Dynamical Processes and Structural Transformations of Supermassive Black Holes in Galaxy Mergers : A Simulation Study

Gidesa Abdena Bekele

A Thesis Submitted to the Department of Applied Physics

School of Applied Natural Science

Office of Graduate Studies

Presented in Partial Fulfillment of the Requirement for the Degree of Master's in Applied Physics (Astrophysics)

Adama Science and Technology University

November, 2024

Adama, Ethiopia

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Declaration

I declare that this Master thesis entitled *"***Dynamical Processes and Structural Transformations of Supermassive Black Holes in Galaxy Mergers: A Simulation Study***"* is my own work and has not been submitted to any university for a similar purpose. The references used in this Thesis are duly recognized by proper citations.

Gidesa Abdena Bekele	____________	____________
Name of student	Signature	Date

Recommendation

We,the advisors of this thesis, hereby certify that we have read the revised version of the thesis entitled“**Dynamical Processes and Structural Transformations of Supermassive Black Holes in Galaxy Mergers: A Simulation Study**” prepared under our guidance by Gidesa Abdena Bekele Submitted in partial fulfillment of the Requirement for the degree of master's of Science in Applied physics. Therefore ,we recommend the submission of the revised version of the thesis to the department following the applicable procedures.

Dr. Anno Kare	________________	________________
Advisor	Signature	Date
Dr. Jima Asefa	________________	________________
Co-advisor	Signature	Date

Approval page

We, the undersigned, members of the Board of Examiners of the final open defense by **Gidesa Abdena Bekele** have read and evaluated his thesis entitled "**Dynamical Processes and Structural Transformations of Supermassive Black Holes in Galaxy Mergers: A Simulation Study**"and examined the candidate. Therefore,this is to certify that the thesis has been accepted in partial fulfillment of the requirement of the Degree of Master of Science in Applied Physics (Astrophysics).

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List of Abbreviations

AGN	Active Galactic Nuclei
FPP	Final Parsec Problem
GPU	Graphics processing Unit
GW	Gravitational Wave
GRAPE	GRAvitational PErformance
INTGC	Intelligent Grid Computing
LIGO	Laser Interferometer Gravitational-Wave Observatory
LISA	Laser Interferometer Space Antenna
PDF	Probability Distribution Function
PM	Particle Mesh
SMBH	Super Massive Black Hole
TDE	Tidal Disruption event

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Abstract

Supermassive black holes (SMBHs) are commonly found at the centers of galaxies and exhibit strong correlations with their host galaxies, indicating a close relationship between the growth of SMBHs and galaxy evolution. According to the hierarchical galaxy formation model, this correlation suggests that SMBH binaries must merge efficiently, facilitated by processes such as dynamical friction, interactions with stars and gas, and eventually gravitational wave emission. If SMBH binaries can merge within a time frame shorter than the Hubble time, they could become significant sources of gravitational waves for detection. However, it has been proposed that SMBH binaries might stall at a separation of around 1 parsec, a phenomenon known as the Final Parsec Problem (FPP). This study uses N-body simulations to explore an improved model for the orbital decay of SMBHs driven by dynamical friction. By running a large number of simulations, the findings show that the FPP does not occur in galaxies formed through mergers. The non-spherical shape of the merger remnants ensures a continuous supply of stars for the SMBH binary to interact with, allowing it to continue shrinking its orbit. As the binary progresses toward merging, it ejects stars of multiple times its mass in stars, leading to the formation of low-density cores at the centers of massive elliptical galaxies. These results support the idea that SMBH mergers are frequent in a cosmological context, suggesting that SMBH binaries are likely sources of gravitational waves both at low and high redshifts.

key words: Supermassive black holes (SMBHs), Final Parsec Problem (FPP), Gravitational waves, N-body simulations.

1

Introduction

1.1 Background of the Study

Galaxies, vast collections of stars, gas, and dust held together by gravity, are fundamental building blocks of the universe. At the center of many galaxies there exist supermassive black holes, (SMBHs) with masses ranging from millions to billions of times that of the Sun. The interconnection between SMBHs and their host galaxies is a crucial aspect of galactic evolution, and understanding the growth, and properties of these enigmatic cosmic objects is a compelling field of research Alonso (2007). One significant process that can profoundly influence the growth and properties of SMBHs is galaxy mergers. Galaxy mergers occur when two or more galaxies gravitationally interact and eventually combine into a single, larger galaxy, Smole (2015). During this transformative event, the dynamics, structure, and composition of galaxies, including their central SMBHs, undergo dramatic changes. Consequently, investigating the effects of galaxy mergers on SMBHs is essential for unraveling the intricate co-evolution of galaxies and the formation of structures in the universe Latif and Ferrara (2016). Galaxy mergers can have a profound impact on SMBH growth through enhanced accretion. As galaxies merge, the interaction of gas and dust in the colliding systems triggers increased accretion onto the SMBHs. The merger process leads to the combination of gas and dust clouds, creating dense regions that serve as abundant fuel for the SMBHs. Consequently, the availability of accretion material increases, resulting in accelerated accretion rates and rapid growth of SMBHs Hopkins et al. (2006). Tidal disruption events (TDEs) are another consequence of galaxy merger that significantly affect SMBH growth. TDEs occur when stars pass too close to the SMBHs and experience tidal forces that tear them apart. These disrupted stars form an accretion disk around the SMBHs, leading to episodic increases in accretion rates and providing additional fuel for SMBH growth Mihos and Hernquist (1995). Furthermore, the merger process can generate asymmetrical gravitational waves, resulting in gravitational wave recoil. When SMBHs coalesce during a merger, the merged SMBH experiences a kick, causing it to be ejected from the center of the newly formed galaxy. This gravitational wave recoil can temporarily halt or reduce the accretion rate onto the SMBH, impacting its growth until it settles back into the galactic center Blecha et al. (2016); Gerosa et al. (2018). Investigating the effects of gravitational wave recoil during galaxy mergers is vital for understanding the dynamics of SMBHs and their subsequent impact on galactic structures. Moreover, the feedback effects of SMBH accretion, known as active galactic nuclei (AGN) feedback, play a significant role in the coevolution of SMBHs and galaxies. AGN feedback occurs when SMBHs release enormous amounts of energy in the form of radiation and outflows of ionized gas. These feedback processes can regulate star formation and prevent excessive gas accumulation in the galaxy, influencing the growth of both the SMBH and the host galaxy itself. Galaxy mergers can trigger or enhance AGN feedback, thereby affecting the subsequent growth and properties of the SMBH and the host galaxy Di Matteo et al. (2008); Fabian (2012). Understanding the intricate interconnection between SMBH accretion and AGN feedback during galaxy mergers is crucial for comprehending the broader context of SMBH-galaxy coevolution. To explore the effects of galaxy mergers on the growth and properties of SMBHs, a comprehensive approach is required. This involves combining observational data, theoretical models, and computational simulations to capture the complex dynamics involved in galaxy mergers and SMBH accretion processes Whalen and Fryer (2012).

1.2 Statement of the Problem

Supermassive black holes (SMBHs) play a critical role in galaxy evolution, particularly during mergers where their dynamics and interactions can significantly alter both the SMBHs and their host galaxies. However, the detailed processes driving the dynamical evolution and structural transformation of SMBHs during these events remain poorly understood due to observational challenges and limitations in theoretical models. Key questions persist regarding SMBH orbital decay, accretion, and the conditions leading to coalescence or displacement, with current models often failing to capture the full complexity of these phenomena or reconcile predictions with observed merger rates and gravitational wave signatures Begelman et al. (1980). N-body simulations offer a powerful tool to investigate these interactions, yet many existing simulations lack the resolution to fully explore SMBH dynamics, especially in the presence of gas dynamics, tidal forces, and feedback effects Springel et al. (2005). This study aims to address these gaps by employing high-resolution N-body simulations to investigate the dynamical processes governing SMBHs in galaxy mergers, analyzing the resulting structural transformations of both the SMBHs and their host galaxies. Through these simulations, this research seeks to uncover new insights into the conditions that lead to SMBH coalescence, orbital evolution, and the broader impact on galactic morphology, contributing to a deeper understanding of SMBH behavior in galaxy evolution and providing valuable predictions for future observations.

1.3 Objectives of the Study

1.3.1 General Objective

The general objective of this study is to investigate the dynamical processes and structural transformations of SMBHs in galaxy mergers.

1.3.2 Specific Objectives

The specific objectives of this study are to:

- study N-dependence of SMBHs hardening rates in galaxy mergers models.
- compare the hardening rates of SMBHs in galaxy mergers models and in isolated spherical galaxy models.
- explore how galaxy mergers affect the central density profiles of the merging galaxies.

1.4 Significance of the Study

The study of dynamical processes and structural transformations of SMBHs in galaxy mergers offers valuable insights into the relationship between galaxies and their central black holes. The study of dynamical processes and structural transformations of SMBHs in galaxy mergers is vital for understanding key astrophysical phenomena. This study investigates how SMBHs evolve during mergers, focusing on the N-dependence of their hardening rates, which reveals how surrounding stars influence their interactions. Understanding these rates is crucial for predicting SMBH merger events, with significant implications for gravitational wave astronomy. Additionally, comparing hardening rates in galaxy merger models and isolated spherical models deepens our understanding of how different environments affect SMBH dynamics. Overall, this study enhances our comprehension of SMBH dynamics and contributes to theoretical models of black hole growth and galaxy evolution.

1.5 Outline of the Thesis

This thesis is divided into five chapters. Chapter One lays the groundwork by introducing the background, identifying the research problem, outlining the objectives, and highlighting the study's significance. Chapter Two provides an extensive literature review, discussing the dynamical processes of SMBH mergers, gravitational wave emission from such mergers, structural changes in merging galaxies, the formation of SMBHs, and the evolution of SMBH binaries. It also consolidates the current understanding in the field. Chapter Three details the methodology, including galaxy models, N-body simulations, spherical system models, and numerical codes used for galaxy modeling. Chapter Four focuses on the results and discussion, emphasizing the key findings from the simulations. Lastly, Chapter Five summarizes the main conclusions, discusses their implications, and proposes directions for future research in the area.

2

Literature Review

2.1 Supermassive Black Holes

Supermassive black holes (SMBHs) are believed to reside at the centre of most massive galaxies in the local Universe and to obey tight relationships between their mass and several quantities of the host spheroid Kormendy and Richstone (1995); Richstone et al. (1998); De Nicola et al. (2019); Gebhardt et al. (2000); Gültekin et al. (2009); McConnell and Ma (2013). A small fraction of these systems has been detected as active galactic nuclei (AGN), which is a consequence of high levels of gas accretion to the central BH. This gas is believed to originate at large (galactic: from 0 to ∼kpc) scales and, through the loss of most of its angular momentum, to flow towards the center of the massive system, down to sub-pc scales, where it can be accreted Bell et al. (2006) and drive the growth of SMBHs. Many possible explanations have been proposed, including minor interactions (and extremely minor mergers) and/or internal (secular) processes, which include instabilities driven by bars and violent gas instabilities at high redshift Gabor and Bournaud (2013). One of the first mechanisms brought forward were the largescale gravitational torques produced by major galaxy mergers. During these encounters, torques generate large-scale gas inflows that drive the gas down to pc-scale.

2.2 Formation of SMBHs

Supermassive Black Holes SMBHs are thought to form through a combination of processes during the evolution of galaxies Latif and Ferrara (2016). The formation of SMBHs begins with the creation of seed black holes. These seeds can emerge from the remnants of massive stars that have undergone gravitational collapse after depleting their nuclear fuel Dittmann and Miller (2020). SMBHs are thought to form through a combination of processes during the evolution of galaxies Latif and Ferrara (2016). The formation of SMBHs begins with the creation of seed black holes. These seeds can emerge from the remnants of massive stars that have undergone gravitational collapse after depleting their nuclear fuel Dittmann and Miller (2020).

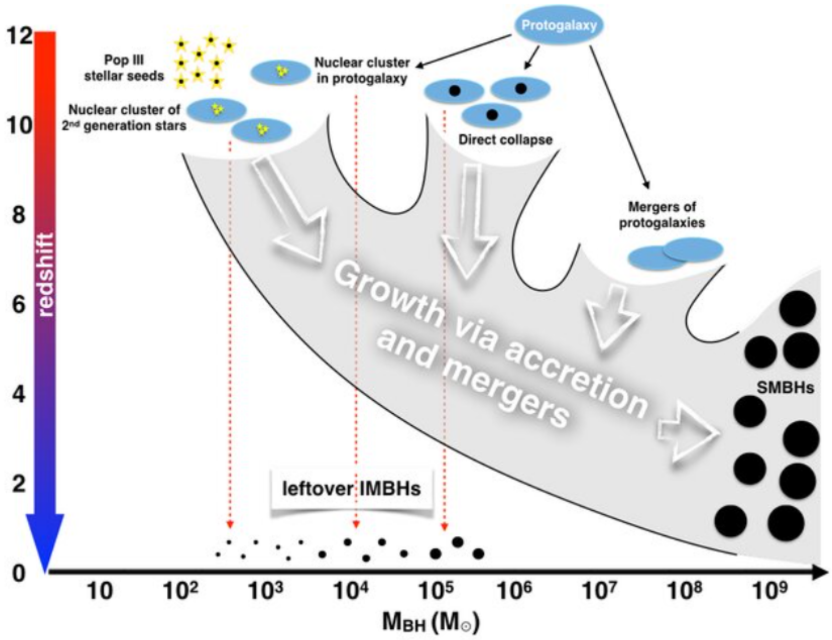

Figure 2.1: Possible channels for the formation of SMBHs.
The figure taken from Mezcua (2017).

Figure: 2.1 shows the possible channels for the formation of SMBHs including the epochs in which their "seeds" are born, their subsequent evolution with redshift, and the expected masses of "left-over" IMBHs to be found in the present era.

