Performance Analysis of Model Predictive Control for Electrical Quadriplegic Wheelchair

Rut Chalchisa Tola

A Thesis Submitted to
The Department of Electrical Power and Control Engineering
School of Electrical Engineering and Computing

Presented in Partial Fulfillment of the Requirement for the Master of Science Degree in Electrical Power and Control Engineering (Control Engineering)

Office of Graduate Studies
Adama Science and Technology University

February,2023
Adama,Ethiopia

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APPROVAL SHEET

We, the advisors of the thesis entitled "Performance Analysis of Model Predictive Controller for Electrical Quadriplegic Wheelchair and developed by Rut Chalchisa, hereby certify that the recommendation and suggestions made by the board of examiners are appropriately incorporated into the final version of the thesis.

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Major Advisor	Signature	Date
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Co-advisor	Signature	Date

We, the undersigned, members of the Board of Examiners of the thesis by Rut Chalchisa have read and evaluated the thesis entitled "Performance Analysis of Model Predictive Controller for Electrical Quadriplegic Wheelchair" and examined the candidate during open defense. This is, therefore, to certify that the thesis is accepted for partial fulfillment of the requirement of the degree of Master of Science in control engineering.

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Finally, approval and acceptance of the thesis is contingent upon submission of its final copy to the Office of Postgraduate Studies (OPGS) through the Department Graduate Council (DGC) and School Graduate Committee (SGC).

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DECLARATION

I hereby declare that this Master Thesis entitled "Performance Analysis of Model Predictive Controller for Electrical Quadriplegic Wheelchair" is my original work. That is, it has not been submitted for the award of any academic degree, diploma or certificate in any other university. All sources of materials that are used for this thesis have been duly acknowledged through Citation.

______________________	______________________	______________
Name of the student	Signature	Date

RECCOMENDATION

we, the advisor(s) of this thesis, hereby certify that we have read the revised version of the thesis entitled "Performance Analysis of Model Predictive Controller for Electrical Quadriplegic Wheelchair" prepared under my/our guidance by Rut Chalchisa submitted in partial fulfillment of the requirements for the degree of Masters of Science in Control engineering.

Therefore, we recommend the submission of revised version of the thesis to the department following the applicable procedures.

Name of major Advisor/supervisor	Signature	Date
Name of co-advisor	Signature	Date

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LIST OF ACRONYMS

AFF	Attractive Force Field
APF	Artificial Potential Field
CV	Control Variable
DC	Direct Current
EPW	Electric-Powered Wheelchair
MATLAB	Matrix Laboratory
MIMO	Multiple Input-Multiple Output
MV	Manipulated Variable
MPC	Model Predictive Control
PWD	Power with Disabilities
PID	Proportional Integral Derivate
RFF	Repulsive Force Field
QP	Quadratic Programming

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LIST OF SYMBOLS

F	Actual force acting to a mass point
FL, FR	Forces causing linear motion by drives and inertial force
FS	Resistance force FO proportional to speed vB
ia	Armature current
if	Field current
i ,R	Left and right current flowing through winding
JB	Moment of inertia with respect to rotation axis in Point
Jm	Moment of inertia
kb	Electro motoric constant
kr	Coefficient of rotation resistance
Kv	Resistance coefficient against linear motion.
Kω	Resistance coefficient against rotary motion
La	Inductance
LL	Distance of the left wheel from point B
LR	Distance of the right wheel from point B
LT	Distance of the center of gravity from point B
L, R	Driving wheels
m	Mass of the Human Body
M	Load torque
M , MR	Load moments on left and right wheels respectively
MGL	Moment of the left drive
MGR	Moment of the right drive
PG	The gear box transmission ratio
r	Semi diameter of the wheels
R, Q	Output weight, input weight.
Ra	Resistance
T	General center of gravity

uR , uL	Control voltages of the right and left motors
va	Voltage supplied by amplifier to move the motor
Va	Terminal voltage
vb	Back EMF voltage
vB	Linear motion speed
vf	Field voltage commands
vL, vR	Peripheral speeds
W	Column vector of N future set points
xB,B,αB	Global coordinates of the wheelchair
vB, ϖB	Linear and angular velocities
ωL	Left motor angular velocity
ωR	Right motor angular velocity
ωB	Angular speed in point B
ϖGL, ϖGR	Angle speeds of gearbox output
τm	Motor torque produced by the motor shaft

ABSTRACT

Quadriplegic: Afflicted by partial or complete paralysis of both arms and legs, often as a result of disease or damage to the neck region of the spinal cord. A quadriplegic wheelchair that travels along a track must adhere to it exactly. The wheelchair chosen for this thesis has two front wheels and one caster situated in the back. The quadriplegic wheelchair's mathematical model includes the dynamics of two identical series DC motors, the wheelchair's chassis, and its kinematics because the model of the plant is crucial for developing the MPC. In MATLAB, the quadriplegic wheelchair model is simulated. For the purpose of improving position and orientation straight path line tracking, the influence of a change in body mass is regarded as a disturbance. This thesis in order to settled the unstability of position and orientation and trajectory track problems. Improve the performance of the quadriplegic wheelchair to track its position and orientation from the ideal track line position, this study focuses on building MPC (Model Predictive Control) for quadriplegic wheelchair operation in track. MATLAB simulates the two DC series motors located in each of the front wheels as well as the controller. The pinhole for the input component is defined. To limit the forward speed and the maximum angle or direction error, the quadriplegic wheelchair's position (x, y), orientation (a), and control signals (uL, uR) are all regulated. By simulation tests, the effectiveness of the suggested controllers for tracking position and orientation set points is assessed. The result showed that the closed loop system without consideration of the disturbance was very good to track the path with cost function $j=0.653*e^{-10.}$ *This shows that the controller tracks the desired path with small angular error* $-0.3*10^{-10}$ *degree. The other simulation result is control system that took disturbances into account. The cost function used to track the desired path was roughly* $j=0.653*e^{-10}$*, but the angular error was* $-0.3*10^{-4}$ *degrees. The lowest error between the desired and actual position and orientation is the lowest cost function. When working in a path line that follows the track due to track restrictions, the MPC technique is particularly favorable and exhibits improved performance.*

Key words: Model Predictive Control, Mathematical model, Quadriplegic wheelchair

CHAPTER ONE

1.INTRODUCTION

1.1Background of the thesis

Several health problems result from prolonged sitting: pressure sores, poor blood pressure circulation, internal cavities tighten up which males bladders do not work as well. Quadriplegic is the result of damage or illness, almost affected backbone. In the world one billion people physically disable (www.worldbank.org/en/topic/disability) that is difficult to move and work properly in daily life activity, so exactly help is needed. In 1933 Harry C. Jennings, Sr. and his disabled friend Herbert Everest, both mechanical engineers, invented the first lightweight, steel, folding, portable modern wheelchair. These technologies gave solutions to problems that are difficult to solve by humans. Today wheelchairs help the quadriplegic person get everywhere, and houses can be equipped with similar systems to operate lights and appliances (www.merriam-webster.com/dictionary/quadriplegic#note-1). To improve the lifestyle of the physically challenged, in this work, to develop a wheelchair system. Elderly and partially quadriplegic people suffering from severe paralysis may not be able to use these technologies since they require accurate control. The first records of wheeled seats being used for transporting disabled person's date to three centuries later in China; the Chinese used early wheelbarrows to move people as well as heavy objects (Wheelchair, 2021).

There are two types of quadriplegic wheelchair, manual and electric wheelchair. Manual quadriplegic wheelchair requires little maintenance and do not depend on charging or technology capabilities. Quadriplegic wheelchair requires another assistance person to carry their daily life. But electric quadriplegic wheelchair needs charging properly, they provide energy and constant power. It does not need any other assistance person. In this thesis electric quadriplegic wheelchair studied.

Figure 1. 1 Electrical Quadriplegic Wheelchair(www.merriam-webster.com)

The advancement and development of technology has always influenced a few parts of our lives since quite a while and will keep on doing as such later on with additional capacity and more unexpected development. Nowadays, a wide range of technologies is available to help the disabled and physically challenged, and many things around us heavily depend on technologies that make life easier and more flexible. Control systems are the most recent technological advancement designed for various purposes, specifically to help the disabled and physically challenged, and to replace conventional manual support systems (Fahd N.Al-Wesabi et al, August 5,2019).

Many researchers proposed motion control of wheelchair by using different recognition systems such as head motion, voice, hand gesture and so. On, but it has a limitation that doesn't consider the position and orientation stability of wheelchair. The quadriplegic wheelchair is design with two-wheel and caster wheel consider to balance the wheelchair that is when compared to three and four wheel unstable, but in terms of cost expensive and construction complexity the two wheel is better. In this thesis proposed the MPC for quadriplegic wheelchair it is possible to move along the path trajectory without any part of the collision with any hazards in the static environment. The control action of MPC is determined as the result of an online optimization problem. Determining the motion commands so the wheelchair will move as planned to satisfy human users, by ensuring that the motion is efficient, safe, and comfortable. Once the system position and orientation balance may command the motion turn left or right, forward, backward.

The system consists of three main parts:

1. **Dynamics of the DC series motor:** moves the system backward or forward to balance left or right of mechanical systems with considering the different forces that affect it.
2. **Chassis:** holds the motors, circuits and any parts required for the system.
3. **Kinematics of the vehicle:** the motion of mechanical systems without considering the forces that affect the motion.

1.2 Statement of the problem

The unstability of position and orientation the wheelchair make trajectory tracking problems. The trajectory tracking problems occurred wheelchair collision and damages. The reason behind the application use of electrical quadriplegic wheelchair in the elderly and disable people in control engineering is its mechanical stability of position and orientation. Comparing with ordinary four wheel, two wheeled wheelchair have many advantages like cost reduction, size reduction, more affordable and easier to operate for paralyzed person, but it's unstable. This unstable is solved by applying model predictive controller. The benefits which are offered by this wheelchair is to solve physical disability challenges and assist the elderly and fully or partially quadriplegic people to perform their daily life activities without others' help.

1.3 Objective of the thesis

1.3.1 General objectives

The main objective of this thesis is to design model predictive control for controlling position and orientation of electrical quadriplegic wheelchair system.

1.3.2 Specific objectives

- To design the mathematical model of the kinematics and dynamics electrical quadriplegic wheelchair system.
- To model the MPC for quadriplegic wheelchair.
- To simulate and analyze the electrical quadriplegic wheelchair in MATLAB/Simulink.
- To make relevant conclusion and recommendation for the systems results.

1.4 Significance of the study

Elderly and quadriplegic people are the group with the highest rates of manual and electric wheelchair use. Today, most of the engineers are participating in electrical quadriplegic wheelchair system. Electric quadriplegic wheelchair is considered to be one of the most efficient ways of improving the quality of life elderly and quadriplegic day to day activities by themselves. So, developing a control mechanism plays a big role and increasing the ability of performance activities for elderly and quadriplegic as youth and ordinary person. This thesis can also be an input for peoples engaged in the work of electrical quadriplegic wheelchair for position and orientation control system.

1.5 Scope of thesis

This thesis provides theoretical studies and simulation of model predictive control for electrical quadriplegic wheelchair with desired trajectory path. This thesis is to investigate how to improve the capability of tracking the desired position (x, y) and orientation (α) of the electrical quadriplegic wheelchair in order to protect the elderly and quadriplegic people from collision and damage, where the wheelchair is controlled by model predictive control then implement the system using the available mathematical computation tools of MATLAB /SIMULINK. Taylor series approximation method is applied to nonlinear kinematics to cancel system of the nonlinearities then augmented with dynamics model. Implementation of the system is not performed which is the limitation of the study.

1.6 Thesis Organization

This thesis organized into five chapters.

Chapter 1: - Presents the introduction of the thesis works, statements of the problem, the objective of the thesis, significance of thesis and scope of the research work.

Chapter 2: - Reviews recent research on the existing Quadriplegic wheelchair based on control strategy implementation and review closely related literature on Model Predictive Control.

Chapter 3: - Describes the methods followed in this thesis and the mathematical model of the two-wheel wheelchair and DC motor are modeled. This chapter also presents the control design (MPC).

Chapter 4: - Presents the proposed results and discussions.

Chapter 5: - Discusses the research work conclusions and recommendations.

CHAPTER TWO

2.LITERATURE REVIEW

This topic explains about the overview of quadriplegic wheelchair, the main classification of quadriplegic, described the model predictive controller with quadriplegic wheelchair, detailed literature study on design a model predictive control for electrical quadriplegic wheelchair.

2.1 Overview of Quadriplegic Wheelchair

When walking is difficult or impossible because of a disease, accident, age related issues, or disability, a quadriplegic wheelchair is a chair with wheels (wheelchair Wikipedia). This wheelchair can be changed with advanced of technology from time to time. Electric wheelchair or electric-powered wheelchair (EPW) is a wheelchair that is propelled by means of an electric motor (usually using differential_steering) rather than manual power. Motorized wheelchairs are useful for those unable to propel a manual wheelchair or who may need to use a wheelchair for distances or over terrain which would be fatiguing in a manual wheelchair (Wheelchair, 2021).

Wheelchair technologies and sensors with relevant materials and technologies have been integrated with microcontrollers used to enhance the confidence, function, and willpower of the elderly and physically challenged people as it will help them to be self-reliant with no need to use any extra devices. (Fahd N.Al-Wesabi et al, August 5,2019)

2.2 Quadriplegic Wheelchair Components

The non-proportional control of Motor d.c. is the most popular speed control system for motorized wheelchairs, with forward, reverse, left, and right units. It runs smoothly and is quite effective (wheel freedom).

2.2.1Seating and Positioning

Seating and positioning are a critical part of the quadriplegic wheelchair and have an important role in comfort, function, safety and health. Seating system should be prescribed and designed specific to medical, functional and personal preference needs, including protecting skin from too much pressure.

2.2.2 The Base

The power wheelchair base is the lower portion of a power wheelchair that houses the motors, batteries, drive wheels, casters and electronics to which a seating system is attached.

2.3.2 Recline and Tilt-in-Space

Recline and tilt-in-space technologies relieve pressure, manage posture, provide comfort and help with personal care activities. Recline which changes the angle between the seat and backrest, helps to stretch hip flexors and makes attending to catheters, toileting and transfers more convenient for caregivers. The addition of tilt-in-space, which tilts the seat and backrest together, keeps the hip and knee angles constant when tilting back. This reduces the possibility of shear when in the recline position. People who cannot independently shift weight or transfer should have a tilt-in-space and recline system on their wheelchair.

2.4.2Seat Elevation & Standing Chairs

Power wheelchairs can also have elevating seats or mechanisms that stand the user upright. Elevating seats can help with transfers, as it is easier to transfer downhill. In addition, elevating seats and standing chairs can make it easier and more functional to perform activities that are above shoulder height when sitting. Because performing activities above shoulder height puts at risk of injuring their arms, recent guidelines recommend that all individuals with SCI who use power wheelchairs and have good arm function be provided with seat elevation.

2.5.2Cushions and Backrests

Cushions and backrests are made out of a variety of materials, including contoured foam, air-filled bladders, combinations of air and foam, and gels. They vary in how well they address pressure distribution, postural stability, airflow, insulation or conduction of heat. Choice of style and material will depend on their individual needs and activities.

2.6.2Rear-Wheel Drive

The drives wheels are behind the user’s center of gravity, and the casters are in the front. It has predictable drive characteristics and stability but can be difficult to maneuver in tight places because of its larger turning radius.

This setup tends to be quite stable and provides a tight turning radius. However, it may have a tendency to fishtail and be difficult to drive in a straight line, especially when traveling fast on uneven surfaces.

2.3Related Works

(Ankit Kunit et al, 2018) intoduces to prepare a Head-Motion Controlled Wheelchair for physically disabled people who cannot move by their own. This wheelchair is suitable for those whose hands or fingers are paralyzed as it requires head motion. Rather than these, the head-motion controlled wheelchair is far cheaper than joystick wheelchairs. It uses MPU6050 accelerometer for movement sensing. In this technology, they connect accelerometer to ear-set, which is connected to Arduino Nano. This Arduino Nano is connected to transmitter side of Radio-Frequency module which uses 433 MHz frequency for communication with Receiver side. This Receiver side is connected with Wheel-Chair Motor for movement control of Wheel Chair. The main drawback of this paper is the wheelchair controlled by only head motion they can't use voice controller and other methods. The other limitation is the controller is Arduino Nano, it's simple controller.

(I.M Sollehuain, P.M. Heerwan,M.I Ishak,M.A.Zakaria, 2021) proposed tends to help Power with Disabilities (PWD) with their daily routine by introducing a path planning method that recognize the trajectory of the route by using the Artificial Potential Field Method (APF) that will act as an algorithm in a control system of an Electric Powered Wheelchair (EPW). Artificial Potential Field Method (APF) are consist of two main force field, Repulsive Force Field (RFF) and Attractive Force Field (AFF). The Repulsive Force Field (RFF) is designated as obstacle on a path whereas influencing the EPW to navigate away from the obstacle. While the Attractive Force Field (AFF) act as goal or the destination that designated for the EPW in which the EPW will be navigated in a way by approaching towards the AFF. The RFF consists of spacing headway that defines the distance from the obstacle to the EPW and while the limit distance is the maximum distance for the EPW to navigate away from the obstacle before the EPW collides with the obstacle. The strategy is to control the trajectory of the EPW during the obstacle avoidance with yaw angular velocity of the EPW. With spacing headway, limit distance and yaw angular velocity results, the trajectory of the EPW during the obstacle avoidance could be determined. The limitation of this paper is design and simulate the wheelchair to avoid obstacle straight forward it is not considering the turning strategy. It doesn't consider the stability of position and orientation.

(Gianluca Bardaro et al, 2018) linear MPC control scheme is proposed to indoor environment address the motion problems of an autonomous wheelchair in a realistic environment. An inner feedback-linearizing loop, the formulation of the model predictive control problem is simplified, allowing for a real-time computationally-efficient implementation. The MPC framework, constraints like obstacle avoidance, actuator limitations, and passenger comfort have been included in the optimization problem. However, the linear MPC control is design for only indoor environments.

(Fen Lin et al, 2019) AMPC controller is designed for path tracking in changeable working conditions. First, on the basis of the general MPC controller, the estimators of tire cornering stiffness and road friction coefficient are added, as well as the control parameter selection module. Moreover, the estimated tire cornering stiffness and road friction coefficient are used to update the vehicle model and road adhesion constraints respectively, which improves path tracking accuracy and stability. Also, the control parameter selector sets the suitable control parameters according to vehicle speed. The simulation results show that the proposed AMPC path tracking controller not only ensures good path tracking accuracy and stability but also has a great adaptive ability to changing working conditions, especially to speed and road conditions. However, the AMPC path tracking in this thesis to control only speed and road adhesion.

(Saverio Bolognani et al, 2008) proposed using a permanent magnet synchronous motor drive as a test bench, the gives an exhaustive description of the design procedure of a Model Predictive Control (MPC) applied to the combined control of the motor speed and current. Suggestions for extending the design to different drive controllers are also included. Simulations, as well as numerous experimental results, highlight the promising features and characteristics of MPC applied to electrical motor drives. Speed and current controllers are combined together in a single MPC that includes all the state variables of the system, instead of keeping the conventional cascade structure. The MPC potentials pointed out in the thesis should stimulate further exploration and study on this type of controllers, in order to achieve the familiarity required to transfer the results to practical applications. However, a permanent magnet synchronous motor drive needs orientation controls.

(Eugenio Ceravolo et al, 2017) proposed a linear MPC control scheme has been proposed to address the motion problems of autonomous wheelchair in a realistic human environment. The introduction of inner feedback linearizing loop simplifies the development of the controller and allows for a real time computationally efficient implementation. The constraints like obstacle avoidance, actuator limitations and passenger comfort can been easily included in the optimization problem.

(Viet Thuan et al, 2018) introduces a path following control approach for the EPW system combining transverse feedback linearization and MPC technique was developed in this thesis. Transverse feedback linearization integrates the desired path in the coordinate transformation which can assure the path invariance property while the constraints in solving the optimal control problem have been taken into account using MPC approach. The effectiveness of the proposed control methodology has been clearly demonstrated through numerical simulation with different driving situations. The desired path is always well followed in both studied cases, circular path and arbitrary path in the office environment while ensuring states and inputs constraints. The constraints coming from the user comfort (acceleration and jerk) and the physical limitations of the motor's torque are respected. Though, the passengers comfort is not added.

Despite the promised robustness, real-time implementation of Proportional Integral Derivate (PID) is limited by a significant drawback known as constraints, multiple input multiple output used. This problem settled by using Model Predictive Controller (MPC). The MPC controllers is optimal control technique which is calculated minimize cost function for constrained dynamics systems and simulated in MATLAB/Simulink and the results are presented.

The main contribution of this thesis is the performance analysis of MPC controller for successful (orientation) and position control of a two-wheel quadriplegic electrical wheelchair. The speed, current, wheel angular speed, control signal and position of wheels combining together by Model Predictive Controller. As far as we know, the MPC using speed, current, position, angle, control signal, and wheel angular speed together to the balancing of position and orientation has never been applied for quadriplegic electrical wheelchair.

CHAPTER THREE

3.SYSTEM MODELING AND CONTROLLER DESIGN

3.1Introduction

In this chapter, the mathematical model of the DC motor, kinematics and chassis of the dynamics wheel and a model predictive controller is applied for balancing position and orientation (angle) the quadriplegic wheelchair. The performance of the controller was analyzed using the cost function taken to the desired track line.

3.2Materials Used in This Thesis

In this research work, software such as MATLAB/2017a, Microsoft Office 2016, and Math Type 6.0 equation editor are used. MATLAB is a computing software which consists of technical toolboxes and Simulink. Microsoft office is used for editing the thesis documentation. Math Type 6.0 is used for writing mathematical formulas and equations in Microsoft office word and PowerPoint presentation tools.

3.3Methods

The methodology of this thesis mainly focuses on software algorithms. The thesis will be developed using a flowchart that lists all of the tasks to be performed. The project will be developed based on a flowchart that determines all the necessary activities that has to be accomplished. Figure3.1Shown the flow chart of the project, which begins with information collection on topics related to the electrical quadriplegic wheelchair through literature review. After reviewing all the resources, next step is to do the mathematical model it provides the fundamental concept for this project. Matlab Simulink will be used as a simulation tools to see the performance of the controller. After that the MPC tuning parameters Kalman filter estimation such as prediction horizon, control horizon and sampling times used for online tuning parameters.

