Synchronization of a Sprott Chaotic System Using Sliding Mode Control

Soreti Tefera Desalegn

A Thesis Submitted to

The Department of Electrical Power and Control Engineering

Collage of Electrical Engineering and Computing

Presented in Partial Fulfillment of the Requirement for the Degree of Master’s in Electrical Power and Control Engineering

Office of Graduate Studies

Adama Science and Technology University

June, 2025

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Co-advisor**:** Dr.Endale Ayenewu

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DECLARATION

I declare that this thesis entitled "**Synchronization of a Sprott Chaotic System Using Sliding Mode Control**" is my own work and has not been submitted to any university for a similar purpose. The references used in this proposal are duly recognized by appropriate citations.

Name of the student	Signature	Date
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RECOMMENDATION OF ADVISOR

We, the advisors of this thesis, hereby certify that we have read the revised version of the thesis entitled "**Synchronization of a Sprott Chaotic System Using Sliding Mode Control"** prepared under our guidance by **Soreti Tefera** submitted in partial fulfillment of the requirements for the degree of Mater's of Science in Electrical Power and Control Engineering (Control Engineering). Therefore, we recommend the submission of revised version of the thesis to the department following the applicable procedures.

Advisor	Signature	Date
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Co-Advisor	Signature	Date
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APPROVAL SHEET

We, the advisors of the thesis entitled "**Synchronization of a Sprott Chaotic System Using Sliding Mode Control**" and developed by **Soreti Tefera**, hereby certify that the recommendation and suggestions made by the board of examiners are appropriately incorporated into the final version of the thesis.

Advisor	Signature	Date
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We, the undersigned, members of the Board of Examiners of the thesis by **Soreti Tefera** have read and evaluated the thesis entitled "**Synchronization of a Sprott Chaotic System Using Sliding Mode Control**" and examined the candidate during open defense. This is, therefore, to certify that the thesis is accepted for partial fulfillment of the requirement of the degree of Master of Science in Electrical Power and Control Engineering

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ACKNOWLEDGEMENT

First and foremost, I would like to thank God Almighty for granting me the strength, patience, and perseverance to complete this thesis. I am sincerely grateful to Adama Science and Technology University, particularly the Collage of Electrical Engineering and Computing and the Power and Control Engineering Department, for giving me the opportunity to be part of the Master's program. I would like to express my deepest appreciation to my advisor, Prof. Gang GyooJin, for his invaluable guidance, continuous encouragement, and unwavering support throughout this research. I am also thankful to my co-advisor, Dr. Endale Ayenew, for his constructive feedback and consistent mentorship. Additionally, I extend my sincere thanks to Dr. Shimeles Demisie for his kind assistance and helpful contributions during the course of this work.

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LIST OF ABBREVIATIONS

Abbreviations	Description
SMC	Sliding Mode Control
PSO	Particle Swarm Optimization
PID	Proportional Integral Derivative
IAEU	Integral of Absolute Error per Unit control
IoT	Internet of Things
OPCL	Open-Plus-Closed-Loop
IAE	Integral of Absolute Error

ABSTRACT

The synchronization of chaotic systems remains a vital area of research due to their broad range of applications in fault detection, secure communications, and power electronics. This thesis proposes a Sliding Mode Control (SMC) strategy to achieve synchronization between a master and a slave chaotic system, where the slave is dynamically guided to follow the behavior of the master system. The work addresses key challenges such as sensitivity to initial conditions, parameter variations, and external disturbance factors that commonly impair synchronization accuracy. To overcome these challenges, a sliding mode control law is formulated to drive the synchronization error to zero, thereby ensuring precise and stable synchronization. To further enhance controller performance, Particle Swarm Optimization (PSO), an evolutionary algorithm inspired by the social behavior of bird flocking, is used to fine-tune the SMC parameters. The optimization process aims to improve synchronization accuracy and reduce convergence time. Simulation results, conducted in MATLAB, demonstrate that the PSO-optimized SMC significantly improves the system's performance. Compared to a conventional PID controller, the proposed method reduces the settling time by 36.6% under normal conditions and by 94.57% in the presence of external disturbances. These results demonstrate the robustness and efficiency of the proposed method, highlighting its strong potential for real-world applications in chaotic system synchronization within the domains of power system monitoring, secure sensor data integration, and cybersecurity for IoT devices.

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Keywords: Chaotic system synchronization, Master chaotic system, Slave chaotic system, Sliding mode control, Particle swarm optimization

CHAPTER ONE

INTRODUCTION

1.1 Background of the study

Chaotic systems are nonlinear dynamical systems which are highly sensitive to initial conditions, where small changes can lead to vastly different outcomes. This sensitivity is popularly known as the ‘butterfly effect’(Alligood et al., 1997; Jagatheesan et al., 2016). In addition to that They are also characterized by Nonlinearity, determinism, order in chaos, the inability to make long-term predictions are characteristics of chaotic systems (Ditto & Munakata, 1995). This make them difficult to predict and control, even though they are governed by deterministic laws (Fiveable, 2024). This characteristic is not merely a mathematical curiosity; it is observable in a wide range of natural and engineered systems, including fault detection and diagnosis in power system, power converter, signal generator design, biology, economics, and many other engineering systems (CHEN & DONG, 1993).

Some examples of chaotic physical systems are the weather, which is well known to be difficult to forecast beyond a few days due to chaotic behavior in atmospheric dynamics; the double pendulum, a mechanical system consisting of two pendulums joined together that swing in a wildly unpredictable way; some nonlinear electronic circuits that produce irregular, chaotic signals; turbulent fluid flow; In turbulence, small changes in initial conditions, such as the velocity or pressure at a specific point in the fluid, can lead to vastly different patterns of flow. This sensitivity to initial conditions becomes a major obstacle in predicting the behavior of turbulent flows, despite the fact that they obey well-known fluid dynamics equations such as the Navier-Stokes equations. Turbulence is one of the most difficult geophysical phenomena to study, and chaos theory has contributed significantly to understanding its basic principles. Similarly, some lasers, which are stimulated light sources, can exhibit chaotic behavior when their emitted light shows irregular fluctuations in intensity (abdatum, 2025; Gurikar, 2024; Sciamanna & Shore, 2015; Strogatz, 2018).

Chaotic systems are classified into two. The first classification is based on system dimensionality, which is low dimension and high dimension. Low dimensional chaotic systems include two or three differential equations and include benchmark examples like the Lorenz system, the Rössler system, and the Sprott jerk system. On the other hand, high-dimensional chaotic systems, like the Chen-Lee system and other types of Hyperchaotic systems, include four or more state variables. The second classification separates chaotic systems into discrete time and continuous time categories according to the type of system. Ordinary differential equations (ODEs) describe continuous time systems, such as Chua's circuit and the Lorenz attractor. Conversely, difference equations or iterative mappings describe discrete time systems. Example of discrete time systems include the Logistic map and the Henon map (Fiveable, 2025; Strogatz, 2018; Wang et al., 2006; Wikipedia, 2025).

The sprott jerk chaotic system originally introduced by J.C. Sprott in 1997. They are class of third order differential equation. They are characterized by their ability to produce complex chaotic behavior using simple algebraic expression. The word jerk is originates from physics which refer to third derivative of position with respect to time (Guo & Li, 2009; Lerescu et al., 2004; Sprott, 1997).

The Sprott jerk system is selected rather than well-known chaotic systems due to several key considerations. Firstly, the jerk form is particularly advantageous for control applications, given its compatibility with real-world systems where higher-order derivatives like acceleration can be directly influenced. Secondly, it has a simpler algebraic expression than the Lorenz and Rössler systems. This leads to lower computational complexity, making it highly efficient for simulation and controller design, while preserving the complexity of its chaotic behavior (Lerescu et al., 2004; Miptahudin, 2021).

The inherent unpredictability and complexity of chaotic systems have sparked significant research interest. The synchronization of chaotic systems is a problem and an opportunity in control theory. The word synchronization is originally comes from Greek, which means “toshare the common time”. Since the 17th century, this idea has drawn attention, most notably when Christian Huygens noticed that two pendulum clocks on the same beam may eventually swing simultaneously. Synchronization has been expanded to encompass chaotic systems in contemporary scientific studies. Because chaotic systems are so sensitive, even slight variations in initial conditions can result in drastically divergent system trajectories, which makes synchronization between chaotic systems very difficult (Boccaletti et al., 2002).

Synchronization of chaotic systems is a process where two (or many) chaotic systems adjust a given property of their motion to a common behavior due to coupling or forcing. This ranges from complete agreement of trajectories to locking of phases(Pecora & Carroll, 2015). This phenomenon can occur in several distinct forms, including complete synchronization (CS), where the system trajectories become identical; phase synchronization (PS), where only the oscillatory phases are locked; lag synchronization (LS), where the states align with a fixed time delay; and generalized synchronization (GS), where a functional relationship exists between the systems' states. Depending on the nature of the interaction, systems may be coupled unidirectionally, commonly referred to as a drive–response or master–slave configuration, or bidirectionally, where mutual influence leads to a shared evolution, as seen in biological or physical systems such as coupled neurons or lasers (Boccaletti et al., 2002; Pecora et al., 1997).

In order to achieve accurate synchronization, despite the inherent complexity and unpredictability of chaotic systems, it is essential to apply appropriate coupling mechanisms. Achieving synchronization is essential in many application areas like fault detection, encryption, power converter and many others (Pecora, L. M., & Carroll, 1990).

However, synchronizing chaotic systems such as the Sprott jerk is difficult because of their inherent nonlinearity, unpredictability, and sensitivity to initial conditions. Even slight variation, like parameter change or presence of external disturbances, can result in a different outcome, which might hinder effective synchronization. Traditional linear control solutions like PID controller often fail in chaotic systems because they are not well suited to handle their dynamic and unpredictable behavior. To overcome these challenges, it is essential to create advanced control algorithms that can ensure reliable synchronization even in the presence of disturbance and parameter change (Mkaouar & Boubaker, 2014; Nguyen et al., 2022; H. Wang et al., 2009).

Sliding mode control (SMC) is one of the effective nonlinear robust control approaches since it provides system dynamics with an invariance property to uncertainties once the system dynamics are controlled in the sliding mode. SMC operates by forcing the system states to slide along a predetermined surface, ensuring that the system behaves in a desired manner despite the inherent non-linearities and disturbances present in chaotic systems. This controller is effective to achieve synchronization not only in normal conditions but also during parameter uncertainties and disturbances (Nazzal & Natsheh, 2007). However, the effectiveness of SMC can be further enhanced through a process of tuning, where the control parameters are optimized to achieve better performance. This is where Particle Swarm Optimization (PSO) comes into play.

Particle swarm optimization (PSO) has received much interest for achieving high efficiency and searching for global optimal solutions in problem spaces. The particle swarm optimization (PSO) is an evolutionary computation technique developed by Eberhart and Kennedy in 1995 , inspired by the social behavior of bird flocking (Wai et al., 2014). It is used to optimize the SMC parameter, like sliding surface gain and coefficient of sliding surface. This optimization in turn reduces control signal oscillation, thereby minimizing chattering and also enhancing controller performance. By combining PSO with SMC, the advantages of both methods are effectively utilized, providing an effective approach to address the challenges of chaotic synchronization while effectively managing issues like chattering (Hashem Zadeh et al., 2016; Mahmoodabadi & Taherkhorsandi, 2017; Wai et al., 2014).

This thesis proposes SMC, which is optimized by PSO to fine tune the parameter effectively to achieve effective synchronization of chaotic systems even in case of parameter change and disturbance. The proposed method aims to increase the robustness and efficiency of synchronization. Through comprehensive simulations and analysis, this research will explore the efficiency of the system under disturbance and parameter variation by including the plot of synchronization error, control input, and sliding surface. It is also compared with a PID controller to clarify the effectiveness of the proposed system.