Alternatively, seed black holes may form through the direct collapse of massive gas clouds, particularly in the early universe. These clouds collapse under certain conditions, such as low metallicity and efficient cooling mechanisms, bypassing the formation of stars and leading to the birth of massive black holes Whalen and Fryer (2012). Once the seed black holes are in place, they can grow in mass through a process called accretion. Surrounding gas and dust are drawn toward the black hole by its immense gravitational pull, forming an accretion disk Whalen and Fryer

(2012). Material in the disk spirals inward, releasing energy in the form of radiation before being consumed by the black hole. This accretion process can significantly increase the mass of the black hole over time Richter et al. (2006). Galaxy mergers also play a crucial role in the formation of SMBHs. When galaxies merge, their central black holes can merge as well, resulting in the formation of a more massive black hole. The merger brings additional gas and dust to the galactic center, fueling the accretion onto the black hole and further augmenting its mass Latif and Ferrara (2016). This hierarchical merging process allows SMBHs to grow to the enormous sizes observed in the centers of galaxies.

2.3 Dynamical Processes of SMBH Mergers

2.3.1 Dynamical Friction and Orbital Decay

Galaxy mergers commence with the gravitational interaction between two colliding galaxies, initiating a complex sequence of dynamical events that ultimately lead to significant structural changes within both galaxies. The gravitational forces exerted by the merging galaxies alter their orbital trajectories, causing their stars, gas, and dark matter to redistribute in response to the evolving gravitational landscape Toomre and Toomre (1972). A key aspect of this merger process is the phenomenon known as orbital decay, which occurs as a result of dynamical friction. Dynamical friction can be mathematically described by the following equation, derived from the theory of stellar dynamics:

$$\frac{dv}{dt} = -\frac{4\pi G^2 M}{v^3}\rho_{star} V, \tag{2.1}$$

where dv/dt the rate of change of speed of the object due to dynamical interactions with surrounding stars, where V is the volume of SMBHs, G is the gravitational constant, M is the mass of the SMBH, v is the speed of the SMBH, and ρ_{star} is the density of the surrounding stars Binney and Tremaine (2011). This equation describes the rate of change of the velocity of the SMBH due to gravitational interactions with surrounding stars. As the SMBHs move through their environments, they exert gravitational influences on the surrounding stars and dark matter, creating a wake of gravitational interactions that leads to energy loss for the SMBHs. This energy dissipation causes the SMBHs to gradually lose momentum and spirals toward the center of the newly formed merged galaxy.

The efficiency of dynamical friction is heavily influenced by the presence of gas in the merging galaxies. In gas-rich mergers, the interaction between the SMBHs and the surrounding gas is particularly pronounced. The inflow of gas can amplify the frictional forces acting on the SMBHs, enhancing the rate at which they lose energy and angular momentum. The total energy loss due to dynamical friction can be expressed as:

$$\frac{d\mathbf{E}}{dt} = -\frac{GM^2}{a^2}\rho_{gas}V, \tag{2.2}$$

where E is the energy of SMBHs, a is semi-major axis of the SMBH's orbit, ρ_{gas} is density of gas and v is the velocity of the SMBH Ostriker et al. (1986). As the gas becomes compressed and heated by the gravitational forces of the merging galaxies, it can lead to the formation of dense inflows that further facilitate the inward migration of the SMBHs. This heightened efficiency of dynamical friction in gas-rich environments significantly accelerates the process of orbital decay, allowing the SMBHs to move closer together in a shorter timescale. Conversely, in gas-poor systems, the dynamics are quite different. The lack of substantial gas reservoirs results in reduced efficiency of dynamical friction. In these scenarios, the SMBHs predominantly interact with the stellar and dark matter components, which may not exert sufficient gravitational drag to drive the SMBHs inward effectively Escala et al. (2005). Consequently, binary SMBHs in gas-poor mergers can remain at larger separations for extended periods, often spanning hundreds of millions to billions of years. This prolonged separation can significantly impact the eventual fate of the SMBHs, as it extends the timescale for coalescence, delaying the onset of processes that would lead to their eventual merging. The timescale, t_{decay} for the orbital decay of binary SMBHs can be estimated using the following expression:

$$t_{decay} \approx \frac{a^4}{G^2 M \rho_{star}}, \tag{2.3}$$

where a is the semi-major axis of the binary orbit, M is the mass of the SMBH, and ρ_{star} is the stellar density in the core of the merging galaxy Merritt (2006). In gas-poor systems, this timescale can be considerably longer, as the density of stars may be lower and the gravitational interactions less effective, resulting in a slower inward migration of the SMBHs. The stage of orbital decay is a pivotal phase in the SMBH merger process. It not only dictates whether the SMBHs will ultimately coalesce but also influences the surrounding environment and the structural dynamics of the host galaxies. The interactions that occur during this phase can lead to various outcomes, such as the ejection of stars from the central regions, which can result in the formation of low-density cores in massive elliptical galaxies Gualandris and Merritt (2008). Additionally, the dynamical processes at play can trigger complex feedback mechanisms within the galaxy, impacting star formation rates and the overall morphology of the galaxy.

2.3.2 Stellar Interactions in Gas-Poor Mergers

In gas-poor mergers, stellar interactions predominantly drive the hardening process. As stars pass near the binary SMBHs, they are subjected to strong gravitational forces that can impart significant ”kicks.” This interaction results in the ejection of stars from the vicinity of the binary system, effectively removing energy and angular momentum from the binary and allowing the SMBHs to move closer together. This can be quantitatively described by the following expression:

$$\Delta E \approx -\frac{GM_1M_2}{r}, \tag{2.4}$$

where ΔE represents the change in energy of the binary system, G is the gravitational constant,M_1 and M_2 are the masses of the SMBHs, and r is the distance between them Merritt (2006). Each gravitational interaction serves to decrease the orbital separation of the SMBHs, promoting the hardening of the binary. However, as the separation between the SMBHs decreases, the number of stars available for interaction diminishes. This reduction in available stars is a critical aspect of the ”final parsec problem.” The final parsec problem describes a scenario where the binary SMBHs become trapped at separations that are too large for efficient gravitational wave emission, thus stalling the merger process Merritt and Poon (2004). At such close separations, the gravitational influence of the binary becomes weaker relative to the surrounding stellar environment, leading to insufficient interactions to further drive down the separation.

2.3.3 Circumbinary Disks in Gas-Rich Mergers

In contrast, gas-rich mergers introduce additional dynamics that facilitate the hardening of the binary. The presence of gas leads to the formation of a circumbinary disk around the SMBHs. This disk acts as a reservoir of gas that can exert gravitational torques on the binary, further aiding in the hardening process. The gravitational interaction of the circumbinary disk with the binary SMBHs is critical in enabling the energy dissipation necessary for the SMBHs to move closer together. The torques exerted by the circumbinary disk can be described by the equation:

$$\boldsymbol{\tau} = \mathbf{r} \times \mathbf{F}, \tag{2.5}$$

where $\boldsymbol{\tau}$ is the torque experienced by the binary, $\mathbf{r}$ is the position vector from the SMBHs to the point of interaction, and $\mathbf{F}$ is the gravitational force exerted by the gas Armitage and Natarajan (2002). These gravitational interactions lead to angular momentum exchange, which effectively removes angular momentum from the binary system and enables the SMBHs to spiral inwards more efficiently. Moreover, the gas dynamics surrounding the binary system can create regions of increased density, which enhances the interactions between the SMBHs and the gas. This increased interaction not only allows for a more effective reduction of binary separation but also facilitates additional mechanisms for energy dissipation, allowing the SMBHs to overcome the limitations faced in gas-poor mergers Escala et al. (2005).

2.3.4 The Final Parsec Problem

The Final Parsec Problem (FPP) is a theoretical challenge in the study of supermassive black hole (SMBH) mergers, particularly in systems that are gas-poor. After two SMBHs form a bound binary system as a result of a galaxy merger, their separation must shrink further for coalescence to occur. Early in the merger, dynamical friction—a process where the SMBH binary loses energy through interactions with surrounding stars and gas—drives the separation to decrease. However, as the binary shrinks to approximately 1 parsec, a stall can occur, known as the final parsec problem. At this scale, gravitational wave emission alone is not strong enough to cause the SMBHs to merge, and the number of stars in the vicinity that could interact with and continue to harden the binary is thought to become depleted Begelman et al. (1980). This stalling presents a major obstacle in the theoretical framework of hierarchical galaxy formation, where SMBH mergers are expected to occur frequently. To overcome the FPP, several solutions have been proposed. One prominent idea is that the triaxial or flattened shape of galaxy cores, as opposed to perfectly spherical models, may provide a solution Berczik et al. (2006).

In non-spherical systems, stars can continue to interact with the binary SMBH even when the local stellar population appears depleted. This occurs because the elongated structure of triaxial galaxies allows for chaotic stellar orbits, which can continuously replenish the loss cone—an area in phase space containing stars with orbits that intersect the binary. These interactions provide the necessary dynamical friction to further harden the binary, allowing it to reach separations where gravitational wave emission can dominate and drive the final coalescence Merritt and Poon (2004).

In gas-rich systems, another proposed solution involves gas inflows and circumbinary disk interactions, which can significantly accelerate the orbital decay of the binary SMBH. In such systems, the presence of cold gas around the binary exerts torques that extract angular momentum from the SMBHs, hastening their coalescence Churchwell et al. (2009). The gas can form a circumbinary disk that drives binary hardening by transferring angular momentum outward, similar to how angular momentum exchange operates in protostellar disks. The interaction between the SMBH binary and the circumbinary disk thus represents an effective mechanism for overcoming the final parsec problem in gas-rich environments. In addition to these solutions, chaotic orbits and resonant relaxation have also been proposed as mechanisms that can prevent stalling in certain stellar-dominated systems.

Chaotic orbits arise in non-axisymmetric potentials where stars do not follow simple, regular paths but instead exhibit irregular, rapidly changing orbits that can bring them into interaction with the binary Vasiliev (2019). Resonant relaxation, a process that occurs in nearly Keplerian potentials like those near SMBHs, allows stars on near-resonant orbits to collectively exchange angular momentum with the binary over relatively short timescales Tremaine et al. (1994). This can help maintain a supply of stars for interactions with the binary, facilitating continued hardening.

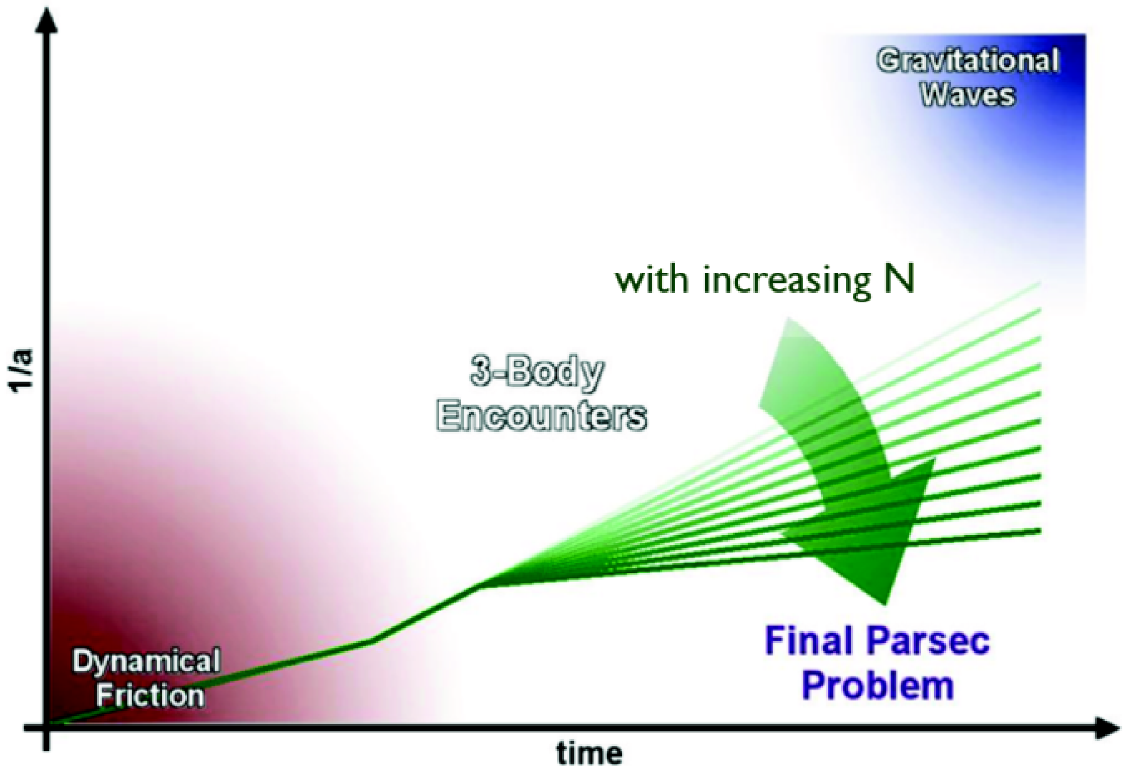

Figure 2.2: sketch of final parsec problem

In figure 2.2, the early phase of binary evolution is governed by dynamical friction. As mass is ejected from the central region and the separation shrinks dynamical friction becomes inefficient. Three body encounters take energy away from the binary. This phase is N-dependent; with an increasing number of particles in numerical experiments the rate at which binary semi-major axis evolves decreases. For realistic number of particles, which are several orders of magnitude larger than current state of the art simulation can accommodate, the binary semi-major axis should stop evolving and the two black holes should not enter the regime where gravitational waves become efficient in carrying energy away from the binary. Figure credit: Dr. Ingo Berentzen.

2.4 Gravitational Wave Emission from SMBH Mergers

2.4.1 Gravitational Wave Radiation from Binary SMBHs

As supermassive black hole (SMBH) binaries approach separations on the order of milliparsecs, gravitational wave radiation becomes the primary mechanism for energy dissipation. Gravitational waves, predicted by Einstein's theory of general relativity, are oscillations in spacetime generated by the acceleration of massive objects. In the context of SMBH mergers, these ripples propagate through space, carrying away both energy and angular momentum from the binary system Barack and Cutler (2004). When the binary separation decreases to milliparsec scales, the emission of gravitational waves intensifies, effectively driving the two black holes closer together. This process leads to the final inspiral and eventual coalescence of the black holes. The emission of gravitational waves from SMBH binaries is particularly significant for astrophysics because it provides a unique observational signature, detectable by space-based interferometers such as the future Laser Interferometer Space Antenna (LISA) Amaro-Seoane et al. (2012). LISA will be sensitive to the low-frequency gravitational waves produced by these massive systems, enabling the direct observation of SMBH mergers for the first time Berti and Volonteri (2008).