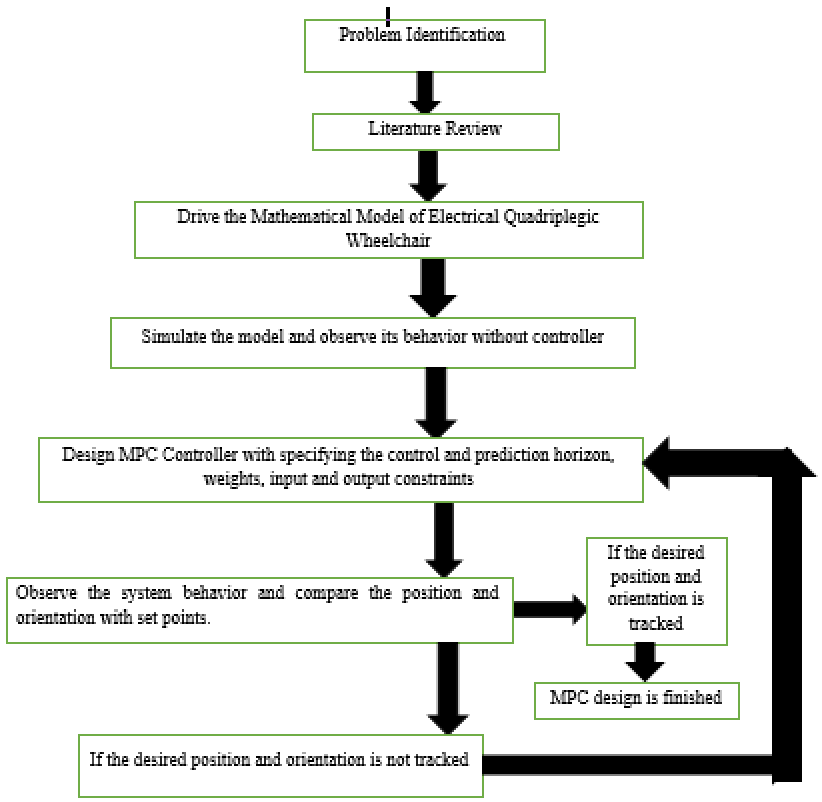

Figure3.1 Flow chart of working principle of electrical quadriplegic wheelchair

3.4Working Principle of Quadriplegic Wheelchair

The working principle of an electrical quadriplegic wheelchair is involved in continuously obtaining the feedback by the receding horizon, compensating the error with respect to the reference and maintaining the entire platform in an upright position. This is necessary to reduce the effects of model/plant mismatch, and also to stabilize the effect of unknown disturbances. In order to recover back to the stable position has been included in the control unit of the quadriplegic wheelchair as it improves the overall safety assurance for the person.

The camera is to identify and estimate a road and its position and orientation relative to the quadriplegic wheelchair. This means that once the road has been identified in an image, the coordinates of the road have to map into base link frame.

Figure 3.2 Quadriplegic Wheelchair Trajectory Path

The quadriplegic wheelchair starts at lower left position follows the middle row until the end is reached. The field has established desired trajectory path that the wheelchair should follow and in between the wheelchair tracks there are three roads as seen in Figure 3.2. The approach used here for navigation is to identify the middle road and attempt to follow it closely. The rear caster wheel is mounted off-center to ensure that no collision occurred during operation.

Finally control system is computing a trajectory of future control inputs that optimizes the future behavior of output, where the optimization is carried out within a limited time window is objective of a model predictive control strategy. Controllability is needed in order for the design of MPC to access all dynamic modes in the process. A good dynamic model gives a consistent and accurate prediction of the future.

3.5Block Diagram of System with its Description

The receding horizon is re-optimized at every time step, as means to obtain some feedback of control signals. The desired position of the trajectory path obtained from the camera and it is used for Precalculation of the reference control signals. The MPC necessary to reduce the effects of model/plant mismatch, and also to counteract the effect of unknown disturbances. The quadriplegic wheelchair is controlled by model predictive control output control signal summed with reference control signal. The output of electrical quadriplegic wheelchair is the motor position and orientation (X_r, Yr and α_r).

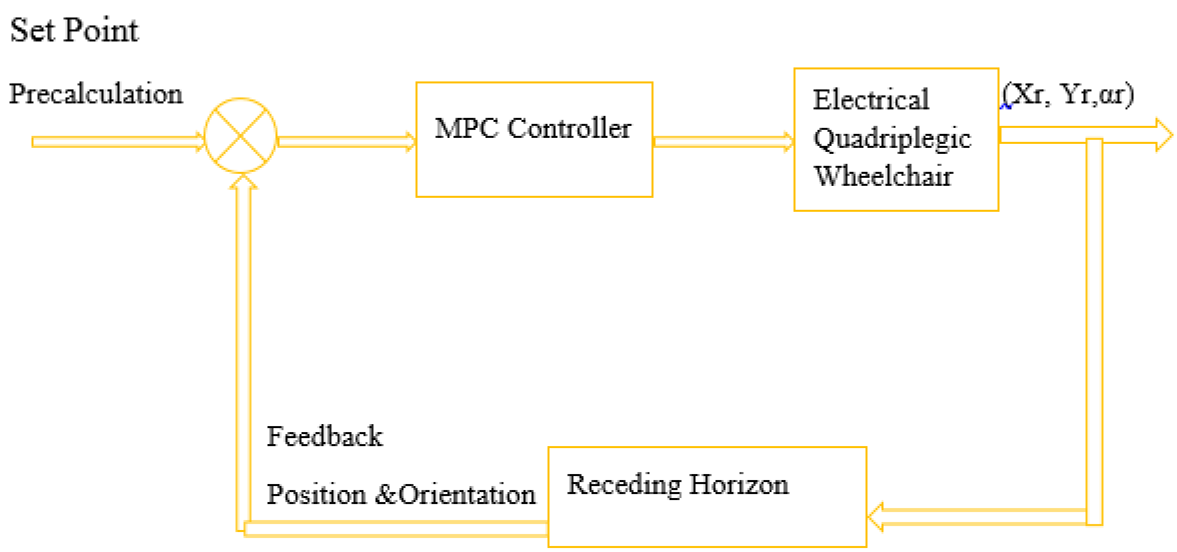

Figure 3.3 Block Diagram of System

3.6Mathematical Modeling

Mathematical model of quadriplegic electrical wheelchair is three independent parts:

- Dynamics model
- Kinematics model
- Chassis dynamics model

Motor supply voltage actuating the wheel causes driving torque and thereby wheel rotation. Inertial and resistance forces act against driving torque. Both driving torques influence each other because of these forces. Planar curvilinear motion of the robot is result of various time variant wheel rotation speeds. Planar curvilinear motion can be decomposed to a sum of linear motion (translation) and rotation motion. Forces balance is starting point for the derivation of motion equations. If *F* is actual force acting to a mass point with weight *m* and with distance *r* from axis of rotation, then it holds for general curvilinear motion that vector sum of all forces acting to a selected point is zero. (F.Dusek,D.Honc and P.Rozsival, June 14-17,2011)

$$F + m\frac{dv}{dt} + m\frac{d\omega}{dt} + 2m\omega \times \frac{dr}{dt} + m\omega \times (\omega \times r) = 0 \tag{3.1}$$

Wheelchair is driven by two DC motors with common voltage source and independent control of each motor. Motors are connected with driving wheels through gear-box with constant gear ratio.

Ideal gear-box means that it reduces linearly angular speed and boosts the moment (nonlinearities are not considered). Loses in motor and also in gear-box are proportional to rotation speed. Chassis is equipped with caster wheel with no influence to chassis motion.

3.7 DC Series Connection

By applying Kirchhoff voltage law around the loop we get the electrical equivalent circuit. Kirchhoff voltage law states that "the algebraic sum potential differences (product of current and resistance) in closed loop equal to zero." Field current is generally kept constant and steady state.

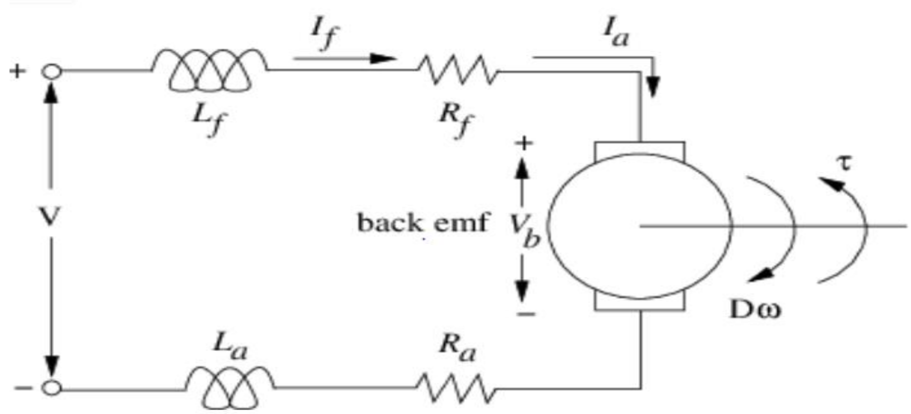

Figure3.4 DC Electrical Equivalent Circuit *(Wudhicai Assawinchaichote and Sing Kiong Nguang, 2009)*

By applying Kirchhoff voltage law,

$$V_a - VR_a - VL_a - V_b = 0 \quad (3.2)$$

Based on ohm's law, the voltage across the resistor can be represented as:

$$VR_a = i_a(t) R_a \quad (3.3)$$

Where, i_a is the armature current. The voltage across the inductor is proportional to the current change through the coil with respect to time and can be written as:

$$VL_a = L_a \frac{dia}{dt} \quad (3.4)$$

Where, La is the inductance of the armature coil. At the end, the back emf can be formulated as:

$$V_b = k_b \omega_m(t) \quad (3.5)$$

Where, k_b is the velocity constant, determined by the flux density of the permanent magnets, the reluctance of the iron core of the armature, and the number of turns of the armature winding ω_m is the rotational velocity of the armature.

Substituting equation (3.2,3.3,3.4and 3.5) gives the differential equation of

$$V_a - i_a R_a - L_a \frac{dia}{dt} - K_b \omega_m = 0 \qquad (3.6)$$

3.7.1Mechanical Equation of DC series motors

The mechanical equation of newton law of motion is obtained as follows;

$$\tau_m(t) = \frac{J_m d^2\theta(t)}{dt^2} + \frac{Krd\theta(t)}{dt} \qquad (3.7)$$

Where, τ_m is the motor torque produced by the motor shaft, J_m and k_r are denoted as the moment of inertia and motor friction coefficient.

Based on equation (3.6 &3.7),when the input voltage $v_a(t)$ is applied, the armature current $i_a(t)$ goes through resistance R_a and inductance L_a producing magnetic flux and causing the motion of the rotor according to the motor torque give as equation 3.8

$$\tau_m(t) = k_t i_a(t) \qquad (3.8)$$

Where K_t is the torque constant and like the velocity constant, it depends on the flux density of the fixed magnets, the reluctance of the iron core, and the number of turns in the armature winding.

The equation (3.2, 3.6,3.7 and 3.8) give as;

$$L_a \frac{d i_a(t)}{dt} + R_a i_a(t) = V_a(t) - K_b \frac{d\theta(t)}{dt} \qquad (3.9)$$

$$\frac{J_m d^2\theta(t)}{dt^2} + \frac{K_r d\theta(t)}{dt} = K_t i_a(t) \qquad (3.10)$$

The mechanical equation does not involve load torque but, the following state space representation consists load torque.

3.7.2State Space Representation of DC Motors

The state space model which consisting load torque that makes the state space to have two inputs (a load torque (M) and terminal voltage Va). Commutator is not considered. Rotor produces electrical voltage with reverse polarity than source voltage electromotive voltage($kb\omega(t)$) which is proportional to rotor angular velocity. Twisting moment of the rotor Mm is proportional to current $ia(t)$. Ideal behavior means that whole electric energy used to magnetic field creating is transformed without any loses to mechanical energy moment of the motor. Loses in magnetic field are not considered, but only electric loses in winding and mechanical loses proportional to rotor speed are considered.

$$R_a i_a(t) + \frac{L_a di_a(t)}{dt} = V_a(t) - K_b \omega_m(t)$$

$$\frac{J_m d\omega_m(t)}{dt} + K_r \omega_m(t) + M = K_t i_a(t)$$

$$\frac{di_a(t)}{dt} = \frac{V_a(t)}{L_a} - \frac{R_a i_a}{L_a} - \frac{K_b \omega_m(t)}{L_a} \tag{3.11}$$

$$\frac{d\omega_m(t)}{dt} = \frac{K_t i_a(t)}{J_m} - \frac{K_r \omega_m(t)}{J_m} - \frac{M}{J_m} \tag{3.12}$$

The rotor position results 3rd order differential eqn of the motor model.

$$\frac{d\theta(t)}{dt} = \omega_m(t) \tag{3.13}$$

The state space representation of the differential equations of (3.11), (3.12) and (3.13)

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_m \\ \theta \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} & 0 \\ \frac{K_t}{J_m} & -\frac{K_r}{J_m} & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J_m} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ M \end{bmatrix} \tag{3.14}$$

$$\begin{bmatrix} \omega_m \\ \theta \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega \\ \theta \end{bmatrix}$$

Where, $\begin{bmatrix} \omega_m \\ \theta \end{bmatrix}$ are outputs and $\begin{bmatrix} V_a \\ M \end{bmatrix}$ are inputs.

Table 3.1 Estimated Parameters of DC Motors *(F.Dusek,D.Honc and P.Rozsival, June 14-17,2011)*

Notation	Values	Unit	Meaning
R_a	2.000	Ohms(Ω)	Motor winding resistivity
L_a	0.05	Henry (H)	Motor inductance
K_a	0.100	$Kg.m^2.s^{-2}.A^{-1}$	Electro-motive constant
R_Z	0.200	Ohms(Ω)	Internal resistance
J	0.025	$Kg.m^2$	Total moment of inertia of rotor and gearbox
K_r	0.002	$Kg.m^2.s^{-1}$	Coffiecent of the resistance against rotating of rotor and gearbox
PG	25	--------	Gearbox transmission ratio
U_O	10.000	Voltage (v)	Source voltage

This value can be insert in the parameter of equation (3.14).

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_m \\ \theta \end{bmatrix} = \begin{bmatrix} -40 & -20 & 0 \\ 4 & -0.08 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta \end{bmatrix} + \begin{bmatrix} 20 & 0 \\ 0 & -40 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_a \\ M \end{bmatrix} \tag{3.15}$$

$$\begin{bmatrix} \omega_m \\ \theta \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \\ \theta \end{bmatrix}$$

The DC series motor represented by the state space above can also represented with two inputs and one output as shown below.

$$\frac{d}{dt}\begin{bmatrix} i_a \\ \omega_m \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_b}{J_m} & -\frac{K_r}{J_m} \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J_m} \end{bmatrix} \begin{bmatrix} V_a \\ M \end{bmatrix} \tag{3.16}$$

$$\omega_m = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} i_a \\ \omega_m \end{bmatrix}$$

Then, transforming the above two equations using Laplace transformation obtained the two equations as follow.

$$L_a \frac{d i_a(t)}{dt} = -R_a i_a(t) - K_b \omega_m(t) + V_a(t) \tag{3.17}$$

Substitute $K_b \omega_m(t)$ into $K_b \frac{d\theta(t)}{dt}$ because of third order differential equation so, we can get

$$L_a \frac{d i_a(t)}{dt} = -R_a i_a(t) - K_b \frac{d\theta(t)}{dt} + V_a \tag{3.18}$$

$$L_a S i_a(s) + R_a i_a(s) = V_a(s) - K_b S\theta(s) \tag{3.19}$$

$$J_m \frac{\omega_m}{dt} = K_b i_a(t) - K_r \omega_m(t) - M \tag{3.20}$$

Simplifying this equation into

$$J_m \frac{d^2\theta(t)}{dt} = K_b i_a(s) - K_r S\theta(s) \tag{3.21}$$

$$J_m s^2 \theta(s) + K_r S\theta(s) = K_b i_a(s) \tag{3.22}$$

The transfer function of the motor speed is:

$$G_{speed}(s)=\frac{S\theta(s)}{V_a(s)}=\frac{K_t}{J_m L_a s^2+(L_a K_r+R_a J_m)s+K_r R_a+K_t R_a} \tag{3.23}$$

In addition, the transfer function of the motor position is determined by multiplying the transfer function of the motor speed by the term $\frac{1}{s}$.

$$G_{position}(s)=\frac{\theta(s)}{V_a(s)}=\frac{K_t}{J_m L_a S+L_a K_r+R_a J_m+K_r R_a+K_t R_a} \tag{3.24}$$

The Dynamics of the two series DC motor used in the system can be derived from balancing of voltages (Kirchhoff's law) and balancing of moments. From Kirchhoff's voltage law as,

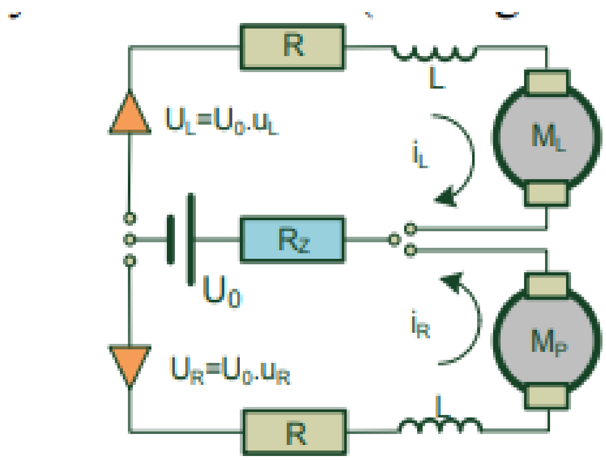

Figure 3.5 DC Motor Wiring Quadriplegic Wheelchair *(F.Dusek,D.Honc and P.Rozsival, June 14-17,2011)*

$$R_a i_L(t)+R_z(i_L(t)+i_R(t)+\frac{L_a di_L(t)}{dt}=U_L V_a-K_b\omega_L(t) \tag{3.25}$$

$$R_a i_R(t)+R_z(i_L(t)+i_R(t))+\frac{L_a di_R(t)}{dt}=U_R V_a(t)-K_b\omega_R(t) \tag{3.26}$$

Where,

Rz [Ω] is internal resistance

Ra [Ω] is motor winding resistance

La [H] is motor inductance

kb [V /m s-1] is the back EMF constant

Va [V] is source voltage

ωL [rad. s-1] is left motor angular velocity

ωR [rad. s-1] is right motor angular velocity

uR and uL are the control voltages of the right and left motors respectively. iL and iR [A] are left and right current flowing through winding. Each motor has its own power supply voltage (UL, UR) disbranched from the common source of voltage Va Control of the supply voltage of both motors using amplifier with control (uR and uL) is shown in Figure.3.5. Because both engines are powered from the common source it was taken into account also effect of the internal resistance RZ. Both motors are considered with the same parameters and the second equation is balance of moments (electric energy) moment of inertia MS, rotation resistance proportional to rotation speed (mechanical loses) MO, load moment of the motor Mx and moment Mm caused by magnetic field which is proportional to current.

$$M_s + M_o + M_x = M_m$$

$$\frac{J_m d\omega_L(t)}{dt} + K_r \omega_L(t) + M_L = K_b i_L(t) \qquad (3.27)$$

$$\frac{J_m d\omega_R(t)}{dt} + K_r \omega_R(t) + M_R = K_b i_R(t) \qquad (3.28)$$

Where,

Jm [kg.m2] is moment of inertia.

kr [N.ms] is coefficient of rotation resistance and Mx [kg.m^2. s] is load moment.

ML and MR are the load moments on left and right wheels respectively.

3.7.3 Chassis dynamics

Let defined the chassis with a vector of linear velocity vB acting on a chassis reference point and with rotation of this vector of angular velocity ωB. The chassis reference point 'B' is the point of the intersection of the axis joining the wheels and center of gravity normal projection see figure below. The point 'T' is common center of gravity which is placed at the center of the axis joining the wheels.

It is possible to calculate trajectory of arbitrary chassis point from these variables. Point ‘B’ for which the equations are derived is center of gravity normal projection to join between wheels. This leads to easiest set of equation for whole model. Forces balances are considered ‘as starting equations. It is possible to replace two forces $'FL'$ and $'FR'$ acting to chassis in left (L) and right (R) wheel ground contact points with one force $'FB'$ and torsion moment Mb acting in point ‘B’. Chassis is characterized with semi-diameter of the driving wheels ‘r’, total weight ‘m’, moment of inertia ‘JT ‘with respect to center of gravity located with parameters LT, LL, LR.

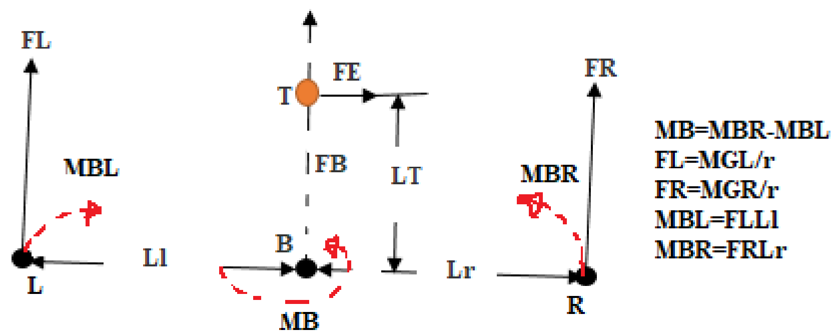

Figure3.6 Chassis Force Scheme *(F.Dusek,D.Honc and P.Rozsival, June 14-17,2011)*

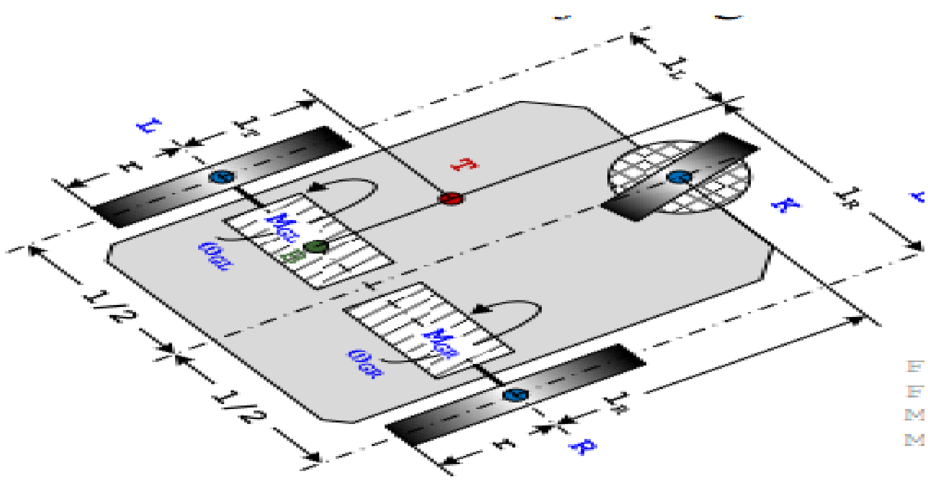

Figure 3.7Chassis Forces Scheme *(F.Dusek,D.Honc and P.Rozsival, June 14-17,2011)*

Position of the center of gravity is constant with respect to axis of rotations. Coriolis force is not needed. Coriolis force has to be considered for example if the chassis moves on rotating surface. Similarly, centrifugal force is not considered. Chassis is supposed to be solid body represented as mass point (center of gravity). Because force vector causing the movement acts in point b and goes through center of gravity. It is enough if we consider inertial force by linear motion. By rotational motion it is necessary to consider moment caused with Euler's force because the axis of rotation does not go through the center of gravity. By the balance of forces causing linear motion, consider except of forces F_L, F_R caused by drives and inertial force 'F$_S$ also resistance force F_O proportional to speed v_B. The balance of all forces influencing linear motion is expressed as fellow.

$$F_L + F_R + F_o + F_s = 0$$

$$\frac{MG_L}{r} + \frac{MG_R}{r} - K_v V_B - \frac{mdV_B}{dt} = 0 \tag{3.29}$$

Where,

m[kg] is robot mass.

