ADAMA SCIENCE AND TECHNOLOGY UNIVERSITY

SCHOOL OF ELECTRICAL ENGINEERING AND COMPUTING

DEPARTMENT OF ELECTRICAL AND COMPUTER ENGINEERING

Thesis on

OPTIMAL REACTIVE POWER COMPENSATION FOR LOSS REDUCTION IN ELECTRICAL DISTRIBUTION SYSTEM

Submitted

IN PARTIAL FULLFILMENT OF THE REQUIRMENTS FOR THE DEGREE OF

MASTER OF SCIENCE

IN

ELECTRICAL POWER ENGINEERING

By

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DECLARATION

I hereby declare that the work which is being presented in the thesis entitled, “Optimal Reactive Power Compensation for Loss Reduction in Electrical Distribution System” in partial fulfillment of the requirements for the award of Master of Science in Electrical Power Engineering submitted in Electrical and Computer Engineering Department of Adama Science and Technology University, Adama, is the result of my own work, except refer the cited references, carried out under the guidance of my Advisor, Molla Biweta(Assistant Professor).

Signature: ___________________________

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This is to certify that the above statement made by the candidate is correct and true as I have read the thesis and guide him as well and hence in my opinion the thesis is fully adequate in scope and quality for the award of Master of Science.

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APPROVAL BY BOARD OF EXAMINATION

The following committee members have examined the final copy of this thesis for form and content, and recommend that it be accepted in partial fulfillments of the requirements for degree of Master of Science in Electrical Power Engineering.

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ABSTRACT

The reactive component of the current in an electrical line causes power losses, voltage drop and thus reduce the transfer of active power. This thesis presents the way of improving the distribution system performance by minimizing the reactive current flowing through the distribution networks with optimally allocating shunt capacitors. The optimal shunt capacitor allocation problem is the determination of the location and size of the capacitor to be placed in distribution networks in an optimal manner to reduce the energy losses and peak power losses of the networks. Single capacitor is connected at each bus and repetitive loss minimization technique is applied to identify the nodes to be compensated. Optimizing the loss saving equations with respect to the capacitor size for multiple capacitors is used to identify the global capacitor size needed for compensation. To obtain the energy savings, a power flow for radial distribution network is executed using MATLAB programs before and after compensation. The study of this work was conducted on primary distribution feeder having 34 buses of Maichew town power distribution system. Through optimal locating and sizing of shunt capacitors on 34 buses radial distribution feeder 35.447854% of the total loss is reduced.

Key words: Radial distribution system, reactive power, power factor, shunt capacitor bank, power loss.

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LIST OF SYMBOLS AND NOTATIONS

IM	Branch to node incidence matrix
I_a	The active component of branch currents
G^c	nxk Matrix derived from G
C	kxk Square matrix
V_i	The line voltage of bus i in KV
I_r	The reactive component of branch currents
p_{La}	The total real power loss due to the active component of branch currents
D	k Dimensional vector
G	Node-to-branch incidence matrix
X	Reactance
R_j	The j^{th} branch resistance
Q_{ci}	The capacitor required in the i^{th} node in Kvar
I_{ij}	The current flows from bus i to bus j (the current flows in branch j)
Q_{copt}	The global optimum capacitor size
I_c	The capacitor current
I'_r	The reactive component of branch currents after a single capacitor placement
S_i	The total complex power demand at bus i
p_{Lr}	The total real power loss due to the reactive component of branch currents

Δp_{LS}	The total real power loss savings due to a single capacitor insertion
S_{Loss}	Total power loss in the feeder
p_{TLoss}	Total real power loss
$\Delta p'_{LS}$	Total real power loss savings due to multiple node compensation
p'_{Lr}	The total real power loss due to the reactive component of branch currents after a single capacitor is connected in the system
I''_{r}	The total reactive component of branch currents after multiple nodes are compensated
ε	Tolerance value
k	Number of capacitor buses
n	Number of branches

1. INTRODUCTION

1.1 Background

Electrical energy is the basic necessity for the economic development of a country. The modern society is so much dependent up on the use of electrical energy that it has become a part and parcel of our life. Electrical energy is produced at the power station from energy resources available in various forms in nature. The basic resources are sun, wind, water, fuels, nuclear, and geothermal energy. The bulk power is transferred by high voltage links between the main load centres called transmission lines and the distribution system is mainly responsible for the conveyance of this power to the consumers by means of lower voltage networks.

At power station electric power is generated in the range of 11 kV to 25 kV, which is increased by stepped up transformers to the main transmission voltage. At substation, the connections between various components are made, for example, lines and transformers and switching of these components is carried out. Transmission level voltages are in the range of 66 kV to 400 kV (or higher). In Ethiopia, high voltage transmissions operate at 400 kV, 230 kV, 132 kV, 66 kV and 45 KV, while the medium voltage distribution is in 33 kV and 15 kV supplying the final consumer feeders at 380 volt three phase, giving 220 volt per phase.