These gravitational wave signals also offer insights into the nature of black holes, the properties of spacetime, and the dynamics of galaxy mergers. As SMBHs are typically found at the centers of galaxies, their mergers are tied to the larger process of galaxy evolution Merritt et al. (2006). The coalescence of SMBH binaries could lead to the formation of even more massive black holes and influence the surrounding galactic environment through feedback mechanisms. Understanding gravitational wave emission in SMBH mergers is therefore crucial not only for testing general relativity but also for tracing the cosmic history of galaxies and black hole growth over time Kocsis et al. (2006). In addition to the observational prospects, the study of gravitational wave radiation in SMBH mergers has theoretical implications. It helps refine models of the final stages of black hole mergers, including the rate at which energy is radiated, the possible recoil velocity of the merged black hole due to asymmetric emission, and the potential formation of gravitational wave echoes

Campanelli et al. (2007). As more SMBH mergers are detected, these observations will deepen our understanding of the universe on both small (relativistic) and large (cosmological) scales Schnittman (2011).

2.4.2 Detection of Gravitational Waves

The detection of gravitational waves from supermassive black hole (SMBH) mergers is one of the primary scientific goals of future space-based observatories, such as the Laser Interferometer Space Antenna (LISA). LISA, which is expected to launch in the 2030s, will be capable of detecting low-frequency gravitational waves, making it ideal for observing the inspiral and merger phases of SMBH binaries, which are too massive to be detected by ground-based interferometers like LIGO or Virgo Amaro-Seoane et al. (2012). These mergers are expected to occur at the centers of galaxies following major galactic mergers, providing a direct observational window into the late-stage evolution of massive galaxies and their central black holes. Simulations have been indispensable in predicting the properties of gravitational waves from SMBH mergers. Numerical relativity and post-Newtonian simulations have helped estimate key parameters of the gravitational waves, including their amplitude, frequency, and waveform Baker et al. (2006). These simulations show that the amplitude of gravitational waves from SMBH mergers can be enormous due to the mass involved, with detectable signals from mergers involving black holes with millions to billions of solar masses. The frequency of these waves is in the millihertz range, which is exactly the frequency band that LISA is designed to observe Gair et al. (2013). Additionally, the waveforms of the emitted gravitational waves provide rich information, revealing not only the masses and spins of the black holes but also the dynamics of the merger itself, including potential asymmetries that could lead to gravitational wave recoilCampanelli et al. (2007). This theoretical groundwork is crucial for designing and optimizing detectors capable of capturing these elusive signals. The precise modeling of waveforms allows scientists to fine-tune the sensitivity of detectors like LISA, ensuring that even weak or distant signals can be picked up. Moreover, accurate waveform predictions are essential for distinguishing SMBH mergers from other sources of gravitational waves and noise Berti and Volonteri (2008).

The detection of gravitational waves from SMBH mergers will open new avenues for astrophysical research. Directly observing these mergers will provide unambiguous evidence of SMBH coalescence, which has been theorized but not yet directly confirmed. It will also enable the study of black hole growth mechanisms, as SMBH mergers are thought to play a critical role in the formation of the most massive black holes in the universe Volonteri (2005). Furthermore, the detection of gravitational waves offers a unique opportunity to test general relativity in the strong-field regime, where spacetime is highly curved and dynamic Sathyaprakash and Schutz (2009). This could lead to new insights into the fundamental nature of spacetime itself and potentially uncover deviations from Einstein's theory.

2.5 Structural Transformations in Merging Galaxies

2.5.1 Core Scouring in Elliptical Galaxies

One of the most significant structural transformations that occurs during supermassive black hole (SMBH) mergers is a phenomenon known as core scouring. This process involves the ejection of stars from the central regions of galaxies due to gravitational interactions with the SMBH binary. As stars pass near the binary system, they undergo strong gravitational slingshot effects. These interactions accelerate the stars, often ejecting them from the galaxy's core, thus carrying away both energy and angular momentum from the binary system Merritt (2006). Over time, this continuous ejection of stars results in the formation of a low-density stellar core at the center of the galaxy, a hallmark of core scouring. Core scouring is particularly relevant to the formation of low-density cores in massive elliptical galaxies, which are often observed in many early-type galaxiesFabian (2012). The depletion of stars in the core alters the overall structure of the galaxy, creating what is sometimes referred to as a "cored" elliptical galaxy. These cored profiles are distinct from the dense central regions found in less massive or younger galaxies, where star formation has not been significantly disrupted.

Numerical simulations have played a critical role in exploring the dynamics of core scouring. These simulations have shown that the size and depth of the scoured core depend on several factors, including the mass of the SMBHs, the properties of the host galaxy, and the evolutionary history of the galaxy merger. Specifically, simulations indicate that the larger the mass of the SMBH binary, the more effective it is at ejecting stars from the core, leading to the formation of larger coresMilosavljević et al. (2009). Additionally, the initial stellar density and velocity dispersion of the galaxy are crucial determinants of how quickly and extensively core scouring occursKormendy and Richstone (1995). In galaxies with high stellar densities and lower velocity dispersions, core scouring can result in significant depletion of stars over time, while in more diffuse galaxies, the effect is less pronounced. The observational evidence of core scouring in massive elliptical galaxies suggests that this process has been a fundamental aspect of galaxy evolution. The presence of large, low-density cores is often interpreted as indirect evidence of past SMBH mergers, offering clues to the history of galaxy interactions and the role of black holes in shaping galactic structures. As such, core scouring not only impacts the structural properties of galaxies but also serves as a key indicator of the dynamical processes driven by SMBH mergers.

2.5.2 Tidal Forces and Galactic Disruption

Tidal forces play a crucial role in driving structural transformations during galaxy mergers. These forces arise due to the differential gravitational pull experienced by different parts of the merging galaxies. When two galaxies approach each other, the side of one galaxy closer to its partner experiences a stronger gravitational attraction than the far side, leading to stretching and distortion. This gravitational tug-of-war results in the stripping of stars and gas from the outer regions of the galaxies, contributing to significant changes in their structureToomre and Toomre (1972). One of the most visually striking outcomes of these tidal forces is the formation of tidal tails, which are elongated structures composed of stars, gas, and dust that extend outward from the merging galaxies. These tidal tails, which can persist for billions of years, serve as clear indicators of past or ongoing gravitational interactions between galaxies. The streams of material ejected from the outer regions of galaxies can eventually fall back into the galaxies or merge with other structures, depending on the dynamics of the system. In addition to tidal tails, other features such as stellar streams and bridges are also common products of tidal interactions and are observed in both early-stage interactions and fully merged systemsSpringel et al. (2008).

Tidal tails are frequently observed in systems undergoing major mergers, where the galaxies involved are of comparable mass. These tails and streams provide crucial insights into the history and dynamics of galaxy interactions. For example, the Antennae galaxies (NGC 4038/NGC 4039), a pair of interacting galaxies, exhibit prominent tidal tails that have been extensively studied. Numerical simulations have been able to reproduce these features, offering a detailed understanding of the role of tidal forces in shaping galactic structures during mergersBarnes and Hut (1986). Simulations of the Antennae galaxies suggest that their tails formed early in the interaction and have persisted as long-lived features throughout the galaxies' ongoing merger Karl et al. (2010). The long-term effects of tidal forces go beyond the formation of these visually distinctive features. Over time, as the galaxies merge and their cores coalesce, tidal forces redistribute stars and gas, altering the galaxies' morphologies. The stripped material often settles into the outer halos of the galaxies or forms new structures such as tidal dwarf galaxies, which are smaller systems formed from the remnants of gas and stars pulled from the merging galaxies Bournaud et al. (2007). Moreover, tidal forces can induce star formation by compressing gas in certain regions, leading to bursts of new star formation, further contributing to the overall transformation of the merging galaxies.

2.5.3 Morphological Evolution: From Disk to Elliptical

Galaxy mergers are one of the most important mechanisms driving the transformation of disk galaxies into elliptical galaxies. During a major merger—where two galaxies of comparable mass collide and merge—the overall gravitational potential of the system undergoes rapid and chaotic changes. This leads to the redistribution of stellar orbits in a process known as violent relaxationLi et al. (2023). As a result, the ordered, rotating disk structure of the progenitor galaxies is disrupted, and the orbits of stars become randomized, creating a pressure-supported system that lacks the coherent rotational motion of disk galaxies. This randomization of stellar orbits is a key feature of elliptical galaxies, which are primarily supported by velocity dispersion rather than rotationToomre and Toomre (1972). Numerical simulations of major mergers have been instrumental in understanding how these interactions produce the spheroidal structures typical of elliptical galaxies. These simulations consistently show that the remnants of major mergers form spheroidal distributions of stars, where the overall shape and properties of the final system depend on several factors, including the mass ratio of the merging galaxies and their internal structures prior to the merger Armitage and Natarajan (2002). For example, mergers between galaxies with roughly equal masses (a 1:1 mass ratio) tend to produce classical elliptical galaxies with smooth, spheroidal profiles, whereas unequal mass mergers (such as those with a 3:1 mass ratio) may result in intermediate structures with features of both disk and elliptical galaxies, often referred to as lenticular galaxies.

The presence of gas in the merging galaxies also plays a significant role in the morphological transformation. Hydrodynamical simulations, which take into account the behavior of gas, have shown that gas can fuel bursts of star formation during the merger process. These bursts can contribute to the central concentration of stars, particularly in the formation of dense stellar cores, a common feature in some elliptical galaxies Springel et al. (2005). The gas is also funneled toward the center of the merging system, where it can contribute to the growth of a supermassive black hole and trigger energetic feedback processes, such as the expulsion of remaining gas through active galactic nucleus (AGN) feedback Di Matteo et al. (2005). This feedback process can effectively shut down star formation, leading to a ”quenching” of the merger remnant, which then evolves into a gas-poor elliptical galaxy over time. The transformation of spiral galaxies into ellipticals during mergers not only affects the structure of the galaxies but also their kinematic properties. In disk galaxies, stars predominantly move along ordered, circular orbits in the plane of the disk, giving rise to strong rotational signatures. However, after a major merger, the randomization of orbits erases this ordered motion, and the stars exhibit more isotropic motion, where there is no preferred direction for stellar movement. This kinematic shift is one of the defining differences between disk galaxies and elliptical galaxies Naab and Burkert (2003).

2.5.4 Starburst Phenomena and AGN Activity

Galaxy mergers are often accompanied by intense bursts of star formation, referred to as starbursts, which are driven by the gravitational forces at play during the merging process. When two galaxies collide, the gravitational interaction triggers the inflow of vast amounts of gas towards the center of the merging system. This rapid inflow compresses the gas, leading to a significant increase in the star formation rate (SFR) compared to what is typically observed in isolated galaxies Barnes and Hut (1986). Starburst events can result in the formation of millions of new stars within a relatively short period, drastically altering the stellar population and dynamics of the merging galaxies. The inflow of gas during mergers not only fuels star formation but also plays a crucial role in triggering active galactic nucleus (AGN) activity. As gas flows toward the central region of the merging system, a fraction of it is accreted by the supermassive black hole (SMBH) at the center of one or both galaxies. The accretion of gas onto the SMBH releases an enormous amount of energy, which is observed as AGN activity—characterized by powerful radiation across the electromagnetic spectrum and, in some cases, the ejection of high-velocity jets or outflows Hopkins et al. (2006). This phase of coexisting starburst and AGN activity is a hallmark of gas-rich galaxy mergers, and many observed AGN are found in interacting systems or remnants of recent mergers. The simultaneous occurrence of starbursts and AGN activity has profound implications for galaxy evolution. AGN activity, in particular, has been shown to regulate star formation through a process known as AGN feedback. This feedback can occur in several forms, including radiation pressure, AGN-driven winds, and jets that can heat or expel gas from the central regions of the galaxy. These feedback mechanisms play a critical role in controlling the availability of gas for star formation Di Matteo et al. (2005). By heating or driving gas out of the central regions, AGN feedback can halt further star formation, effectively quenching the galaxy. This transformation from a star-forming system to a quiescent, or ”red and dead,” system is a key aspect of the morphological and evolutionary changes driven by galaxy mergers.

Starburst phenomena during galaxy mergers are often short-lived but highly efficient, consuming the available gas in rapid star formation episodes. These starbursts typically last for around 100 million years, during which time the SFR can increase by a factor of 10 to 100 compared to pre-merger levels Mihos and Hernquist (1995). The massive stellar populations formed in these bursts can dominate the galaxy's light for billions of years after the merger. The intense radiation and stellar feedback from young, massive stars can also drive powerful galactic winds, further contributing to the loss of gas from the system and reinforcing the quenching process Volonteri (2005). Hydrodynamical simulations of galaxy mergers have been crucial in exploring the complex interplay between starbursts and AGN activity. These simulations show that the merger-triggered gas inflows are key to both phenomena, as they funnel gas toward the galaxy's center, where it fuels both star formation and SMBH accretion. The timing and intensity of starbursts and AGN activity are closely linked to the dynamics of the merger, with starbursts typically peaking shortly after the galaxies' first close encounter and AGN activity becoming more prominent as the galaxies' nuclei coalesce Springel et al. (2005).