Kv [kg. s^{-1}] is resistance coefficient against linear motion.

MGL [kg.m2. s^{-2}] is moment of the left drive.

MGR [kg.m2. s^{-2}] is moment of the right drive.

vB [m/s] is linear motion speed.

r[m] is semi-diameter of the wheels.

Balance of moments is slightly more complicated because the rotation axis does not lie in center of gravity. That's why it is necessary to take into account except chassis momentum MT also moment $ME = ltFE$ caused by Euler's force FE. Similarly, as by linear motion is considered moment MO caused with resistance against rotation to be proportional to angular velocity ω_B.

$$MB_L + MB_R + M_O + M_T + M_E = 0$$

$$-\frac{MG_L L_L}{r} + \frac{MG_R L_R}{r} - K_\omega \omega_B - \frac{J_T d\omega_B}{dt} - \frac{L_T m d\omega_B}{dt} = 0 \tag{3.30}$$

Where

LR [m] is distance of the right wheel from point B,

LL [m] is distance of the left wheel from point B,

LT [m] is distance of the center of gravity from point B,

$K\omega$ [kg.m^2. s^{-2}] is resistance coefficient against rotary motion

JT [kg.m^2] is moment of inertia with respect to rotation axis in center of gravity and ωB [rad. s^{-1}] is angular speed in point B. Resulting moment of inertia JB with respect to rotation axis in Point B is given by equation below which is parallel axis theorem.

$J_B = J_T + m_B L_T$

Where, JT [kg.m2] is moment of inertia with respect to center of gravity and LT [m] is distance between center of gravity and point B.

The equation describing the behavior of the two motors (currents and angular velocity) and the behavior of the chassis (the speed of the linear movement and speed of the rotation) are connected only through moments of engines. Equations express the law of conservation of energy which is conversion of electrical energy to mechanical including one type of losses but represent only one relationship between the speed of the two motors (peripheral speed of the drive wheels) and rates of movement and rotation of the chassis. Additional relation is given by design of the drive and chassis. Both drive wheels are firmly linked to rotors of relevant engines over ideal gearbox with gear ratio PG without nonlinearities and any flexible members. Gearbox decreases output angular velocity ωGx with relation to the input angular speed ωx according to the transmission ratio PG and simultaneously in the same proportion increases output torque MGx with relation to the input torqueMx.

$$\omega_{GL} = \frac{\omega_L}{P_G}, \omega_{GR} = \frac{\omega_R}{P_G} \tag{3.31}$$

$$MG_L = P_G M_L, MG_R = P_G M_R \tag{3.32}$$

The chassis dynamics can be expressed by the combination of balance of the forces and balance of the moment.

$$\left(\frac{P_G M_L}{r} + \frac{P_G M_R}{r}\right) - K_v V_B - \frac{mdv}{dt} = 0$$

$$M_L + M_R - r_G K_V V_B - r_G \frac{mdv}{dt} = 0 \tag{3.33}$$

$$-\frac{L_L P_G M_L}{r} + \frac{L_R P_G M_R}{r} - K_\omega \omega_B - \frac{\left(J_T + MLT^2\right) d\omega_B}{dt} = 0$$

$$-L_L M_L + L_R M_R - r_G K_\omega \omega_B - \frac{r_G J_B d\omega_B}{dt} = 0 \tag{3.34}$$

Where, PG is the gear box transmission ratio, $k\omega$ is the resistance coefficient against rotational motion.

$$r_G = {}^{r}\!/_{P_G} \tag{3.35}$$

$$J_B = J_T + mLT^2 \tag{3.36}$$

Both drive wheels have the same radius r and their peripheral speedsvL, vR depend on the angle speeds of gearbox output ωGL, ωGR according to relations:

$$V_L = r_\omega G_L = {}^{r\omega_L}\!/_{P_G} \tag{3.37}$$

$$V_R = r_\omega G_R = {}^{r\omega_R}\!/_{P_G} \tag{3.38}$$

we can recalculate the peripheral velocities of the wheelsvL, vR to the linear velocity vB and angular velocity ωB at point B by theorem of similar triangles.

$$V_B = {}^{(V_L L_R + V_R V_L)}\!/_{V_L + V_R} = {}^{r}\!/_{P_G}(L_L + L_R)(L_R\omega_L + L_L\omega_R)$$

$$\omega_B = {}^{V_B}\!/_{(x + L_L)} = {}^{(V_R - V_L)}\!/_{L_L + L_R} = {}^{r}\!/_{P_G}(L_L + L_R)(-\omega_L + \omega_R)$$

$$V_B = \frac{r_G}{L_L + L_R}(L_R\omega_L + L_R\omega_R) \tag{3.39}$$

$$\omega_B = \frac{r_G}{L_L + L_R}(-\omega_L + \omega_R) \tag{3.40}$$

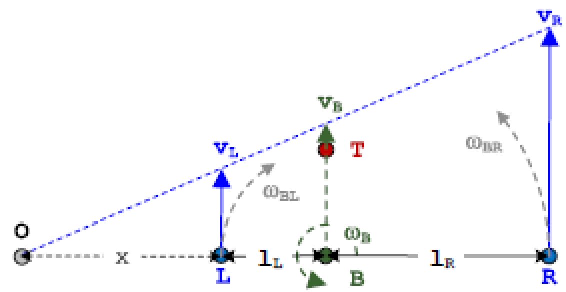

Figure 3.8 Linear and Angular Triangles *(F.Dusek,D.Honc and P.Rozsival, June 14-17,2011)*

3.8Kinematics model

Let the global coordinates of the wheelchair be (x, y), the orientation of the wheelchair be α, and v, ω are the linear and angular velocities.

$$\frac{dx}{dt} = V_B \cos \alpha_B \quad (3.41)$$

$$\frac{dy_B}{dt} = V_B \sin \alpha_B \quad (3.42)$$

$$\frac{d\alpha_B}{dt} = \omega_B \quad (3.43)$$

The simple model as

$$\frac{dx_B}{dt} = \mathrm{f}(xB, uB) \quad (3.44)$$

Where state variables $XB = [xB\ yB\ \alpha B]^T$ and control inputs,$UB = [vB\ \omega B]^T$

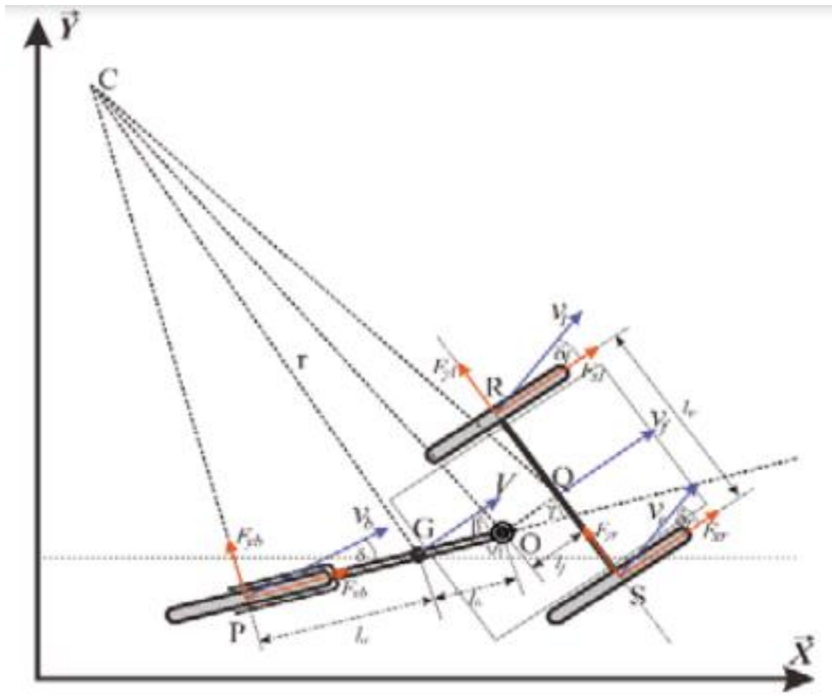

Figure3.9 Coordinate System

A reference wheelchair can be considered with reference (desired) parameters of the wheelchair to follow a trajectory which can be represented as

$$\frac{dx_r}{dt} = f\left(x_r, u_r\right) \tag{3.45}$$

The reference parameters are [$xr\ yr\ \alpha r\ vr\ \omega r$]. The linear velocity, orientation angle and angular velocity of the reference robot can be derived from Equation (3.40 ,3.41, 3.42) as

$$V_r(t) = \sqrt{\left(\frac{dx_r(t)}{dt}\right)^2 + \left(\frac{dy_r(t)}{dt}\right)^2} \tag{3.46}$$

$$\alpha_r = \arctan^2\left(\frac{dy_r(t)}{dt}, \frac{dx_r(t)}{dt}\right) \tag{3.47}$$

$$\omega_r(t) = \frac{d\alpha_r r(t)}{dt} = \frac{\frac{dx_r(t)}{dt} \times d^2 y_r(t)^2 - d^2 x_r(t)^2 \times \frac{dy_r(t)}{dt}}{\left(\frac{dx_r(t)}{dt}\right)^2 + \left(\frac{dy_r(t)}{dt}\right)^2} \quad (3.48)$$

Applying the Taylor series approximation to Equation (3.44), around the reference points (xr,r)

$$\frac{dx(t)}{dt} = f\left(x_r, u_r\right) + \frac{\partial f(x,u)}{\partial x}\left(x_B - x_r\right) + \frac{\partial f(x,u)}{\partial x}\left(u_B - u_r\right) \quad (3.49)$$

Evaluated at $u=ur$, $x=xr$ Subtracting Equation (3.49) from Equation (3.45) gives

$$\frac{d\dot{x}}{dt} = f x_r \cdot x + f u_r \cdot u \quad (3.50)$$

$\dot{x}$ is the error vector of state variables and u is the error vector of control variables with respect to the reference. The approximation of $\frac{d\dot{x}}{dt}$ in Equation (3.50), by the forward differences gives the following discrete-time linear time-variant (LTV) state-space model.

$$X_K(K+1) = AK(k)XK(k) + BK(k)UK(k) \quad (3.51)$$

$$YK(k) = CKXK(k) \quad (3.52)$$

Where

$$AK(k) = \begin{bmatrix} 1 & 0 & -V_r(k)\sin\alpha_r(k)T \\ 0 & 1 & V_r(k)\cos\alpha_r(k)T \\ 0 & 0 & 1 \end{bmatrix}$$

$$B_k(k) = \begin{bmatrix} \cos\alpha_r(k)T & 0 \\ \sin\alpha_r(k)T & 0 \\ 0 & T \end{bmatrix}$$

$$C_K(k) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$X_K = \begin{bmatrix} y_B(k) - x_r(k) \\ y_B(k) - y_r(k) \\ \alpha_B(k) - \alpha_r(k) \end{bmatrix}$$

$$u_K = \begin{bmatrix} V_B(k) - V_r(k) \\ \omega_B(k) - \omega_r(k) \end{bmatrix}$$

Where T is the sampling period and xk is deviation state vector which represents the error with respect to the reference robot, and uk is associated with the control input. The reference values, vr, αr, ωr are the reference linear velocity, orientation angle and angular velocity respectively which can be calculated from Equations (3.46-3.48).

3.9Computational Form of Model

The last part is the calculation of coordinates of the position of arbitrary points of the chassis. Linear part of the model can be modified into simpler form for control design purposes to reduce number of differential equations from six to four. Substitute equations (3.39,3.50) into (3.33,3.34) and eliminate moments ML and MR by substitution of (3.27, 3.28) to (3.33, 3.35) we can able to reduce four differential equations (3.27, 3.28 to (3.33, 3.34) .These six differential Equations (3.25, 3.26) to (3.33, 3.34), and two algebraic Equations (3.39, 3.40) containing eight state variables represent a mathematical description of the dynamic behavior of ideal quadriplegic wheelchair with losses linearly dependent on the revolutions or speed. Control signals, uL and uR, that control the supply voltages of the motors are input variables. To put the equation in state space, new parameters are introduced as follows,

$$a_r = K_r + \frac{K_v \times L_L \times r^2 G}{L_L + L_R}$$

$$a_l = K_r + \frac{K_v \times L_R \times r_G{}^2}{L_L + L_R}$$

$$b_l = J_m + m \times L_R \times \frac{r_G^2}{L_L + L_R}$$

$$b_r = J_m + m \times L_L \times \frac{r_G^2}{L_L + L_R}$$

$$c_l = K_v \times L_L + \frac{K_\omega \times r_G^2}{L_L + L_R}$$

$$c_r = K_r \times L_R + \frac{K_\omega \times r_G^2}{L_L + L_R}$$

$$d_l = J_m \times L_L + \frac{J_b \times r_G^2}{L_L + L_R}$$

$$d_r = J_m \times L_R + \frac{J_b \times r_G^2}{L_L + L_R}$$

$$\frac{dx_d}{dt} = A_d X_d + Bd_u; Y_d = C_d X_d \qquad (3.53)$$

Where,

$$u = \begin{bmatrix} u_L \\ u_R \end{bmatrix}$$

$$X_d = \begin{bmatrix} i_L \\ i_R \\ \omega_L \\ \omega_R \end{bmatrix}$$

$$Y_d = \begin{bmatrix} v_B \\ \omega_B \end{bmatrix}$$

$$A_d = \begin{bmatrix} -\frac{(R_a+R_z)}{L_a} & \frac{-R_z}{L_a} & \frac{-K_b}{L_a} & 0 \\ \frac{-R_z}{L_a} & \frac{-(R_a+R_z)}{L_a} & 0 & \frac{-k_b}{L_a} \\ K_b\frac{d_r+b_rL_L}{b_ld_r+b_rd_l} & K_b\frac{d_r-b_rL_R}{b_ld_r+b_rd_l} & \frac{d_ra_l+b_rc_l}{b_ld_r+b_rd_l} & \frac{-d_ra_r-b_rc_r}{b_ld_r+b_rd_l} \\ K_b\frac{d_l-b_lL_L}{b_ld_r+b_rd_l} & \frac{d_l+b_lL_R}{b_ld_r+b_rd_l} & \frac{-d_la_l-b_lc_l}{b_ld_r+b_rd_l} & \frac{-d_la_r+b_lc_r}{b_ld_r+b_rd_l} \end{bmatrix}$$

$$B_d = \begin{bmatrix} \frac{V_a}{L_a} & 0 \\ 0 & \frac{V_a}{L_a} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_d = \begin{bmatrix} 0 & 0 & r_G\frac{L_R}{(L_L+L_R)} & r_G\frac{L_L}{L_L+L_R} \\ 0 & 0 & -\frac{r_G}{L_L+L_R} & \frac{r_G}{L_L+L_R} \end{bmatrix}$$

A linearized discrete error model is obtained from the kinematic model and the dynamic model is converted to a discrete error based model. The following equation is the discrete time state-space dynamic model.

$$Xd(k+1) = Ad(k)\,Xd(k) + Bd(k)\,Ud(k) \tag{3.54}$$

$$Yd(k) = Cd(k)\,Xd(k) \tag{3.55}$$

The matrices Ad, d and Cd are discretized matrices of the dynamic model (Equation (3.52)). The state variables and control inputs are deviation variables from the reference points, xdr and udr, as $xB, B, \alpha, iL, iR, \omega L, \omega R, vB, \omega B$.

Xd(k)= iL(k)−iLr(k)

iR(k)−iRr(k)

ωR(k)−ωRr(k)

ωL(k)−ωLr(k)

ud(k)=uL(k)−uLr(k)

uR(k)−uRr(k)

This dynamics model and linearized kinematic time variant model, Equation (3.51, 3.52), can be combined into a single state-space time-variant model with five states (currents, wheel speeds, linear and angular velocities and coordinates), two control variables (left and right motor voltage control input) and three outputs (position in x and y direction and orientation measured from x direction).

$$Xk(k+1) = Ak(k)\,Xk(k) + Bk(k)\,uk(k) \tag{3.56}$$

$$yk(k) = C(k)\,X(k) \tag{3.57}$$

Where,

$$Ak(k) = \begin{bmatrix} Ad(4\times4) & 0(4\times5) \\ Cd(2\times4) & 0(2\times5) \\ 0(3\times4)\,Bk(3\times2) & Ak(3\times3) \end{bmatrix}$$

$$Bk(k) = \begin{bmatrix} Bd(4\times2) \\ 0(5\times2) \end{bmatrix}$$

$$Ck(k) = \begin{bmatrix} 0(3\times6) & Ck(3\times3) \end{bmatrix}$$

$$X = \begin{bmatrix} Xd(k) \\ uk \\ Xk \end{bmatrix}, u = ud(k)$$

Table3.2Chassis parameter *(F.Dusek,D.Honc and P.Rozsival, June 14-17,2011)*

Notation	Value	Unit	Meaning
l_L	0.040	M	Distance of the left wheel from point B.
l_R	0.060	M	Distance of the right wheel from point B.
l_T	0.020	M	Distance of the Centre of gravity from join between wheels
l_K	0.040	M	Distance of caster wheel from join wheel between wheels
R	0.050	M	Semi-diameter of driving wheels
M	25	Kg	Total weight of the wheelchair
K_v	0.100	$Kg.s^{-1}$	Coefficient of the resistance against wheelchair linear motion
J_B	0.550	$Kg.m^2$	Moment of inertia of wheelchair with respect to center of gravity
k_ω	1.350	$Kg.m^2.s^{-1}$	Coefficient of resistance against wheelchair

By inserting the chassis parameter values into state space model of wheelchair to verified the above state matrix, output and input state matrix are shown below.

$$A=\begin{bmatrix} -40.4000 & -0.4000 & -2.000 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.4000 & -40.4000 & 0 & -2.000 & 0 & 0 & 0 & 0 & 0 \\ 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0840 & 0.0760 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.4600 & 0.4600 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.000 & 0 & 1.000 & 0 & -0.0004 \\ 0 & 0 & 0 & 0 & 0.0006 & 0 & 0 & 1.0000 & 0.8000 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 10.000 \end{bmatrix}$$

$$B=\begin{bmatrix} 200 & 0 \\ 0 & 200 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ & \end{bmatrix}$$

$$C=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, D=0$$

3.10Disturbance affecting quadriplegic wheelchair

The previous state space of quadriplegic wheelchair is representation without considering disturbance. The model state space with disturbance written as:

$$\begin{aligned} x_m(k+1) &= A_m x_m(k) + B_m u(k) + B_d \omega(K) \\ y(k) &= C_m x_m(k) \end{aligned} \qquad (3.58)$$

Where ω(k) is input disturbance, assumed to be sequence integrated white noise Ɛ(k).

$$\omega(k) - \omega(k-1) = \varepsilon(k) \qquad (3.59)$$

From equation (3.58) the following difference equation is also true.

$$\triangle x_m(k) = A_m x_m(k-1) + B_m u(k-1) + B_d \omega(K-1) \qquad (3.60)$$

By defining $\triangle x_m(k) = x_m(k) - x_m(k-1)$ and $\triangle u(k) = u(k) - u(k-1)$, then subtracting equation (3.57) from (3.59) get to

$$\triangle x_m(k+1) = A_m \triangle x_m(k) + B_m \triangle u(k) + B_d \varepsilon(k) \qquad (3.61)$$

To relate the output of y(k) with Δxm(k), that deduce $Bd\omega(k)$

$$\triangle y(k+1) = C_m \triangle x_m(k+1) = C_m A_m \triangle x_m(k) + C_m B_m \triangle u(k) + C_m B_d \varepsilon(k) \tag{3.62}$$

Where $\triangle y(k+1) = y(k+1) - y(k)$.

Choosing the new state variable $x(k)=[\Delta x_m(k)^T \;\; y(k)^T]^T$,so the new augmented state space is

$$\begin{bmatrix} \triangle x_m(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A_m & 0_m^T \\ C_m A_m & I_{qxq} \end{bmatrix} \begin{bmatrix} \triangle x_m(k) \\ y(k) \end{bmatrix} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \triangle u(k) + \begin{bmatrix} B_d \\ C_m B_d \end{bmatrix} \varepsilon(k) \tag{3.63}$$

$$y(k) = \begin{bmatrix} 0_m & I_{qxq} \end{bmatrix} \begin{bmatrix} \triangle x_m(k) \\ y(k) \end{bmatrix}$$

Without disturbance the ideal system is impossible to design. In the disturbance tests, the mass change of human being has been used as disturbance and added in to input. Testing disturbance is very important as control engineers.

$m=m_b+m_w$

Where m is total mass, m_b is mass of human and m_w is mass of wheelchair.