1.2. Statement of the problem

Chaotic systems are inherently sensitive to initial conditions and system parameters, making their long-term behavior unpredictable and synchronization challenging. External disturbances and parameter variations further complicate synchronization, where traditional control methods like PID controllers often fall short, particularly under uncertain conditions. In contrast, Sliding Mode Control (SMC) effectively handles system nonlinearities and uncertainties by driving synchronization error to zero with minimal corrective effort. However, SMC suffers from the chattering phenomenon, undesirable high-frequency oscillations in the control signal that can cause practical issues like system wear. Moreover, its performance is highly sensitive to the precise tuning of parameters such as the sliding surface gain and its coefficients. To address these challenges, this study integrates Particle Swarm Optimization (PSO) with SMC to fine-tune its parameters. The optimized controller improves key performance metrics such as settling time, rise time, and maximum overshoot, enabling more accurate and faster synchronization. Despite its potential, the PSO-SMC combination remains underexplored in the context of chaotic system synchronization. Therefore, this research aims to fill that gap by developing a PSO-tuned SMC framework that reduces synchronization error, mitigates chattering effects, accelerates convergence, and enhances overall system performance.

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1.3. Research question

To address the problem arose, this research work intends to answer the following primary research questions:

- How can master and slave chaotic systems be modeled?
- How can Sliding Mode Control (SMC) be applied to chaotic systems to achieve reliable synchronization?
- How can Particle Swarm Optimization (PSO) be applied to optimally tune the parameters of the Sliding Mode Controller for enhancing synchronization performance?
- How does the PSO tuned SMC compare with existing synchronization methods?

1.4 Objective of the thesis

1.4.1 General Objective

This thesis aims to develop and simulate a sliding mode control(SMC) strategy which is optimized by Particle swarm optimization(PSO) to enhance synchronization performance of chaotic systems.

1.4.2 Specific Objectives

The specific objectives of this proposal are to achieve the following points:

- To formulate the mathematical models of master(transmitter) and slave(receiver) chaotic systems.
- To develop a sliding mode control law for synchronization of master and slave chaotic systems
- To tune the parameters of the proposed method optimally using Particle Swarm Optimization (PSO).
- To investigate the performance and robustness of the proposed method to maintain synchronization in the presence of external disturbances.
- To compare the proposed system with PID controller.

1.5 Scope of the Study

This study explores the use of PSO based Sliding Mode Controller for synchronization of chaotic systems. The research focuses on optimizing the parameter of SMC to ensure reliable synchronization in the presence of disturbances and parameter changes.

1.6 Limitation of the Study

This research is limited to the simulation and design of the SMC approach optimized by PSO only to the Sprott jerk chaotic system. Although the controller shows promising synchronization results, it was not applied to other widely known chaotic systems such as the Lorenz, Chen, or Rössler systems.

1.7 Significant of the Study

The study aims to improve the synchronization of chaotic systems by using SMC, which is optimized by PSO. The proposed approach optimize the parameter of SMC automatically, which enhance the synchronization quality and robustness against disturbance and parameter variation. The results of this research will contribute to more reliable systems that utilize chaotic signals, with potential applications in fault detection, power converters, and chaotic watermarking and fingerprinting.

1.8 Organization of the Thesis

There are six chapters in this thesis. Chapter One presents the background of chaotic systems and their synchronization, along with the objectives, focus, significance, limitations, and research problem. Chapter 2 reviews existing literature, focusing on chaotic system and its synchronization methods, and areas of application. Chapter 3 presents system modeling and research material. Chapter 4 presents the architecture of the used controller in the selected chaotic system. Chapter 5 presents results from the simulation and a comparison of the performance of the used controller with other techniques. Chapter Six presents the primary conclusions and suggests areas of further research.

CHAPTER TWO

LITRATURE REVIEW

2.1 Overview

The "butterfly effect" refers to the complex behavior of chaotic systems, which are characterized by their nonlinear and highly sensitive dynamics. These systems are particularly sensitive to beginning conditions (Alligood et al., 1997; Jagatheesan et al., 2016). Nonlinearity, determinism, order in chaos, sensitivity to initial conditions, and the inability to make long-term predictions are characteristics of chaotic systems (Ditto & Munakata, 1995). Because of these features, chaotic systems are perfect for applications that need unpredictable and secure behaviors, such as encryption, chaotic communications, and fault detection, power converter. The idea of synchronization, the alignment of two or more chaotic systems in behavior or trajectory, was a major breakthrough in the field of chaotic systems. When these chaotic systems' paths converge or strongly resemble one another, we say synchronization is achieved (Pecora, L. M., & Carroll, 1990).

Many techniques have been developed to accomplish chaotic synchronization since Pecora and Carroll's work in 1990, each with unique benefits. By reconstructing the chaotic dynamics of the master system within the slave, estimators are used in chaotic synchronization based on observer-based techniques (Morgül & Solak, 1996; Yu, 2005; Zhao et al., 2020) to match the state trajectories of the systems. Adaptive synchronization techniques (Delavari & Mohadeszadeh, 2016; Fotsin & Daafouz, 2005; Mohadeszadeh & Pariz, 2022; Pourmahmood et al., 2011; T. Wang et al., 2018) enable robust synchronization in unpredictable circumstances by enabling online control parameter adjustment in response to changes in the system dynamics.

Active control techniques (Mohadeszadeh& Delavari, 2018) which directly inject a control signal into the chaotic system. Feedback synchronization uses a feedback signal that is based on the difference between the state of two systems and is injected into the slave system in order to minimize synchronization error (Guan et al., 2003; Louodop et al., 2014; Mobayen & Ma, 2018; H. Wang et al., 2009). Similarly, backstepping methods are a technique developed for designing stabilizing controls using recursive Lypunov-based control design method iteratively, which achieve synchronization through sequential loops (Vaidyanathan et al., 2015). The other controller like impulse technique is use instantaneous correction, rather than continuous control, in order to update the slave system at discrete interval (T. Yang & Chua, 1997).

Sliding mode control, or SMC (Chan et al., 2022; Mohadeszadeh & Pariz, 2021; Xiang & Huangpu, 2010; Xiu et al., 2021), is one of the most popular techniques in recent years. It uses a high-frequency switching control law to move the system in the direction of a predetermined sliding surface. SMC is preferred for chaotic synchronization because it is resistant to uncertainty, parameter change, and disturbance .The overarching goal in these methods is to synchronize the slave chaotic system with the master chaotic system by applying a control signal derived from the master. This control signal is designed to ensure that the slave system closely follows the master's dynamics, enabling synchronized behavior.

Chaotic synchronization is used in many application areas like chaotic communication, fault detection, image encryption, power system specifically power converter and robotic and control system. Chaotic synchronization is widely utilized for chaotic communication (Chan et al., 2022; Chen et al., 2008; Gupta et al., 2017; Jugale et al., 2019; Kwon et al., 2011; Lakshmanan et al., 2018; Li et al., 2016; Messadi et al., 2023; Mofid et al., 2021; T. Wang et al., 2018). The message is first encrypted by injecting it into a chaotic system in a transmitter that has been synchronized to a receiver system. If the controller effectively designs control input and drives the synchronization error to zero and the message is successfully recovered from the receiver, then chaotic communication is said to have occurred. Chaotic synchronization presents a highly sensitive and effective approach for detecting faults in dynamic systems like power grids, electrical machines, and various industrial processes. A master chaotic model simulates expected power system behavior. The live system is modeled by a slave chaotic system. Then This technique works by linking a chaotic observer system (the slave) to a physical process (the master) and keeping an eye on how well they synchronize with each other (Hsieh et al., 2014; Lu et al., 2020). Image encryption based on chaotic synchronization is a modern cryptographic technique. First, an image is ejected into the transmitter for encryption purposes, and then design control input at the receiver side for decryption of image data (Singh et al., 2017; Volos et al., 2013). Chaotic synchronization is also used in power converters because power converter exhibit chaotic behavior under certain conditions, like high switching frequency and specific load variation. In order to maintain this behavior, it design master chaotic system from known chaotic types and make converter to act as slave system, which is forced to synchronize with the master using a control law (Iu & Tse, 2000).

To overcome the limitations of existing methods, the integration of Sliding Mode Control (SMC) with Particle Swarm Optimization (PSO) has been proposed as an effective solution. PSO enhances the performance of SMC by optimizing its control parameters, thereby reducing the chattering effect, improving synchronization precision, accelerating convergence rates, and minimizing computational overhead. To address these issues comprehensively, this thesis proposes a novel approach that integrates the robustness of SMC with the adaptive optimization capabilities of PSO. This method reduces computational demands, enhances synchronization accuracy, and ensures efficient message recovery, offering a scalable and reliable framework for different application areas while addressing existing gaps in the field.

2.2 Related Work

The study of chaotic systems has evolved significantly, particularly in the context of synchronization. Early work introduced the Open-Plus-Closed-Loop (OPCL) method as a strategy for achieving master–slave synchronization in Sprott's collection of chaotic systems. This work was critical in demonstrating that even simple chaotic systems can be synchronized reliably and with low computational overhead, but its work which is OPCL is limited only valid when the system parameter is well known and it doesn't consider disturbance in its implementation(Lerescu et al., 2004).

Active Sliding Mode Control (ASMC) for synchronization of chaotic systems has demonstrated robustness against nonlinearities and disturbances. The technique effectively reduces synchronization errors and improves system robustness. However, it typically suffers from the chattering phenomenon undesirable high-frequency oscillations in the control signal that can cause wear and practical limitations in real applications. Additionally, ASMC relies heavily on manual tuning of control parameters, which is time-consuming and may not achieve optimal performance under varying system dynamics or parameter uncertainties. Notably, early work on ASMC for synchronizing different chaotic systems did not incorporate optimization techniques for parameter tuning (Haeri & Emadzadeh, 2005)

To address the challenge of parameter tuning in chaotic synchronization, it proposed an Evolutionary Programming (EP)-based PID control approach. Their method effectively minimized the Integrated Absolute Error (IAE) and demonstrated improved synchronization performance and it also applied synchronization into secure communication both in simulation and practically. Nevertheless, EP is computationally intensive and lacks real-time responsiveness. Moreover, classical PID controllers despite optimization struggle with highly nonlinear and rapidly varying chaotic dynamics, suggesting the need for more structurally robust solutions(Chen et al., 2008).

Building upon control and communication integration, the study on the design and implementation of the Sprott chaotic secure digital communication systems explores secure communication through the synchronization of Sprott chaotic systems. This approach involves converting chaotic analog signals into pulse-width modulated digital sequences and applying XOR-based masking for message encryption and decryption. While practical and cost-effective, this design relies on ideal synchronization the conditions and does not account for disturbance like noise, parameter drift, or real world circuit imperfections(Hou et al.,2012)

Bifurcation Analysis and Electronic Circuit for Sprott Jerk System is focused on the bifurcation analysis and electronic realization of the Sprott jerk system. While this work is valuable for characterizing chaotic behavior and demonstrating circuit feasibility, it does not extend to control or synchronization which is critical aspects for enabling on different practical applications (Miptahudin, 2021).

To overcome the limitations identified in previous studies such as sensitivity to parameter variations and external disturbances, insufficiently robust synchronization methods, and the lack of optimized parameter tuning this research proposes the integration of Sliding Mode Control (SMC) with Particle Swarm Optimization (PSO). This combined approach is designed to significantly improve synchronization accuracy while effectively suppressing the chattering phenomenon, a common drawback of traditional SMC that causes undesirable high-frequency oscillations and practical implementation challenges. Furthermore, the PSO-based optimization ensures precise tuning of control parameters, enhancing convergence speed and overall system stability. Unlike earlier works, this study rigorously evaluates key performance metrics, including settling time, rise time, and overshoot, across a range of conditions normal operation, presence of disturbances, and parameter variations providing a comprehensive assessment of synchronization effectiveness and robustness. This integrated approach is designed to improve the reliability and performance of synchronization methods, making them well-suited for a wide range of practical applications.

