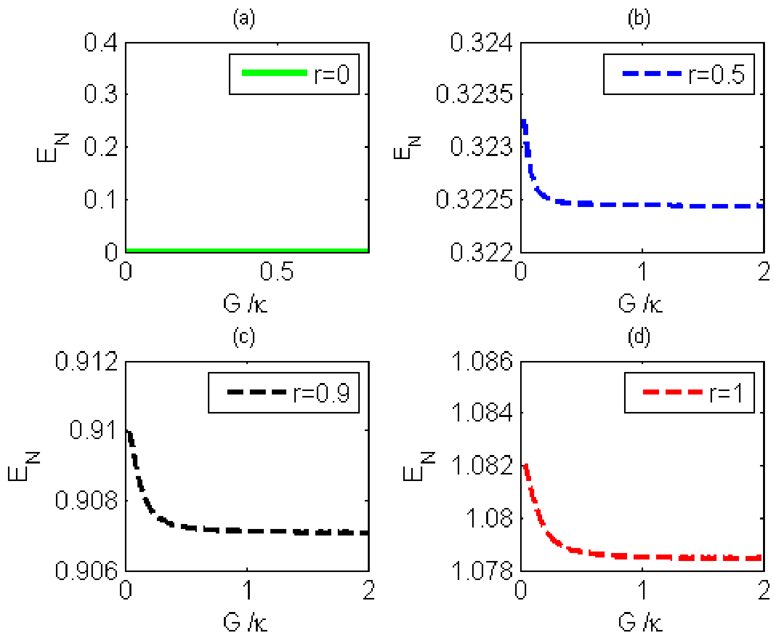

Figure 8: plot of mechanical entanglement versus the ratio of parametric gain to the cavity decay for different value of squeezing parameter r, L=5mm and G= 0.8 κ

metric gain to the cavity decay, the entanglement can be enhanced due to the increase of the squeezing parameter. Furthermore upon comparing expressions in fig. 8(a,b,c,and d) one can observe that at steady-state the entanglement of a signal mode squeezed vacuum reservoir are dependent on squeezed vacuum reservoirs.

The optical cavity and mechanical oscillator are coupled by radiation pressure and by the mechanical oscillator physically changing the length of the optical cavity. In this case the coupling comes from the cavity frequency actually depends on the position of the mirror. This indicate with increasing coupling strength of

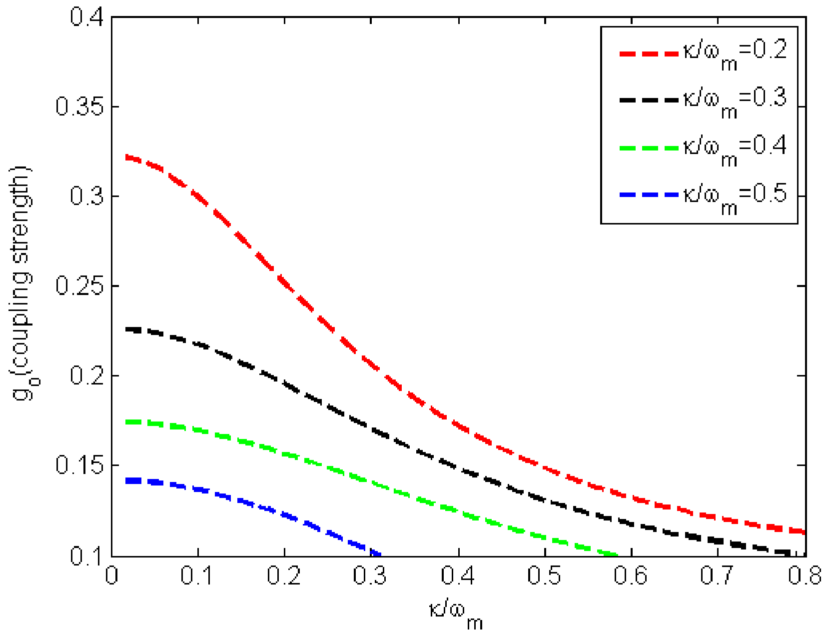

Figure 9: Plot of the coupling strength versus ration of cavity decay to resonance frequency G= 1.6κ, G= 1.2κ, G= 1.0κ, and G= 0.8κ, respectively and r=0.5

the coupled optomechanical system the entanglement is increasing as cavity decay decreasing. Indeed, such couplings can be made very large as the effective length in the coupling can be much smaller than the physical dimensions of the cavity involved. Generally, above fig. (6, 7, 8, **??** and 9) are our key result which tells how the macroscopic entanglement would depend on the parameters of input laser power, cavity decay rate κ, parametric gain G, coupling strength g, squeezing parameters r, temperature T and cavity length and etc.

5 CONCLUSION AND RECOMMENDATION

5.1 Conclusion

In this thesis, we have investigated that the mechanical entanglement can be enhancing by optical parametric amplifier coupled to squeezed vacuum reservoir with optomechanical system. For the purpose of measuring the enhancing entanglement at the steady state, we have proposed a model of a cavity field and a movable mirror filled with an optical parametric amplifier coupled to squeezed vacuum reservoir interacts with a nonlinear crystal inside a cavity. It is found that the strong mechanical entanglement is induced by optical parametric amplifier, which can be coupled by squeezed vacuum reservoir. Using a Heisenberg equations, the dynamics of our model incorporating new features, will be derived based on the linearized quantum Langevin equations. The enhancing entanglement measurement is quantifying by using the logarithmic negativity, which characterizes the entanglement between the cavity can be increase with the help of OPA coupled to squeezed vacuum reservoir.

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In conclusion, Our study shows that the entanglement of the cavity optomechanical system can be pronouncedly enhanced due to the introduction of the OPA coupled to squeezed vacuum resersoir and the larger entanglement corresponds to the larger input laser power as well as parametric gain. Our key result which tells how the macroscopic entanglement would depend on the parameters of the cavity decay rate, input laser power, temperature, gain of the OPA, coupling strength, squeezing parameters and cavity length, etc. We have also shown that the degenerate parametric amplifier could be entangled for a squeezing parameter above a certain value when it is coupled to a squeezed vacuum reservoir.

5.2 Recommendation

In this work, we have studied the bipartite entanglement of a cavity radiation from optomechanical system with optical parametric amplifier (OPA) and coupled to a squeezed optical reservoir through logarithmic negativity. Furthermore, studies will be conducted on dynamics of quantum feature with optical parametric amplifier coupled to a squeezed optical reservoir by using different quantification of inseparability criteria.

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