As these galaxies approach each other, the gravitational pull between them becomes increasingly pronounced, setting off a cascade of interactions that affect their individual components Lotz et al. (2011). These interactions can result in mergers if the galaxies get close enough to combine their structures. Mergers can bring about significant morphological transformations Mihos and Hernquist (1995), altering the overall shape and appearance of the galaxies involved. For instance, spiral galaxies can be transformed into elliptical galaxies through major mergers. The merging galaxies also experience tidal effects, where the gravitational interaction generates tidal forces that distort their shapes. This can create observable features such as tidal tails, bridges, and shells, providing evidence of past or ongoing mergers Pearson et al. (2019). Additionally, mergers trigger intense bursts of star formation. The gravitational interaction causes gas and dust within the galaxies to collide and compress, leading to the formation of new stars Comerford and Greene (2014). These star formation episodes can give rise to the creation of young, massive star clusters. Moreover, merging galaxies can contribute to the growth of their central supermassive black holes. As the galaxies merge, their central black holes can merge as well, resulting in the formation of a more massive black hole Omori et al. (2023). The merger process can also fuel the black holes, leading to increased accretion of matter and the generation of active galactic nuclei (AGN). Galactic cannibalism involves the interaction between a larger galaxy, commonly referred to as the primary or host galaxy, and a smaller neighboring galaxy, known as the satellite or accreted galaxy Treister et al. (2012). During this process, the gravitational forces of the primary galaxy can strip stars, gas, and dust from the satellite galaxy, a phenomenon called tidal stripping. As a result of tidal stripping, the stripped material can form tidal streams or diffuse features around the primary galaxy, providing observable evidence of the interaction Dittmann and Miller (2020).

2.6 Supermassive Black Hole Binary Evolution

Galaxy mergers, which are an essential part of galaxy evolution, are considered to be the predominant path leading to the formation of SMBH binaries. An understanding of the formation and evolution of SMBH binaries is thus important in understanding the galaxy formation and evolution. The dynamical evolution of pair of supermassive black holes in galaxy mergers can be described broadly in three phases Begelman et al. (1980):

Supermassive black holes are embedded in the central cores/cusps of their host galaxies. The galaxy cores/cusps sink to the center of the merger product due to the dynamical friction caused by background dark matter + stars + gas. The galaxy cores/cusps merge, undergo violent relaxation and form a new galaxy nucleus in a characteristic galactic dynamical time scale. The two SMBHs form a binary system once their separation shrinks below the influence radius. The binary continues to decay under the combined effects of dynamical friction and the gravitational slingshot effect. As the binary's orbital time decreases the dynamical friction becomes inefficient and the binary's further evolution is governed by its interaction with stars on orbits intersecting the binary, carrying away energy and angular momentum from binary. These stars are then ejected at the velocities comparable to binary's orbital velocity. For a sufficiently small separation the emission of the gravitational waves become dominant and carries efficiently the last remaining energy and angular momentum, resulting in a rapid coalescence of supermassive black holes.

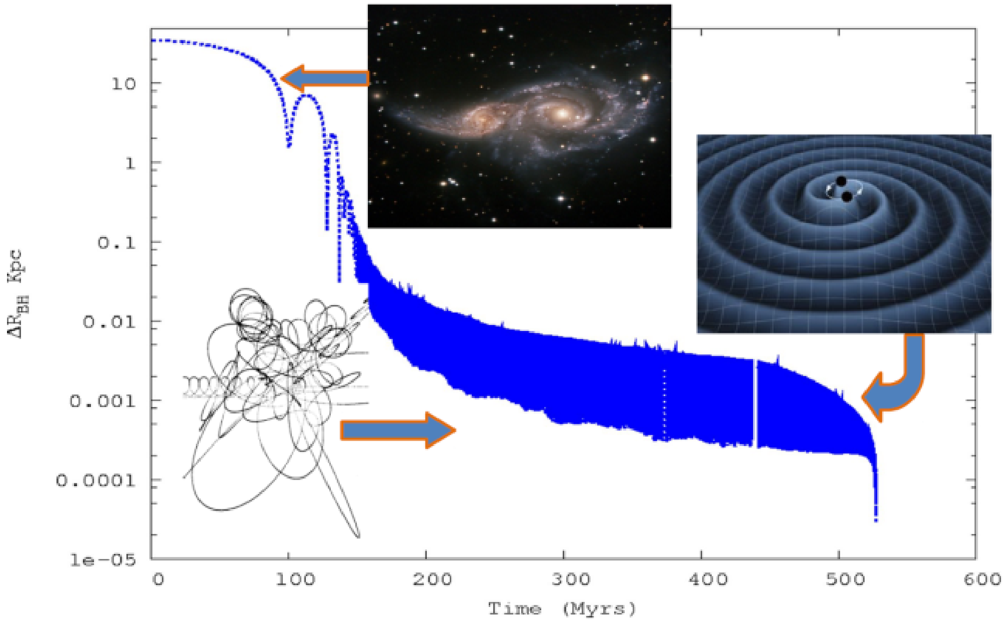

Figure 2.3: Different phases of SMBH binary evolution.

Figure 2.3 shows the relative separation of the two black holes during galaxy merger. As the galaxies merge the two SMBHs sink towards the center and form a binary system due to dynamical friction (phase I). The binary gets harder by flinging stars away on orbits intersecting the binary (phase II). At small separations ($\sim$ milliparsec) SMBH binary shrink rapidly and SMBHs coalesce due to emission of gravitational waves (phase III).

2.6.1 Galaxy Mergers and Dynamical Friction

Galaxy mergers can be divided mainly in two classes: the first, in which the merging galaxies have almost equal masses, are termed as major mergers while the second, which involve mergers of galaxies with large mass ratio (typically 1:10 or larger), are referred to as minor mergers. In major mergers as the galaxies merge, the two SMBHs centered inside the individual cusps fall towards the center of the merger remnant and form a close pair surrounded by individual cusps. Once the individual cusps are merged and a new galactic nucleus is formed the two SMBHs move as individual entities, loose angular momentum due to dynamical friction and form a bound pair. In the case of minor mergers, the less massive galaxy will be tidally disrupted. The black hole of the less massive galaxy looses its surrounding cusp and spirals in due to dynamical friction. The dynamical friction phase in minor mergers is shortened in the case of satellite galaxies which have compact center since these galaxies can survive against the tidal disruption for a longer period of time. If the merging galaxies (especially the less massive one) have a considerable fraction of the gas, the gas can be funneled towards the center and undergo a strong star-burst, making the central part of the galaxy more compact Callegari et al. (2009).

2.6.2 Stellar Dynamical Hardening

Let us define the mass of the two black holes as M_1 and M_2 for larger and smaller black holes respectively. M = M_1 +M_2 and q = M_2/$M_1$is the mass ratio of the two black holes. For equal mass binary q = 1 and for unequal mass binaries q is always less than 1. Also we define the reduced mass, μ = M_1M_2=M and a is the semi-major axis of the SMBH binary. Following the dynamical friction phase the two black holes become bound and form a binary system. The separation at the time of formation of the SMBH binary is typically $\sim r_h$, the gravitational influence radius of the black holes, defined as Merritt and Milosavljevic (2004)

$$r_h = \frac{GM}{\sigma^2} \approx 11pc(\frac{M}{10^8 M})(\frac{\sigma}{200kms^{-1}})^{-2} \tag{2.11}$$

Here σ is the one dimensional velocity dispersion of the stellar bulge. An alternate definition of r_h is the radius which encloses the stellar mass around the SMBH equal to $2M$,

$$r_h = M_*(r) = 2M \tag{2.12}$$

Here we define the binary's parameters according to approximate relative Keplarian orbit ignoring the force contribution from the stars. The binding energy is

$$E_b = -\frac{GM_1M_2}{2a} = \frac{G\mu M}{2a} \tag{2.13}$$

and the binding energy per unit mass of the SMBH binary is

$$\epsilon_b = E_b/M = G\mu/2a \tag{2.14}$$

We define the semi-major axis a and eccentricity e via the standard relations:

$$\frac{1}{a} = \frac{2}{R} - \frac{v^2}{GM} \tag{2.15}$$

$$e = \sqrt{1 + \frac{2h^2}{GM^2}\left[\frac{v^2}{2} - \frac{GM}{R}\right]} \tag{2.16}$$

where v is the relative speed, and h is the specific angular momentum of the relative motion. For a circular orbit, the relative velocity of two SMBHs and the orbital period is given by

$$v = \sqrt{\frac{GM}{a}} \tag{2.17}$$

and

$$P_{orb} = 2\pi\sqrt{\frac{a^3}{GM}} \tag{2.18}$$

respectively.

2.6.3 Gravitational Waves Hardening

Gravitational waves are generated by accelerated mass-energy distributions. If M is the mass of a system with size R at a luminosity distance D, the periodic motion of such a system generates gravitational waves having a strain amplitude:

$$h \sim (\frac{GM}{Rc^2})^2\frac{R}{D} \tag{2.19}$$

and the frequency of the GWs is determined the frequency of motion of the system. The strongest gravitational field systems (GM/R), generate the most powerful GWs. SMBHs have large masses and small sizes $(GM/Rc^2) \approx 1$, and hence are very efficient at emitting gravitational waves. If stellar dynamical processes or gas dynamics manage to reduce the binary separation sufficiently that gravitational waves emission become dominant mechanism in extracting energy and angular momentum then the two black holes coalesce in a time T_{gw} given by Peters (1964),

$$T_{GW} = \frac{5}{256}\frac{c^5 a^4}{G^3 \mu M^2} F(e) \tag{2.20}$$

where

$$F(e) = \left[1 - e^2\right]^{-7/2} \left[1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right]^{-1} \tag{2.21}$$

$$T_{GW} = 7.6 \times 10^{-5} \frac{G\mu^3 c^5}{\sigma^8 M^2} (\frac{a}{a_h})^4 F(e) \tag{2.22}$$

3

Methodology

3.1 Galaxy Models and N-body Simulations

Despite the fact that gravity is the weakest force by far of the four fundamental forces (strong nuclear force, weak nuclear force, electromagnetic force and gravitational force), it dominates all other forces at large distances. Gravitational systems, ranging from planetary systems and star clusters to galaxies and the large-scale structure of the universe, can be modeled as collections of point masses that interact with one another through gravitational forces. In order to study the dynamical evolution of such systems, one needs to solve a large number of coupled differential equations. Numerical modeling has become a basic method, alongside observations and pure theory, to study the dynamical evolution of astrophysical objects. It permits astrophysicists to provide theoretical insights into physical processes at work in these objects. Gravitational systems can be classified in two classes "collisional systems" and "collision-less systems"determined by whether or not the two-body relaxation" timescale $T_{\rm r}$ is shorter than lifetime of the system according to Smole (2015):

$$T_r = \frac{0.13\sigma^3}{G^2 m_* \rho \ln}\Lambda \tag{3.1}$$

where: G the universal gravitational constant, m_* is the mean mass of the stellar population, σ is the velocity dispersion, ρ is the density and Λ is the Coulomb logarithm.

Examples of collisional systems are dense objects like globular star clusters and galactic centers. Systems such as dark matter halos and galaxies have relaxation times much longer than the age of the Universe and hence can be treated as collision-less systems. The dynamical modeling of stellar system dates back to the very first light bulb experiment of Holmberg (1941) who followed the evolution of a 37 particle system. The computer simulations started with the first such calculation performed by von Hoerner (1960) at Astronomical Rechen Institute (ARI) in Heidelberg with 16 particles. Aarseth (1963) studied the dynamical evolution of galaxy clusters using up to 100 particles. For collision-less system fast and efficient algorithms such as particle-mesh (PM) codes and tree codes Barnes and Hut (1986) were developed in 1980's. For tree codes and PM codes the computational cost increases roughly as $NlnN$. With recent advances in computer power and massive parallelization on thousands of cores collision-less N-body codes can handle very large particle number simulations Springel et al. (2008).

3.2 Models for Spherical System

In order to setup a galaxy for numerical investigations, a smooth equilibrium model is needed for the probability distribution to have a system in dynamical equilibrium. In isotropic spherical systems, the gravitational potential and density depend only on dimension, the distance "r" from the center of the system. Spherical symmetry leads to $\rho(\overrightarrow{r}) \to \rho(r)$ and $\Phi(\overrightarrow{r}) \to \Phi(r)$. The gravitational potential and density are related through Poisson's equation:

$$\Delta\Phi(r) = 4\pi G\rho(r) \tag{3.2}$$

The amount of mass enclosed by a sphere of radius r is called the cumulative mass distribution, M(r)

$$M(r) = \int_0^r 4\pi(r')^2\rho(r')dr' \tag{3.3}$$

The phase space distribution function depends only on the energy per unit mass $E = \frac{1}{2}v^2 + \Phi(r)$ and can be calculated from the Eddington formula Binney and Tremaine (2011),

$$f(E) = \frac{1}{\sqrt{8}\pi^2}\frac{d}{dE}\left[\int d\Phi \frac{1}{\sqrt{\Phi - E}}\frac{d\rho}{d\Phi}\right]. \tag{3.4}$$

Here we describe commonly used analytical models for star clusters, elliptical galaxies and the bulges of spiral galaxies.

3.2.1 The Plummer Model

The Plummer (1911) model is widely used to generate initial conditions for star cluster simulations. This model is an analytic solution of the Lane-Emden equation. In this chapter and others to follow we shall use gravitational constant G = 1, total mass of the system M = 1 and scale radius (plummer radius) a = 1 for convenience, unless otherwise is mentioned. The density distribution, cumulative mass profile and potential of the Plummer model are :

$$\rho(r) = \frac{3}{4\pi}.\frac{1}{(1+r^2)^{5/2}}, \tag{3.5}$$

$$M(r) = r^3(1+r^2)^{-3/2}, \tag{3.6}$$

$$\Phi(r) = -(1+r^2)^{-1/2}, \tag{3.7}$$

respectively.

3.2.2 The Hernquist Model

One of the most successful model for elliptical galaxies and bulges of disk galaxies is the Hernquist model Hernquist (1990). The density profile, mass profile and potential are

$$\rho(r) = \frac{1}{2\pi}.\frac{1}{(1+r)^3} \tag{3.8}$$

$$M(r) = r^2(1+r)^{-2} \tag{3.9}$$

$$\Phi(r) = -(1+r)^{-1} \tag{3.10}$$

respectively.