CHAPTER THREE

MATERIALS AND METHODS

3.1 Overview

This chapter presents the materials used in the system, along with the system's flowchart and block diagram. Additionally, it provides a detailed discussion of the plant modeling, including the master chaotic system and its phase portrait, as well as the corresponding slave system used for synchronization.

3.2 Materials

The MATLAB software package will be used in this thesis to simulate, test, and analyze the system model. This software offers a wide range of toolboxes for numerical computations, system simulations, and graphical visualization, making it a powerful tool for analyzing complex systems. Additionally, Microsoft Office and will be utilized to draft and format the thesis documentation, ensuring clarity and a professional presentation and also Math Type equation is utilized to write mathematical formulas and equations in the Microsoft Office Word and PowerPoint presentation.

3.3 Methodology of the System

In order to attain general and specific objectives of the study, the following methods and techniques will be followed.

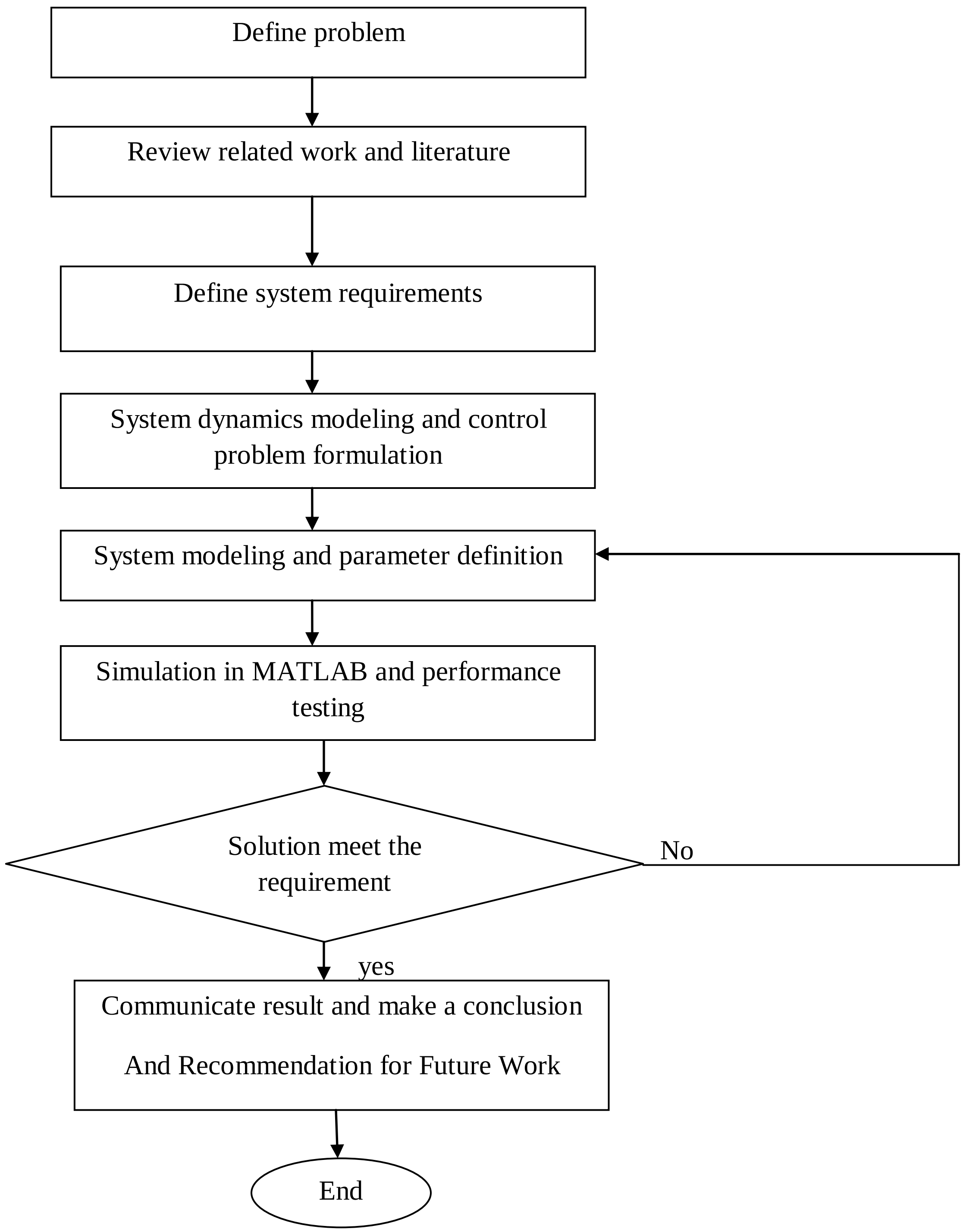

Figure 3.1 Flowchart of the research methodology

3.4 Block Diagram of the System

This section presents the thesis methodology using the block diagram in Figure 3.2. The process starts with a master chaotic system generating a reference signal. The slave system is then synchronized with the master to follow the same trajectory over time. To achieve this, a Sliding Mode Controller (SMC) is designed to force the state of the slave system to follow the master system. The synchronization error, defined as the difference between the states of the master and slave systems, is continuously monitored and used to guide the control action. During the synchronization process, the system is subjected to possible disturbances and parameter uncertainties. The SMC is chosen because it handle disturbance and parameter uncertainties and result in effective and robust synchronization performance. It operates by driving the system trajectory onto a predefined sliding surface, and maintaining motion along this surface until synchronization is achieved. in order to attain accurate synchronization and faster convergence, the SMC parameter are optimized by PSO. it is optimizing the SMC parameter based on the synchronization error. it optimize the parameter of SMC until complete synchronization is attained. By adapting the controller's behavior based on the error dynamics, PSO not only minimizes errors but also enhances the overall synchronization process, Ensuring consistent and reliable performance in the presence of disturbances, uncertainties, or parameter variations.

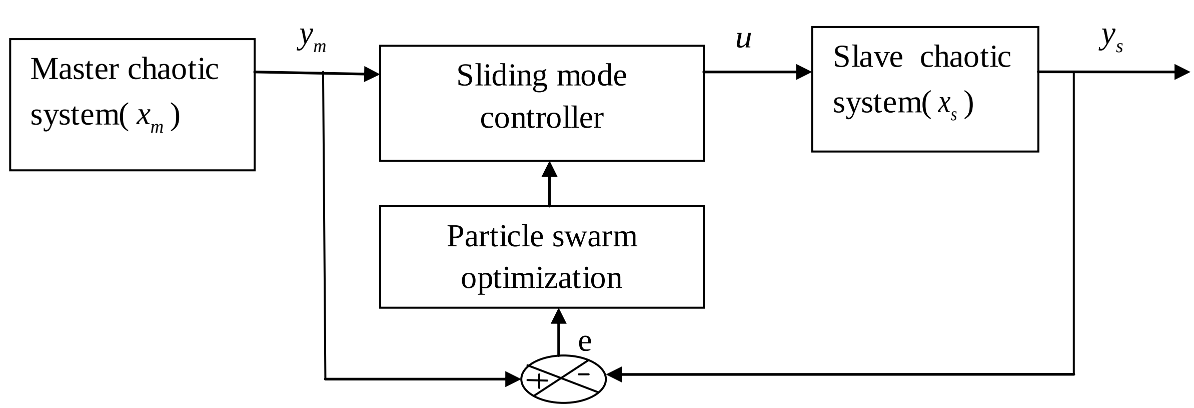

Figure 3.2 General Block diagram of the Proposed System

3.5 State Space Model of the Master and Slave Systems

3.5.1Master System

Master chaotic system is referred to as driving system because it generate the chaotic signal that have to be followed by response system. It is used as a reference for the response system. It provide chaotic behavior to which the response system must track. Figure 3.3 is used to illustrate the master chaotic system, that is designed using the method described in (Chen et al., 2008) .The master system follows the Sprott Jerk chaotic model**,** which is use electronic circuits with operational amplifiers, resistors, and capacitors in order to construct the equations.

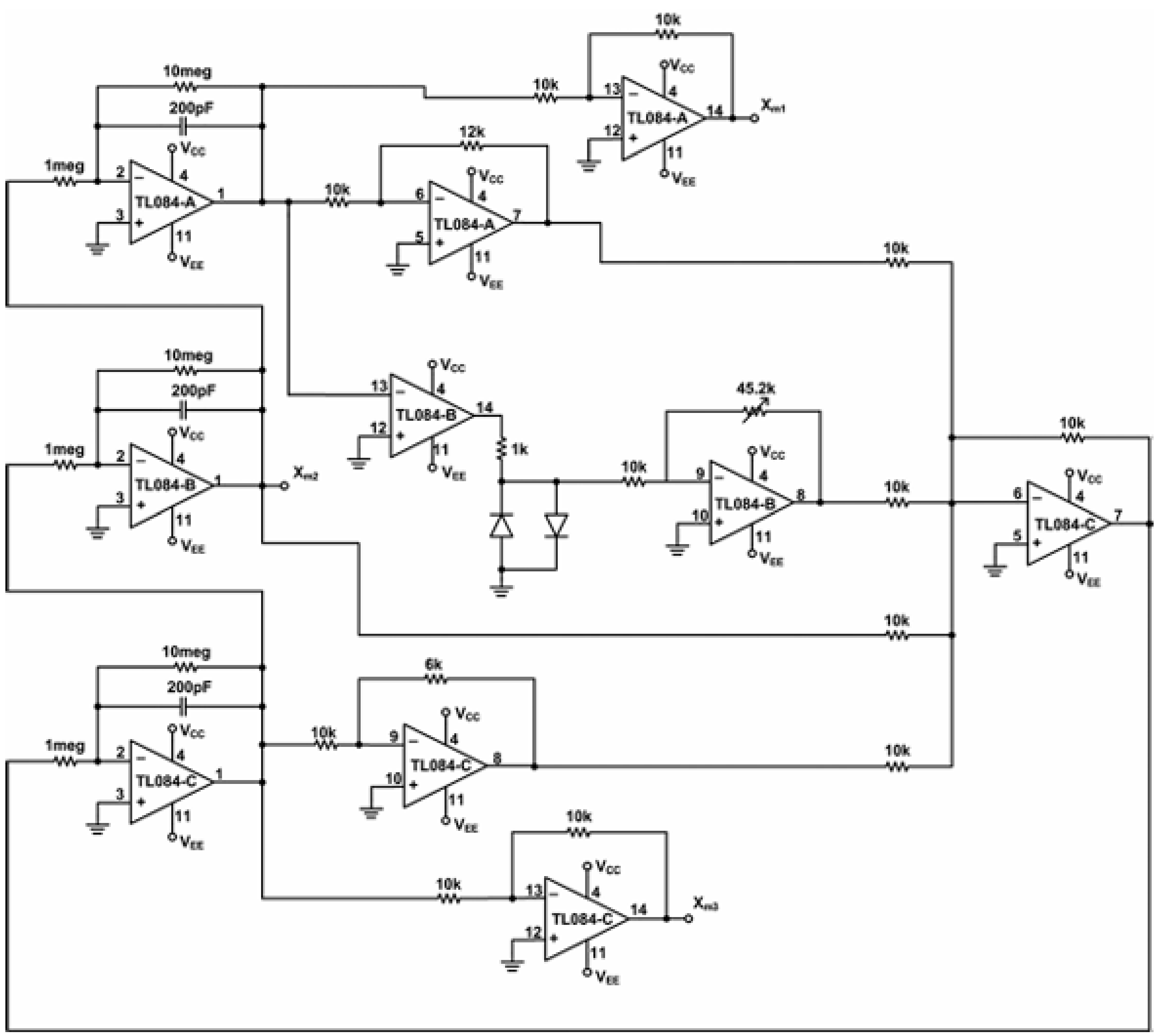

Figure 3.3 Electronic implementation of master sprott’s circuit(Chen et al., 2008)

After substituting the capacitor and resistor values the master is become like below:

$$
\begin{aligned}
\dot{x}_{m1} &= x_{m2} \\
\dot{x}_{m2} &= x_{m3} \\
\dot{x}_{m3} &= -ax_{m1} - x_{m2} - bx_{m3} + 2\,\mathrm{sgn}(x_{m1}) + d \\
y_m &= x_{m1}
\end{aligned}
\tag{3.51}
$$

Where a= 1.2 and b= 0.6, d is an unknown external disturbance or model uncertainty with a known upper bound for all $\boldsymbol{x}_m$ and t.sgn(·) is the signum function defined as

$$
\mathrm{sgn}(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}
\tag{3.52}
$$

It is represented in state-space form, as shown in Equation (3.53):

$$
\begin{bmatrix} \dot{x}_{m1} \\ \dot{x}_{m2} \\ \dot{x}_{m3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -1 & -b \end{bmatrix} \begin{bmatrix} x_{m1} \\ x_{m2} \\ x_{m3} \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} d
\tag{3.53a}
$$

$$
y_m = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \boldsymbol{x}_m
\tag{3.53b}
$$

Where $\boldsymbol{x}_m = [x_{m1}\, x_{m2}\, x_{m3}]^T$ is the state vector of the master system.