For the stability of wheelchair, the more centered and lower the mass of human. Knowing precisely which state the disturbance is impacting is crucial before including it in the dynamic equation. The equation is below.

$$\begin{bmatrix} d\omega_L/dt \\ d\omega_R/dt \end{bmatrix} = \begin{bmatrix} k_b \frac{d_r + b_r L_L}{b_l d_r + b_r d_l} & k_b \frac{d_r - b_r L_R}{b_l d_r + b_r d_l} & \frac{a_r a_l + b_r c_l}{b_l d_r + b_r d_l} & \frac{-a_r a_r - b_r c_r}{b_l d_r + b_r * d_l} \\ k_b \frac{d_l - b_l L_L}{b_l d_r + b_r d_l} & \frac{d_l + b_l L_R}{b_l d_r + b_r d_l} & \frac{-d_l a_l - b_l c_l}{b_l d_r + b_r d_l} & \frac{-d_l a_r + b_l c_r}{b_l d_r + b_r d_l} \end{bmatrix} \begin{bmatrix} i_L \\ i_R \\ \omega_L \\ \omega_R \end{bmatrix}$$

Where, $b_l = J_m + m * L_R * \frac{p_G^2}{L_L + L_R}$, $b_r = J_m + m * L_L * \frac{r_G^2}{L_L + L_R}$.so, it can replace the equation of total mass (m) with m_b+m_w.

$$b_l = J_m + (m_b + m_w) * L_R * \frac{p_G^2}{L_L + L_R}, b_r = J_m + (m_b + m_w) * L_L * \frac{r_G^2}{L_L + L_R}$$

$$b_l = J_m + \left(m_b * L_R * \frac{P_G^2}{L_L + L_R} \right) + m_w * L_R * \frac{r_G^2}{L_L + L_R}$$

$$b_r = J_m + \left(m_b * L_L * \frac{p_G^2}{L_L + L_R} \right) + m_w * L_L * \frac{p_G^2}{L_L + L_R}$$

m_b is mass of the human, the wheelchair can be used different humans. The disturbance equation is as follows:

$$\begin{bmatrix} d_l \\ d_r \end{bmatrix} = \begin{bmatrix} L_L * \frac{p_G^2}{L_L + L_R} \\ L_R * \frac{p_G^2}{L_L + L_R} \end{bmatrix}$$

$$xm(k+1) = A_m x_m(k) + B m_u(k) + B d_\omega(k)$$

$y(k) = C_m x_m(k)$,where $B_d \omega(k)$ is input disturbance.

$$Am = \begin{bmatrix}
\frac{-(R_a+R_z)}{L_a} & -\frac{R_z}{L_a} & -\frac{k_b}{L_a} & 0 & 0 & 0 & 0 & 0 & 0 \\
-\frac{R_z}{L_a} & \frac{-(R_a+R_z)}{L_a} & 0 & -\frac{k_b}{L_a} & 0 & 0 & 0 & 0 & 0 \\
k_b \frac{d_r + b_r L_L}{b_l d_r + b_r d_l} & k_b \frac{d_r - b_r L_R}{b_l d_r + b_r d_l} & \frac{d_r a_l + b_r c_l}{b_l d_r + b_r d_l} & \frac{-d_r a_r - b_r c_r}{b_l d_r + b_r * d_l} & 0 & 0 & 0 & 0 & 0 \\
k_b \frac{d_l - b_l L_L}{b_l d_r + b_r d_l} & \frac{d_l + b_l L_R}{b_l d_r + b_r d_l} & \frac{-d_l a_l - b_l c_l}{b_l d_r + b_r d_l} & \frac{-d_l a_r + b_l c_r}{b_l d_r + b_r d_l} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & r_G \frac{L_R}{(L_L + L_R)} & r_G \frac{L_L}{(L_L + L_R)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -\frac{r_G}{(L_L + L_R)} & \frac{r_G}{(L_L + L_R)} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & \cos(ar*T) & 0 & 1 & 0 & -vr*\sin(ar*T) \\
0 & 0 & 0 & 0 & \sin(ar*T) & 0 & 0 & 1 & \cos(ar*T) \\
0 & 0 & 0 & 0 & 0 & 0.1 & 0 & 0 & 1
\end{bmatrix}$$

$$Bm=\begin{bmatrix} \frac{v_a}{L_a} & 0 \\ 0 & \frac{v_a}{L_a} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, B_d=\begin{bmatrix} 0 \\ 0 \\ L_l * \frac{r_G^2}{L_l + L_r} \\ L_r * \frac{r_G^2}{L_l + L_r} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, Cm=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, Dm=[0]$$

The following state space of the plant with its disturbance can be obtained by inserting robot parameter values.

$$Am=\begin{bmatrix} -40.4000 & -0.4000 & -2.000 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.4000 & -40.4000 & 0 & -2.000 & 0 & 0 & 0 & 0 & 0 \\ 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0.0001 & 0.0001 & 0.0001 & 0.0001 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0840 & 0.0760 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.4600 & 0.4600 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1.000 & 0 & 1.000 & 0 & -0.0004 \\ 0 & 0 & 0 & 0 & 0.0006 & 0 & 0 & 1.0000 & 0.8000 \\ 0 & 0 & 0 & 0 & 0 & 1.0000 & 0 & 0 & 10.000 \end{bmatrix}$$

$$Bm=\begin{bmatrix} 200 & 0 \\ 0 & 200 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, Bd=\begin{bmatrix} 0 \\ 0 \\ 1.0240e\text{-}04 \\ 1.5360e\text{-}04 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, Cm=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3.11Model for Reference Generator

The states are the differences between the reference wheelchair and the actual wheelchair. Once the position (xr,yr) is detected by the camera, an equation can be used to determine the reference wheelchair linear velocity, orientation angle, and angular velocity. The steady-state values for constant engine power voltages are calculated and shown below in matrix form.

$$\begin{bmatrix} U_L \\ U_R \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} R_a+R_Z & R_Z & K & 0 & 0 & 0 & 0 & 0 \\ R_Z & R_a+R_Z & 0 & K & 0 & 0 & 0 & 0 \\ K & 0 & -k_r & 0 & -1 & 0 & 0 & 0 \\ 0 & K & 0 & -k_r & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -r_G k_v & 0 \\ 0 & 0 & 0 & 0 & -l_L & l_R & 0 & -r_G k_\omega \\ 0 & 0 & l_R & l_L & 0 & 0 & -\frac{l_R+l_L}{r_G} & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & -\frac{l_R+l_L}{r_G} \end{bmatrix} \begin{bmatrix} i_L \\ i_R \\ \omega_L \\ \omega_R \\ M_L \\ M_R \\ V_B \\ \omega_B \end{bmatrix} \tag{3.64}$$

The steady state equation is displayed below after entering the values for the wheelchair parameter settings.

$$\begin{bmatrix} 10U_L \\ 10U_R \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2.0200 & 0.0200 & 0.1000 & 0 & 0 & 0 & 0 & 0 \\ 0.0200 & 2.0200 & 0 & 0.1000 & 0 & 0 & 0 & 0 \\ 0.1000 & 0 & -0.0020 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0.1000 & 0 & -0.0020 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & -0.0016 & 0 \\ 0 & 0 & 0 & 0 & -0.0400 & -0.0600 & 0 & -0.02160 \\ 0 & 0 & 0.0600 & 0.0400 & 0 & 0 & -6.2500 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & -6.2500 \end{bmatrix} \begin{bmatrix} i_L \\ i_R \\ \omega_L \\ \omega_R \\ M_L \\ M_R \\ V_B \\ \omega_B \end{bmatrix}$$

After obtaining the appropriate position (xr,yr), orientation (r), r, and vr. The desired control signals(uL,uR) and the rest states are produced from equation (3.64).

3.12Controller design

Design controllers is important that will maintain closed-loop stability in spite of erroneous design models and uncertainties in parameter values. To design controllers consists of five basic components input, plant, output, sensing elements and controller. One of the controller is model predictive controller(MPC), in this chapter deals electrical quadriplegic wheelchair of MPC. Predictive control is a control system based on a predictive model of the process. Based on historical data about the process and anticipated future input, the model is used to estimate the future output. It emphasizes the model's purpose rather than its structure. In contrast to PID, which has a single input and one output, MPC considers many inputs and plans several control outputs, hence MPC is used in this situation.

MPC is a good controller for practically any type of issue, but it excels when used to address issues involving:

- A large number of manipulated and controlled variables.
- Constraints imposed on both the input and output variables.
- Changing control equipment (sensor/actuator) failure.
- Time delays Indeed, in its basic unconstrained form MPC is closely related to linear quadratic optimal control. (AUNG, 2019)

Using Model Predictive Control (MPC), explicitly uses models to forecast future plant behavior; respects input, output, and state constraints; By resolving an optimization issue for each sample and combining it with state estimation, the control sequence is established.

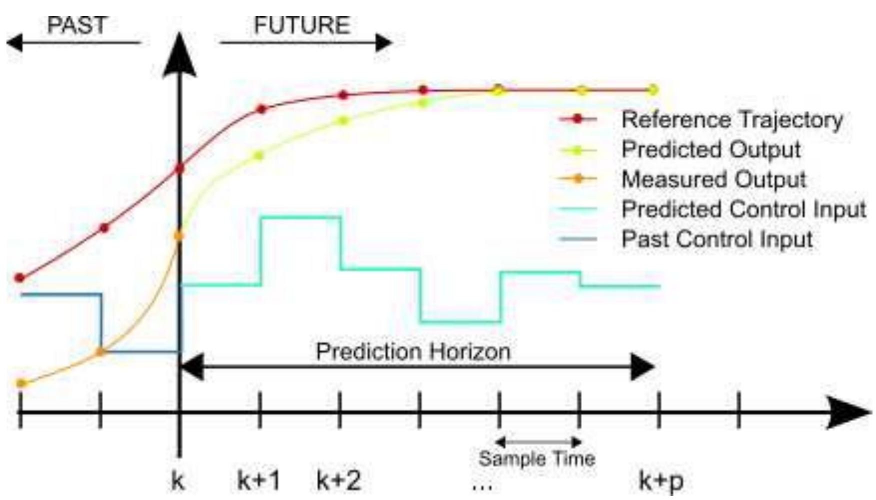

Figure3.10 Model Predictive Control

This figure shows that control horizon is the section of the time horizon where manipulated variable (MV) moves are allowed. In order to anticipate the final control variable (CV) outcomes without manipulated variable (MV) movement, the prediction horizon extends past the control horizon. An input that improves the future plant's recommended path for the controlled variable to converge on the set-point in a particular way is known as a reference trajectory in a quadriplegic wheelchair.

At time k, apply the first input and solve an open loop optimum control problem over a predetermined horizon. The prior optimal solution can be utilized as a starting point at time k + 1.Model predictive control (MPC) has received a great deal of interest in automotive applications over the past ten years due to its ability to handle states and input restrictions in a systematic manner. A mathematical model of the plant serves as the basis for the design of model predictive control systems. State-space modeling is assumed to be the model to be used in the design of the control system. Assume the plant has n1 states, m inputs, and q outputs.

Assumed as well is that there are fewer or the same number of outputs as inputs (i.e., $q \leq m$). It is impossible to regulate all of the measured outputs separately with zero steady-state errors if there are more outputs than inputs. The plant noise and disturbance are taken into account in the general formulation of the predictive control problem. A good design of an observer requires both observability and controllability in order for the predictive control system to achieve the necessary closed-loop control performance. They developed the new augmented state space because it is also required.

3.13Augmented of State Space Representation

Integrator effects are taken into consideration using an enhanced state space. This model is error-based, with deviations from the reference variable serving as the state variable. wheelchair for a quadriplegic as a reference, following a time-varying reference trajectory. With the reference inputs, the reference velocities may be determined (positional coordinate of wheelchair). The equation of augmented state space is:

$$\begin{bmatrix} \Delta x_m(k+1) \\ y(k+1) \end{bmatrix} = \begin{bmatrix} A_m & 0_m^T \\ C_m A_m & I_{q\times q} \end{bmatrix} \begin{bmatrix} \Delta x_m \\ y_k \end{bmatrix} + \begin{bmatrix} B_m \\ C_m B_m \end{bmatrix} \Delta u(k)$$

$$y(k) = \begin{bmatrix} 0_m & I_{q\times q} \end{bmatrix} \begin{bmatrix} \Delta x_m(k) \\ y(k) \end{bmatrix}$$

The state space is the augmented model of quadriplegic wheelchair written in equation with the parameters of table

A_m

$$= \begin{bmatrix} -7.132e-09 & -6.975e-09 & -0.003468 & 3.142e-05 & 0 & 0 & 0 & 0 & 0 \\ -6.975e-09 & -7.153e-09 & 3.141e-05 & -0.003468 & 0 & 0 & 0 & 0 & 0 \\ 2.078e-06 & 2.033e-06 & 1.001 & 0.0008434 & 0 & 0 & 0 & 0 & 0 \\ 2.033e-06 & 2.084e-06 & 0.0008434 & 1.001 & 0 & 0 & 0 & 0 & 0 \\ 3.28e-06 & 3.81e-06 & 0.8407 & 0.7607 & 1 & 0 & 0 & 0 & 0 \\ -2.061e-07 & 2.332e-07 & -4.6 & 4.6 & 0 & 1 & 0 & 0 & 0 \\ 2.715e+29 & -3.075e+29 & 7.56e+36 & -7.56e+36 & 2.203e+04 & -1.644e+38 & 2.203e+04 & 0 & -1.644e+39 \\ -3.948e+32 & 4.471e+32 & -1.099e+40 & 1.099e+40 & 15.15 & 2.389e+41 & 0 & 2.203e+04 & 2.389e+42 \\ -4.441e+33 & 5.029e+33 & -1.237e+41 & 1.237e+41 & 0 & 2.688e+42 & 0 & 0 & 2.688e+43 \end{bmatrix}$$

B_m

$$= \begin{bmatrix} -4.951 & -0.04903 \\ -0.04903 & 4.951 \\ 0.004143 & 0.004053 \\ 0.004053 & 0.004155 \\ 0.003271 & 0.003272 \\ -0.0002055 & 0.0002327 \\ 5.431e+30 & -6.15e+30 \\ -7.896e+33 & 8.941e+33 \\ -8.883e+34 & 1.006e+35 \end{bmatrix}$$

C_m

$$= \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The augmented state space matrix is not controllable. Minimal realization is used for controllable model order reduction. Then the controllable model of the wheelchair is used in the design of MPC is shown below.

$$\mathbf{A}=\begin{bmatrix} -28.91 & -18.28 & -2.888e-05 & -.827e-05 & 0.3412 \\ -18.18 & -11.49 & -1.768e-05 & -1.73e-05 & 0.2088 \\ -0.2855 & -0.2909 & 0.9966 & -0.003348 & 40.4 \\ -34.15 & -21.59 & -3.444e-05 & 1 & 0.4069 \\ 0.2856 & 0.2909 & 0.003504 & 0.00343 & -40.4 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} -0.002073 & 169.3 \\ -0.02887 & 106.5 \\ 200 & 4.667e-08 \\ 0.01657 & 200 \\ -200 & -0.0005513 \end{bmatrix}$$

C=

$$\begin{bmatrix} 7.107e-10 & 1.204e-08 & 1 & -7.013e-09 & 8.464e-05 \\ 6.957e-10 & 1.179e-08 & -7.013e-09 & 1 & 8.285e-05 \\ -8.396e-06 & -0.0001423 & 8.464e-05 & 8.285e-05 & 8.468e-05 \end{bmatrix}$$

3.14 Constraints

As an optimization issue that takes into account the current constraints, the predictive control problem. It's important to parameterize the constrained variables using the same parameter vector as when designing the predictive control model. Consequently, a series of linear equations based on the parameter vector are used to represent the restrictions. In literature on optimization, the vector is frequently referred to as the decision variable. The restrictions are taken into account for each moving horizon window because the predictive control problem is stated and addressed within the context of receding horizon control. The constraints can been used for passengers comfort. As the optimal solutions will be obtained using quadratic programming, the constraints need to be decomposed into two parts to reflect the lower limit, and the upper limit with opposite sign.

$$\Delta U^{min} \leq \Delta U(j) \leq \Delta U^{max}, j \in \{K, K+N-1\} \quad (3.65)$$

$$Y^{min} \leq Y(j) \leq Y^{max}, j \in \{K+1, K+N\} \quad (3.66)$$

Where U^{min} is lower limit of input control

U^{max} is upper limit of input control

Y^{min} is lower limit of process

Y^{max} is upper limit of process

Table 3.3 output constraints

Output constraint	Values
Lower limit α_B orientation	0
Upper limit α_B orientation	180 deg
x_B position lower limit	-0.5m
x_B position upper limit	20m
y_B position lower limit	-0.5m
y_B position upper limit	20m

Table 3.4 Input constraint

Input constraint	Values
Lower limit of right motor control signal	-0.5v
Upper limit of right motor control signal	0.5v
Lower limit of left motor control signal	-0.5v
Upper limit of left motor control signal	0.5v

3.15 Cost Function

The method of assessing the effectiveness of the algorithm or model is the cost function. Calculating how much off the model was in its forecast, it takes into account both the model's projected and actual outputs.

$$J = \sum_{i=1}^{n_y} \sum_{j=1}^{n2} \left\{ w_{i,j}^{y} \left[r_i(k+j|k) - y_i(k+j|k) \right] \right\}^2 + \sum_{i=1}^{n_u} \sum_{j=0}^{n3} \left\{ w_{i,j}^{u} \left[u_i(k+j|k) - u_i(k+j-k) \right] \right\}^2 \quad (3.67)$$

Where $w_{i,j}^{y}$ tuning weight of optimization

$r_i(k+j|k)$ reference value of i th output at j th optimization

$y_i(k+j|k)$ is predicted value i th plant output at j th optimal

n_y number of plant output variables

n_u number of manipulated variables

n_2 is control horizon error and n_3 is control horizon

$\Delta u_i(k+j-k)$ is the control increments of i th plant output.

The quadriplegic wheelchair has three outputs and two inputs. The input and output weights are diagonal are Q=diag (1,1) and R=diag (1,1,1).

3.16 Prediction and control horizon

The number of upcoming control intervals that the MPC controller must evaluate via prediction in order to optimize its MVs (manipulated variables) during control interval K(gain) is known as the prediction horizon (p). To ensure closed-loop stability and resilience, the prediction horizons are extended such that regulated output predictions can comprise a considerable portion of the data.

The number of MV (manipulated variable) motions that must be optimized at control interval k is known as the control horizon, or m. The robustness of the control system decreases as the control horizons increase, while the aggressiveness, capacity to stabilize unstable plants, and computational cost (size of the optimization problem that must be addressed each instant) of the controller all rise. Six states, two inputs, and three outputs are present in the plant. Control at zero steady state is difficult if there are more outputs than inputs.

The design parameters weights N=2;P=5 where N is control horizon, P is prediction horizon .

3.17 Manipulated variable rates

The signal that modifies the controller to meet its goals is known as the modified variable. The modified variable's increases are positive-definite Hessian increases. A delayed controller reaction could be an issue.

The following table shows the controller's minimum MPC object, weight of left motor control signal and right motor control signal of MV rate.

Table 3.5 penalty of manipulated variable rates weight

MV	Weights MV rate
Right motor control signal	0.2
Left motor control signal	0.2

3.18 weights of output variable

The Hessian is impacted by the orientation (aB) and position output (xB,yB) penalty weights. A non-zero value highlights the significance of (OV) output variable target tracking and increases the likelihood of an original QP (Quadratic programming) solution.

The following table lists the minimum weight for each output variable along the prediction horizon.

Table 3.6 manipulated output variable

Output variable	Weights output variable
X_B position	3000
Y_B position	3000
aB orientation	*3000*

3.19 Overall System of Structure

The electrical quadriplegic wheelchair mathematical model is designed by 2nd newton's law of motion. This model created an unstable, nonlinear system. In order to stabilize the system, an appropriate controller is needed like model predictive control. The system is MIMO (three output and two input) as shown below in figure. The first input is x_B and y_B position used for position control of the wheelchair. The output is compared with reference position and fed into the controller (MPC). The second input a_B orientation angle to balance the wheelchair. The angle is compared with the reference angle and fed into controller to balance the wheelchair. This controller is optimizing and tuned to get the satisfactory result needed. The controller output is applied to the

DC motor (actuator) through a decoupling unit to generate the necessary torque, which determines the wheelchair balance or position control.

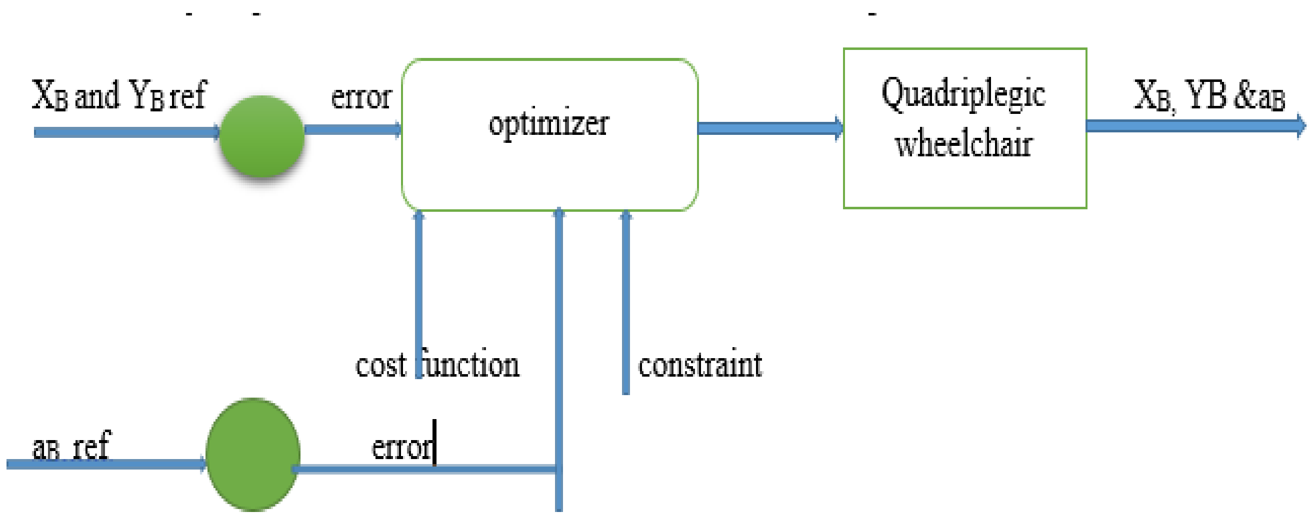

Figure 3. 11 Overall System Structure

CHAPTER FOUR

4.1RESULTS AND DISCUSSIONS

4.1Introduction

This chapter discuss about simulation was performed based on the table of 3.1. Model predictive control is used for good performance of balancing (orientation) the wheelchair and direction control. The overall performance of the quadriplegic wheelchair is done using MATLAB/Simulink.

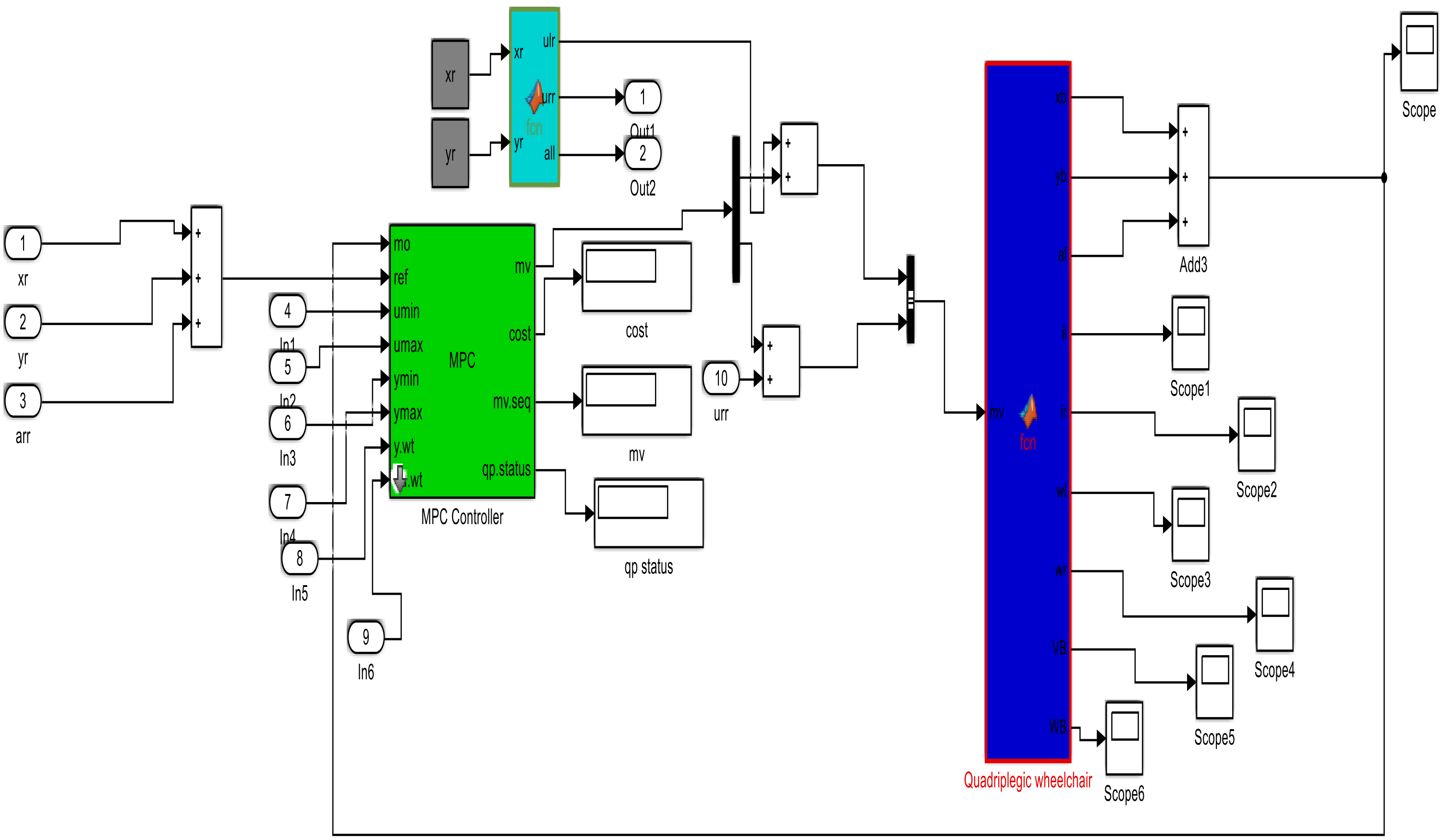

Figure4.1 MATLAB/ SIMULINK of electrical quadriplegic wheelchair

Discerption of MATLAB/Simulink model of figure 4.1.

a) The grey color is the line of wheelchair.
b) The green color is the controller of the system (MPC Controller).
c) The blue one is the plant of the system (Quadriplegic Wheelchair).
d) The cyan color is reference of the plant.