3.2.3 The Dehnen and Tremaine Model

Dehnen (1993) and Tremaine et al. (1994) presented a one-parameter family of isotropic spherical models for stellar systems. Here we describe the η- models of Tremaine et al. (1994)in more detail. The density distribution of the η-model is given by

$$\rho(r) = \frac{\eta}{4\pi}.\frac{1}{r^{3-\eta}.(1+r)^{1+\eta}}, \tag{3.11}$$

Well outside scale radius (1 in our model units), all η-models have $\rho \propto r^{-4}$ and well inside scale radius density goes with $r^{3-\eta}$. Hernquist model corresponds to $\eta = 2$. The cumulative mass profile for the η-models is given below

$$M(r) = \frac{r^{\eta}}{(1+r)^{\eta}} \tag{3.12}$$

and the gravitational potential is

$$\Phi(r) = \frac{1}{\eta - 1}\left[\frac{r^{\eta-1}}{(1+r)^{\eta-1}} - 1\right] , \; \eta \neq 1 \tag{3.13}$$

and for η=1,the potential is $-\ln(1+1/r)$. Some of the η-models (η=0.5,1,1.5,2,2.5,3) have analytic expressions for the distribution functions (DF).

3.2.4 η-Models With a Central Black Hole

Here we describe the η-model modified to contain a central supermassive black hole of mass M. The total mass in stars is still unity and the stellar density profile is still described by equation (3.11). The main effect of the black hole is to modify the potential:

$$\Phi_*(r) = \Phi(r) - \frac{M}{r} \tag{3.14}$$

We used a numerically computed distribution function in order to reproduce $\rho(r)$ (equation 3.11) in the combined potential of the SMBH and the stars in dynamical equilibrium. These models are used to set up initial conditions for galaxy models used to study SMBH binary evolution in galaxy mergers. Matsubayashi (2007) modified the η- model with a central SMBH. The stellar density distribution of their model is given as

$$\rho(r) = \frac{\eta}{4\pi}.\frac{1}{r^{3-\eta}(1+r^2)^{1+\eta/2}}, \tag{3.15}$$

One main difference of this model with original η- model is that it has an outer slope of -5 instead of -4. This sharp cutoff guarantees high particle resolution inside core radius as compared with the models having outer slope of -4. Thus this model is useful for studying the dynamics of black holes at the center of single a galaxy. We use it to study the orbital decay of a less massive SMBH inside the sphere of influence of a more massive black hole due to dynamical friction. The mass profile and distribution function of the model (equation 3.15) are

$$M_\eta(r) = M_\eta \frac{r^\eta}{(1+r^2)^\eta/2} \tag{3.16}$$

and

$$f(\epsilon) = f_0(\epsilon)^{7/2}(\epsilon_0^s + \epsilon^s)^{-(\eta+2/\eta)} \tag{3.17}$$

respectively. Hence ϵ =E,

$$\epsilon_0 = (f_1/f_0)^{-1/(\eta+2)} \tag{3.18}$$

$$f_0 = \frac{\eta M\eta\Gamma(4-\eta)}{2^{7/2}\pi^{5/2}M_\bullet^{3-\eta}\Gamma(5/2-\eta)} \tag{3.19}$$

$$f_1 = \frac{8\sqrt{2}}{7\pi^3}\eta M_\eta \tag{3.20}$$

Here with s = 5 and η = 5/4, these equations can be used to construct BW cusp in approximate equilibrium. The constant f_0 represents the BW cusp, f_1 the Plummer model and ϵ_0 is the transition threshold in energy.

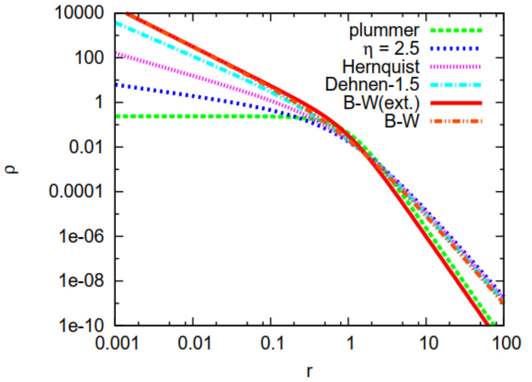

Figure 3.1: Density profiles of Models

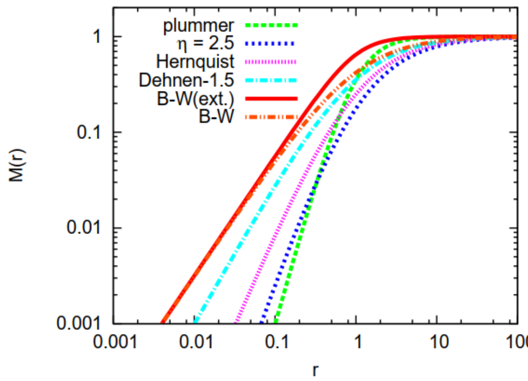

Figure 3.2: Mass profiles of Models

Figure 3.1: shows the density profile for Plummer model, shallow cusp ($\eta = 2.5$),Hernquist model(η =2), Dehnen model ($\eta = 1.5$), Bahcall-Wolf cusp (equation 3.15 with $\eta = 1.25$) and Bahcall-Wolf cusp (equation 3.11 with $\eta = 1.25$) respectively. Figure 3.2 shows the mass interior to radius(r) for Plummer model, shallow cusp ($\eta = 2.5$), Hernquist model, Dehnen model ($\eta = 1.5$), Bahcall-Wolf cusp (equation 3.15 with η = 1.25) and Bahcall-Wolf cusp (equation 3.11 with $\eta = 1.25$) respectively.

3.3 Initial Setup for Representing Galaxy Model

In this section we shall describe how to generate initial positions and velocities for the particles representing a model galaxy. We shall present the procedure only for model described by equation (3.15). Following same steps one can generate initial conditions for all the models described above. In order to generate positions for the particles one needs the cumulative mass profile of the model while in order to assign the velocities of the particles, the energy distribution function is required. We choose $\eta = 5/4$ in equation (3.16) which corresponds to a Bahcall-Wolf cusp. The cumulative mass profile M(r) is essentially a probability distribution function (PDF). The PDF can be realized using Monte Carlo sampling. We invert the function M(r),

$$M(r) = \frac{r^{1.25}}{(1+r^2)^{0.625}} \tag{3.21}$$

into $r(M)$

$$r = \frac{1}{(M^{-1.6}(r) - 1)^{0.5}} \tag{3.22}$$

In order to give a distance r to the star, we generate a random number $\mathbf{X}_1$ and equate M(r) to this random number $\mathbf{X}_1$. So the distance is selected as:

$$r = \frac{1}{({X_1}^{-1.6} - 1)^{0.5}} \tag{3.23}$$

Three dimensional positions $(x; y; z)$ of the star are selected on the sphere of radius r. Again two random numbers X_2 and X_3 are generated and the coordinates x; y; z are computed as follow:

$z = z(1 - 2X_2)r$, $x = (r^2 - z^2)^{1/2}\cos(2\pi X_3)$, and $y = (r^2 - z^2)^{1/2}\sin(2\pi X_3)$.

We compute the potential for the star at distance r due to SMBH and stellar distribution,

$$\Phi_* = \left[\frac{M}{4} + \left[4 - \frac{4}{(1 + \frac{1}{r})^{1/4}}\right]\right] \tag{3.24}$$

The escape velocity at distance r from the center is,

$$V_e = \left[\frac{2M}{r} + 2\left[4 - \frac{4}{(1 + \frac{1}{r})^{1/4}}\right]\right]^{1/2} \tag{3.25}$$

and the energy distribution function of equation (3.17) is proportional to

$$f(q) \propto \frac{q^2(1 - q^2)^{3.5}}{\left[0.58 + \frac{(V_e)^{10}}{32}(1 - q^2)^5\right]^{13/20}} \tag{3.26}$$

Here q = V/V_e. The maximum value of $f(q)$ is f_{max} = 0.14 and q has a range between 0 and 1. We use Neumann's acceptance-rejection method to assign velocities to the particles. We generate two more normalized random numbers X_4 and X_5. If $f_{max} \times X_5$ is less than X_4, we adopt q = X_4. Otherwise we generate a new pair of random numbers until the condition is fulfilled. The absolute velocity of the particle is then obtained by V = q × V_e. Two new random numbers X_6 and X_7 are generated to assign the velocity components V_x,V_y and V_z.

$$V_x = (V^2 - V_z^2)^{1/2}\cos(2\pi X_7), V_y = (V^2 - V_z^2)^{1/2}\sin(2\pi X_7), V_z = (1 - 2X_6)V$$

The procedure is repeated until positions and velocities are assigned to all N stars in galaxy model.

3.4 Numerical Codes for Galaxy Models

In this section we briefly describe the numerical codes that we use to study the dynamics and evolution of SMBHs in galaxy mergers. SMBHs reside in the galactic nuclei, which are collisional systems, so we used direct N-body codes $\phi - GRAPE$ and $\Phi - GPU$ to carry out our simulations. For the orbital decay of SMBHs due to dynamical friction in outskirts of galaxies the system is collision-less and furthermore, the density drops rapidly r^{-4} or $r^{-}5$ as we move far away from scale radius in all our models and in order to gain sufficient resolution higher particle numbers are necessary. The particle mesh code, SUPERBOX, which can accommodate a much higher number of particles when compared to direct N-body code such as ϕ-GRAPE or Φ-GPU, is used.

3.4.1 ϕ-GRAPE

According to Harfst et al. (2007) ϕ-$GRAPE$ is a parallel, direct-summation N-body code that uses special purpose hardware to compute the pairwise gravitational forces between all particles:

$$F_i = -m_i \sum_{j=1, j\neq i}^{N} \frac{m_j(r_i - r_j)}{(|r_i - r_j|^2 + \epsilon^2)^{3/2}} \tag{3.27}$$

Here m_i and r_i are the mass and position of the i^{th} particle and ϵ is a force softening parameter. ϕ-GRAPE integrates the equations of motion using a fourth-order Hermite integrator with individual block time steps .

3.4.2 Φ-GPU

The Φ-GPU code is also used for direct N-body simulations, and has a higher order Hermite integration scheme and individual block time steps (the code supports time integration of particle orbits with 4^{th}, 6^{th} and even 8^{th} order schemes). The code is fully parallelized with the use of MPI library. On each node, the code uses many cores of the GPU hardware. The code is based on an earlier C code version[1] for GRAPE6a clusters Harfst et al. (2007). The new code is written from scratch in C++ and based on Nitadori and Makino (2008) earlier CPU serial N-body code . The

MPI parallelization was done in the same "j" particle parallelization mode as in the earlier $\phi - GRAPE$ code Harfst et al. (2007). All the particles are divided equally between the working nodes and in each node fractional forces are calculated only for the, so called, "active-i" particles at the current time step. The current version ofΦ-GPU code uses native GPU support and direct code access to the GPU using only the NVIDIA native CUDA library[2]. The multi GPU support is achieved through MPI parallelization. Each MPI process uses only a single GPU, but usually two or more MPI processes per node are started (to use effectively the multi core CPU's and the multi GPU's on our clusters) are. This code uses different softening parameter ϵ (if required) for different components like SMBHs, dark matter and stars.

3.4.3 SUPERBOX

The particle mesh (PM) code Superbox Bien and Cheng (1991)is a highly efficient code with fixed time step for galaxy dynamics, where more than 10 million particles per galaxy in co-moving nested grids for high spatial resolution at the galaxy centers can be simulated. The code is intrinsically collision-less, which is necessary for long-term simulations of galaxies. Another advantage of a PM code is the large particle number which can be simulated in a reasonable computing time (up to a few days or a week per simulation). Three grid levels with different resolutions are used which resolve the core of each component/galaxy, the major part of the component/galaxy and the whole simulation area.

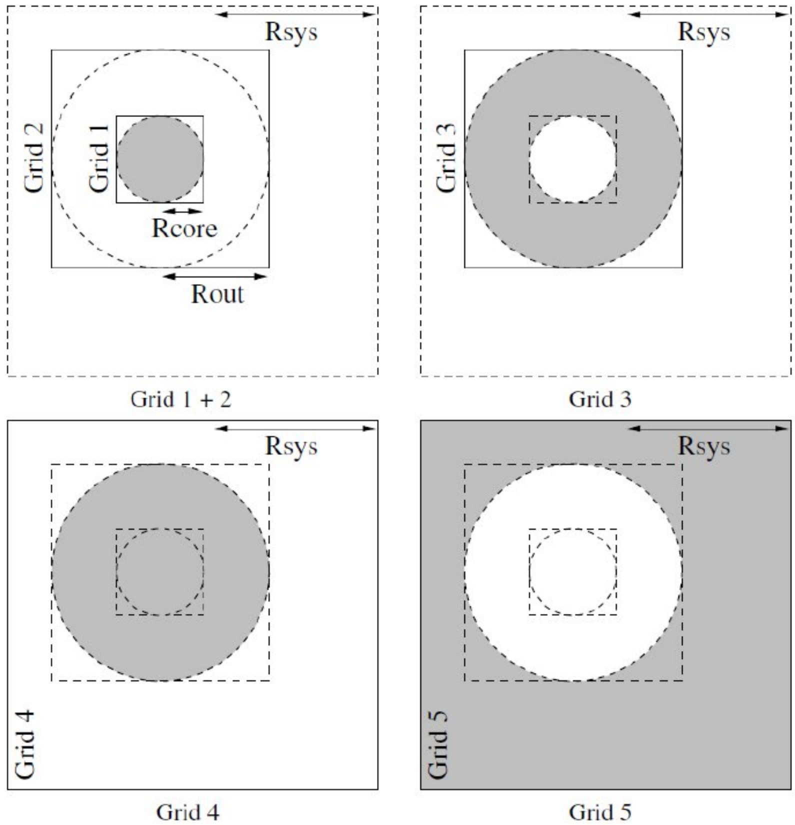

Figure 3.3: The five grids that are used in SUPERBOX. Figure taken from Bien and Cheng (1991).

The spatial resolution is determined by the number of grid cells per dimension $N_c = 2^m$ and the size of the grids. The SMBH is included as a moving particle with own sub-grids for high spatial resolution in its vicinity. The grid sizes are chosen such that the full orbit of the secondary BH falls into the middle grid. We also tried to use SUPERBOX to study the orbital decay of SMBH due to dynamical friction in Bahcall Wolf cusp around a primary SMBH. We noticed that the cusp was not stable and the Lagrange radii (radius enclosing fraction of cusp mass) were expanding soon after the start of the run. The reason could be the use of fixed time steps for each particle. In the center, close to primary SMBH, one needs very small time steps to integrate the orbits of stars correctly. Gray section in each grid shows the region filled with particles in respective grid.