The power supply network can be divided in to two parts, which are transmission and distribution systems. The transmission systems may be divided in to primary and secondary transmission system. Distribution systems can also be divided in to primary and secondary distribution system. Most of the distribution networks operate radially for less short circuits current and better protective coordination. In addition to voltage magnitude, distribution networks are different from transmission networks in many ways. The general structure or topology of the distribution system is different and the number of branches and sources is much higher.

A typical distribution system consists of a step down transformers at a bulk supply point feeding a number of lines with varying length from a few hundred meters to several kilometres. Several three phase step-down transformers, e.g. 15 kV/380 V are spaced along the feeders and from these, three phase four wire networks of consumers are supplied which give 220 volt single phase supply to houses and similar loads.

Many communities, in both developed and developing countries are seriously concerned with the problems stemming from energy production and use. Distribution losses are inevitable consequence of the transport of electrical energy from power stations to consumers. Losses occur firstly at power stations and then across the transmission grid and distribution networks. Distribution system losses are the largest contributor to the total losses. According to the discussions held on January 2008 by electricity commission on " reducing distribution system loss", by the year 2005, the percentage losses on its transmission and distribution networks of New Zealand, Sweden, USA, United Kingdom, and Finland is 8.6%, 7.6%, 6.5%, 7%, and 4.5% respectively.

Nowadays, it is acknowledged that the reduction of losses especially in the distribution networks, leads to power and energy cheaper than building new capacities of production and transmission. Energy efficiency is accepted by the specialists as the most available, the least pollutant and cheapest resource of all. In order to explain this it was proved that 1 installed KW is four times more expensive than 1 saved KW [1]. In practice, energy transfer efficiency reduced due to the phase shift between voltage and current at the fundamental frequency. The phase shift arises largely due to energy flows characterizing electric motors that dominate the aggregated load. Reactive currents in an electric utility distribution system produce losses and result in increased ratings for distribution components. Shunt capacitors are commonly used in distribution systems for several reasons, in particular in order to reduce power losses, to improve the voltage profile along the feeders and to increase the maximum flow through cables and transformers. These benefits depend greatly on how capacitors are placed in the system. The general capacitor placement problem is how to optimally determine the locations to install capacitors and sizes of capacitors to be installed in the buses of radial distribution systems.

1.2 Motivation

It is well known that there are an all round energy crises all over the world and efforts are being made to conserve energy during the stages of conversion, transmission, distribution and at end use equipment. Studies have showed that reduction of losses in the power networks is much more beneficial than the increase of generating capacities. Distribution networks are well known for their high R/X ratio and significant voltage drop that could causes substantial power losses along the feeders. It is estimated that as much as 13% of the total power generation is lost in the distribution networks. Portion of this loss is caused by the reactive current flowing in the network [2]. Reactive currents in an electrical utility distribution system produce losses, result in increased ratings for distribution components, reduced system voltage and capacities. As the industrial and overall economy of the country grows, the electrical energy demand increases proportionally. To fill the gap between energy demand and supply, the country has to go for both additional power generation and improving the overall efficiency at both utility and consumer sides. This thesis is expected to increase our knowledge base about the importance of loss reduction on distribution networks both on the utility and consumers side for efficient utilization of power.

1.3 Problem Statement

Most of the electrical equipments connected to a power supply not only required active power but also certain amount of reactive power. Magnetic fields in motors and transformers are maintained by reactive currents and series inductance in transmission lines implies consumptions of reactive currents. Circulation of reactive energy has major technical and economical consequences. Keeping active power constant, a higher reactive power results a higher apparent power and hence a higher current must be supplied from the substation.

Thus, due to a higher supplied current, circulation of reactive energy on distribution networks results in:

- Higher investment costs, because generators, transformers, power lines, switchgear, and cables must be sized for a greater power ratings than it would have been needed for loads on active power
- Additional power losses proportional to the square of the current
- Reduce equipment life due to temperature rise of the main components such as: cables, transformer, etc
- Increased voltage drops along the transmission and distribution lines

For this reasons, reactive energy must be produced as close as possible to the loads, to prevent the unnecessary circulation of currents in the network. One of the most effective and useful methods in reducing the power losses of distribution networks due to reactive energy, is utilization of optimal capacitor placement. By using shunt capacitors, the reactive power needed for loads is provided so that besides the reduction of losses the voltage profile of nodes is also improved.