A quadriplegic wheelchair is highly output response is non-linear and unstable without any controller.

4.2Discussions result of Quadriplegic Wheelchair without controller

4.2.1 kinematics model of quadriplegic wheelchair result discussions

With the considerations of test open loop system, the source voltage and reference voltage is constant at the given time is increases. The output of two voltage signal is at rest within ten seconds of operating time.

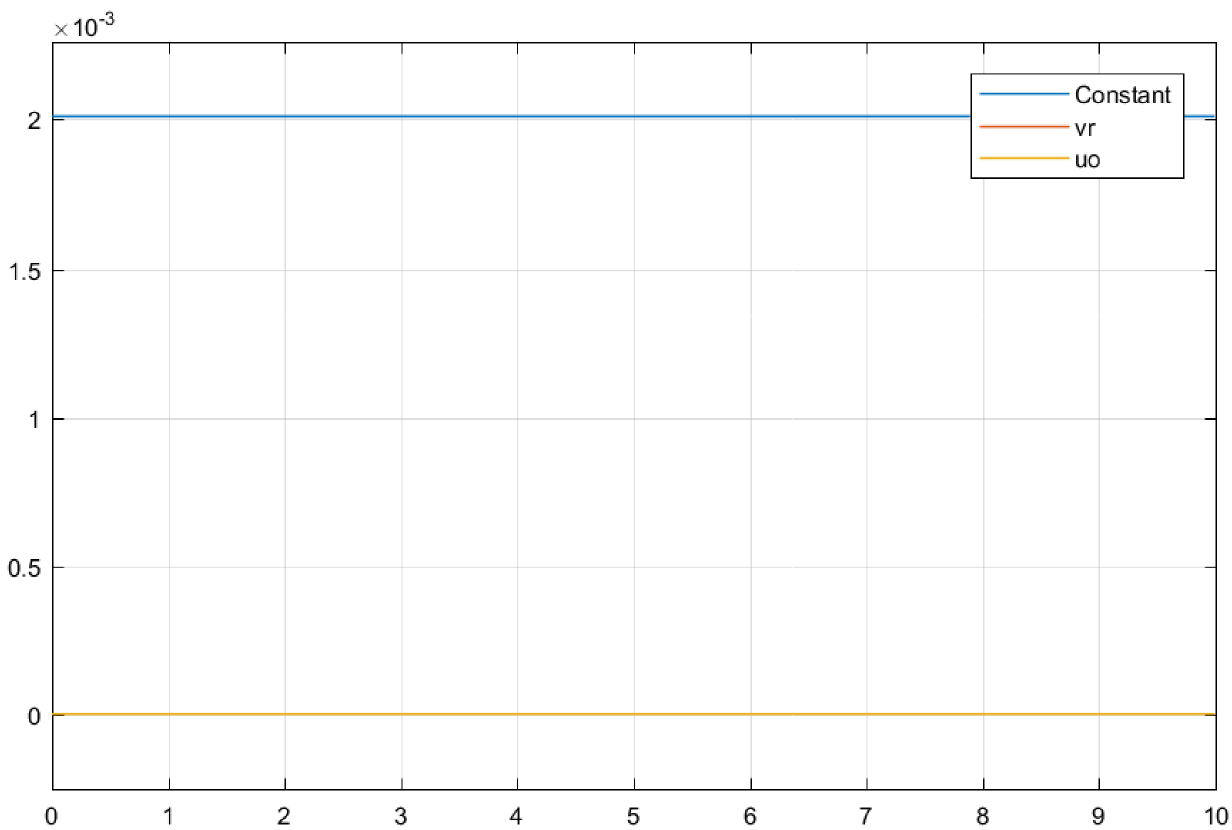

Figure 4.2 test input voltage signal of quadriplegic wheelchair

In the figure above shows that control signal applied for the DC series motor installed in the right wheel of the quadriplegic wheelchair. It shows left motor control signal that actuated the wheel to move the quadriplegic wheelchair as right motor does. Both signals are a unit step test inputs for a given ten seconds operating time. Once the motor actuated the left and right motor current increases positively for one second. The current kept constant until the end of the time in seconds. The quadriplegic wheelchair involves two DC series motors with common power supply. The two motors actuated differentially.

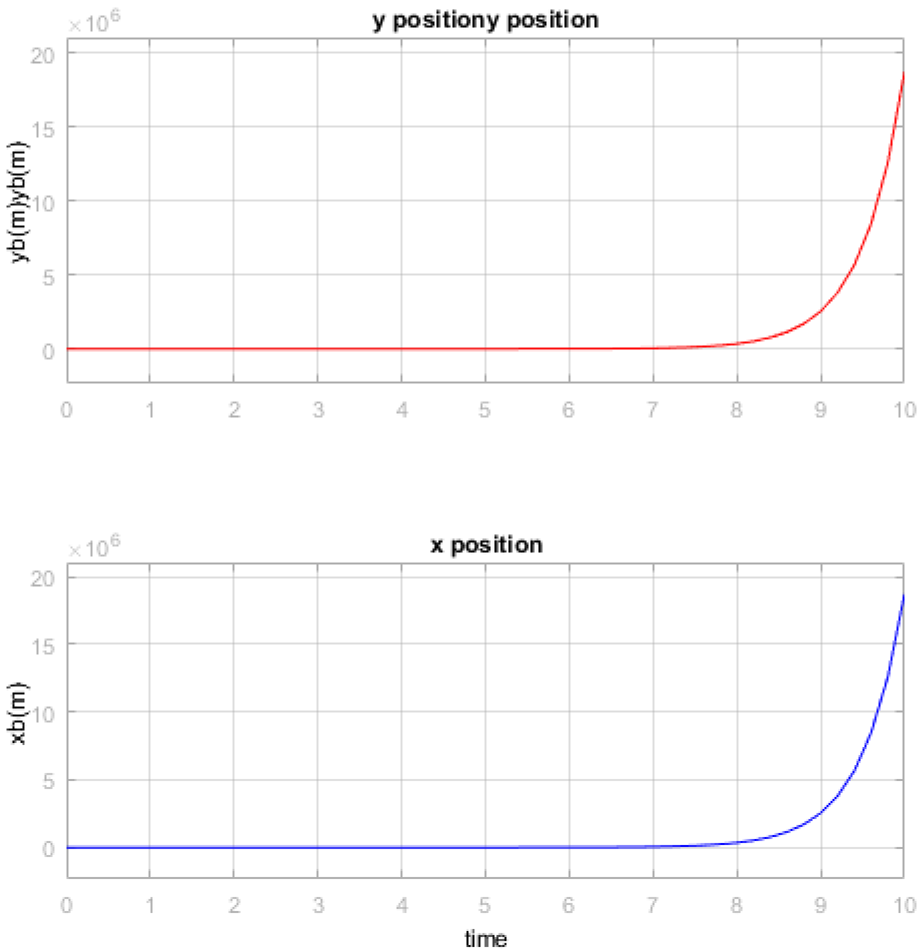

Figure4.3position of quadriplegic wheelchair

The output position of quadriplegic wheelchair starts from zero at initial value suddenly increases at 7.5 seconds. This graph indicates that the wheelchair moving the position between x and y at B chassis reference. Figure 4.6 bellow shows the quadriplegic wheelchair position and orientation vs time graph. The quadriplegic wheelchair starts from zero initial values of the states (xB, yB, α, iL,

iR, ωL, ωR, vB, ωB). The output of the quadriplegic wheelchair is the position (xB, yB) and orientation or heading angle (α). The graph indicates that the robot is moving in between x and y position with small angle at chassis point B.

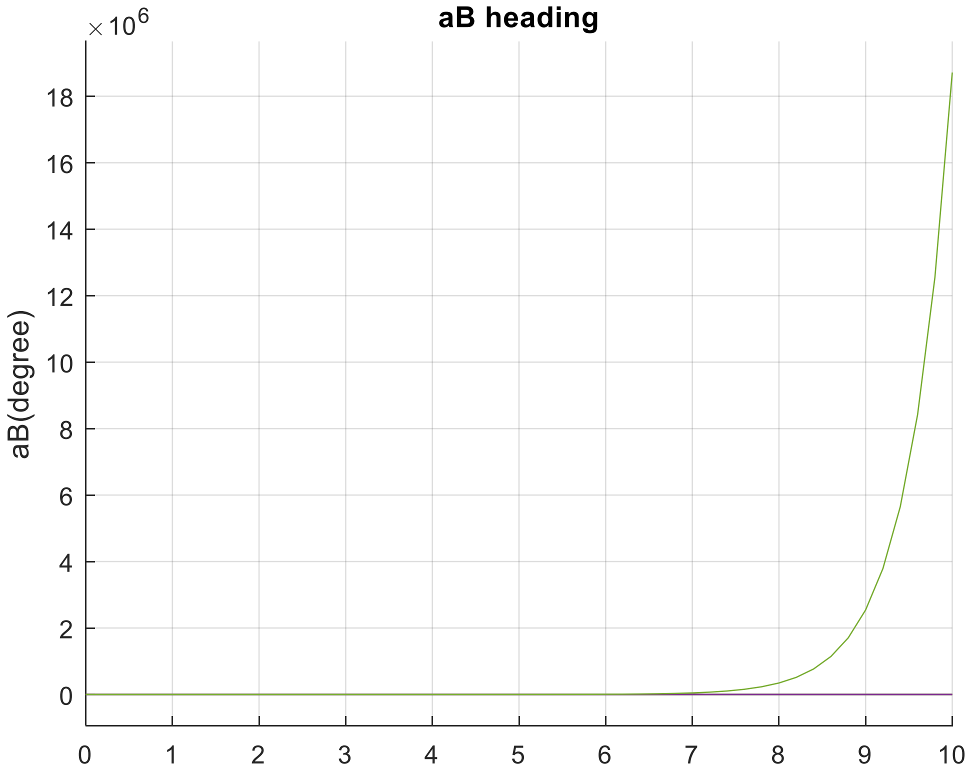

Figure 4.4 heading angle of quadriplegic wheelchair

The open loop simulation results the following heading angle of the robot. It shows that the quadriplegic wheelchair is moving at angle of different magnitude with time. One how want to stir the robot in straight line it is necessary to make the angle zero at all time during the movement. To test open loop ab heading kinematics model the result is zero at initial value. It shows the figure of angle wheelchair moving different magnitude with time. The wheelchair moving in straight line it is necessary to make the angle zero at all time during the movement.

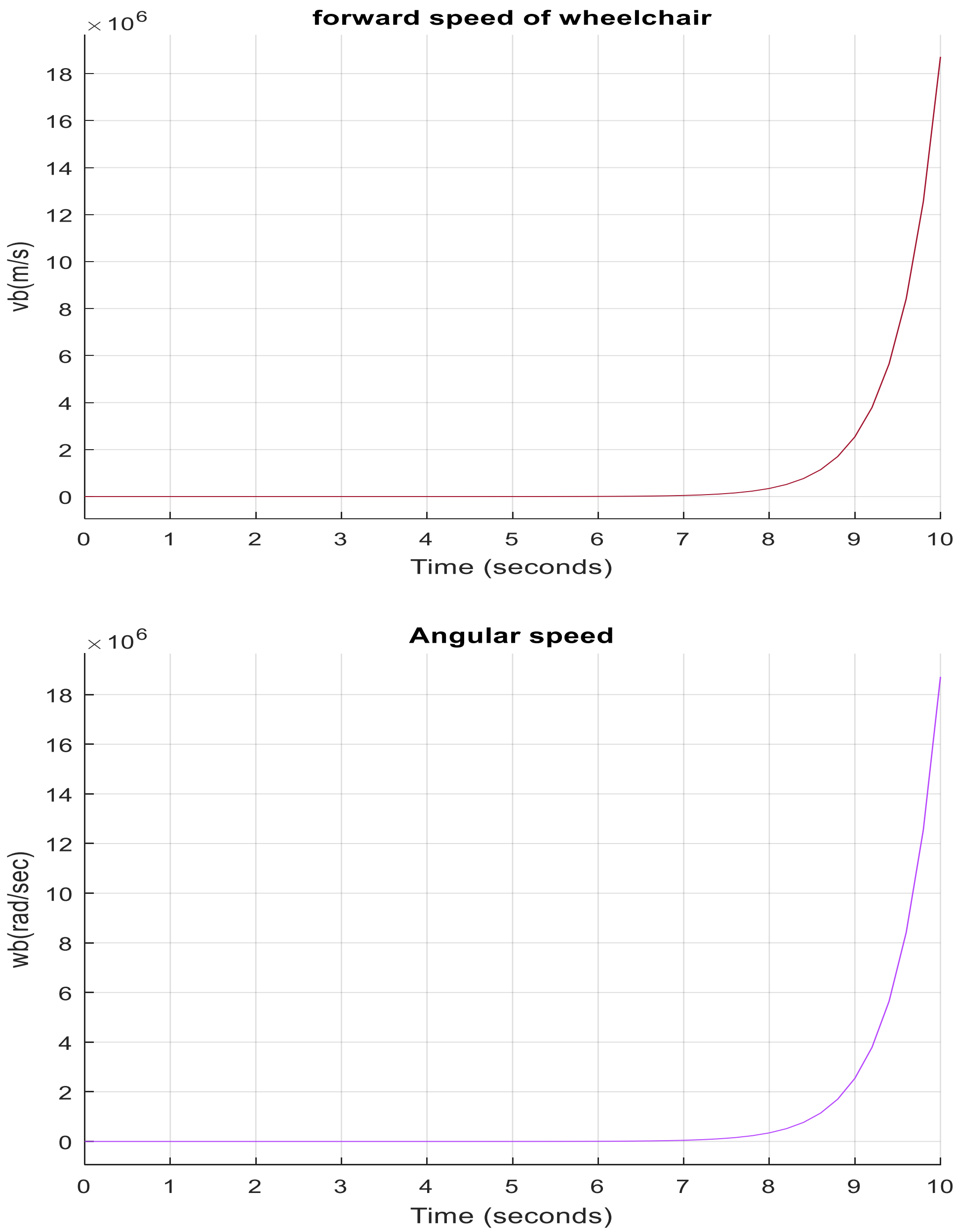

Figure 4.5 angular speed and forward speed of wheelchair

Fig 4.5. above shows the quadriplegic wheelchair angular speed and forward speed which indicates its total movement. The quadriplegic wheelchair is moving with same magnitude of angular and forward speeds at a different time. Figure 4.5 first shows the quadriplegic wheelchair angular speed. It is varying with very small magnitude with time. This indicates that the quadriplegic wheelchair is moving in straight path.

Figure 4.5 the second shows the quadriplegic wheelchair forward speed. It is increasing with time hence someone who wants to fix the quadriplegic wheelchair to the desired value a controller must be designed.

4.2.2Dynamics model of quadriplegic wheelchair result discussions

Dynamic behavior is demonstrated on the time courses of currents and angular speeds of the motors starting from zero initial conditions. The output (vB,B) of the dynamic model is an input to the kinematic model of the robot.

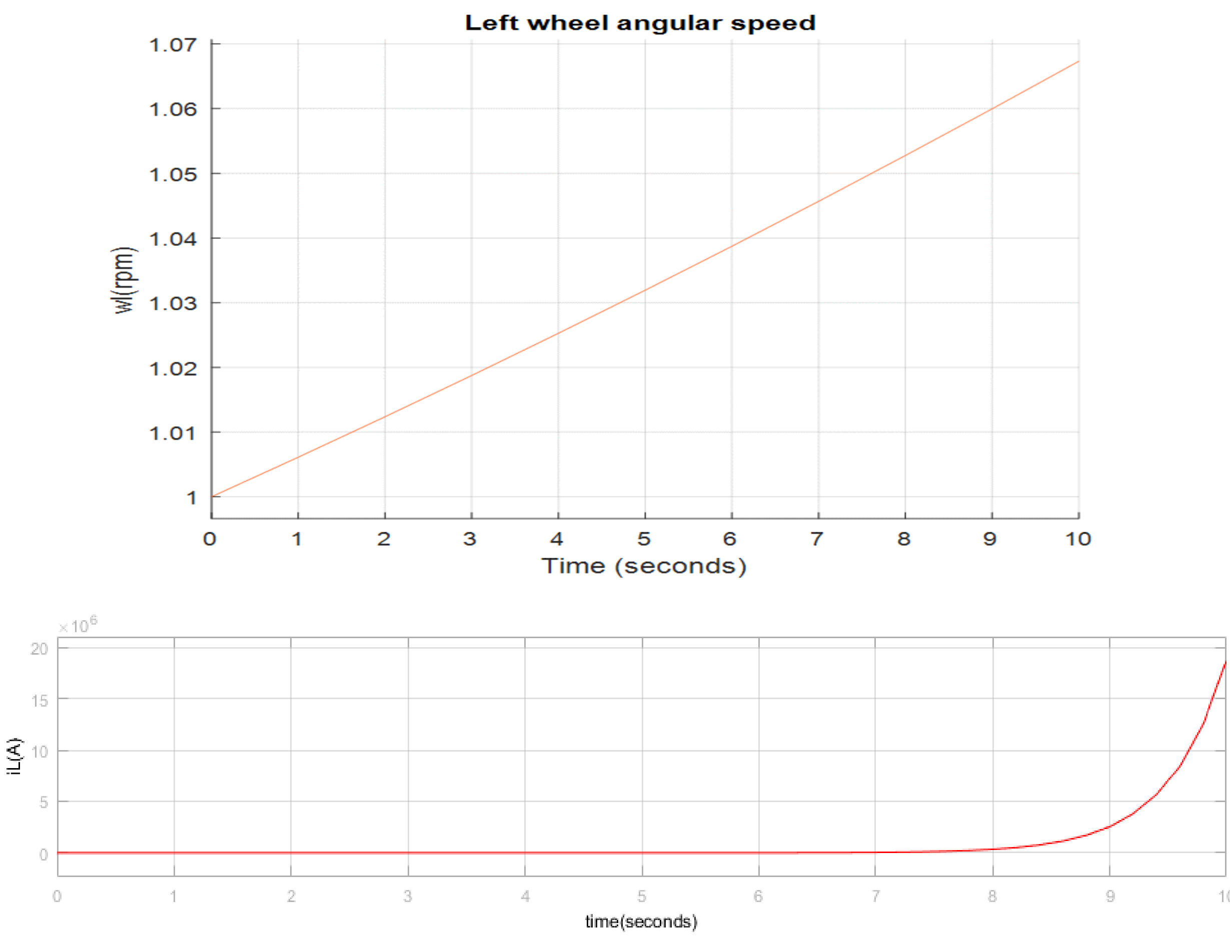

Figure 4.6 Left DC Motor Current and its Speed

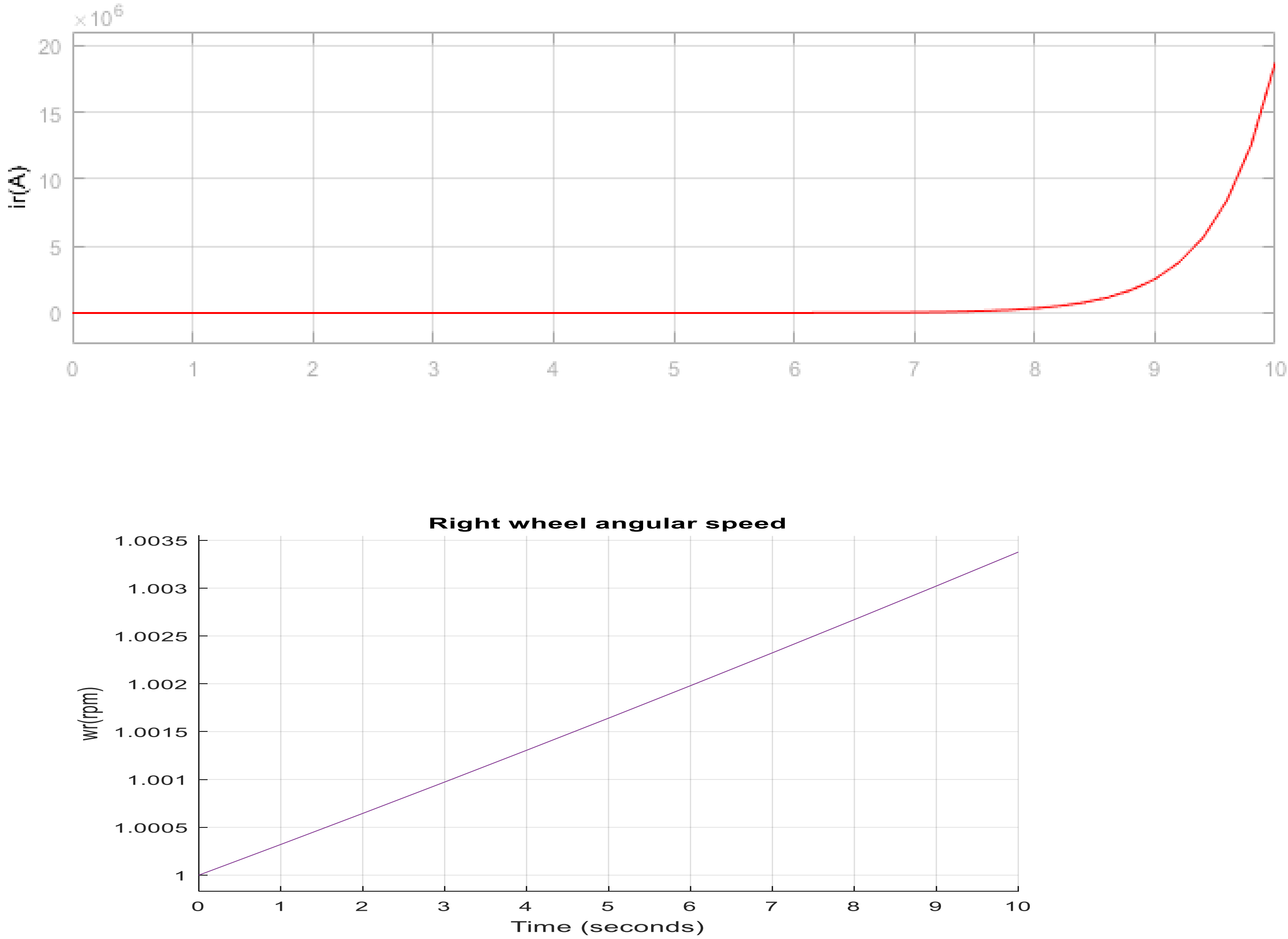

Figure 4.7 Right DC Motor and its Speed

First figure shows left motor current and speed. The second one shows that right motor current and motor speed. Both motors have the same current and speed for the unit step actuating signal. The maximum current and speed for both motors $3*10^{4}$A and 1.0033 rad/s respectively. As final goal of this thesis is to control the position and orientation, a good controller is needed to simulate the quadriplegic wheelchair.