4

Result and Discussion

In this chapter, we present the results of our study on the dynamical processes and structural transformations of SMBHs in galaxy mergers. The specific objectives of our study were to investigate N-dependence of SMBHs hardening rates in galaxy mergers models, to compare the hardening rates of SMBHs in galaxy mergers models and in isolated spherical galaxy models as well as explore how galaxy mergers affect the central density profiles of the merging galaxies, specifically focusing on the preservation of Hernquist-like profiles in steep density galaxies using N-body simulation. To achieve these objectives, we used different spherical system models like the Plummer model, the Hernquist model, the Dehnem and Tremaine model and γ-model with a central black hole. To set up these galaxy models and study the dynamics processes and structural transformation of SMBHs in galaxy mergers we incorporated a numerical codes like ϕ-GRAPE, Φ-GPU, SUPERBOX, and INTGC.

4.1 SMBHs in Equal Mass Galaxy Mergers

4.1.1 Numerical Methods and Initial Conditions

Our initial conditions are based on spherical galaxy models following Dehnen's (1993) density law. Here we use the power law index γ for density profile and it is related to η through γ = 3-η.

$$\rho(r) = \frac{(3-\gamma)M_{gal}}{4\pi}\frac{r_0}{r^{\gamma}(r+r_0)^{4-\gamma}} \tag{3.28}$$

and with cumulative mass profile

$$M(r) = M_{gal}(\frac{r}{r + r_0})^{3-\gamma}, \tag{3.29}$$

where M_{gal} is the total mass of the galaxy, r_0 is the scale radius, and γ defines the slope of inner(insider_0)density profile. We adopted γ = 1, corresponding to a Hernquist model Hernquist (1990), for all initial models. A massive particle representing a SMBH was placed at the center of each galaxy model. The isotropic distribution function reproducing $\rho(r)$ in the combined potential of the stars and the SMBH in dynamical equilibrium was computed numerically and used to generate Monte-Carlo positions and velocities for the stars. In what follows, we adopt "model units" such that $M_{gal} = G = r_0 = 1$. The models can be scaled to physical units by fixing galaxy mass M_{gal} and scale radius r_0 combined with the relations

$$[T] = \left[\frac{GM_{gal}}{r_0^3}\right]^{-1/2}$$

$$= 1.5Myr\left[\frac{M_{gal}}{10^{11}M\odot}\right]^{-1/2}\left[\frac{r_0}{kpc}\right]^{3/2} \tag{3.30}$$

$$[V] = \left[\frac{GM_{gal}}{r_0}\right]^{1/2}$$

$$= 655kms^{-1}\left[\frac{M_{gal}}{10^{11}M}\right]^{-1/2}\left[\frac{r_0}{kpc}\right]^{-1/2} \tag{3.31}$$

We use the following relation Gualandris and Merritt (2008) to scale our models to the physical systems:

$$R_{eff} \approx 1.2kpc\left[\frac{M_{gal}}{10^{11}M}\right]^{0.075} \tag{3.32}$$

In a Hernquist model the effective (projected half-mass) radius R_{eff} is $\sim$ 1.81 r_0. For a typical, luminous ($M_B \approx -20$) elliptical galaxy or bulge, $R_{eff} \approx 1.5$ kpc, $M_{gal} \approx 10^{11}M$ and $[T] \approx 1.1$ Myr. Effective radii scale very weakly with galaxy luminosity (though with considerable scatter; e.g Ferrarese et al. (2006) and in the case of 10^9M galaxy, whose SMBH would have a mass more suited to detection by LISA, the physical unit of time would be closer to 6 Myr. In the latter case, the length of our longest integrations amounts to ~ 1.5 Gyr. Two sets of simulations were carried out. In the first set, a single spherical galaxy was created, with a central

SMBH of mass M_1 = 0.001 (Model A) or M_2= 0.01 (Model B). A second SMBH, of the same mass (M_2) = (M_1), was then placed at a distance of 0.5 from the center on a circular orbit. Integrations were continued until a time t_{max} = 100.In the second set, two identical spherical models were created, each containing a single central SMBH, and the two galaxies were merged. Here again, two values for SMBH mass were used: M_1 = 0.001 (Model C) or 0.01 (Model D) and the integrations were continued until a time t_{max} = 250. In order to test the dependence of the results on particle number N, all simulations were carried out with N = $32{\times}10^3$, $64{\times}10^3$, $128{\times}10^3$, $256{\times}10^3$ and $512{\times}10^3$. Tables 4.1 and 4.2 summarize the parameters of the two sets of runs. The N-body integrations were carried out using $\phi - GRAPE$. The time-step parameter η was set to 0.01 for all our runs.

Run	N	M_1
A1	32k	0.001
A2	64k	0.001
A3	128k	0.001
A4	256k	0.001
A5	512k	0.001
B1	32k	0.01
B2	64k	0.01
B3	128k	0.01
B4	256k	0.01
B5	512k	0.01

Table 4.1: Parameters of the single-galaxy integrations

Note. Columns from left to right read: the galaxy model; number of particles used in galaxy model; The mass of the supermassive black hole used in the model; galaxy model; particle number N; SMBH mass.

The choice of parameters was motivated by Berczik et al. (2005) who found that integration accuracy is more sensitive to η than to softening length ϵ. Those authors adopted η = 0.01 and ϵ = 1×10^{-4} for their study of SMBH binary evolution. In order to resolve the orbit of the massive binary in our simulations down to a semi-major axis $a \approx 10^{-4}$ we adopted the smaller softening length $\epsilon = 10^{-5}$. With these choices of η and ϵ, the relative energy error in our simulations was $\sim 10^{-8}$.

4.1.2 Isolated Models

Evolution of a binary SMBH in a gas-free galaxy can be divided into three phases Begelman et al. (1980). In first phase, the SMBHs are unbound to each other and move independently in the galaxy potential. The evolution of the individual SMBH orbits during this phase is governed by dynamical friction against the background stars. The second phase begins when the two SMBHs are close enough together to form a bound pair. In this phase, the separation between the SMBHs decreases very rapidly due to the combined effects of dynamical friction and ejection of stars by the gravitational slingshot. (For a detailed discussion of this phase, we refer the reader to the careful analysis in Merritt and Milosavljevic (2004).

At the end of the second phase, the two SMBHs form a hard binary, defined as a binary that ejects passing stars with positive (unbound) energies. Phase three consists of slow evolution of the binary as depleted orbits are repopulated, via two-body scattering or some other process. In a spherical galaxy with large N, the timescale for orbital re-population is essentially the two-body relaxation time which scales as $\sim N = lnN$.

Here we can see the three phases of binary evolution described above.

1. In the first phase, the separation between the two SMBHs decreases due to dynamical friction. This phase ends when $R \approx r_h$, the gravitational influence radius. r_h can be defined as the radius of a sphere around the binary SMBH that encloses a stellar mass equal to twice the binary mass. For Model A, $M_1 = 0.001$ and $r_h \approx 0.07$. From Figure 4.1, we can see that the first phase ends indeed when $R \approx 0.07$. Here we want to emphasize that this estimate does not take into account the changes in the galaxy that are induced by the presence of the second SMBH. For Model B, the radius of influence is ~ 0.25. From the panel of Figure 4.2, it is clear that the first phase ends when $R \approx 0.25$. Here the dynamical friction phase is very short-lived because the initial separation of the two SMBHs is only $\sim 2r_h$.

2. In the second phase, the separation between the two SMBHs decreases very rapidly. Dynamical friction, and the ejection of stars by the gravitational

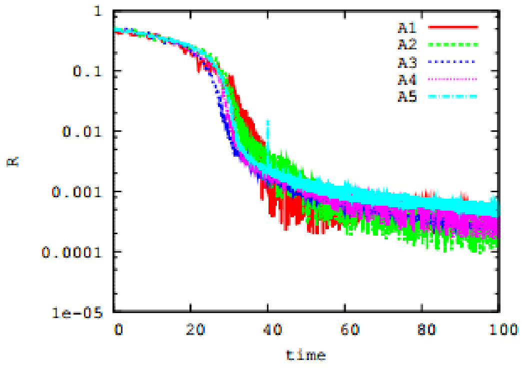

Figure 4.1: Models A, $M_1 = M_2 = 0.001$

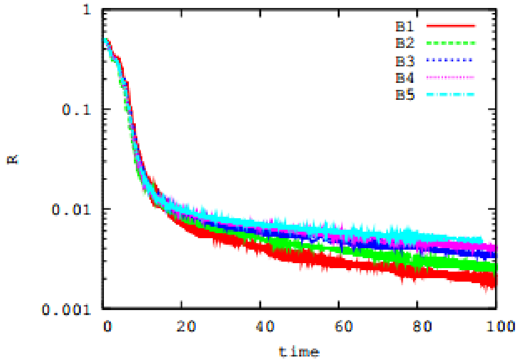

Figure 4.2: Model B, $M_1 = M_2 = 0.01$

slingshot, act together to efficiently extract angular momentum from the massive binary. These processes are not well defined in the regime where the massive binary is neither very hard nor very soft; equations of Merritt and Milosavljevic (2004) give approximate expressions for the rate at which energy is transferred to stars by the two mechanisms. For Model A this process continues until $t \approx 40$ while for Model B, this phase ends at $t \approx 20$. The motion of two SMBHs in this phase is approximately Keplerian. We define the semi-major axis a and eccentricity e via the standard relations:

$$\frac{1}{a} = \frac{2}{R} - \frac{v^2}{\mu} \tag{3.33}$$

$$e = \sqrt{1 + \frac{2h^2}{\mu^2}\left[\frac{v^2}{2} - \frac{\mu}{R}\right]} \tag{3.34}$$

where $\mu = G(M_1 + M_2)$, v is the relative speed, and h is the specific angular momentum of the relative motion.

3. The rapid phase of binary hardening comes to an end when $a \approx a_h$, where ah is the semi-major axis of a ”hard binary”:

$$a_h = \frac{q}{(1+q)^2}\frac{r_h}{4} \tag{3.35}$$

Here, $q = M_2/M_1$ is the mass ratio of two SMBHs; for both Models A and B, q = 1. For Model A, $a_h \approx 0.004$ with $a_h{}^{-1} \approx 250$ and for Model B, $a_h \approx 0.016$ with $a_h{}^{-1} \approx 65$.

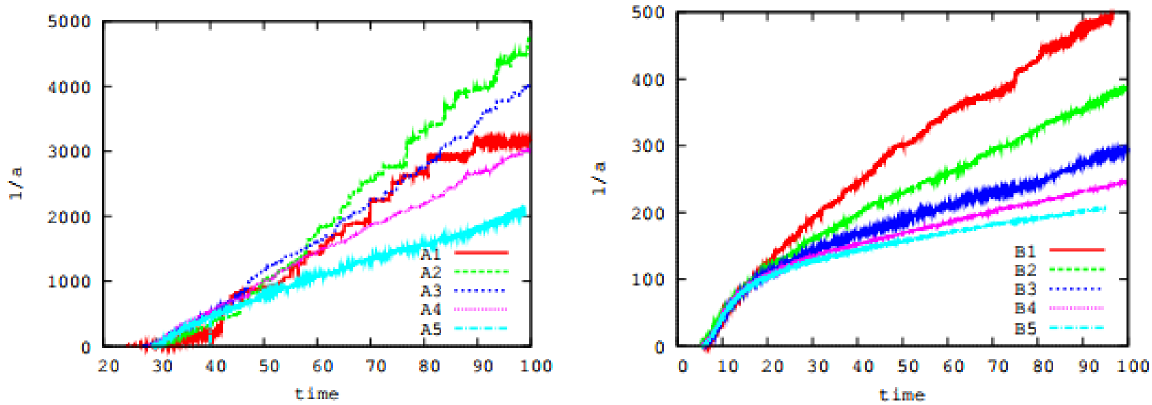

Figure 4.3: Evolution of the inverse semi-major axis of the massive binary in the isolated models for models A and B.

From Figure 4.3, we see that there is no clear dependence of the binary hardening rate on N until the binary becomes hard, $a < a_h$. For Models A this happens around $t = 25$ and for Models B around $t = 30$. Beyond these times the binary hardening rate exhibits a clear N dependence in the sense that hardening is slower for larger N. Similar N-dependence has been seen in other studies of binary evolution in spherical galaxies. For real spherical galaxies, with much larger N, the binary would stop evolving beyond this point.

We estimated the hardening rate s in the N-body integrations, where

$$s \equiv \frac{d}{dt}(\frac{1}{a}) \tag{3.36}$$

by fitting straight lines to $a^{-1}(t)$ in an interval $\Delta t = 50$ from $t = 50$ to $t = 100$. Figure 4.4 shows the N-dependence of s. We see that for $N > 50k$, the hardening rate is a decreasing function of N; scaling as $\propto N^{-0.6}$; this is consistent with the scalings found by other authors in the same N-range, e.g. Berczik et al. (2005), Merritt et al. (2007). The Figure 4.4 also shows lower hardening rates for Model B (right panel) when compared with Model A (right panel), by roughly an order of magnitude. For large N, the two-body relaxation time is long compared with orbital periods of stars near the massive binary, and the loss cone around the binary is nearly empty. In this regime, and in the asymptotic (large-N) limit, the rate of binary evolution is predicted to scale as $\sim[(M_1 + M_2)]^{-1}$. Given that a is roughly a factor ten smaller, at

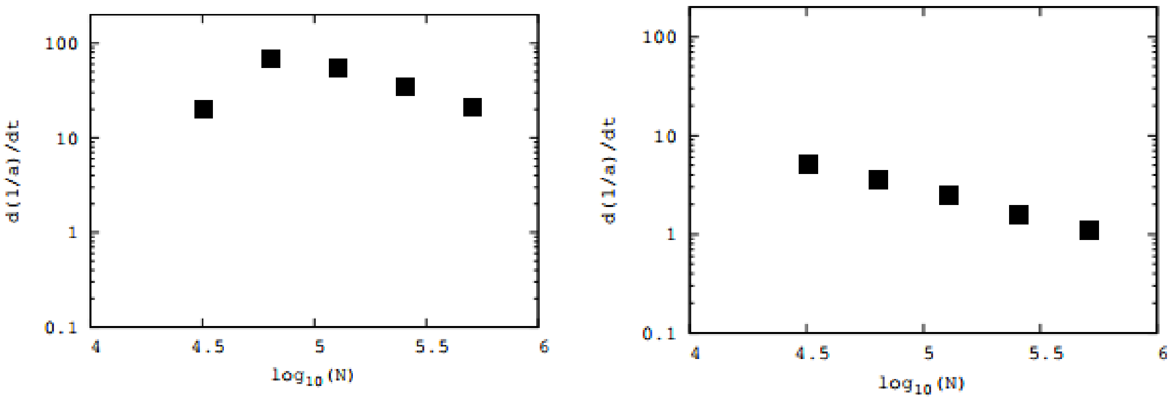

Figure 4.4: N-dependence of the binary hardening rate in the isolated galaxy models. Left: Models A; Right: Models B.

a given time, in Models A as compared with Models B, there is an additional delay in the early phase of the hard black hole evolution. However this delay is negligible for the decay time t_0 until which the semimajor axis reaches a_0, when energy loss by GWs start to dominate.