In context of economic activities increasing in recent years in Ethiopia, results in rapid growth in electrical demand, especially in commercial and industrial sectors. Most industrial and commercial loads and delivery apparatus are inductive in nature and therefore operate at lower power factor angles. Therefore, the power system requires additional reactive power flow, which results in higher I^2R loss in distribution networks, reduced the system capacity to supply the load and the system voltage at the load end and in its turn, result in miss operation of equipments and decrease working life time.

Nowadays, the few remnant natural forests are cut down and used as fire woods for cooking in most rural and many urban families due to lack of available or affordable supply of electric power. Burning wood or charcoal, without reforestation, increases the carbon dioxide emission into the atmosphere. Atmospheric carbon dioxide is also increased from use of fossil fuel such as diesel in generators during power failure or at places where electric power is not available from the national grid. The carbon dioxide released into atmosphere contributes to the global warming (climate change).

1.4 Objectives

Reduction of total I^2R losses in distribution system is very important to improve the overall efficiency of power delivery. This can be achieved by placing the optimal value of capacitors at proper locations in radial distribution feeder.

General Objective

The general objective of this thesis is to study and analyze distribution network to reduce power loss owing to increasing of reactive power demand.

Specific Objectives

Power and energy losses due to high load demand in distribution system are need to be reduced using shunt capacitors on primary distribution system. Thus the specific objective of this thesis is mainly focused on the following points.

- To determine type, size and electrical parameters of the feeder line,
- To determine the optimal location of the capacitors,
- To determine the appropriate size of capacitors needed,
- To analyze the system with and without capacitors and
- To make recommendation and conclusion for implementation

1.5 Methodology

In order to make on analysis on distribution networks and determine the parameters for capacitor placement, primary and secondary data have been used in the course of this thesis work.

Ladder iterative power flow algorithms are implemented in the radial distribution feeder before compensation using MATLAB computer programs to determine the total power losses, the voltage magnitude and angles at each bus and the currents flowing through each branch. To identify the nodes to be compensated repetitive applications of loss minimization technique is applied. Global size of capacitors for multiple nodes for capacitor placement is determined by optimizing the loss saving equations with respect to the capacitor kilo volt ampere reactive (Kvar) sizes.

This technique is easy to apply and helps to identify the size of capacitors and locations for maximum loss reductions. To determine the benefits obtained from the installation of capacitors in primary distribution networks, the power flow analysis programme run after the identified nodes compensated with the appropriate capacitor size selected.

1.6 Description of Study Area

This thesis is conducted in Maichew town southern part of Tigray region. The study area is selected for the following reasons

- There is demand growth in industrial and commercial sectors which results high reactive current flows.
- There is no study conducted to reduce loss in distribution networks
- Measures taken to reduce reactive power flow from the generation by supplying it near to the load using capacitors are less.

Maichew town is located in northern part of Ethiopia, in Tigray National Regional State, Southern Tigray Zone at a Distance 665 Km from Addis Ababa, and 127 Km from Mekelle the regional capital. Its astronomical location is $12^0$47’ North Latitude & 39^0 32’ East Longitude.

Maichew town was founded in 1892. The town is a populated place and located at an elevation of 2,340 meters above sea level and its population amounts to 30, 186.