Figure 3.4 shows the phase planes of the master system with an initial vector $\boldsymbol{x}_m$= $[0.1, 0.1, 0]^T$. The plot (a) up to (c) in the Figure 3.4 is used to show the 2D projection of the system dynamic which is x_2 along x_1, x_3 along x_2 and x_3 along x_1 which is used to show how the first variables evolves in relation to the second one and plot (d) show the 3D attractor. It is used to show sensitive dependence of chaotic system on initial conditions, it clearly illustrate bounded yet periodic nature of the attractors and also it is used to show how the system evolves unpredictability through the plane. It is used to show unfolding behavior of the chaotic phenomena.

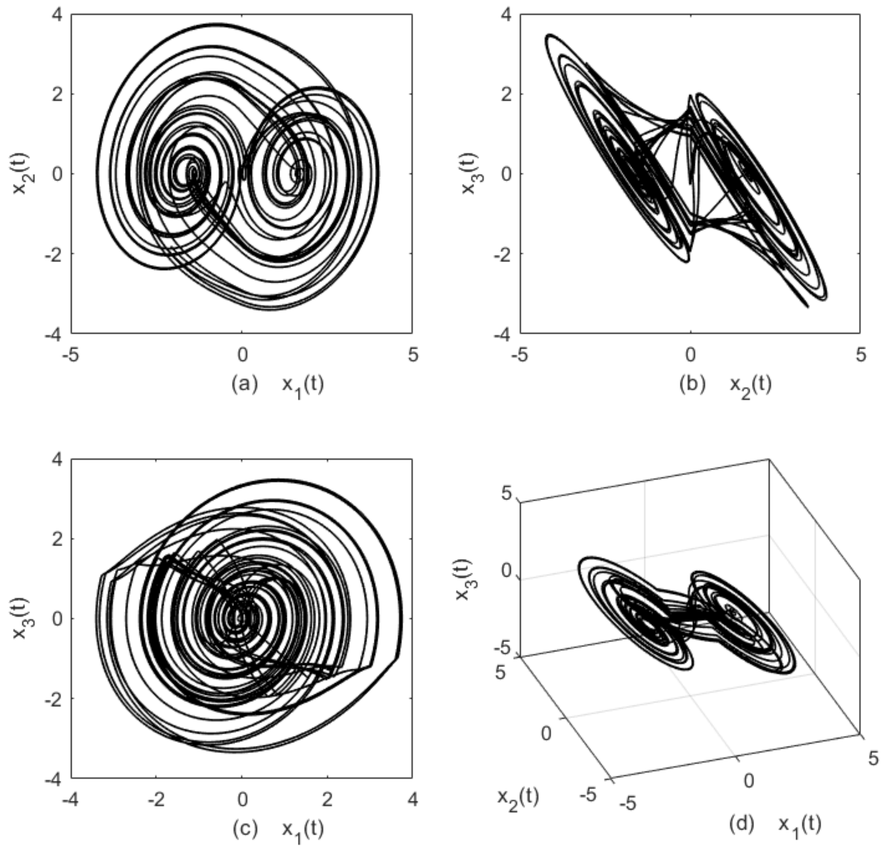

Figure 3.4 Phase plane plots of the master system

3.5.2 Slave System

It is known as response system. it use control input to synchronize its dynamic with that of the master system. Slave chaotic system sometimes referred to as driven system because it is driven by the master system to replicate its dynamic behavior. Figure 3.5 is used to illustrate the slave chaotic system, that is designed using the method described in (Chen et al., 2008) . The slave system follows the Sprott Jerk chaotic model**,** which uses electronic circuits with operational amplifiers, resistors, and capacitors in order to construct the equations.

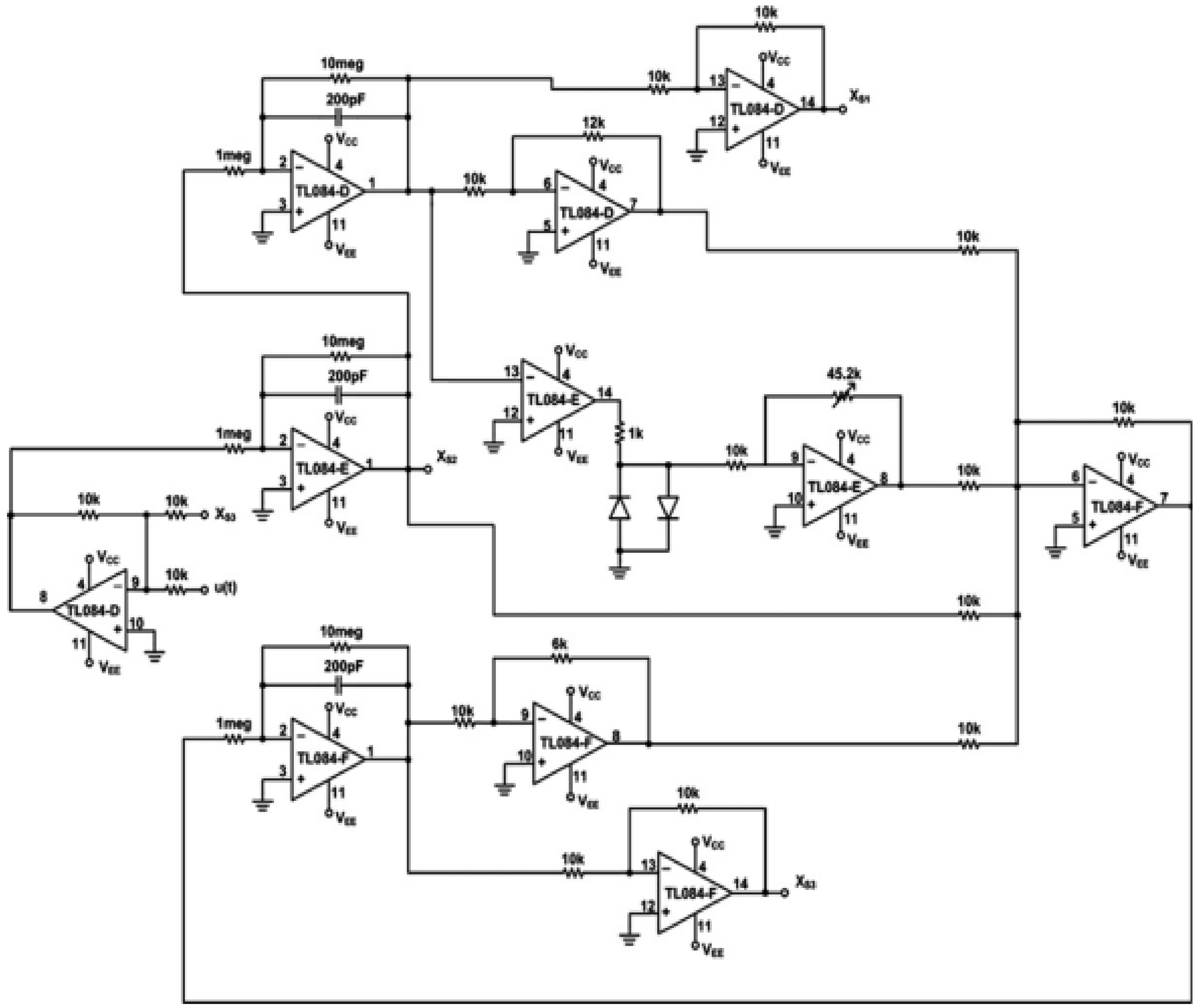

Figure 3.5 Electronic implementation of slave sprott's circuit(Chen et al., 2008)

After substituting the capacitor and resistor values the master is become like below:

$$\begin{aligned}
\dot{x}_{s1} &= x_{s2} \\
\dot{x}_{s2} &= x_{s3} + u \\
\dot{x}_{s3} &= -ax_{s1} - x_{s2} - bx_{s3} + 2\,\mathrm{sgn}(x_{s1}) \\
y_s &= x_{s1}
\end{aligned} \tag{3.54a}$$

Where u is the control input used to synchronize the slave system to the master system and is added to the second equation.

It is represented in state-space form, as shown in Equation (3.55):

$$\begin{bmatrix} \dot{x}_{s1} \\ \dot{x}_{s2} \\ \dot{x}_{s3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a & -1 & -b \end{bmatrix} \begin{bmatrix} x_{s1} \\ x_{s2} \\ x_{s3} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u \tag{3.55a}$$

$$y_s = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} x_s \tag{3.55b}$$

Where $x_s = [x_{s1}\ x_{s2}\ x_{s3}]^T$ is the state vector of the slave system.

CHAPTER FOUR

CONTROLLER DESIGN

Chaotic systems, due to their characteristics like sensitivity to initial conditions and system parameters, Inability to predict long-term behavior, present significant challenges in achieving synchronization. Therefore to synchronize chaotic systems effectively, an appropriate chaotic system is required. For this thesis, a sliding mode controller optimized by PSO is utilized to achieve efficient synchronization.

4.1 Sliding Mode Control System

Chaotic systems are nonlinear dynamical systems characterized by extreme sensitivity to initial conditions, where even minor variations can lead to drastically different outcomes. To harness this complexity for different applications, an effective control strategy is essential. Sliding Mode Control (SMC) emerges as a powerful solution, offering robustness against nonlinearities and disturbance, ensuring the desired synchronization of chaotic systems.

Sliding-mode control (SMC) is a highly effective and robust nonlinear control approach. The first step in designing an SMC is to choose a sliding surface, which essentially acts as a model for the desired closed-loop performance within the state variable space. then design the control system to ensure the system's state trajectories are forced towards this sliding surface and remain on it. The period before the system reaches the sliding surface is called the reaching phase. One of its biggest benefits is that, once it kicks in, the system becomes completely unaffected by certain types of disturbances, so it can essentially ignore them and keep going smoothly.(Utkin, 1992).

The proposed control system, based on sliding mode variable structure control, is described as follows.

$$\dot{x} = f(x, u, s) \quad \text{where} \quad x \in R^n, u = R^m \text{ and } t \in R$$

Where u is the control law, s is time, and x is the state variable.

Design a switching function for the system as follows:

$$s(x) \quad s \in R^m$$

Solve for the system control rate function u as follows:

$$u = \begin{cases} u^+ & s(x) > 0 \\ u^- & s(x) < 0 \end{cases} \tag{4.1}$$

Equation (4.1) highlights a defining feature of variable structure systems: the discontinuity in the control input u around the switching surface. For successful Sliding Mode Control (SMC) design, three key conditions must be met. First, the existence of sliding mode ensures that the system trajectory converges to and remains on the switching surface. Second, Lyapunov stability guarantees that the system dynamics on the sliding surface exhibit asymptotic stability. Finally, finite-time reachability ensures that the system trajectory reaches the switching surface within a finite time. When these conditions are satisfied, SMC effectively mitigates nonlinearities, enabling precise synchronization of chaotic systems.