4.3Reference of Quadriplegic Wheelchair

As the final goal of the control design is to driving the quadriplegic wheelchair without occurring accident by the wheels, the controller must know the track of the quadriplegic wheelchair. The pinhole camera receives the position of the quadriplegic wheelchair in pixel in the form of (xB, yB) coordinate.

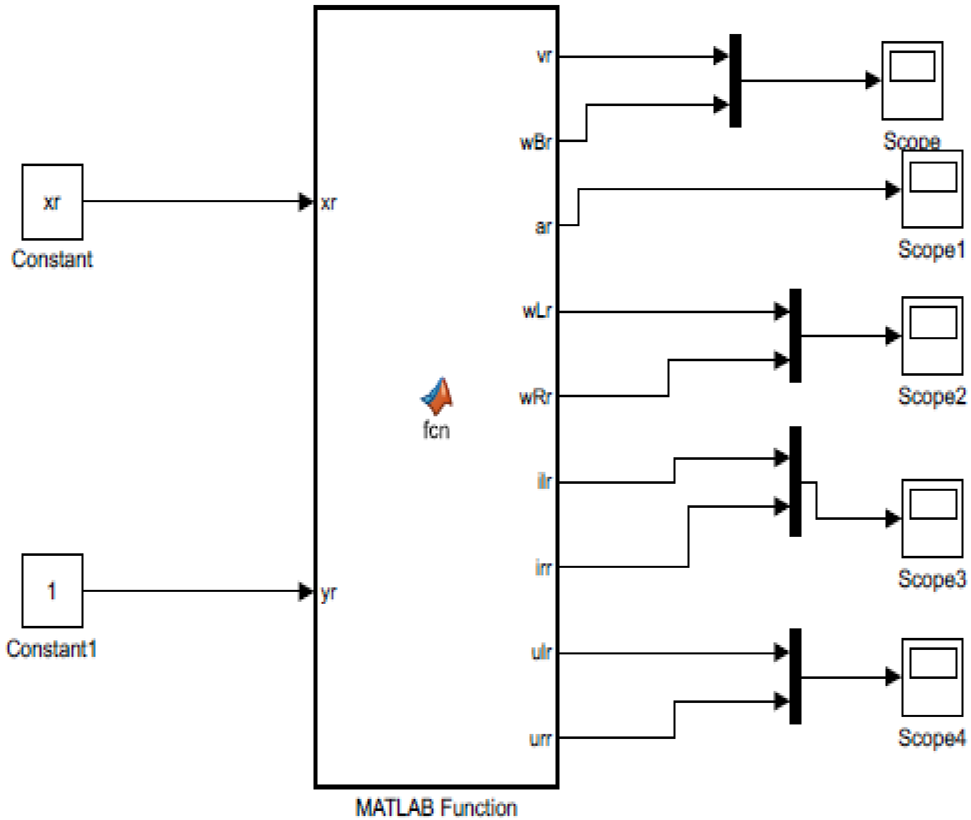

Figure 4.8 References of Quadriplegic Wheelchair

Since the plant Model used in the model predictive controller is an error model the nine states (iL, iR, ωL, ωR,vB ,ωB ,xB,,yB,αB) and two control signals or manipulated variables (uL,uR) needs a reference to form error states as explained mathematically in the previous chapter.

The figure bellow is the graph generated using MATLAB to act as desired quadriplegic wheelchair forward speed and angular speed has to be actuated to fellow the desired path.

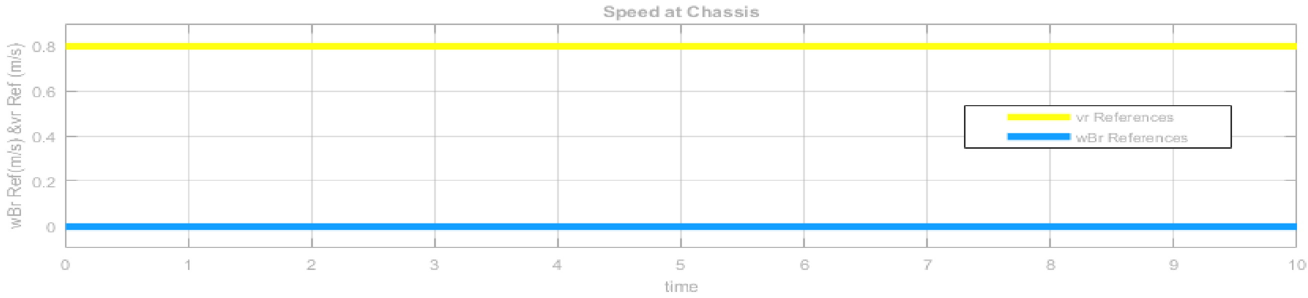

Figure4.9 Quadriplegic Wheelchair Speed

The control signal of the DC motor has left and right motor control signals that the actual quadriplegic wheelchair has to be actuated to fellow the line.

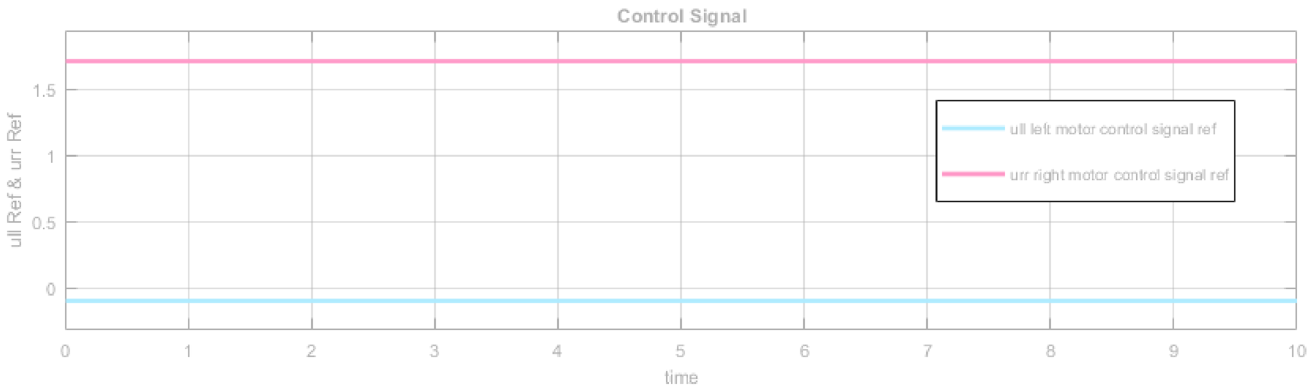

Figure 4.10 Desired Control Signal References

The result below is the desired motor current that the quadriplegic wheelchair has to follow for the best trajectory tracking of the wheelchair.

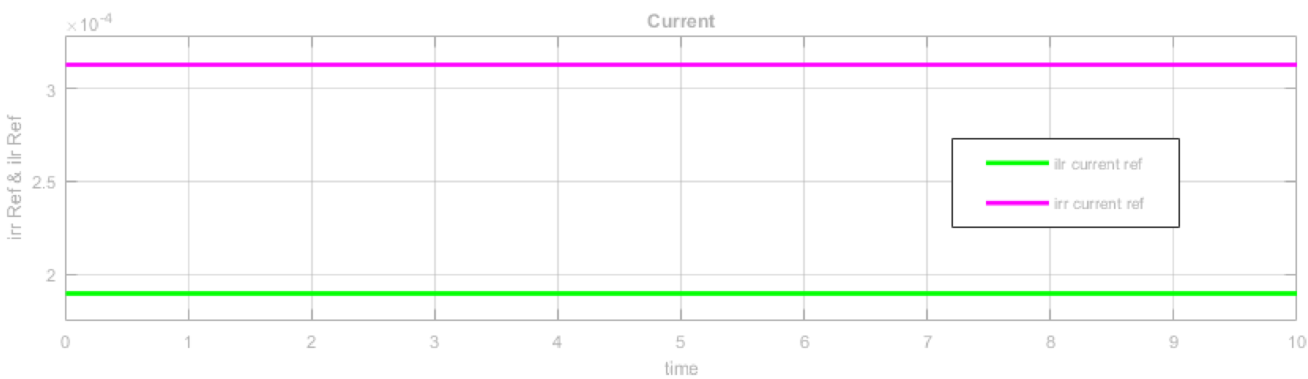

Figure 4.11 References Current Motor

The left and right DC series motors angular speed that the quadriplegic wheelchair has to follow obey for the best trajectory tracking of wheel line. Equal angular speed of the quadriplegic wheelchair shows that it is moving in straight path.

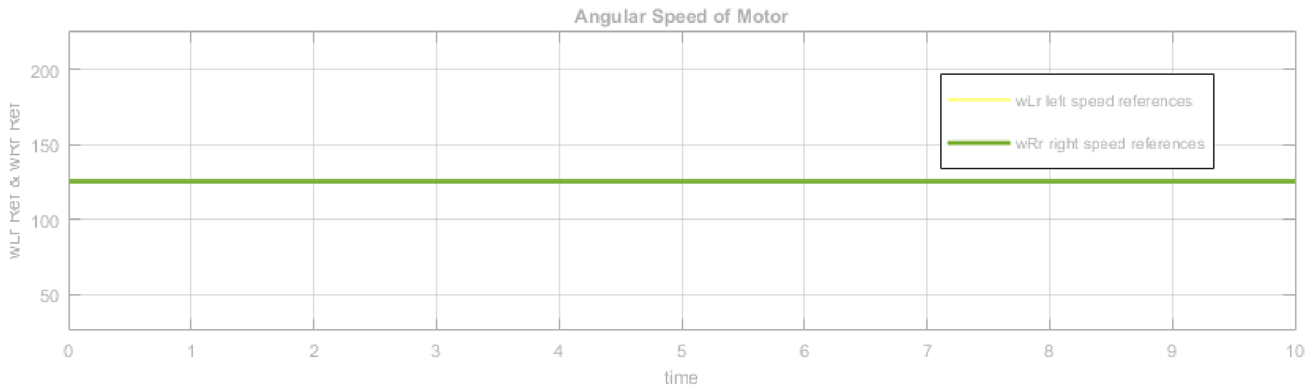

Figure 4.12 References of Motor Speed

4.4System response with model predictive control

A unit step input control is used as test unit, as explained above with open test. The position of (Xr, Yr) and the angle (a_{B}) is increased with time increased. The angle is very small and negligible its good but the moving speed and the angular speed is varying with time. But the main point of this thesis to control the position and angle of quadriplegic electrical wheelchair and fixing the maximum value of angle with model predictive controller.

4.4.1Closed loop response without disturbance

This section describes with the analysis of controller performance without disturbance. A Simulink block is describes shown below based on model predictive controller. The block is drawn MATLAB Simulink environment which has model predictive control block. It consists the plant dynamic and kinematics model inside the block. The controller input ports involves minimum and maximum constraint of the outputs ($xB, B, \alpha B$), minimum and maximum constraints of the input control signals or manipulated variables (uL, uR), weights on the manipulated variables, output variables and quadratic program switch. The output port consists manipulated variable and cost displayer port. The model predictive control performance is measured based on cost function.

The plant model of quadriplegic wheelchair used in the design of MPC is an error model which mean the state variables are the difference between desired states and actual states where the actual states are obtained from the real wheelchair and the desired states are obtained from the reference that assumed to exactly with the wheelchair track.

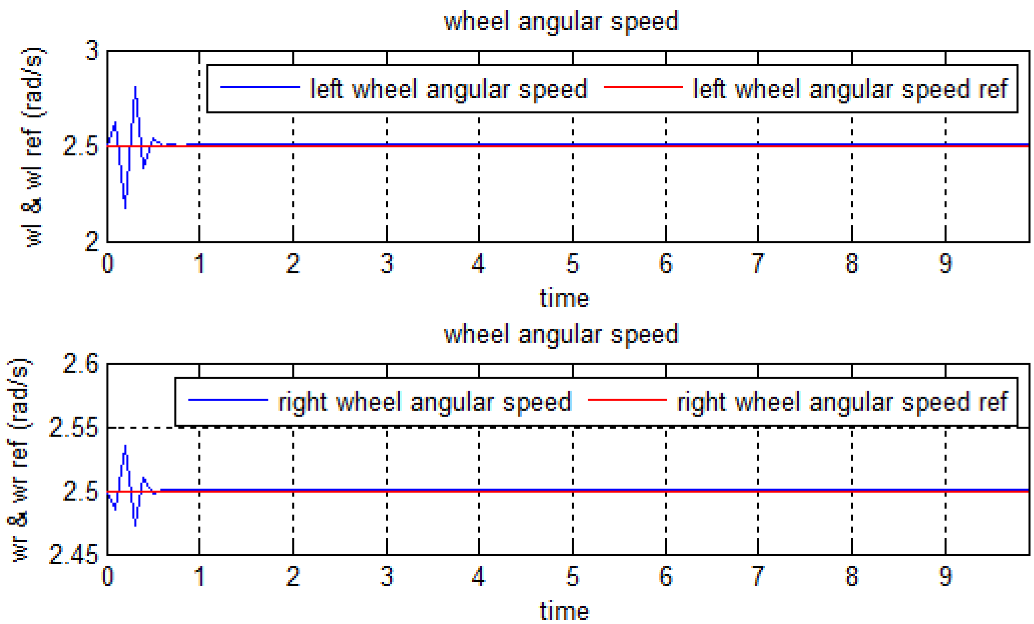

Figure 4.13 Left and Right Angular Speed of Quadriplegic Wheelchair

The desired value is obtained at 0.3 second after the control signals are applied. The left and right motors are moving with (125,125) rad/s respectively until the end of the simulation time. This shows that the wheelchair is moving in straight line.

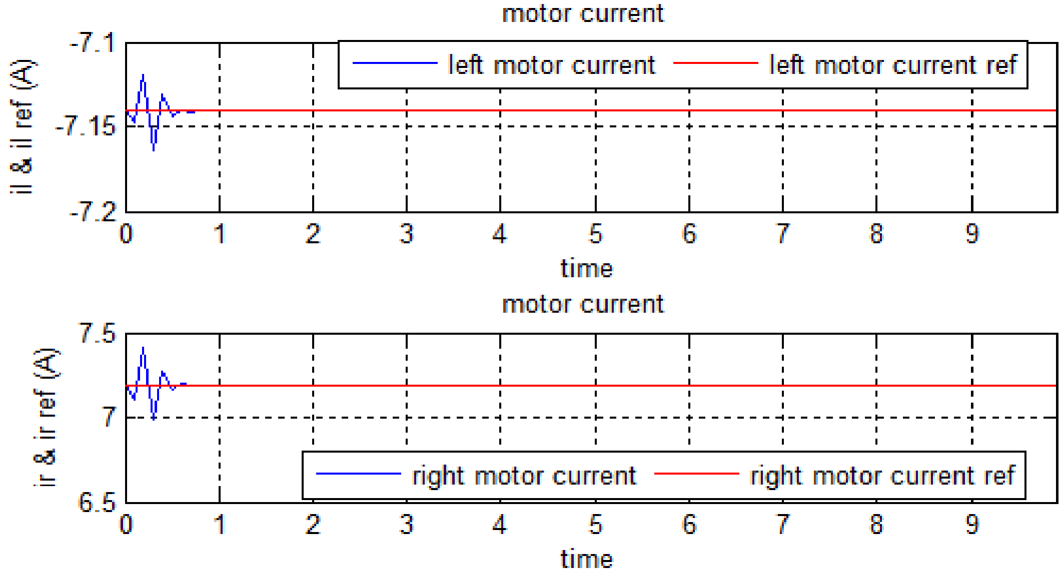

Figure 4.14 Left and Right Current Motor with Comparison of Desired and Actual

The desired left and right currents are obtained at 0.3 second exactly after the control signal is applied. Its magnitude is varying for 0.3 second with the order of the manipulated variable or control signals.

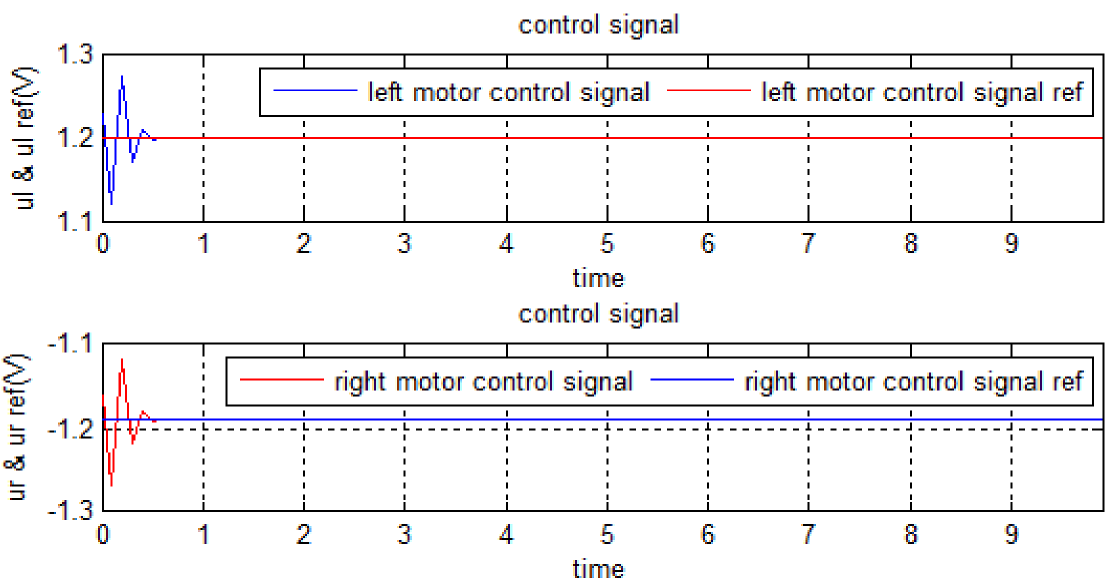

Figure4.15 Comparison of Desired and Actual Control Signal of The Motor

The right and left control signals are varying between (-0.095, -0.085) and (1.5,2) volt for 0.3 second respectively. The left and right control signals have almost similar magnitude but different polarity. This situation is happened because both motors are supplied from common supply 24 V.

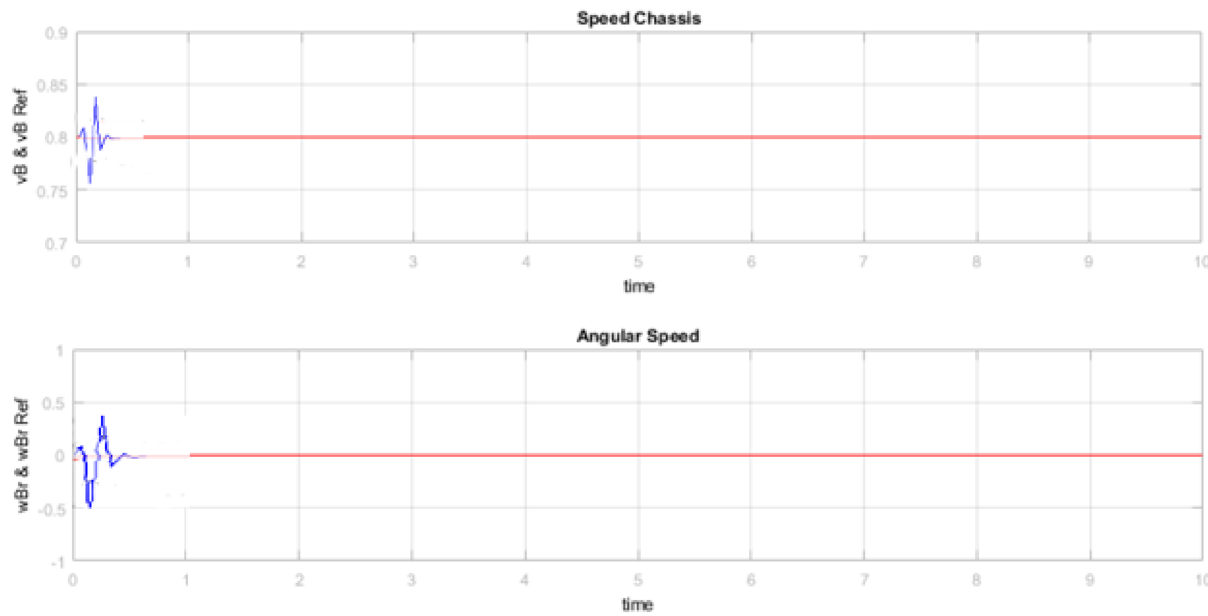

Figure 4.16 Speed of Chassis and Angular Speed of Quadriplegic Wheelchair

The chassis speed and angular speed of the robot are kept constant with value 0.8 m/s and 0rad/s respectively after 0.3 second of applying the control signal. Unlike the open loop system, the closed response shows the quadriplegic wheelchair is moving with constant forward velocity.

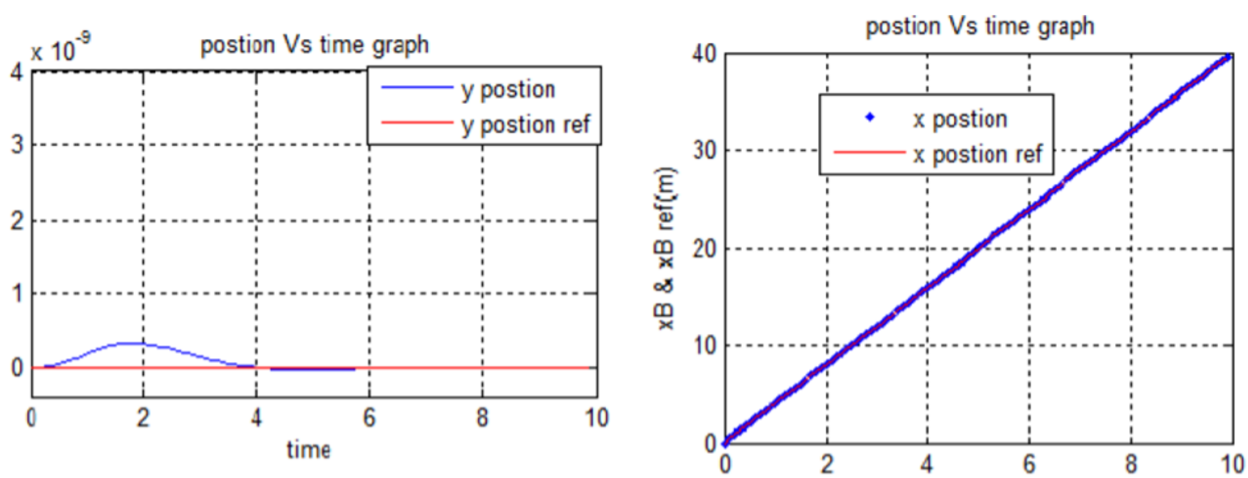

Figure 4.17 Quadriplegic Wheelchair Position

The outputs of the x_B position is exactly fits to the desired value whereas y_B position does not track the reference value. It deviates by $0.3*10^{-9}m$ from its reference but this is negligible in wheelchair operating in the track path.