4.1.3 Galaxy Mergers set up

We studied the evolution of binary SMBHs in merging galaxies by creating two identical galaxy models, each with a central SMBH, and placing the two galaxies on bound relative orbits. Table 3.2 summarizes the parameters of the galaxy merger models.

The initial separation of the two galaxy centers was $20r_0$. The initial relative velocity of the two galaxies was chosen such that the SMBH separation at first peri-center passage was $\sim r_0$; in other words, the initial orbit of the binary galaxy was substantially eccentric. This was done in order to reduce the computational time required to bring the two SMBHs close together; in fact, a galaxy merger from such nearly ”head-on” initial conditions is unlikely.

$Note$: Columns from left to right read: the galaxy merger model; Total number of particles used in galaxy merger; The mass of the supermassive black hole used in each of the merging galaxies.

Run	2× N	M_1
C1	64k	0.001
C2	128k	0001
C3	256k	0.001
C4	512k	0.001
D1	64k	0.01
D2	128k	0.01
D3	256k	0.01
D4	512k	0.01

Table 4.2: Parameters of the galaxy merger simulations

For Figure 4.5, First row: t = (0, 70, 90). The two black holes spiral in centered at their respective cusps. Second row: t = (110, 150, 200). The two cusps undergo violent relaxation and merge into one. The density of the newly-formed cusp drops rapidly as the stars in the central region gain energy from SMBH.

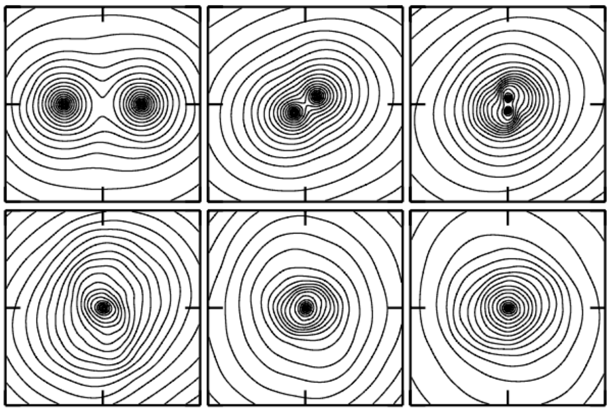

Figure 4.5: Density contours projected onto the initial orbital plane for Model C3 ($M_\bullet = 0.01$).

From figure 4.6, Left: Spatial density profile for model A5 averaged over five different time steps (t = 40, 48, 56, 64, 72). The figure also shows the initial density profile (γ = 1) and a fit to the data using Eq. 3.28 with γ = 0.6 and r_0 = 0.8. The central density drops as the binary ejects mass from the core. Right: Spatial density profile for model C4 averaged over five different time steps (t = 100, 120, 140, 160, 180). The figure also shows the initial density profile (γ = 1) and a fit to the data using equation 3.28 with γ = 0.3 and r_0 = 0.6.

Figure 4.5 illustrates the merging of two galaxies. The two galaxies merge due to violent relaxation. In the initial phase (top panels) the two SMBH particles remain

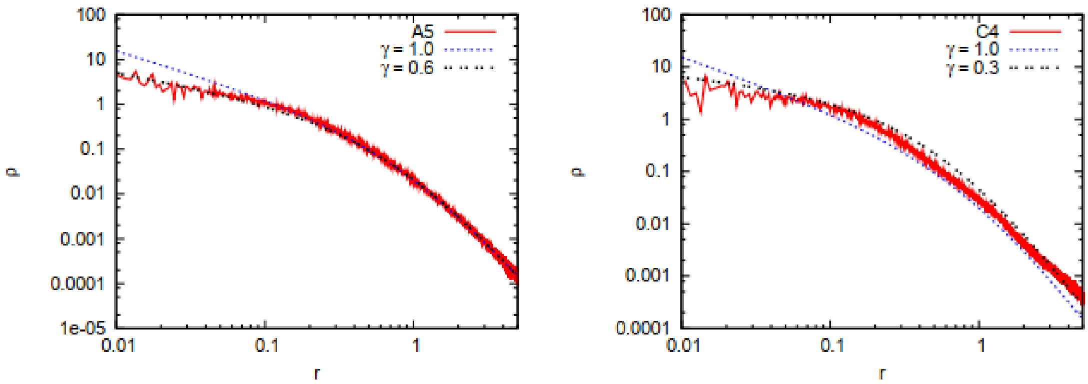

Figure 4.6: Spatial density profile for model A5 and C4 at different time steps.

strongly associated with their respective density cusps. As a consequence dynamical friction is much more efficient in bringing the two black holes together than if they were naked. This is because the effective mass that goes into the dynamical friction is higher. Dynamical friction cannot be distinguished from violent relaxation due to the deformation of the galaxies. The two cusps merge and form a new galactic center. The density profile of the newly formed galaxy as a result of the merger is identical to those of initial merging systems (both merging galaxies have Hernquist profile). Homology following a merger of galaxies with SMBHs at their centers was already found by Merritt and Milosavljevic (2004). The homology is short lived and the profile changes drastically in the sphere of influence of SMBHs after they form the binary. This change is attributed to the transfer of energy from the SMBH binary to surrounding stars.

Figure 4.6 shows the density profile for model A5 where core scouring is done by the binary in individual galaxy and model C4 where we notice same effect for the cusp which is formed by the galaxy merger. This depleted cusp is already reported by (e.g. Makino and Funato (2004), Merritt and Milosavljevic (2004) studying the evolution of binary in a single spherical galaxy model and by Merritt and Milosavljevic (2004) while studying binary SMBH evolution in merger of steep cusps ($\rho \propto r^{-2}$). The central logarithmic slope of the density profile has dropped from -1 to ~ -0.3 (model C4).

In figure 4.5: The time for first row is t = (0, 70, 90). The two black holes spiral in centered at their respective cusps. For Second row: t = (110, 150, 200). The two cusps undergo violent relaxation and merge into one. The density of the newly-formed cusp drops rapidly as the stars in the central region gain energy from SMBH binary.

In figure 4.6, $Left$: Spatial density profile for model A5 averaged over five different time steps (t = 60, 68, 76, 84, 92). The figure also shows the initial density profile $(\gamma = 1)$ and a fit to the data using Equation 3.28 with $\gamma = 0.6$ and $r_0 = 0.8$. The central density drops as the binary ejects mass from the core. $Right$: Spatial density profile for model C4 averaged over five different time steps (t = 140, 160, 180, 200, 220). The figure also shows the initial density profile $(\gamma = 1)$ and a fit to the data using equation 3.28 with $\gamma = 0.3$ and $r_0 = 0.6$.

4.1.4 SMBHs in Galaxy Mergers

Figure 4.7 shows the separation between the two SMBHs in the merger models. The first peri-center passage occurs around $t \approx 80$. As the galaxies merge, the two SMBHs remain centrally located in their respective density cusps, until coming close enough together that a binary SMBH forms, at $t \approx 100$, or slightly earlier in the case of Model D (phase one). The separation between the two SMBHs then decreases very rapidly due to the combined effects of dynamical friction and slingshot ejection of stars (phase two). Once the hard binary is formed, the hardening rate of the binary decreases (phase three). This behavior is qualitatively similar to what was seen in the isolated galaxy models. However there is one important difference between the binary evolution in the isolated and merging galaxies. For models C the peri-center of the bound SMBH orbit is just above 10^{-5} which is the softening used in the code. For models D the peri-center is well above 10^{-5}.

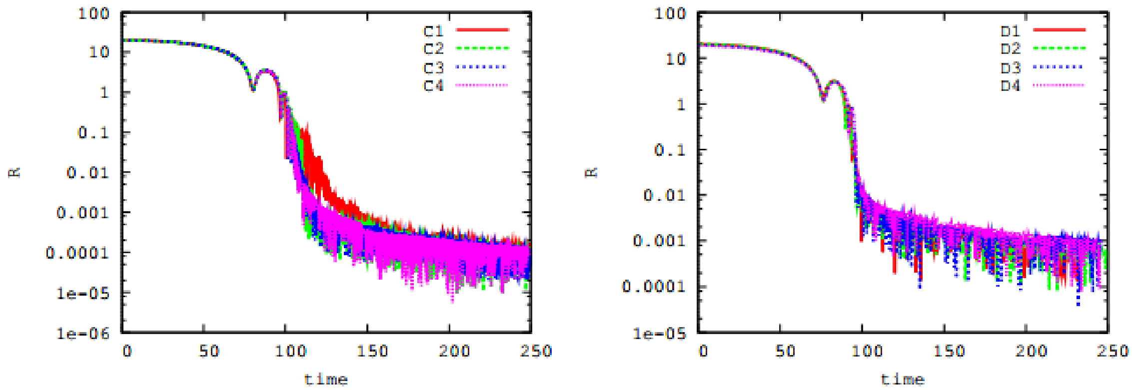

Figure 4.7: Evolution of the relative separation between the two SMBHs in the galaxy merger simulations.

Figure 4.8 shows the binary hardening rates in the galaxy merger simulations; the latter were computed in the same way as in the isolated galaxy models, by linear fit of $1/a$ in the range $150 \leq t \leq 200$. In the galaxy merger models, there is essentially no dependence of the binary hardening rate on particle number. Furthermore, in Models C and D, the hardening rates are much higher - more than five times higher than in the Models A and B, respectively, when $N = 512k$. Also the $(M_1 + M_2)^{-1}$ dependence of binary hardening rates noticed in the spherical galaxy models is still present for the galaxy mergers. The N-independent hardening in galaxy mergers simulations imply that our results can be extrapolated to the much larger N of real galaxies. The binary hardening rates are calculated by fitting straight lines to the inverse semi-major axis in final phase of the binary evolution. For both models C and D the hardening rates are N-independent. Also the hardening rates are approximately 10 times smaller for models D which has 10 times more massive SMBH binary than for models C.

A likely explanation for this difference is that the non-spherical shapes of the merger remnants provide an additional source of torque on the stellar orbits, allowing them to interact with the central binary on a timescale shorter than the two-body relaxation time. It has been argued (Merritt and Poon (2004), Berczik et al. (2006)) that allocating even a small fraction of the stellar orbits to a ”centrophilic” family could lead to binary hardening rates that are much larger than those that result from two-body relaxation alone. In galaxies with non spherical shapes the angular

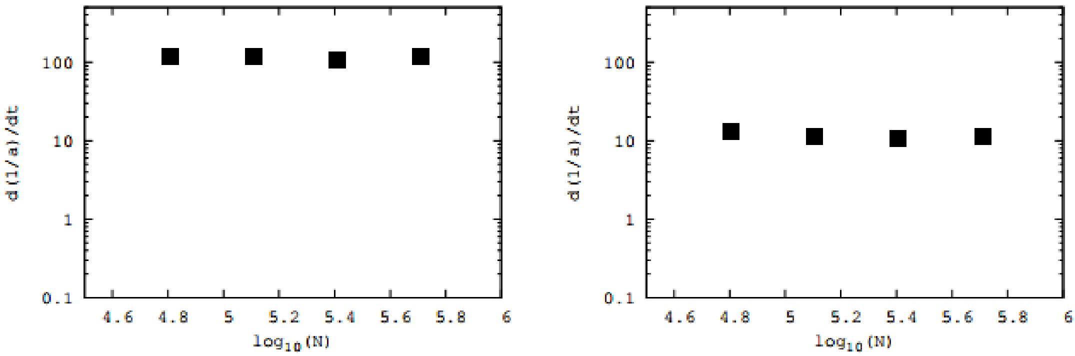

Figure 4.8: Binary hardening rates in the galaxy merger simulations (left panel: Model C, right panel: Model D).