To design an effective Sliding Mode Controller (SMC), we first establish the foundation by defining the master and slave chaotic systems. These systems form the basis for synchronization, where the master system serves as the reference and the slave system adapts through control inputs.

$$\begin{aligned} \dot{x}_{m1} &= x_{m2} \\ \dot{x}_{m2} &= x_{m3} \\ \dot{x}_{m3} &= -ax_{m1} - x_{m2} - bx_{m3} + 2\,\mathrm{sgn}(x_{m1}) + d \\ y_m &= x_{m1} \end{aligned} \tag{4.2}$$

Equation(4.2) represents the dynamics of the master system, serving as the reference model in the synchronization process. The master system's behavior is crucial in guiding the slave system, ensuring that both systems synchronize effectively.

$$
\begin{aligned}
\dot{x}_{s1} &= x_{s2} \\
\dot{x}_{s2} &= x_{s3} + u \\
\dot{x}_{s3} &= -ax_{s1} - x_{s2} - bx_{s3} + 2\,\mathrm{sgn}(x_{s1}) \\
y_s &= x_{s1}
\end{aligned} \tag{4.3}
$$

Equation(4.3) defines the dynamics of the slave system, which is controlled to synchronize with the master system. By applying an appropriate control law, the slave system is guided to track the master system's trajectory, ensuring precise synchronization. where u is control input.

4.2 Sliding Surface

It is referred to as switching manifold or switching surface. it is geometrical locus of points in state space where the control law switch its structure and along which the system's motion exhibit sliding behavior and also it is a function of system's state variable that represents the desired system dynamics. it is used to drives the system to desired dynamics and enhance robustness against disturbance and model uncertainties during sliding . The goal of sliding mode controller is to design control law that force the system state to reach and then stay on sliding surface. when sliding mode controller force the system state to reach and remain on sliding surface, the system is said to be in sliding mode. In order to design the sliding surface first we have to define the synchronization error as shown in below:

$$e_1 = x_{m1} - x_{s1} \tag{4.4a}$$

$$e_2 = x_{m2} - x_{s2} \tag{4.4b}$$

$$e_3 = x_{m3} - x_{s3} \tag{4.4c}$$

Once the error dynamics are derived, the sliding surface is defined as follows:

$$s = c_1 e_1 + c_2 e_2 + c_3 e_3 \tag{4.5}$$

Where c_1, c_2 and c_3 are user-defined positive constants.

4.3 Sliding Model Control Law

To design the control law, differentiate Equation (4.4), and then substitute Equation (4.3) into the result. This yields the following error dynamics:

$$\begin{aligned}\dot{e}_1 &= \dot{x}_{m1} - \dot{x}_{s1} \\ &= x_{m2} - x_{s2} \\ &= e_2\end{aligned} \tag{4.6a}$$

$$\begin{aligned}\dot{e}_2 &= \dot{x}_{m2} - \dot{x}_{s2} \\ &= x_{m3} - x_{s3} - u \\ &= e_3 - u\end{aligned} \tag{4.6b}$$

$$\begin{aligned}\dot{e}_3 &= \dot{x}_{m3} - \dot{x}_{s3} \\ &= -ax_{m1} - x_{m2} - bx_{m3} + 2\,\mathrm{sgn}(x_{m1}) + d - [-ax_{s1} - x_{s2} - bx_{s3} + 2\,\mathrm{sgn}(x_{s1})] \\ &= -a(x_{m1} - x_{s1}) - (x_{m2} - x_{s2}) - b(x_{m3} - x_{s3}) + 2\,\mathrm{sgn}(x_{m1}) - 2\,\mathrm{sgn}(x_{s1}) + d \\ &= -ae_1 - e_2 - be_3 + 2\,\mathrm{sgn}(x_{m1}) - 2\,\mathrm{sgn}(x_{s1}) + d\end{aligned} \tag{4.6c}$$

Differentiating both sides of the sliding surface equation gives:

$$\dot{s}_1 = c_1\dot{e}_1 + c_2\dot{e}_2 + c_3\dot{e}_3 \tag{4.7}$$

Substituting Equation (4.6) into Equation (4.8), we obtain:

$$\begin{aligned}\dot{s} &= c_1 e_2 + c_2(e_3 - u) + c_3[-ae_1 - e_2 - be_3 + 2\,\mathrm{sgn}(x_{m1}) - 2\,\mathrm{sgn}(x_{s1}) + d] \\ &= -c_3 a e_1 + (c_1 - c_3)e_2 + (c_2 - c_3 b)e_3 + 2c_3[\mathrm{sgn}(x_{m1}) - \mathrm{sgn}(x_{s1})] - c_2 u + c_3 d\end{aligned} \tag{4.8}$$

To ensure the existence of the sliding mode, the reaching condition is enforced as follows:

$$s\dot{s} = s\{-c_3 a e_1 + (c_1 - c_3)e_2 + (c_2 - c_3 b)e_3 + 2c_3[\mathrm{sgn}(x_{m1}) - \mathrm{sgn}(x_{s1})] - c_2 u + c_3 d\} < 0 \tag{4.9}$$

Define a control law u of the form

$$u = u_n + u_d \tag{4.10}$$

Where u_n is the nominal control that is designed to achieve desired nominal performance while u_d is a nonlinear part to induce a sliding mode and is chosen to reject the disturbance $d(t)$.

Substituting Equation (4.10) into Equation (4.9) yields as:

$$s\dot{s} = s[-c_3ae_1 + (c_1 - c_3)e_2 + (c_2 - c_3b)e_3 + 2c_3(\mathrm{gn}(x_{m1}) - \mathrm{sgn}(x_{s1})) - c_2(u_n + u_d) + c_3d] < 0 \tag{4.11}$$

The nominal control u_n is of the form

$$u_n = \frac{1}{c_2}[-c_3ae_1 + (c_1 - c_3)e_2 + (c_2 - c_3b)e_3 + 2c_3(\mathrm{gn}(x_{m1}) - \mathrm{sgn}(x_{s1})) + c_3d] \tag{4.12}$$

The discontinuous control u_d is of the form η

$$u_d = \frac{1}{c_2}\eta\,\mathrm{sgn}(s) \tag{4.13}$$

Finally, the control input is designed by rearranging equation (4.11) and solving for the control term, resulting in:

$$u = u_n + u_d$$

$$u = \frac{1}{c_2}[-c_3ae_1 + (c_1 - c_3)e_2 + (c_2 - c_3b)e_3 + 2c_3(\mathrm{sgn}(x_{m1}) - \mathrm{sgn}(x_{s1})) + c_3d] + \frac{1}{c_2}\mathrm{sgn}(s) \tag{4.14}$$

4.4 Stability Analysis

The system's stability is then thoroughly established using a Lyapunov function, which is defined for the sliding mode control system as follows:

$$V = \frac{1}{2}s^2 \tag{4.15}$$

Differentiation of the expression leads to the following outcome:

$$\dot{V} = s\dot{s} \tag{4.16}$$

Substituting Equations (4.9) and (4.14) into Equation (4.16) yields

$$\dot{V} = s\left(-\eta\,\mathrm{sgn}(s) + c_3 d\right) \tag{4.17}$$

From the fact that $s \cdot \mathrm{sgn}(s) = |s|$ and the upper bound D of d,

$$\begin{aligned} \dot{V} = -\eta s \cdot \mathrm{sgn}(s) + s c_3 d &\le -\eta\,|s| + |s c_3 d| \\ &\le -\eta\,|s| + |s| \cdot |c_3| \cdot D \end{aligned} \tag{4.18}$$

Grouping Equation (4.18) by common element |s| gives

$$\dot{V} \le -|s|(\eta - |c_3| \cdot D) \tag{4.19}$$

Choosing η such that the following conditions, we have

$$\eta \ge |c_3| \cdot D + \varepsilon \tag{4.20}$$

Where ε is a small positive constant. This confirms the fulfillment of the sliding mode reaching condition. Therefore, the feedback control system with the control law in Equation (4.14) becomes asymptotically stable.

$$\dot{V} \le -|s|\,\varepsilon < 0 \tag{4.21}$$

4.5 PID controller

A proportional–integral–derivative controller (PID controller or three-term controller) is a feedback-control loop mechanism that generates a control action which is proportional to the error which is the difference between a desired setpoint and a measured process variable. It consists of three components: the proportional term (K_p), the integral term (

K_i), and the derivative term (K_d). Within a feedback loop, the PID controller's output adjusts the plant’s input, while the plant’s output is continuously measured and fed back to compute the error, thereby establishing a closed-loop control system (geeksforgeeks, 2024).PID controller parameter is tuned by PSO. In this thesis, the PID controller is utilized solely for comparison purposes.

4.6 PSO-Based Optimization of Sliding Mode Control Parameters

To enhance the performance of the sliding mode controller and ensure robust synchronization of chaotic systems, optimization of controller parameters is crucial. In this research**,** PSO is employed to fine-tune the parameters of the SMC, including the sliding surface coefficients and the switching gain. In this thesis the PSO parameter is used to optimize the four parameters which is c_1 , c_2 , c_3 and η .PSO is Inspired by nature**,** PSO was developed by simulating the natural behavior of bird flocks searching for food in a multi-dimensional space. PSO is a population-based optimization technique in which individual solutions, called particles, move through the search space to discover the optimal parameters. Each particle adjusts its trajectory based on its own experience (pBest) and the experience of the best-performing particle in the swarm (gBest). In every iteration, particles update their velocities and positions by learning from both individual and collective experiences, thereby efficiently exploring the solution space.

$$v^i(t+1) = w(t)v_i^k(t) + C_1 r_1(pBest_i^k(t) - x_i^k(t)) + C_2 R_2(gBest^k(t) - x_i(t)) \tag{4.20}$$

$$x_i^k\left(t+1\right) = x_i^k\left(t\right) + v_i^k\left(t+1\right) \tag{4.21}$$

$$w\left(t\right) = \frac{\left(t_{\max} - t\right)\left(w_{\max} - w_{\min}\right)}{\left(t_{\max} - 1\right) + w_{\min}} \tag{4.22}$$

Here, x_i^k and v_i^k represent the position and velocity of the i-th particle at iteration k, while r_1 and r_2 are uniformly distributed random numbers in [0, 1], and C_1, C_2 are acceleration factor. The inertia weight w(t) controls the balance between exploration and exploitation.

In this research, PSO is employed to optimize the controller parameters of the Sliding Mode Control (SMC) scheme for chaotic system synchronization. Specifically, PSO is used to tune The sliding surface parameters that define the dynamics on the surface and The switching gain**,** which ensures robustness and fast reaching to the sliding surface. The optimization objective is to reduce synchronization error between the master and slave chaotic systems. The cost function J is chosen based on error performance criteria, particularly the Integral of Absolute Error and Absolute Control (IAEU)**,** given as:

$$J = \int_0^\infty (|e(t)| + w|u(t)|)dt \tag{4.23}$$

Where $e(t)$ is the difference between the master and slave system states and w is the weighting factor. The IAEU criterion is particularly suited for systems where small tracking errors must be minimized continuously over time. It penalizes errors uniformly and avoids overly aggressive control action. The optimization problem has the following short form when expressed mathematically.

Objective function:

$$\min J = \int_0^\infty (|e(t)| + w|u(t)|)dt \tag{4.23a}$$

Subjected to:

$$\boldsymbol{p}^{(L)} \leq \boldsymbol{p} \leq \boldsymbol{p}^{(U)} \tag{ 4.23b}$$

Where $\boldsymbol{p}$ is the tuning parameter vector defined as $\boldsymbol{p}= [c_1 c_2 c_3]^T$ for SMC control and $\boldsymbol{p}= [K_p K_i K_d]^T$ for PID control.