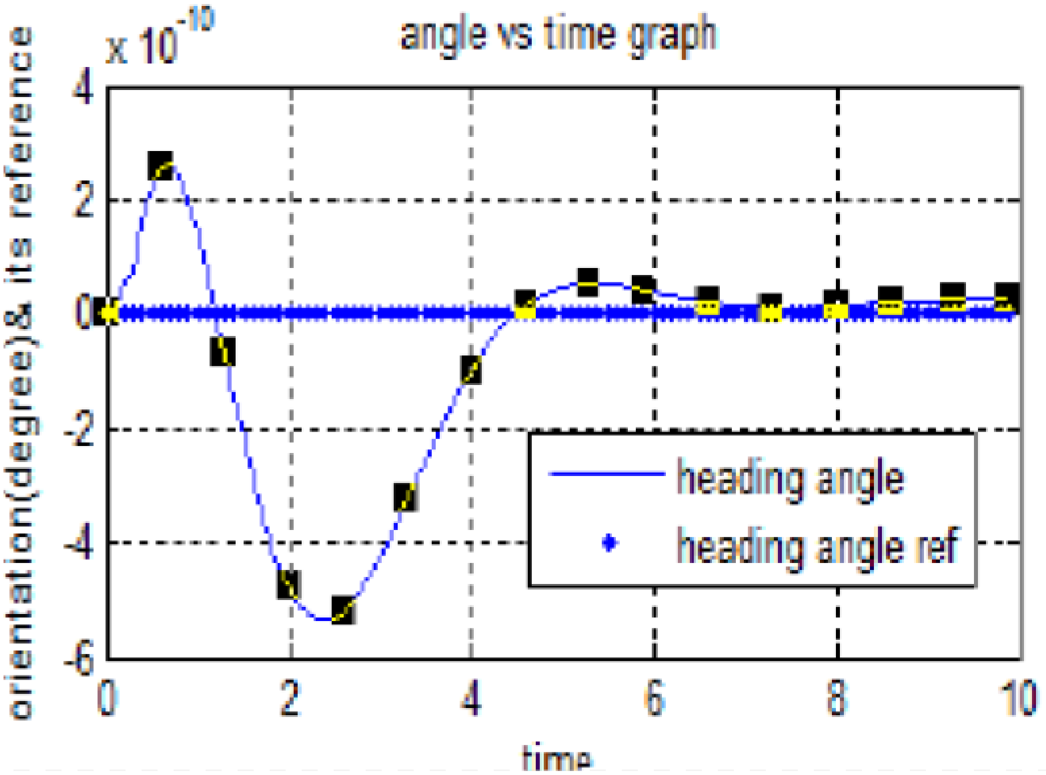

Figure 4.18 Quadriplegic Wheelchair Orientation

The maximum heading or orientation error is $-0.3*10^{-10}$ d$egree$.The negative sign indicates that rotation for the error is to the left of the desired path line.

4.4.2 Closed Loop Response with Disturbance

This part discusses the impact of disturbance on the quadriplegic wheelchair's output (xB, yB, and B) as described in chapter three's disturbance model. The impact of the disruption is demonstrated in this subtopic. The wheelchair's starting values for each state are set to zero, i.e., xB = 0, yB = 0,α =0, iL=0, iR=0, L=0, R=0, vB=0, and B=0. As can be seen, the disruption has no impact on the x position. Comparing the response without the disturbance described above to the response, only the y position and heading error is raised. Even though it takes longer for the control signal to stabilize at zero, the quadriplegic wheelchair is still travelling down the intended course with little deviation. The angular error peak change is $5*10^{-10}$ degrees. Upon receipt of the error control signal The wheelchair motor is powered by the model predictive controller's sum of the error signal and the desired values or reference control signals. By altering the output variables' weight, the steady state error can be reduced. Yet, this is made to determine the impact of a disturbance without altering the controller's parameters.

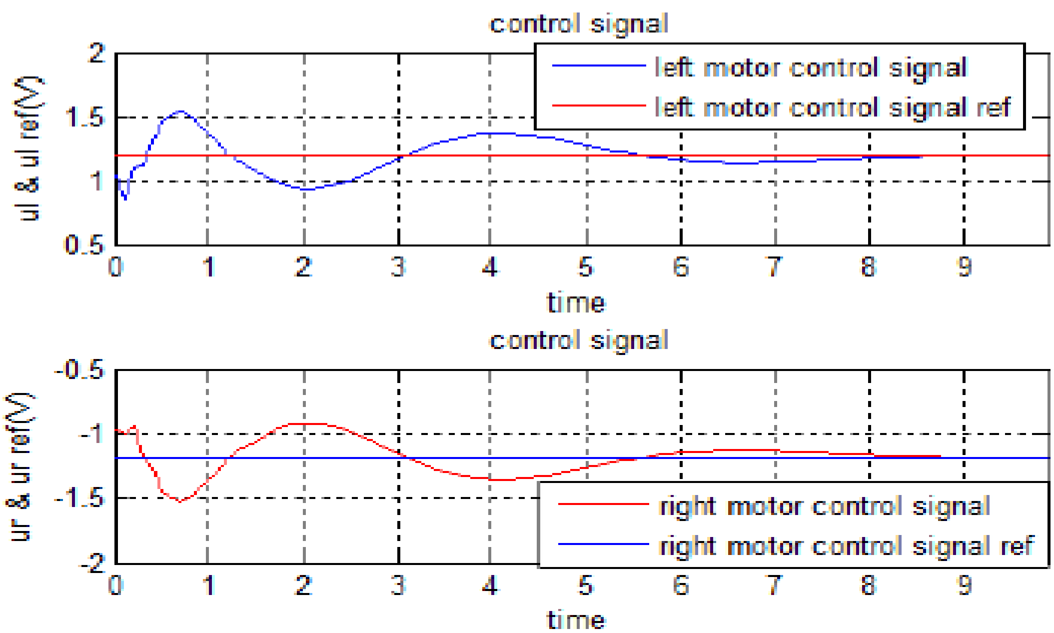

Figure 4.19 Comparison of Desired and Actual Control Signal

The actual control signal and desired control signal comparison graph is shown in figure. Once the error control signal is generated from the model predictive controller the quadriplegic wheelchair motor actuated by sum of the error signal and the desired values or references control signals. It is possible to decrease the steady state error by changing the weight of the output variables.

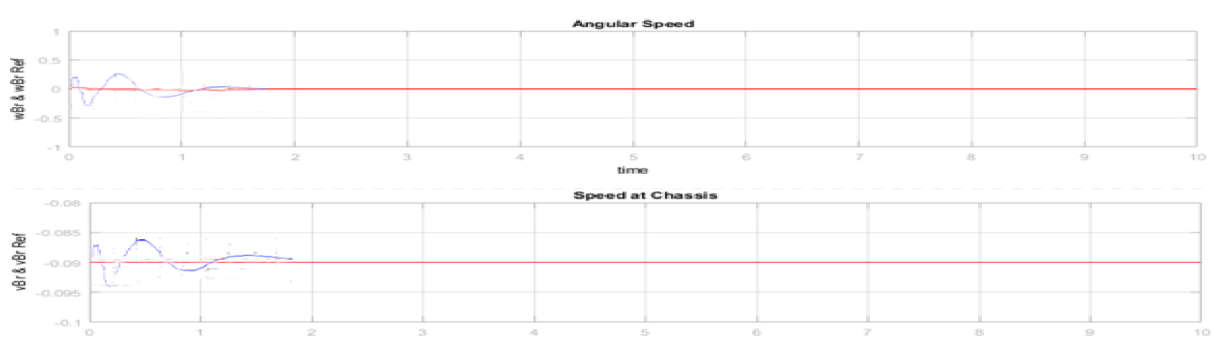

Figure4.20 Angular Speed and Speeds at Chassis

The quadriplegic wheelchair speed at chassis, and angular speed, left motor current, right motor current and the track of the quadriplegic wheelchair in the path. The maximum forward speed error is -0.02 m/s. Almost the wheelchair is moving with constant forward speed. The angular speed of the robot varies between -0.3 and 0.3 rad /s for the first three seconds but, it settles to the desired value after the perturbation.

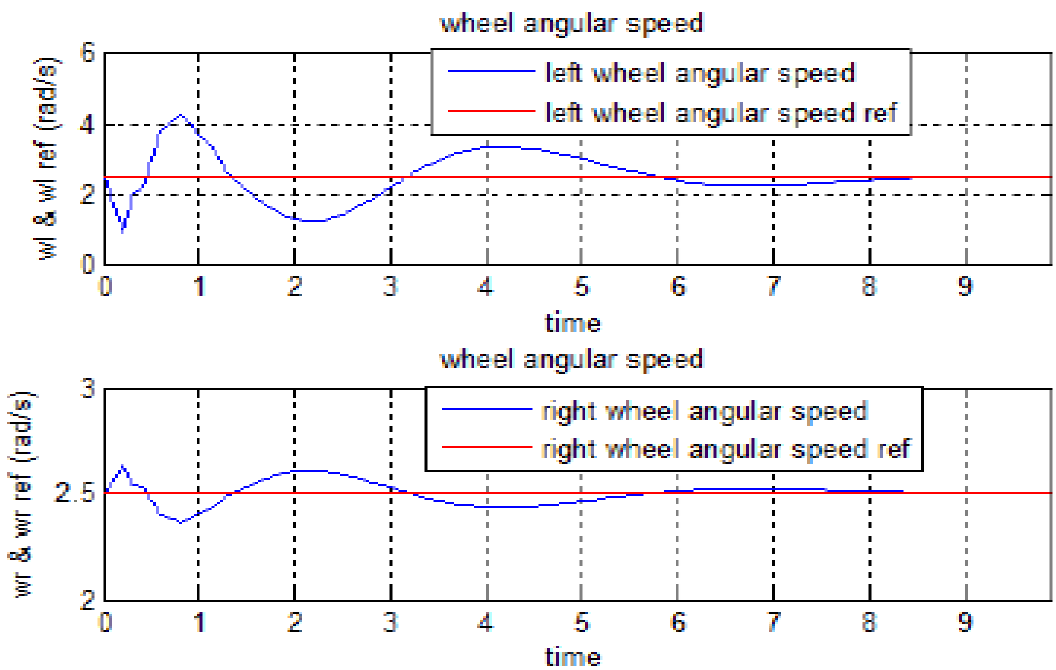

Figure4.21 Comparison of Desired and Actual Angular Speed of Motor

It is possible to decrease the steady state error by changing the weight of the output variables. But this is designed to know the effect of disturbance without any change to controller parameters. As the right and left wheels angular speed is varied with time for three seconds.

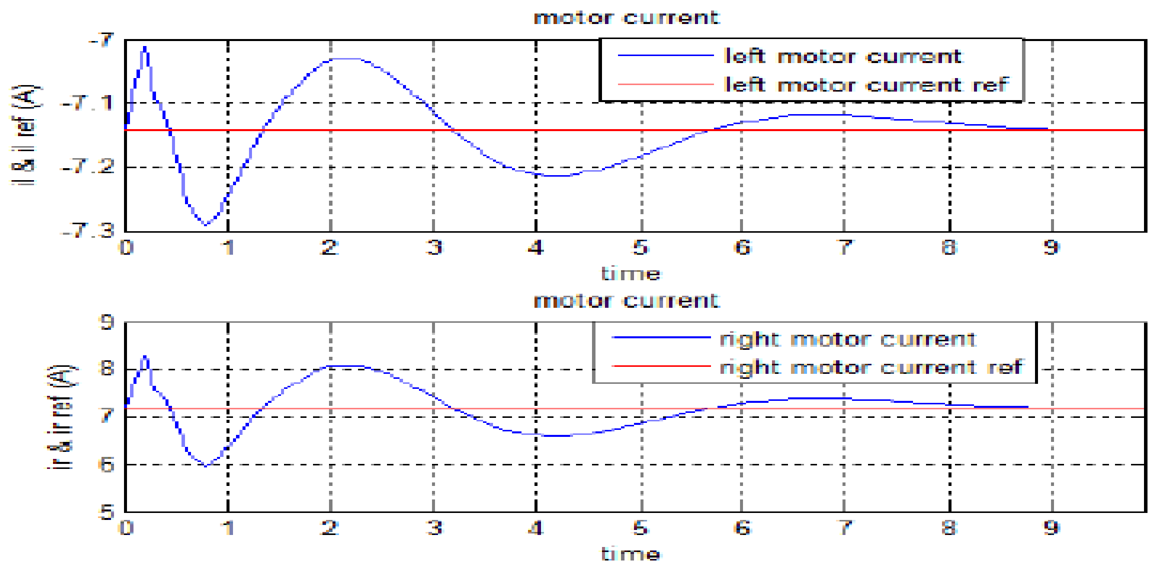

Figure4.22 Comparison Desired and Actual Current Motor

The right and left current motor is varying within three seconds.

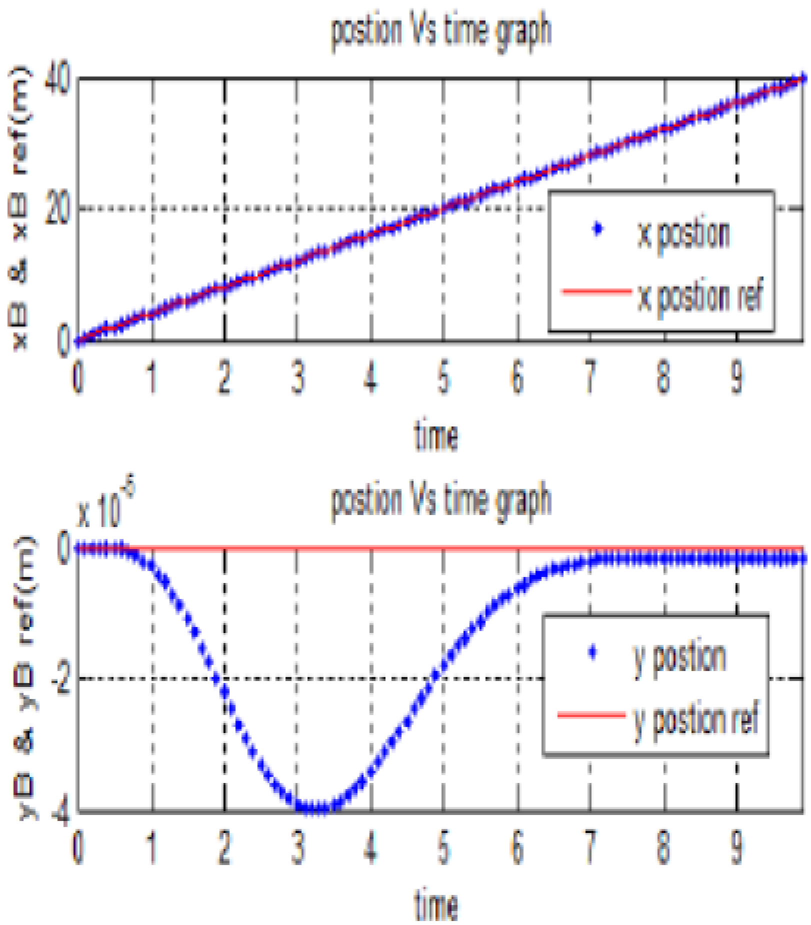

Figure4.23 Quadriplegic Wheelchair Position with Disturbance

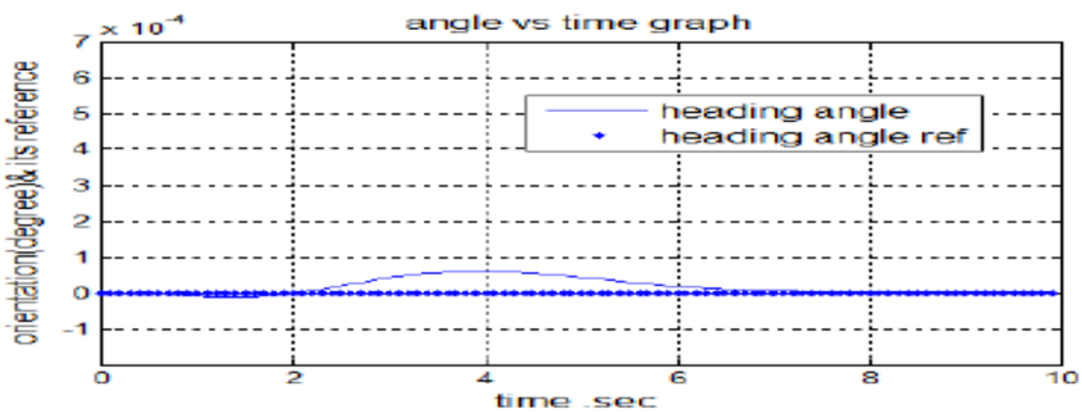

Figure4.24 Quadriplegic Wheelchair Orientation with Disturbance

From the simulation of both the system with disturbance and without disturbance it is showing that the desired path is tracked with very small position error and angular error. Since the operation of the wheelchair is for transportation it has to move with constant forward speed. Both simulation verifies that the quadriplegic wheelchair is moving with $0.8m/s$ in the track path line.

As the target of the quadriplegic wheelchair is to follow the track line error minimization are the measures of the performance of the controller. The maximum angular error for the system with disturbance is $0.3 * 10^{-4}$ degree, but without considering the disturbance the controller achieves $0.3 * 10^{-10}$ degree angular error for the straight line track path line following the wheelchair. It will be assumed as no error during move the quadriplegic wheelchair.

5.CONCLUSIONS AND RECOMMENADATIONS

5.1 Conclusions

Without a controller, a quadriplegic wheelchair is unstable and non-linear. Model Predictive Controllers (MPCs), a non-linear and unstable controller, are favored to address this issue. The air pressure, center of gravity, and weights of the back and center of the wheels all have an impact on the open loop response outcome.

The model predictive controller for an electronic quadriplegic wheelchair is discussed in this study. As a key component of the MPC design, a mathematical model of the DC motor, chassis dynamics, the kinematics of a quadriplegic wheelchair, and the disturbance was created. The model of the quadriplegic wheelchair and its augmented model are described in state space form. Weights are chosen and restrictions are placed on the quadriplegic wheelchair drive's input and output. N=2, Control Horizon, P=5, and the controller 0.1 sampling period for the prediction horizon.

The main focus of this thesis is how to design model predictive controller for electrical quadriplegic wheelchair. The performance of the controller was analyzed using the cost function taken to the desired path or reference of the outdoor line. An error minimization ability was also taken as performance minimization of the controller.

The result showed that the closed loop system without consideration of the disturbance was very good to track the path with cost function $j = 0.653 * e^{-10}$.This shows that the controller tracks the desired path with small angular error -$0.3 * 10^{-10}$ degree. Unlike the open loop response, the wheelchair able to move with constant forward speed 0.8m/s. The wheelchair was able to move with constant forward speed with a maximum error of 0.02 m/s from the desired forward speed of 0.8 m/s, according to the control system that took disturbances into account. The cost function used to track the desired path was roughly $j=0.653*e^{-10}$, but the angular error was $-0.3*10^{-4}$ degrees.

5.2 Recommendations

Today’s, many quadriplegic people have been done their daily life activities without dependent of other person. This happen because of quadriplegic wheelchair. The proposed system is implemented only in MATLAB/SIMULINK software; the hardware implementation work is recommended. In this work the controller tracks straight line and in the future by adding some feature the quadriplegic wheelchair turning strategy will be consisted. Nowadays, several quadriplegic wheelchairs are built in different parts of the world. Even though the quadriplegic wheelchair is suitable for quadriplegic person, they walk without help of other person's needs a better control system. Accordingly, it is also recommended that the student practice, update, and implement by doing experiments.