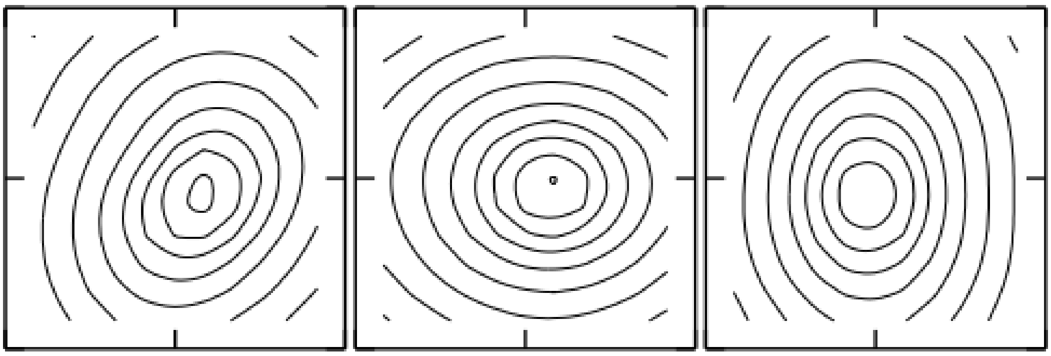

Figure 4.9: Density contours of the central region for galaxy merger model C3 at $t = 150$, projected on x-y plane(left), x-z plane(center), y-z plane(right).

momentum of the stars is not conserved. This produces a whole wealth of stellar orbits including the orbits that can pass very close to the center. We evaluated the shapes of the merged galaxies in a number of ways. The central density contours for Model C3 are shown in Figure 4.9. Departures from axisymmetry are evident. Figure 4.10 shows the principle axis ratios of the merger remnant in Model C3 as a function of time; these were defined as the axis ratios of a homogeneous ellipsoid with the same inertia tensor. The departures from spherical symmetry are modest, but definite, and they appear to be nearly independent of time toward the end of the simulation. Presumably, the non-spherical shapes of merger remnants reported in this study result in a large population of stars on centrophilic orbits. Inside the influence radius of SMBH, the centrophilic orbit family includes saucer or cone orbits in the axisymmetric potentials Sridhar et al. (1999) and pyramids orbits

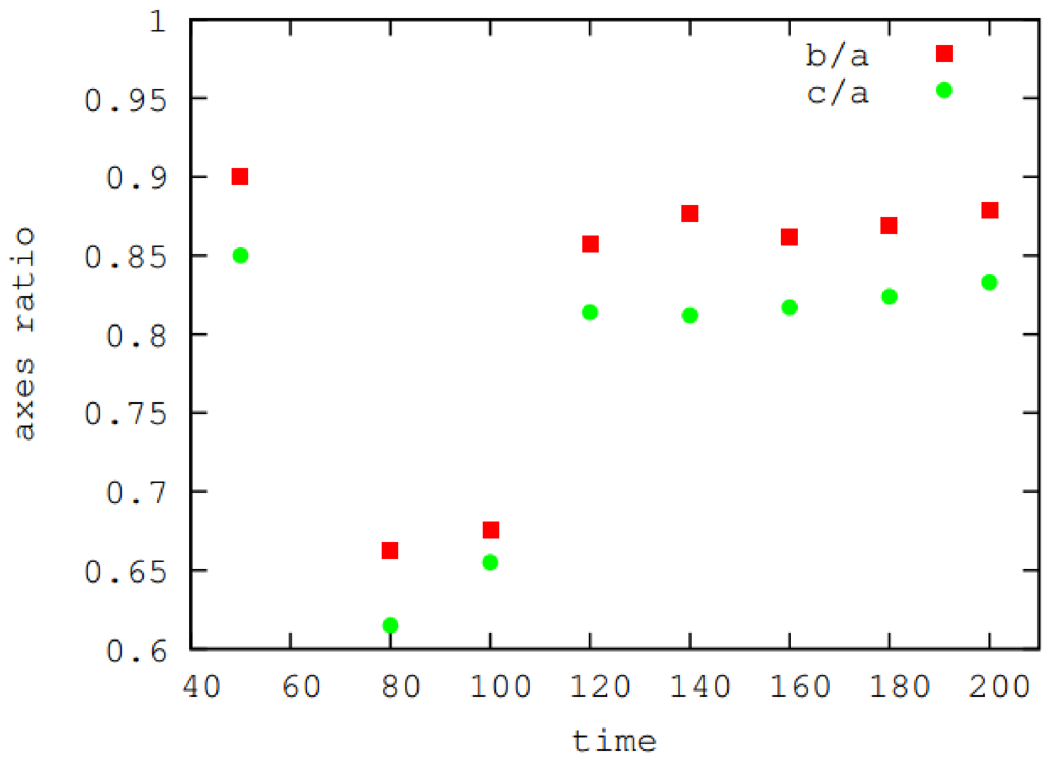

Figure 4.10: Ratio of intermediate to major (b/a) and minor to major (c/a) axes for Run C3.

(angular momentum close to zero) in triaxial potentials Merritt and Vasiliev (2012). Outside the influence radius, most of the centrophilic orbits are chaotic in a triaxial potential Merritt and Poon (2004).

4.2 SMBHs in Unequal Mass Galaxy Mergers

4.2.1 Galaxy Merger Setup

The initial center of mass positions and velocities for the two galaxies are calculated from the Keplerian orbit of the equivalent two-body problem, with given apo- and peri-centers, r_a and r_p, respectively. The two galaxies start at the apo-center of their relative orbit. The initial separation between the center of mass of the two galaxy is $\Delta r = 15r_{0,p}$; and since the half-mass radius of each nucleus is $R_{1/2} \leq 2.5$, this choice ensures that the galaxies are initially well separated while, at the same time, we minimize computing time. The initial relative velocity of the two galaxies is chosen such that the SMBH separation at first peri-center passage is $\approx 2r_{0,p}$.

Run	N_{total}	γ	q	$r_{0,s}$
A1	256k	0.5	0.1	0.316
A2	256k	0.5	0.25	0.5
A3	256k	0.5	0.5	0.707
A4	256k	0.5	1.0	1.0
B1	256k	1.0	0.1	0.316
B2	256k	1.0	0.25	0.5
B3	256k	1.0	0.5	0.707
B4	256k	1.0	1.0	1.0
C1	256k	1.5	0.1	0.316
C2	256k	1.5	0.25	0.5
C3	256k	1.5	0.5	0.707
C4	256k	1.5	1.0	1.0
D1	128k	1.75	0.1	0.316
D2	128k	1.75	0.25	0.5
D3	128k	1.75	0.5	0.707
D4	128k	1.75	1.0	1.0

Table 4.3: Parameters of the galaxy mergers study

$Note$: Col(1) Galaxy merger model, Col(2) Number of particles, Col(3) Central density profile index γ, Col(4) Mass ratio of merging galaxies and their black holes and Col(5) Scale radius of secondary galaxy.

Figure 4.11 shows the relative separation R between the two black holes during a galaxy merger. According to the equivalent two-body trajectory used to set up the initial galaxy orbital parameters, the galaxy centers, and hence the SMBHs, should reach a separation of $2r_{0,p}$ during the first peri-center passage. However, as figure 4.11 shows, in case $q \geq 1/2$, the SMBH separation shrinks well below $2r_{0,p}$ during the galaxy merger phase. As q decreases, and hence the mass of the secondary galaxy decreases as well, the galaxies reach the first peri-center passage after following more closely the corresponding two-body orbit. In the same figure the color of the arrows signals the time T at which the two SMBHs in a particular model form a binary system.

As q decreases, the arrows move from left to right signaling the longer interval of time between the start of merger and formation of the binary. Neither the time T nor R_{peri} are as sensitive to γ as they are to q. When the SMBHs become bound their separation is $R \approx r_h$. Once a SMBH binary is formed, dynamical friction and

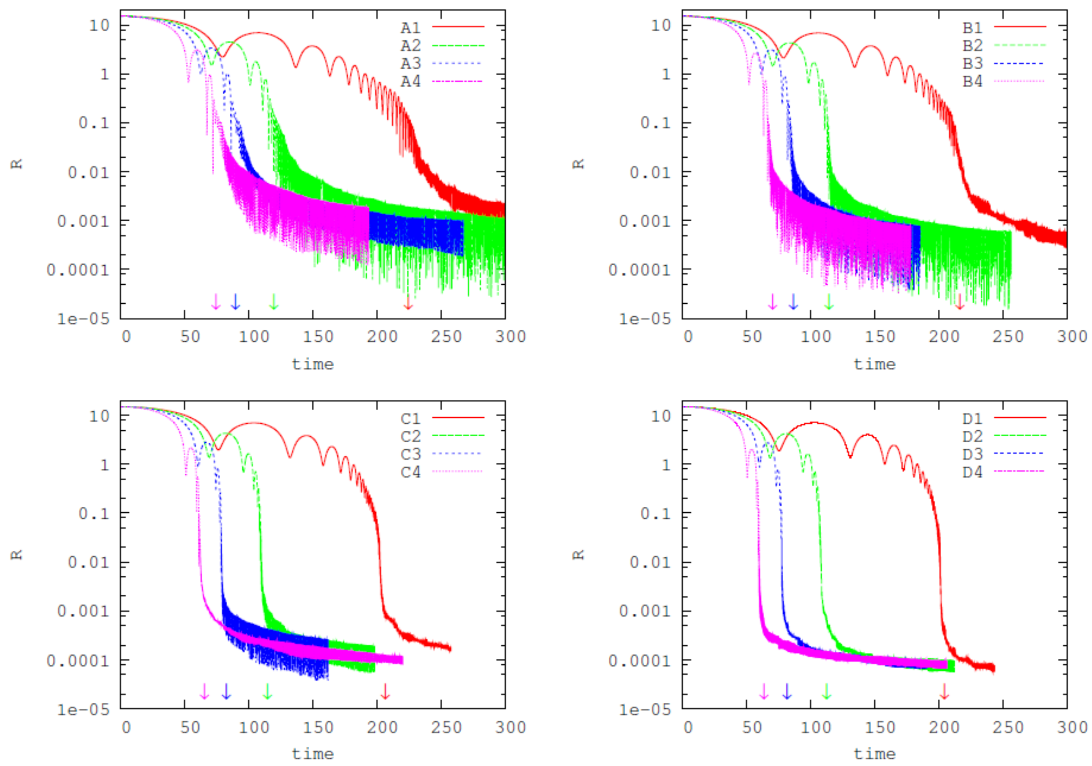

Figure 4.11: Evolution of the separation between the SMBHs, in N-body integrations of galaxy mergers for Central density profile index γ.

the ejection of stars by the gravitational slingshot, act together to efficiently extract angular momentum from the massive binary and the binary separation shrinks rapidly.

Evolution of the separation between the SMBHs, in N-body integrations of galaxy mergers for = 0.5 (top left), 1.0 (top right), 1.5 (bottom left) and 1.75 (bottom right) according to Table 3.3. The arrows represent the time (T = 0) at which the two black holes become gravitationally bound for each model. Time and R are measured in N-body units.

5

Conclusion and Recommendation

5.1 Conclusion

This study makes considerable strides in advancing our understanding of the evolution and eventual merger of supermassive black hole (SMBH) binaries, particularly in galaxies formed through mergers. Earlier models proposed that SMBH binaries could stall at a separation of around 1 parsec due to the depletion of nearby stars available for interaction. This stalling would prevent the binaries from losing sufficient orbital energy, thereby halting their progression toward a merger. However, the results of this research, which employs detailed N-body simulations, challenge this assumption and suggest that the FPP is unlikely to occur in galaxies shaped by mergers. In these systems, the non-spherical structure of the merger remnants plays a critical role by ensuring a continuous supply of stars that interact with the SMBH binary. These interactions facilitate ongoing energy loss, allowing the binary's orbit to decay steadily. As a result, the SMBH binary can overcome the stalling predicted by the FPP and proceed toward a final merger.

A significant byproduct of this process is the ejection of stars from the central region of the galaxy as the SMBH binary loses energy. This stellar ejection leads to the formation of low-density stellar cores in the galactic center, a feature that is commonly observed in massive elliptical galaxies. The results of this study provide a clear physical explanation for the origin of these low-density cores, linking them directly to the dynamical processes that occur during SMBH binary mergers. Thus, the research not only addresses and resolves the FPP in the context of galaxy mergers but also sheds light on the formation of key structural features in galaxies, particularly the central cores of massive ellipticals. Furthermore, the findings suggest that SMBH mergers are frequent events on cosmological timescales, likely occurring within periods shorter than the Hubble time. This has important implications for our understanding of the role of SMBHs in galaxy evolution and the larger universe. The continuous orbital decay of SMBH binaries in merger remnants implies that these mergers are significant sources of gravitational waves. As these binaries eventually coalesce, they emit gravitational waves, contributing to the gravitational wave background detectable across different epochs of cosmic history. The study suggests that SMBH mergers are potential sources of detectable gravitational waves at both low and high redshifts, making them an essential target for future observations by gravitational wave detectors such as LISA.

Finally, this research provides a critical solution to the Final Parsec Problem in the context of galaxy mergers and highlights the broader implications of SMBH binary dynamics. By showing that SMBH binaries can efficiently merge, this study emphasizes their role as key players in shaping the central regions of galaxies and their importance as contributors to the gravitational wave background. These findings offer a deeper understanding of SMBH evolution and open new avenues for future research and observational efforts, particularly in the growing field of gravitational wave astronomy.

5.2 Recommendation

There are several recommendations to build upon the findings of this study. First, further simulation studies should explore additional factors, such as gas dynamics, which could play a significant role in the orbital decay of supermassive black hole (SMBH) binaries. Gas accretion and feedback mechanisms may either enhance or hinder the merging process, and as SMBH binaries shrink to sub-parsec separations, relativistic effects become increasingly important. Incorporating general relativity and gravitational wave emission into simulations would provide a more comprehensive understanding of the merger process and the full range of environments under which SMBH binaries can merge. Next, advancements in gravitational wave observations are critical, as SMBH mergers are likely to be significant contributors to the gravitational wave background. Space-based detectors, such as the Laser Interferometer Space Antenna (LISA), are expected to be particularly sensitive to these waves, especially from high-redshift SMBH mergers. These observations will offer direct evidence of SMBH mergers and improve our understanding of their population and frequency over cosmic time, thereby helping confirm theoretical predictions about SMBH dynamics.

Additionally, incorporating SMBH mergers into galaxy evolution models will improve predictions about the structure and formation of galaxies, particularly massive ellipticals. The ejection of stars and the formation of low-density cores due to SMBH mergers should be integrated into models of galaxy evolution, which will offer more accurate descriptions of the morphological and dynamical properties of galaxies. This integration will enhance understanding of the co-evolution of SMBHs and their host galaxies within the framework of hierarchical galaxy formation theories. Finally, stellar distribution studies are important for understanding how interactions between SMBH binaries and their stellar environment facilitate orbital decay. Observational studies of stellar densities and velocity dispersions in post-merger galaxies, using high-resolution imaging and spectroscopy, will deepen insights into how these stellar dynamics influence SMBH evolution. Further theoretical work should also investigate how variations in stellar distributions, such as triaxiality or anisotropic velocities, affect the long-term behavior of SMBH binaries.

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