The particle swarm is initialized with random values for $\boldsymbol{p}^{(L)}$and $\boldsymbol{p}^{(U)}$ within predefined ranges. Each particle's performance is evaluated by running simulations of the synchronized system using these parameters. The particles iteratively converge to an optimal set that minimizes synchronization error. The performance of each candidate solution (particle) is evaluated using the Integral of Absolute Error (IAEU) as the cost function.

CHAPTER FIVE

RESULT AND DISCUSSION

5.1 Chapter Overview

This chapter presents comprehensive analysis and comparative evaluation of simulation results obtained from the proposed SMC and PID controllers, both optimized via PSO for chaotic synchronization. To rigorously validate the effectiveness of the proposed control algorithm, the simulation is structured into two distinct sections. the first section evaluates and compare synchronization response of SMC along with PID controller. The second section is used to verify the robustness of controller under disturbance and parameter change. The synchronization performance of each controller was comparatively assessed based on error minimization plots for the output, control input, and sliding surface. The overall simulation is carried out using MATLAB software .

5.2 Initial states of the master and slave systems

In synchronization master and slave chaotic system are initialized with different states to examine the controller's capacity to force the slave system to match the master overtime.

Table 5.1 initial state of master and slave chaotic system

Initial state	
Master chaotic system	Slave chaotic system
$x_m = \|0.1 \quad 0.1 \quad 0\|^T$	$x_s = \|0 \quad 0 \quad 0\|^T$

5.3 Settings of the Controller Parameters

5.3.1 Parameter Settings for the Proposed SMC Controller

The designed SMC has three parameters which are represented by c_1, c_2, and c_3 The particle swarm optimization technique is used for adjusting the controller parameters in order to get better tracking performance. Table 5.2 provides a comprehensive overview of the PSO technique parameter settings utilized in the simulation(Kennedy & Eberhart, n.d.; Shi & Eberhart, n.d.).

Table 5.2 Parameter settings of the PSO

Parameter		Value
Swarm size		40
Maximum iteration		30
Number of parameter		3
Inertia weight	ω_{min}	0.4
	ω_{max}	0.9
Acceleration constant	c_1	2
	c_2	2
Lower bound		[0 0 0]
Upper bound		[20 10 10]

The graphical result of the PSO technique with its best function value is shown in Figure 5.1. The minimum best function value is 0.0836645.

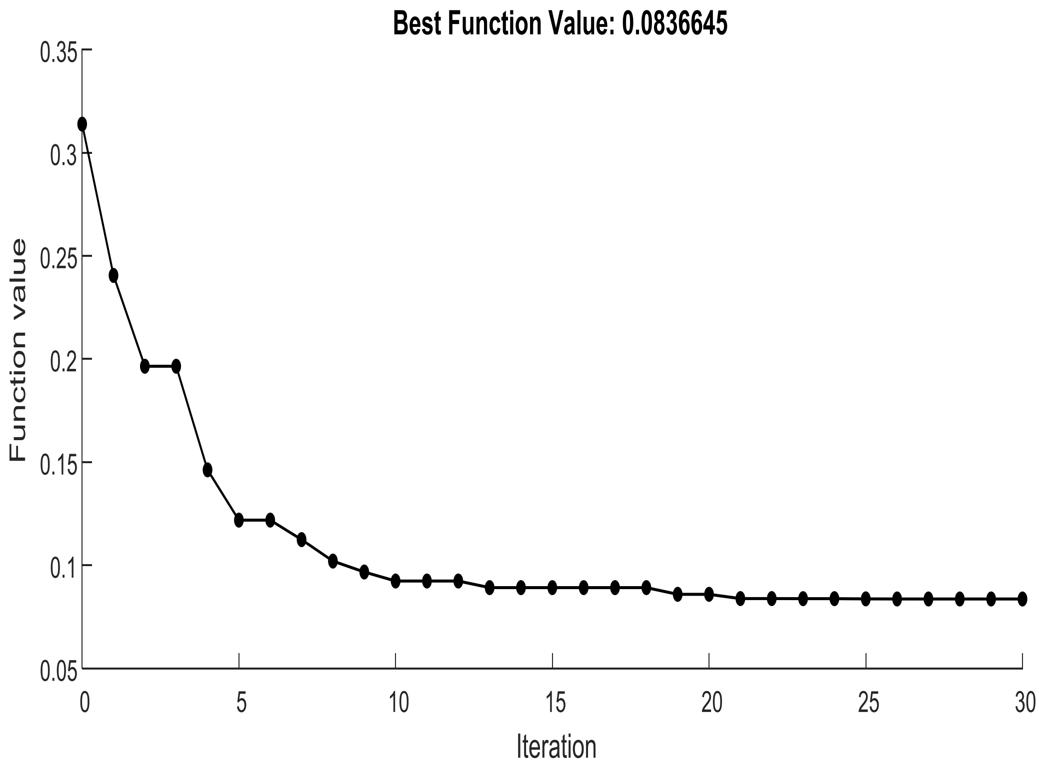

Figure 5.1 Convergence curve of the PSO algorithm

To get minimum value for the proposed objective function the optimization algorithm run for 30 iteration. Figure 5.1 illustrates a graph of the objective function vs. a number of iterations. The PSO reaches the best solution at the 21^{th} iteration. The parameter values of SMC tuned by PSO is shown in Table 5.3.

Table 5.3 Optimal values of the SMC parameters

Optimization technique	SMC parameters		
	c_1	c_2	c_3
PSO	3.4307	0.0344	0.0004

The value of 0.5 was chosen for eta based on heuristic estimation of disturbance

5.3.2 Parameter Settings for the PID Controller

The designed SMC has three parameters which are represented by K_p, K_i and K_d. The particle swarm optimization technique is used for adjusting the controller parameters in order to get better tracking performance. By using PSO The minimum best function value it get is 0.0960176**.** The parameter values of PID tuned by PSO is shown in Table 5.4.

Table 5.4 Optimal values of the PID gains

Optimization technique	PID gain		
	K_p	K_i	K_d
PSO	2.9020	0.001	11.4357

5.4 Synchronization performance Test

In this section, the synchronization performance of the SMC is compared to that of the PID controller. Their performance is evaluated by plotting the synchronization error of the output system. In addition to that the synchronization performance of the system is evaluated under disturbance and varying parameter to show robustness of the controller.

5.4.1 Synchronization Performance Under Normal Condition

This section discusses the synchronization performance of the proposed SMC and PID controller under normal conditions, which is, without external disturbance and parameter variation. This simulation result illustrate the synchronization performance of chaotic system under two control strategies which is SMC and PID. The system performance is tested in terms of synchronization error ,control input and sliding surface as show in Figure 5.2. the first plot represent the synchronization error dynamic over time. Both of them effectively synchronize master and slave by driving the synchronization error toward zero. However SMC achieve faster convergence than PID controller. The SMC drive the synchronization error at the output to negligible value within 0.6610s where as the PID controller takes around 1.0390 which clearly indicate the faster dynamic response and higher accuracy of SMC in achieving synchronization. The second plot show how the SMC quickly applies a strong control input at the beginning to quickly synchronize the system, then the control input settles with minimal fluctuation. In contrast the PID controller exhibit prolonged and more oscillatory behavior, indicating higher energy consumption and slower synchronization performance. This highlights the efficiency of SMC in delivering precise control with less input energy and faster convergence. The third plot present the sliding surface for SMC. it clearly illustrate how the surface converge to zero, This in turn, demonstrates the efficacy of the sliding mode design in achieving the desired system dynamics.

Figures 5.2 illustrate the simulation response of synchronization error, control input and sliding surface under normal condition.

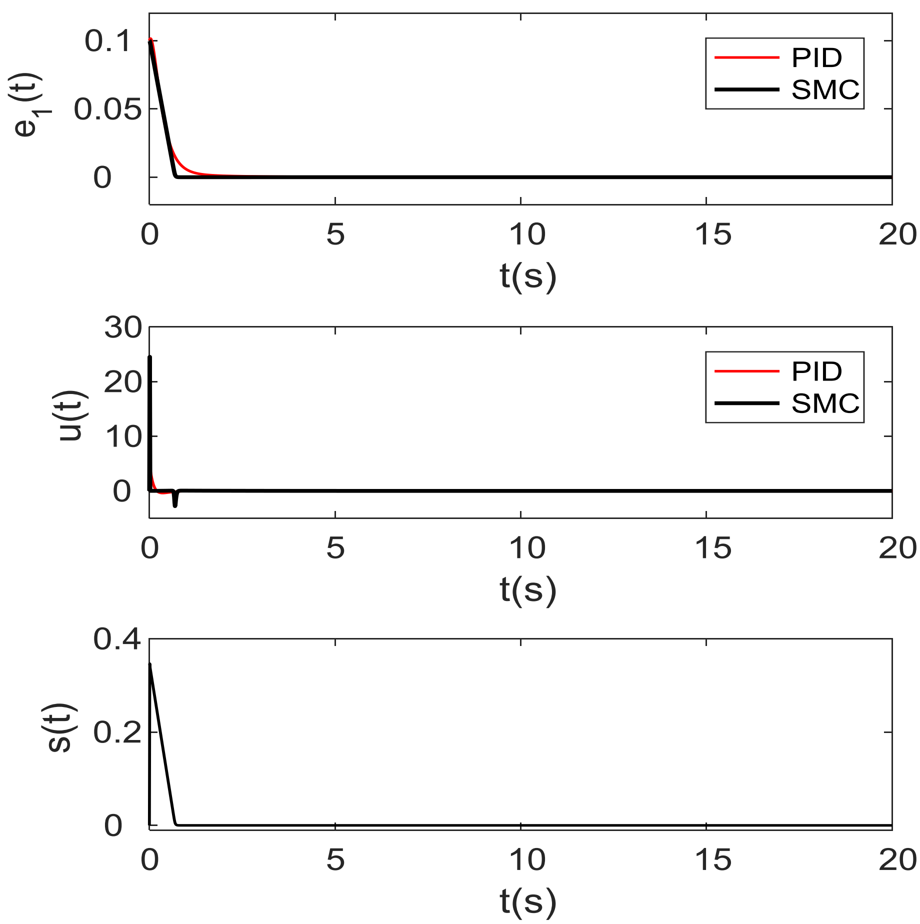

Figure 5.2 Synchronization performance comparisons between PID and SMC controllers

The synchronization performance of the proposed SMC and PID control methods is given in terms of IAEU and transient response characteristics overshoot, rise time, settling time as shown in Table 5.5.