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APPENDIXES

Appendixes A

A.1 MATLAB Codes for MPC

```
%%QUADRIPEGIC WHEELCHAIR
Ra=2; %R [ohm] is motor winding resistance.
U0=10; %U0 [V] is source voltage.u
Rz=0.02; %Rz[ohm] is internal resistance.
La= 0.05; %L [H] is motor inductance.
Ka=0.100;%[K] [kg.m2. s-.2A-1] is the back EMF constant
K=1;% gain constant
J=0.025; %J [kg.m2] is moment of inertia.
ar=180; %desired angle
kr=0.002; %Kr [kg.m2. s-1] is coefficient of rotation resistance
Ll=0.04; %Ll [m] is distance of the left wheel from point b,
Lr=0.06; %Lr [m] is distance of the right wheel from point b
kw=1.350; %K? [kg.m2. s-1] is1- resistance coefficient against rotary motion
kv=0.100; %Kv [kg. s-1] is resistance coefficient against linear motion
Jb=0.550; %JT + mlT.JT [kg.m2] is moment of inertia with respect to center of
% gravity and lT [m] is distance between center of gravity and point b.
Jr=0.03; %rotational moment of inertia
T=0.1; %control interval or sample time
vr=0.8; %reference velocity
PG=25; %PG is the gear box transmission ratio
r=0.4; %r [m] is semi-diameter of the wheels
m=25; %m [kg] mass of body
rG=r/PG
% Introducing new parameters
al=kr+(kv*Lr*rG^2)/(Ll+Lr)
aR=kr+(kv*Ll*rG^2)/(Ll+Lr)
bl=J+(m*Lr*rG^2)/(Ll+Lr)
```

```
bR=J+(m*Ll*rG^2)/(Ll+Lr)
cl=kr*Ll+(kw*rG^2)/(Ll+Lr)
cR=kr*Lr+(kw*rG^2)/(Ll+Lr)
dl=J*Ll+(Jb*rG^2)/(Ll+Lr);
dR=J*Lr+(Jb*rG^2)/(Ll+Lr)
A1=Ka*(dR+bR*Ll)/bl*dR+bR*dl
A2=Ka*(dR-bR*Lr)/bl*dR+bR*dl
A3=-(dR*al+bR*cl)/bl*dR+bR*dl
A4=-(dR*aR-bR*cR)/bl*dR+bR*dl
B1=Ka*(dl-bl*Ll)/bl*dR+bR*dl
B2=Ka*(dl+bl*Lr)/bl*dR+bR*dl
B3=-(dl*al-bl*cl)/bl*dR+bR*dl
B4=-(dl*aR+bl*cR)/bl*dR+bR*dl
C1=rG*Lr/Ll+Lr
C2=rG*Ll/Ll+Lr
C3=rG/Ll+Lr
bl=Ll*rG^2/(Ll+Lr)
br=Lr*rG^2/(Ll+Lr)
MR=(Ll*rG*kv*vr-Ka*ar)/Ll+Lr;
ML=rG*kv*vr-MR;
iL=(ML+kr*al/Ka)*0.001;
iR=(MR+kr*ar/Ka)*0.001;
ul=((Ra+Rz)*iL+Rz*iR+Ka*al)/U0;
ur=((Ra+Rz)*iR+Rz*iL+Ka*ar)/U0;
%% Actual State Space Repersentation of Quadriplegic Wheelchair
A=[-(Ra+Rz)/La -Rz/La -Ka/La 0 0 0 0 0 0; -Rz/La -(Ra+Rz)/La 0 -Ka/La 0 0 0 0 0;
A1 A2 A3 A4 0 0 0 0 0; B1 B2 B3 B4 0 0 0 0 0;0 0 C1 C2 0 0 0 0 0;
0 0 -C3 C3 0 0 0 0 0;0 0 0 0 cos(aR*bR*T) 0 1 0 -vr*sin(aR*bR*T);
0 0 0 0 sin(aR*bR*T) 0 0 1 vr*cos(aR*bR*T);0 0 0 0 0 1 0 0 T]
B= [U0/La 0;0 U0/La;0 0;0 0;0 0;0 0;0 0;0 0;0 0]
C=[ 1 0 0 0 0 0 0 0 0; 0 1 0 0 0 0 0 0 0; 0 0 1 0 0 0 0 0 0]
```

```
D=0;
Q=[1 0 0 0 0 0 0 0 0;0 1 0 0 0 0 0 0 0;0 0 1 0 0 0 0 0 0;0 0 0 0 0 0 0 0 0;
   0 0 0 0 0 0 0 0 0;0 0 0 0 0 0 0 0 0;0 0 0 0 0 0 0 0 0;0 0 0 0 0 0 0 0 0;0 0 0 0 0 0 0 0 0]
R=[1 0;0 1]
sys3=ss(A, B,C, D)
[sysa,K]=c2d(sys3,T)
eig(A)
%sys controllability
co=ctrb(sys3)
controllability = rank(co)
isstable(sys3)
% observability
ob = obsv(sys3);
observability=rank(ob)
eig(A)
isstable(sys3)

K={A,B,Q,R}

%% augmented state space repersentation
[m1, n1] =size(C)
[n1, n_in] =size(B)
A_e=eye (n1+m1, n1+m1)
A_e (1: n1,1: n1) =A
A_e (n1+1: n1+m1,1: n1) =C*A;
B_e=zeros (n1+m1, n_in)
B_e (1: n1, :)=B
B_e (n1+1: n1+m1, :)=C*B;
C_e=zeros (m1, n1+m1)
C_e (:,n1+1 :n1+m1) =eye(m1,m1)
```

```
sys3=ss(A_e, B_e, C_e,D,T,'statename',{'ildot' 'irdot' 'wldot' 'wrdot' 'VB' 'WB' 'xb' 'yb' 'al' 'xbdot'
'ybdot' 'aldot'}, 'inputname', {'right motor control signal'; 'left motor control
signal'},'outputname',{'xb' 'yb' 'al'})
sys=d2d(sys3,T)
sysreal=ss(sys,'min')
%sys controllability
co= ctrb(sysreal);
controllability = rank(co)
% stablity
isstable(sysreal)
% observability
ob = obsv(sysreal)
observability=rank(ob)
modlaa=sysreal
%% Assigns Names and I/O Variables
modlaa. inputname= {'rightmotorcontrolsignal';'leftmotorcontrolsignal'}
modlaa. outputname= {'xposition' 'yposition' 'headingangle'}
modlaa. inputunit = {'V'; 'V'}
modlaa. outputunit = {'m'; 'm'; 'rad'}
%% Define Constarints on I/O
Modlaa.Plant1. outputGroup = {[3],'Measured'};
clear inputspecs outputspecs
inputspecs(1)=struct('Min', -0.1,'Max',0.1,'RateMin', -10,'Ratemax',10)
inputspecs(2)=struct('Min', -0.1,'Max',0.1,'RateMin', -10,'Ratemax',10)
outputspecs(1)=struct('Min', {-2},'Max',{20})
outputspecs(2)=struct('Min', {-2},'Max',{20})
outputspecs(3)=struct('Min',0,'Max',180)
Weights=struct('ManipulatedVariables',[2 2],...
'ManipulatedVariablesRates',[2 2],...
'OutputVariables',[4000; 4000 ;4000])
%% Setup of MPC of Controller
```

```
clear Model
Modla.Plant=modlaa;
PredictionHorizon =5;
ControlHorizon =2;
MLLn =mpc(modlaa,T,PredictionHorizon, ControlHorizon,[],inputspecs, outputspecs)
%% Closed-loop MPC Simulation Using the Command SIM
Tstop=10; % simulation time
Tf=round(Tstop/T) % number of simulation steps
r1= [0.0003 0  0]; % Reference trajectory
% Specify noise signals. In order to do this,
% I create the MPC simulation object 'SimOptions'.
xmpc=mpcstate(MLLn);
d= [];
v= [];
x=zeros(9,1)
SimOptions=mpcsimopt(MLLn);
SimOptions.plantInitialState =x; % plantInitialState
SimOptions.ControllerInitialState=xmpc; %Controller Initial State
SimOptions.RefLookAhead='on';
SimOptions.unmeasuredDisturbance =d;
SimOptions.outputNoise= []; % output measurement noise
SimOptions.inputNoise=[v]; % noise on manipulated variables
% Run the closed-loop simulation and plot results.
% Run the closed-loop simulation and save the results to workspace.
[y2,t2,u2,Tf,d,SimOptions,all]=sim(MLLn,Tstop,r1,v)
[A_e,B_e,C_e,D]=ssdata(modlaa);
YY= [];
UU= [];
XX= [];
jj= [];
cost=[];
```

```
% Main simulation loop
for k=0: round(Tstop/T)
XX= [XX, x];
% Define measured disturbance signal
v=0.03;
d=0.03;
Q=diag (1,1);
R=diag (1);
% Compute MPC law
y=C*x
YY= [YY, y];
% Plant equations state update
ud=mpcmove(MLLn,xmpc, y, r1)
x=A*x+B*ud
UU= [UU, ud];
j=(y-r1')'*R*(y-r1') +ud'*Q*ud;
jj= [jj, j];
end
plot ( jj(1) *size(Ka))
legend('cost')
[~, Info] =mpcmove (MLLn, xmpc, y, r1)
topt=Info. Topt
yopt=Info. Yopt
uopt=Info.Uopt
%% referneces system
xr=2*t2
yr=zeros(size(t2))
vBr=2*ones(size(t2))
aref=zeros(size(t2))
wBr=zeros(size(t2))
wRr=(vBr+Lr*wBr)/rG;
```

```
wLr=-(wBr*(Ll+Lr)-wRr*rG)/rG
MRr=(Ll*rG*kv*vBr+kw*wBr)/Ll+Lr
MLr=rG*kv*vBr-MRr
ilr=((MLr+kr*wLr)*0.001)
irr=((MRr+kr*wRr) *0.001)
ulr=(R+Rz+ilr+Rz+irr+wLr)/U0
urr=(R+Rz+irr+Rz+ilr+wRr)/U0
All= [wRr wLr MRr MLr ilr irr ulr urr]'
%% error with references
udr=urr+u2(:,2);
udl=ulr+u2(:,1);
uhg=[udl udr];
xrd=xr+y2(:,1)
yrd=yr+y2(:,2)
ard=aref+y2(:,3)
yuubb=[xrd yrd ard]
ild=all(:,1)+ilr
ird=all(:,1)+irr
wld=wLr+all(:,1)
wrd=wRr+all(:,1)
vBd=vBr+all(:,1)
wBd=wBr+all(:,1)
%% Analysis
get(MLLn)
display(MLLn)
X=trim (MLLn, y, ud)
DC=cloffset(MLLn)
review(MLLn)
```

Appendix B

B.1 Simulink Blocks

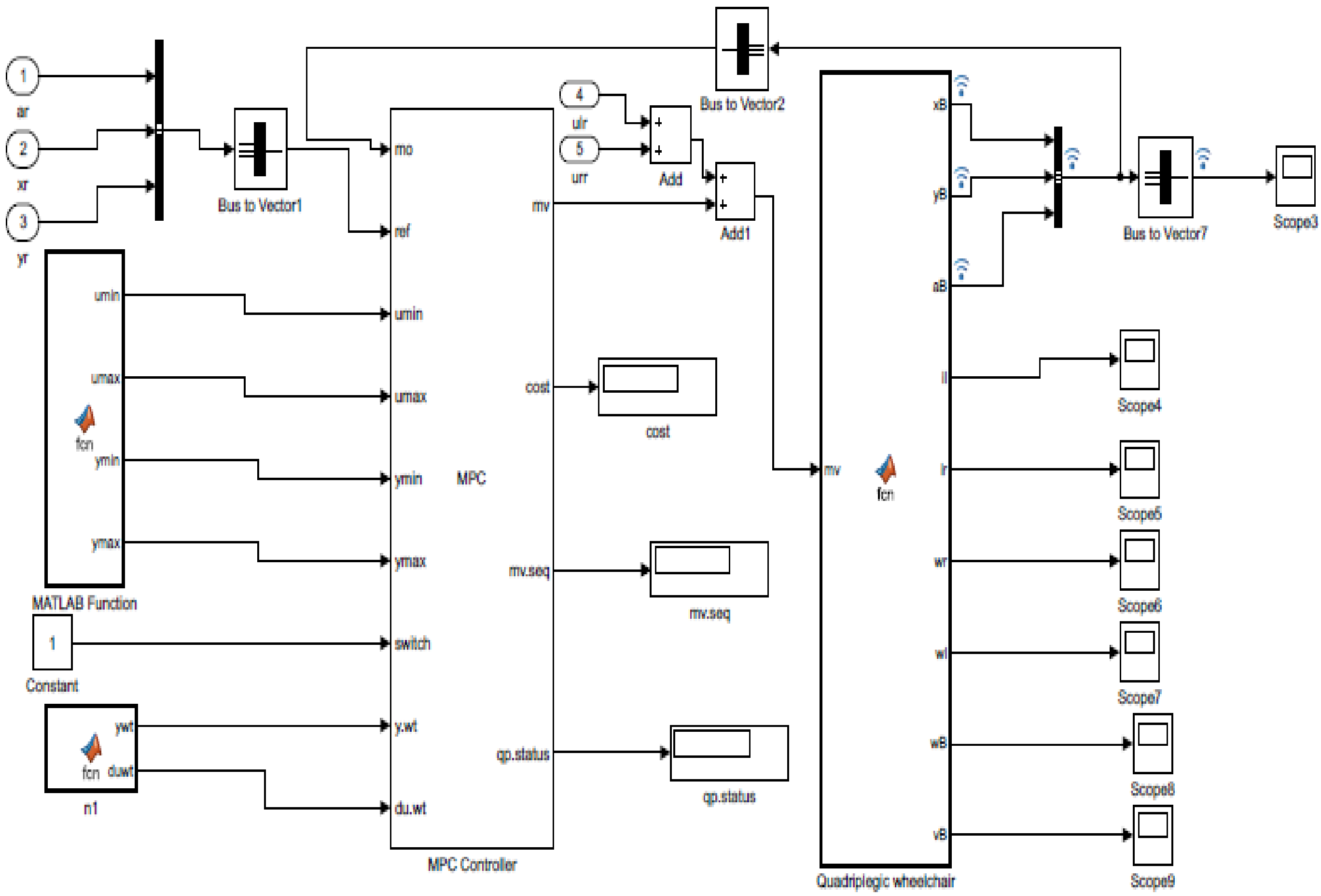


Figure 1- Model Predictive Control Without Disturbances

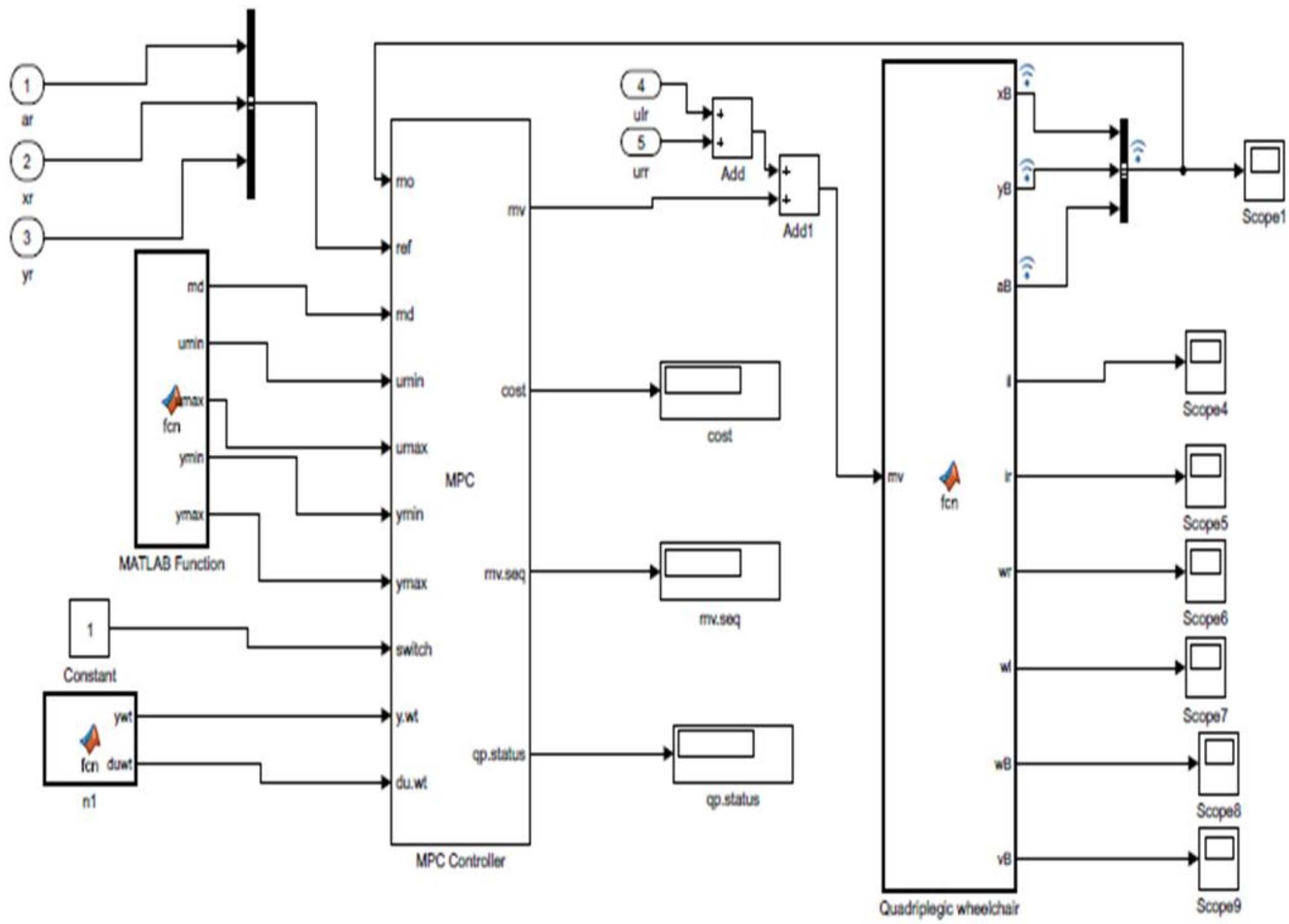


Figure2- Model Predictive Control With Disturbance

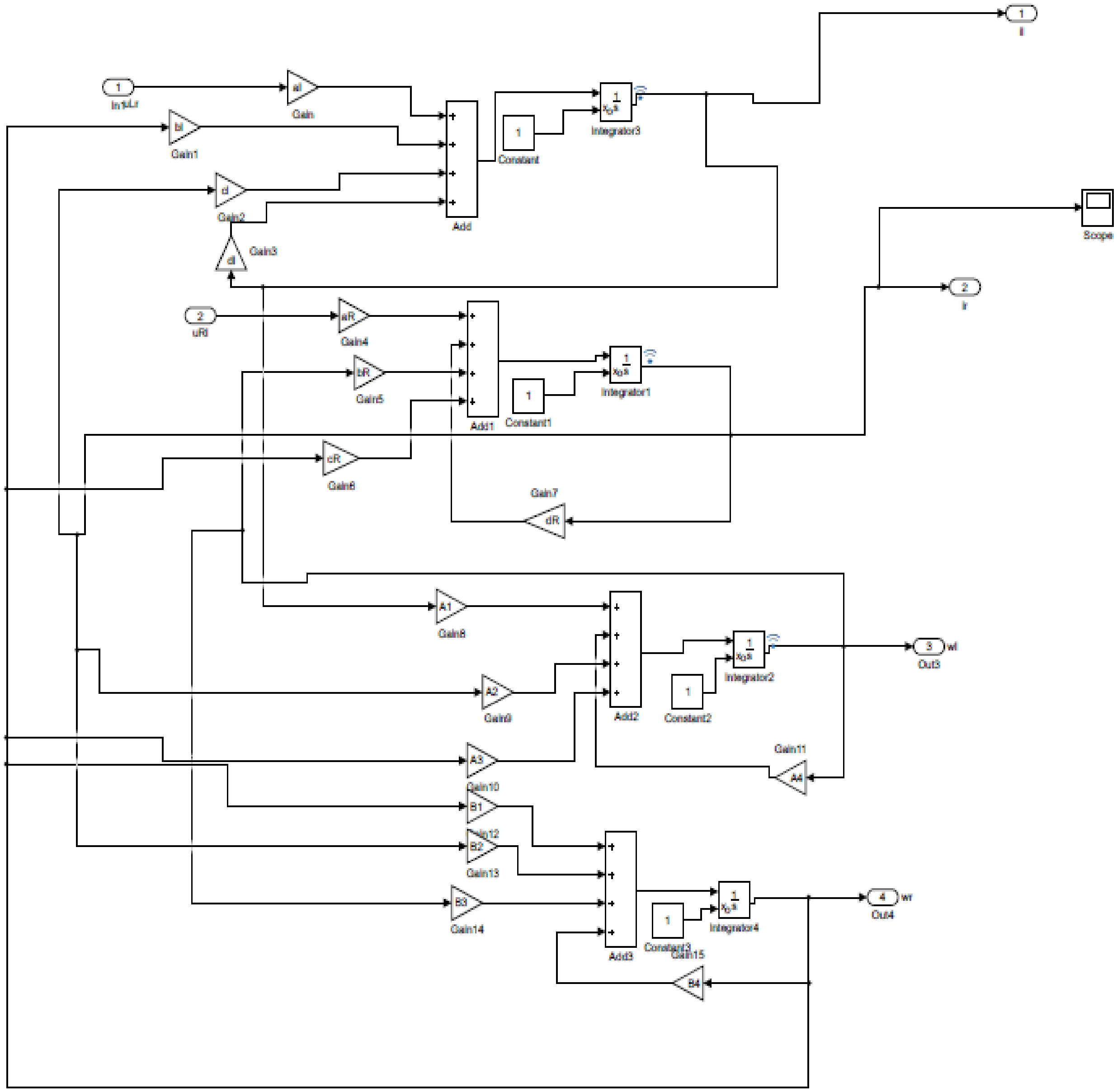
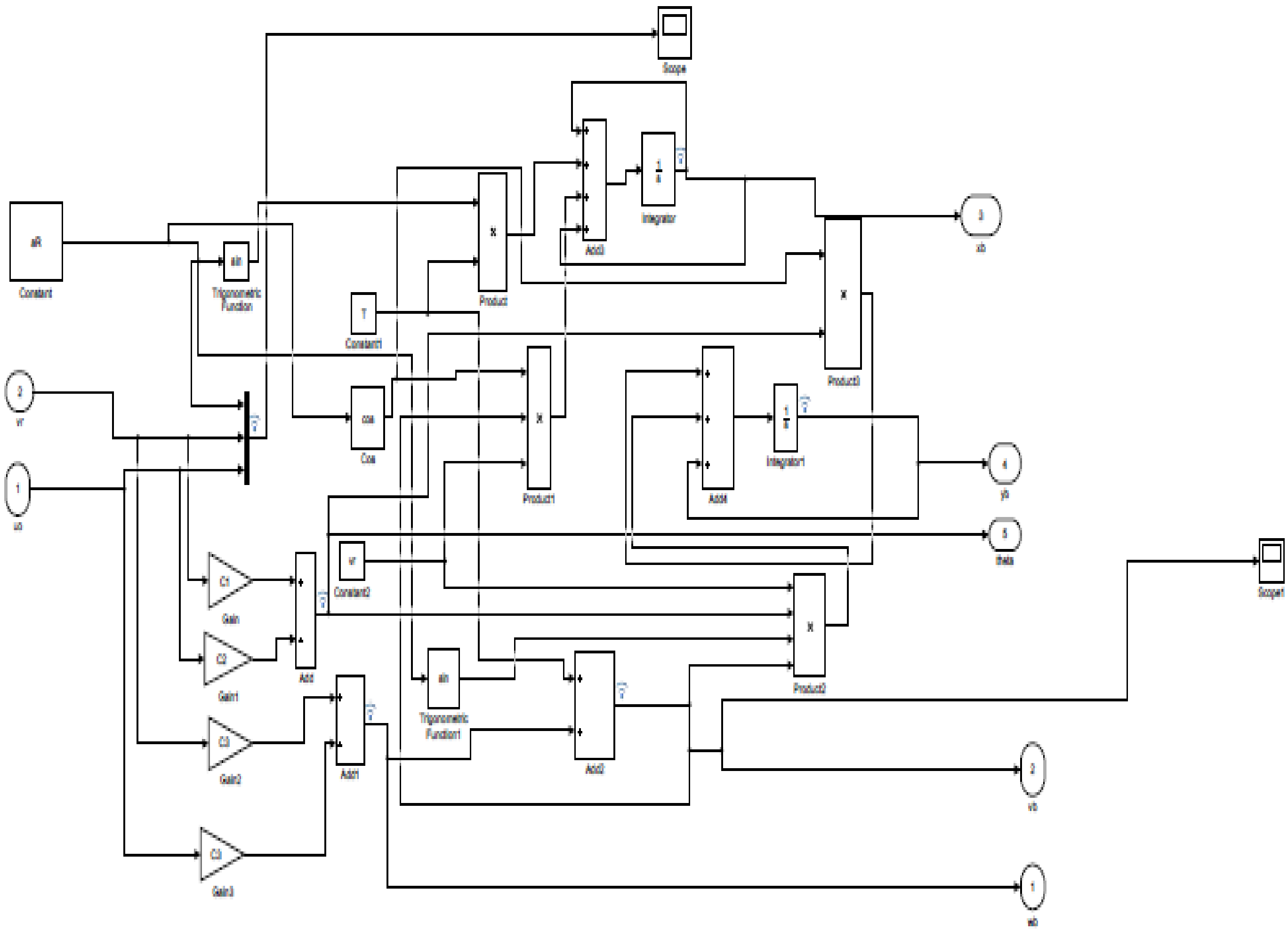


Figure 3-Dynamics Model of Quadriplegic Wheelchair



Figur4 - Kinematics Model of Quadriplegic Wheelchair

APPENDIX C

C.1 Appendixes related to some constraints variables

Closed -Loop Steady-State Gain

Disturbed OV	Affected OV	Gain
heading angle	xposition	-0.0107203
headingangle	yposition	-0.000123512
xposition	headingangle	-0.000106814
yposition	headingangle	-7.90254e-05
headingangle	headingangle	0.999999

Soft Constraints

Constraint	Assumed Violation	Impact Factor	Sensitivity Ratio
Lower limit: headingangle	18	66.94	1000
Upper limit: headingangle	18	66.94	1000
Lower limit: xposition	2.2	1	14.94
Upper limit: xposition	2.2	1	14.94
Lower limit: yposition	2.2	1	14.94
Upper limit: yposition	2.2	1	14.94