Table 5.5 synchronisation performance specifications under normal condition

Synchronisation performance under normal condition	Performance characteristics			
	$M_p(\%)$	$t_r(s)$	$t_s(s)$	$IAEU$
SMC	0.0057	0.5489	0.6610	0.0836
PID	0.0014	0.6855	1.0390	0.0960

Table 5.5 summarize the synchronization performance of each controller in terms of their performance characteristics which is IAEU and transient response characteristics overshoot, rise time, settling time. The SMC (0.0057) exhibit higher overshoot than PID controller (0.0014) because of its aggressive initial control action, which rapidly reduce synchronization error. Specifically, it reduces the rise time by 19.92%, settling time by 36.36% and IAEU by 12%. These comparisons clearly show the accuracy of SMC in terms of achieving faster and more efficient synchronization performance despite a minimal tradeoff in overshoot. Figure 5.3 depicts the graphical representation which compares the settling time and rise time of both controllers

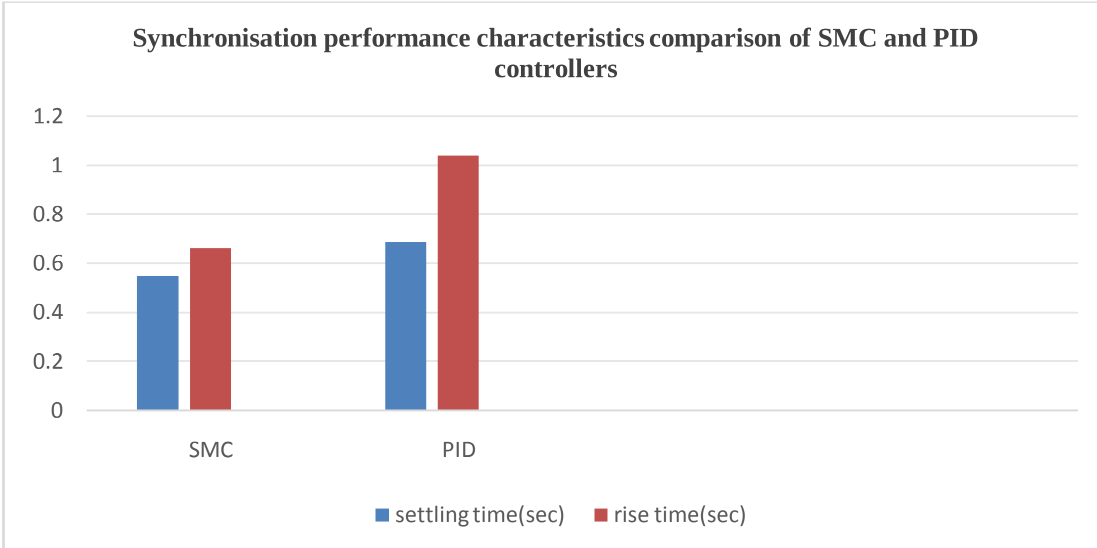

Figure 5.3 Graphical representations of SMC and PID controllers

5.4.2 Responses to Disturbance Change

This section compares the synchronization performance of both controllers under external disturbance, specifically a sinusoidal type disturbance. In practical applications, systems are often subjected to periodic disturbance due to environmental or operational factors. In this research, a sinusoidal disturbance $d= 0.3\sin(2\pi ft)$, where f=0.3Hz,is introduced at t=10s to examine the disturbance handling capacity of the proposed control scheme. Sinusoidal disturbance are used to test the system performance because of their strong practical relevance; various physical systems are subjected to periodic influence such us mechanical vibration, power line fluctuation or environmental cycles which are naturally modeled as sinusoidal signals. Furthermore, sinusoidal inputs are also applied in control theory to analyze robustness and frequency response and hence are most appropriate for controlled and reproducible testing. The selected frequency of 0.3 Hz is itself a low-frequency disturbance, common in physical systems and more difficult to reject because of its sustained nature.

Figures 5.4 illustrate the simulation response of synchronization error, control input and sliding surface under disturbance.

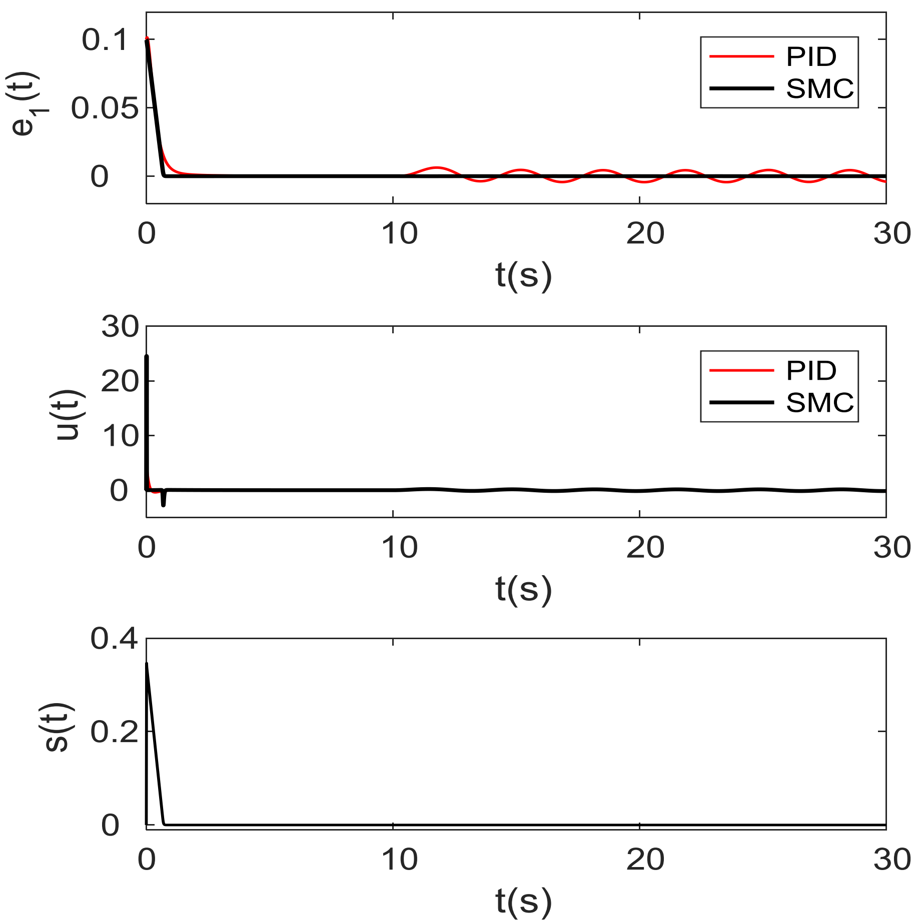

Figure 5.4 Comparison of Synchronization Responses of PID and SMC Controllers Under Disturbance

In the first plot the PID controller manage to reduce error, however, as the sinusoidal disturbance is applied at 10s, it falls to fully reject the periodic input, resulting in continuous oscillation around zero this degrade quality or synchronization precision. in contrast SMC demonstrate outstanding disturbance rejection capability. The system error converge to zero and remain at that point despite the ongoing sinusoidal disturbance .strong convergence result from the discontinuous control in SMC which forces the system back to the sliding surface despite matched disturbance. The second plot is the plot of SMC and PID controller. in order to compensate the fluctuation seen on the error the control effort required is high. This reflects reactive nature of PID controller. Such behavior, if seen in real world implementation it lead to actuator stress and inefficient energy usage. However the SMC, control input is smoother even with the presence of sinusoidal disturbance result in less control effort. it clearly indicate its ability to handle disturbance effectively without excessive control effort. The third plot represents the behavior of the sliding surface used in SMC design. The system converges to zero and remain on the sliding manifold which ensures that the controller continues enforcing the desired dynamics. Once the system is on sliding surface, the system dynamics are governed by reduced order equivalent control, ensuring robustness and accuracy in synchronization.

The synchronization performance of the proposed SMC and PID control under disturbance is given in terms of transient response characteristics overshoot, rise time, settling time as shown in Table 5.6.

Table 5.6 synchronisation performance specifications under disturbance

Synchronisation performance under disturbance	Performance characteristics		
	$M_p(\%)$	$t_r(s)$	$t_s(s)$
SMC	0.0244	0.5489	0.6610
PID	4.3502	0.6855	12.1804

Table 5.6 summarizes the synchronization performance of each controller in terms of transient response characteristics overshoot, rise time, settling time. In terms of overshoot, the proposed SMC controller significantly outperforms the PID controller by reducing the overshoot from 4.3502 to just 0.0244, which corresponds to a remarkable improvement of approximately 99.44%. It also reduces the rise time by 19.92%, settling time by 94.57%. These comparisons clearly show the accuracy of SMC in terms of achieving faster and more efficient synchronization performance and the ability of SMC to handle disturbance .Figure5.5 depicts the graphical representation which compares the overshoot, settling time and rise time of both controllers.

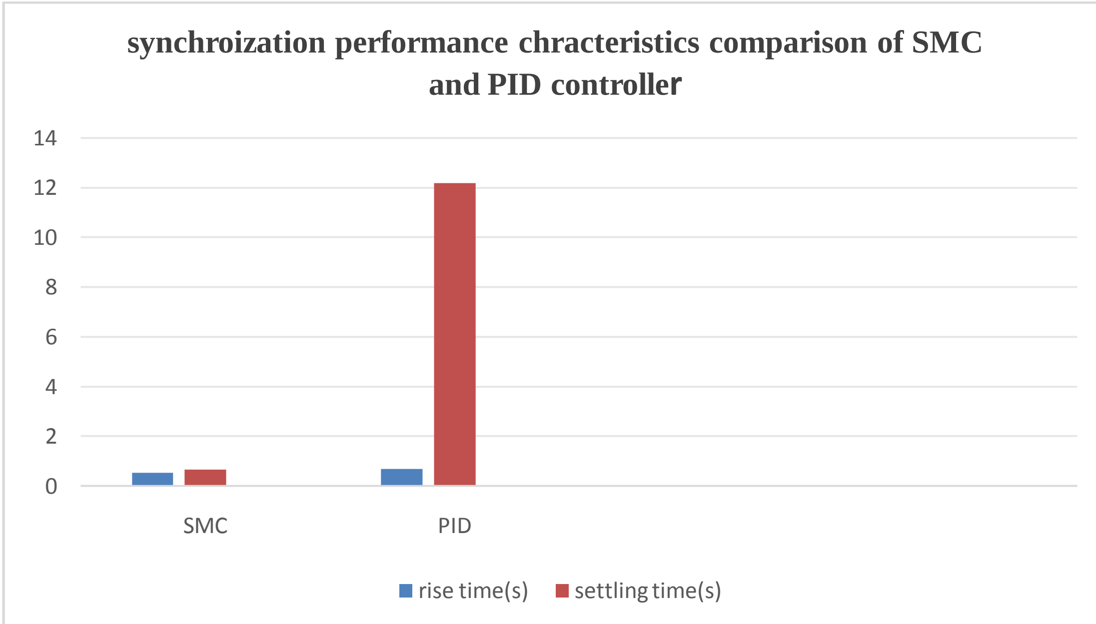

Figure 5.5 Graphical representations of SMC and PID controllers

5.4.3 Response to System Parameter Variation

5.4.3.1 Response to Increased Parameter Value

This section examines synchronization performance of proposed SMC and PID controllers when the system parameter is increased. In practical application, the parameter a can increase due to mechanical stiffening caused by vibration or thermal contraction. aging components such as capacitors or temperature dependent resistors can alter the dynamics, which effectively increasing the feedback gain represented by a. The controller's performance under such condition is important to assess its adaptability and effectiveness. The analysis considers an increase in parameter of a from 1.2 to 1.5. Figures 5.6 illustrate the simulation response of synchronization error, control input and sliding when the system parameter is increased.

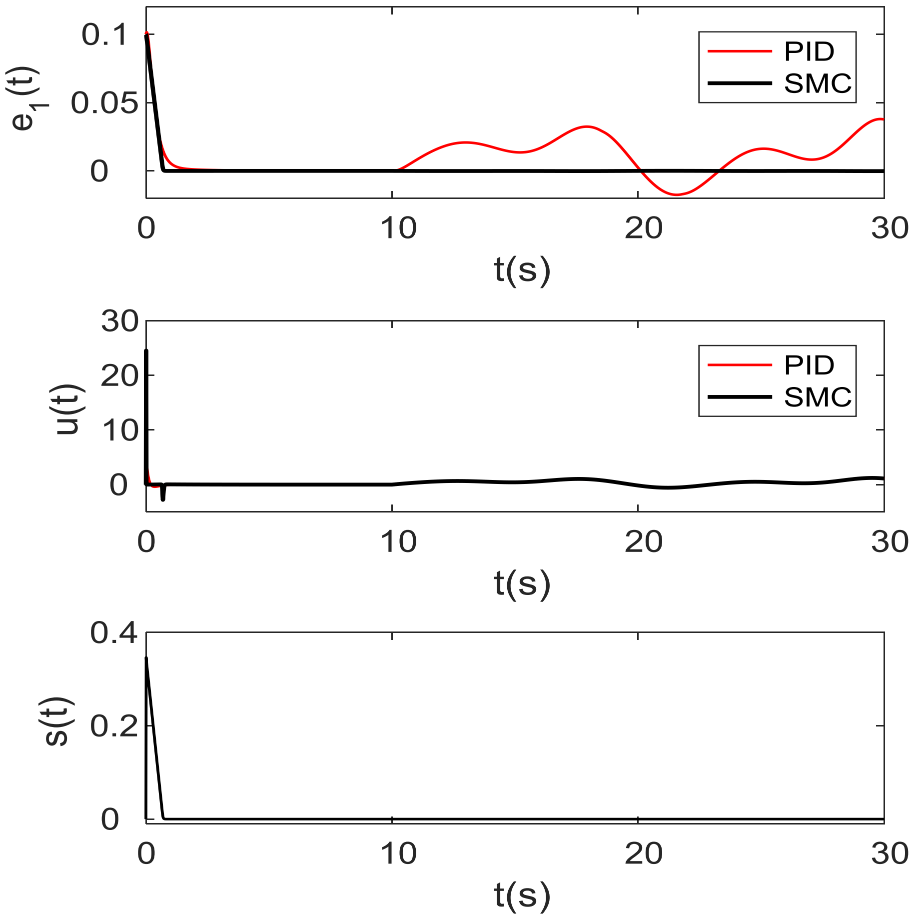

Figure 5.6 Synchronization Performance Comparisons Between PID and SMC Controllers Under Increased Parameter Value.

In the first plot is able to track desired trajectory up to 10sec. but after 10sec it fails to track the desired trajectory. The errors begin to oscillate and grow in amplitude which clearly indicating the controller is no longer able to keep the system output close to the desired trajectory. This fluctuation get worse with time, possibly due to fixed gain values in PID which fails to adapt to the parameter variation. In contrast the SMC bring the error to zero even after parameter change which lead the controller to achieve synchronization effectively. This demonstrates that SMC provides strong robustness and maintain tracking performance even when system parameter varies. The second plot show the control input of both controller .The PID controller keep injecting larger corrective inputs in order to return the system back on track. This behavior, as observed in simulation, it implies increased actuator demand, which is undesirable in physical system. However SMC produced well regulated control effort that show the controller is in control, it adjust the system effectively without overreacting or wasting energy. The third plot show sliding manifold of SMC. The SMC forces the sliding surface close to zero, and remain on that desired trajectory. This reflects robust enforcement of sliding mode condition which guarantees effective synchronization. The synchronization performance of the proposed SMC and PID control when the system parameter is increased which is given in terms of transient response characteristics overshoot, rise time, settling time as shown in Table 5.7.

Table 5.7 synchronisation performance specifications under increased Parameter Value.

Synchronisation Performance Under increased Parameter Value	Performance characteristics		
	$M_p(\%)$	$t_r(s)$	$t_s(s)$
SMC	0.1408	0.5489	0.6610
PID	17.5068	0.6855	NaN

Table 5.7 summarizes the synchronization performance of each controller in terms of transient response characteristics overshoot, rise time, settling time. In terms of overshoot, the proposed SMC controller significantly outperforms the PID controller by reducing the overshoot from 17.068 to just 0.1408, which corresponds to a remarkable improvement of approximately 99.2%. it also reduce the rise time by 19.93%.PID controller fails to settle within the defined tolerance band, resulting in an undefined settling time (NaN).Unlike the PID controller, which fails to settle under parameter variation, the SMC controller achieves a well-defined settling time of 0.6610 s. These comparisons clearly show the accuracy of SMC in terms of achieving faster and more efficient synchronization performance and the ability of SMC to parameter variation.

5.4.3.2 Response to Decreased Parameter Value

This section examines synchronization performance of proposed SMC and PID controllers when the system parameter is decreased. In practical systems, a decrease in the parameter a may arise due to mechanical wear, thermal expansion, or environmental effects like corrosion. In electrical systems, aging of components such as capacitors can lead to reduced feedback gain. Examining the controller's performance under such condition is important to assess its adaptability and effectiveness. The analysis considers a decrease in parameter of a from 1.2 to 1. Figures 5.7 illustrate the simulation response of synchronization error, control input and sliding when the system parameter is decreased.

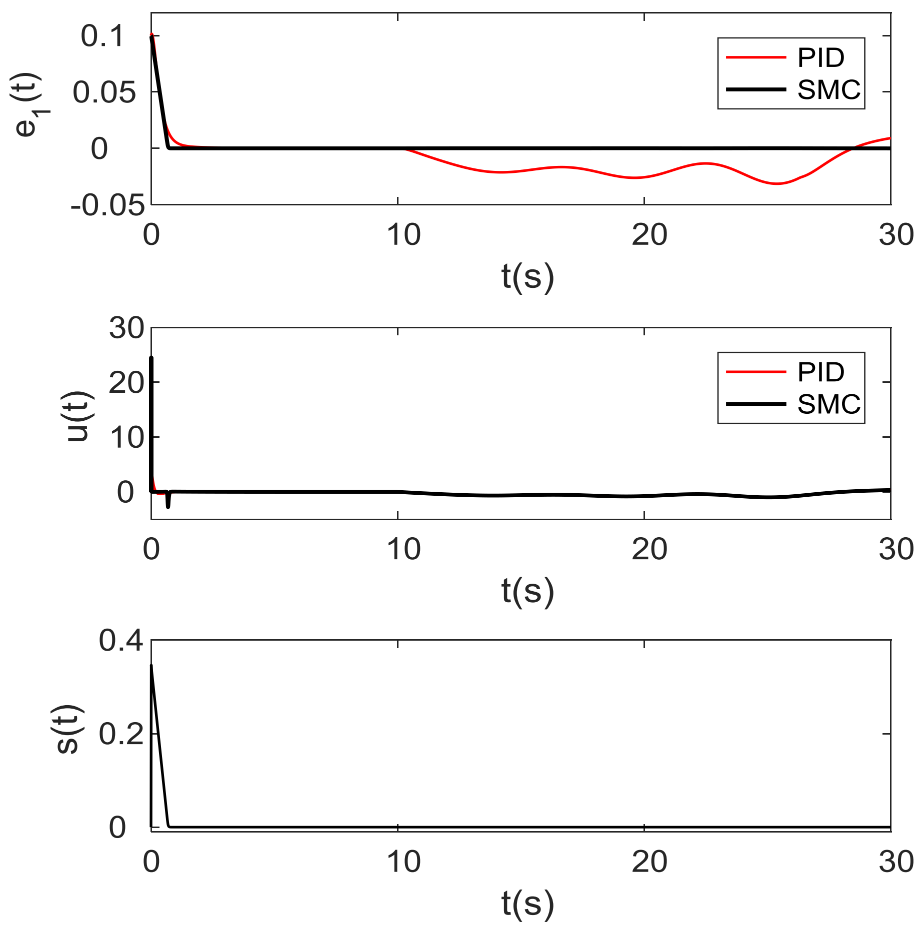

Figure 5.7 Synchronization Performance Comparison between PID and SMC Controllers Under decreased Parameter Value.

In the first plot the PID controller initially decrease parameter after 10s. it oscillates below zero line with increasing amplitude .this show poor performance of controller under parameter degradation. The system fails to settle which indicate the loss of synchronization. Whereas the SMC force the error to zero rapidly and remain in that trajectory, even in parameter reduction shows effective synchronization. The second plot show the control input of both controller .the PID controller keep injecting larger corrective inputs in order to return the system back on track. This behavior, as observed in simulation, it implies increased actuator demand, which is undesirable in physical system. However SMC produced well regulated control effort that show the controller is in control, it adjust the system effectively without overreacting or wasting energy. The third plot show sliding manifold of SMC. The SMC forces the sliding surface close to zero, and remain on that desired trajectory. This reflects robust enforcement of sliding mode condition which guarantees effective synchronization. The synchronization performance of the proposed SMC and PID control when the system parameter is decreased which is given in terms of transient response characteristics overshoot, rise time, settling time as shown in Table 5.8.

Table 5.8 synchronisation performance specifications Under decreased Parameter Value.

Synchronisation Performance Under decreased Parameter Value	Performance characteristics		
	$M_p(\%)$	$t_r(s)$	$t_s(s)$
SMC	0.0373	0.5489	0.6610
PID	31.4058	0.6855	NaN

Table 5.8 summarizes the synchronization performance of each controller in terms of transient response characteristics overshoot, rise time, settling time. In terms of overshoot, the proposed SMC controller significantly outperforms the PID controller by reducing the overshoot from 17.068 to just 0.1408, which corresponds to a remarkable improvement of approximately 99.88%. it also reduce the rise time by 19.93%.PID

controller fails to settle within the defined tolerance band, resulting in an undefined settling time(NaN).Unlike the PID controller, which fails to settle under parameter variation, The SMC controller achieves a well-defined settling time of 0.6610 s. Moreover, its ability to maintain a rapid settling response, even in adverse conditions, makes it particularly suitable for application areas where a settling time on the order of one second or less is acceptable, such as fault detection in power systems, chaos-based sensor fusion, and IoT device security. These comparisons clearly show the accuracy of SMC in terms of achieving faster and more efficient synchronization performance and the ability of SMC to parameter variation.

CHAPTER SIX

CONCLUSION AND RECOMMENDATION

6.1 Conclusion

This thesis focuses on synchronizing chaotic systems via Sliding Mode Control where parameters are optimally tuned by particle swarm optimization. The Sprott jerk chaotic system was chosen due to its lower algebraic complexity and more straightforward physical implementation.

The thesis presents a detailed mathematical model of the master and the slave system. The synchronization controller designed is based on a third-order sliding surface where the PSO is utilized to minimize the synchronization error with a cost function derived from the Integral of the Absolute Error and Absolute Control (IAEU).

The simulation results confirmed that the PSO-tuned SMC controller outperforms the classical PID controller in several aspects. The SMC achieved faster convergence, with a 36.36% improvement in settling time, allowing the system to synchronize more quickly and making it suitable for application areas that require rapid stabilization, such as secure communication and real-time signal processing. It also provided higher synchronization accuracy, with a 12% reduction in IAEU, indicating more precise alignment between the master and slave systems. The controller demonstrated strong robustness against disturbances, achieving a 99.44% reduction in overshoot and maintaining a fast settling time with an improvement of 94.57% under disturbance conditions, ensuring rapid recovery and stability even when disturbances were introduced. Additionally, it exhibited excellent resilience to parameter variations, successfully preserving synchronization even in cases where the PID controller failed.

These findings underscore the proposed controller's capacity to deliver consistent and reliable synchronization in chaotic systems, even under challenging conditions such as disturbances and parameter variations. Its robust performance and adaptability suggest that it can be broadly applied in various real-world scenarios that demand stability, precision, and rapid dynamic response. This highlights the general applicability and practical utility of the PSO-tuned SMC strategy across a wide range of nonlinear control and advanced control applications, including fault detection in power systems, chaos-based sensor fusion, and IoT device security.

6.2 Recommendation

For future research, it would be better if they investigate the proposed control scheme on such classic chaotic systems as Lorenz, Rössler, and Chen to check its generality and scalability. The mentioned systems exhibit different nonlinear dynamics and chaotic behaviors, making them ideal for checking the robustness and flexibility of the controller in switching between the chaotic regimes. Besides, installing the proposed approach in real time on hardware platforms such as microcontrollers, FPGAs, or DSPs is a must to make the simulation-based outcomes applicable. Such hardware-level validation would hence provide evidence for the effectiveness of the controllers in real-world scenarios and assess further their computational efficiency, reliability, and adaptability towards different application areas.

Research Fund Acknowledgement

This thesis work is funded by ASTU under grant number ASTU/SM-R/1111/25.

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