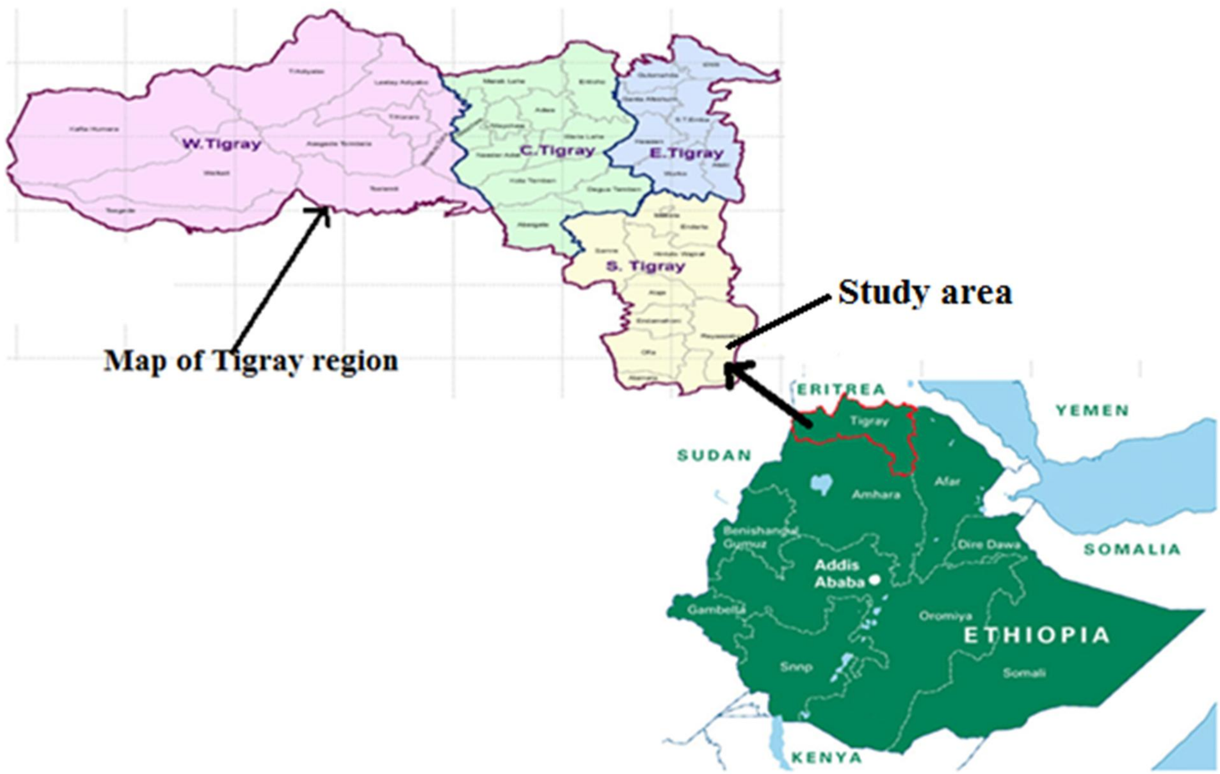


Figure 1.1 Map of the study area

Maichew town is electrified in the year 1990 with installed substation capacity of 6.3MVA. The substation receives 66KV from Alamata town substation. There is one step-down power transformer in the substation. This transformer steps down 66KV to 15KV for primary distribution. From this transformer four feeders are emanated, named as ‘Adesho’, ‘Mehoni’, ‘Chipud’, and ‘Maichew’.

For the study of this thesis the Maichew town feeder is selected because it

- has larger demand at the substation
- contains most industrial and commercial loads which causes much reactive power flow through the feeder
- incorporates all kind of loads (industrial, commercial and domestic loads)

The feeder in the town is configured radially with voltage level of 15KV. This feeder is connected to a total of more than 35 distribution power transformer; most of them are pole mounted, for further step down to 380V three-phase and 220V single-phase for secondary distribution purpose.

In the town there is expansion of loads (industrial, commercial, residential and street lights) on the existing substation. The town gets electric power from hydropower only. Out of the total load supplied by the feeder about 21.7% is industrial, 38.3% is commercial, 39.4% is domestic and 0.6% is street lightings.

1.7 Thesis Organization

This thesis is organized as follows: chapter 1 provides description about background, motivations, problem statement, objectives, research methodology, and the study area. Literature review is obtained in the second chapter which includes descriptions about electrical power distribution system, classification of distribution system, connection schemes of distribution system, line impedances of overhead distribution system, requirements of a distribution system and distribution system loss reduction techniques. Reactive power compensation and installation of shunt capacitor to distribution system are also discussed in chapter 2. In chapter three data collection and analysis are provided. Here distribution feeder analyses including power flow analysis and distribution system components models are presented. In this chapter the ladder iterative power flow algorithms are given. Chapter 4 considers optimal placement of capacitors on radial distribution system. In this chapter the methods used to identify the optimal location and size of capacitors are given. In chapter 5 results along with explanation for electrical energy savings and socio-economic benefits obtained as a result of loss reduction using optimally located and size of shunt capacitor is presented. In the last chapter conclusions and recommendations for implementation and further research are given. Resources used for this work are given on the references section. Finally the appendices which include graphs, derivations and mathematical calculations are provided.

2. LITERATURE REVIEW

2.1 Electrical Power Distribution System

An electric supply system consists of three principal components. That is, the power station, the transmission lines and the distribution system. Electric power is produced at the power stations which are located at suitable places, generally away from the customers. It is then transmitted over large distances to load centers with the help of conductors known as transmission lines. Finally, it is distributed to a large number of small and big customers through a distribution network.

The electrical energy produced at the generating station is conveyed to the consumers through a network of transmission and distribution systems. That part of power systems which distributes electric power for local use is known as distribution system. In general, the distribution system is the electrical system between the substation fed by the transmission system and the consumer meters.

2.2 Elements of Distribution System

It generally consists of feeders, distributors and the service mains. Figure 2.1 shows the single line diagram of a typical low tension distribution system.

(i) Feeders. A feeder is a conductor, which connects the substation (or localized generating station) to the area where power is to be distributed. Generally, no tapping are taken from a feeder so that currents in it remain the same throughout. The main consideration in the design of a feeder is the current carrying capacity.

(ii) Distributors. A distributor is a conductor from which tapping are taken for supply to the consumers. In figure 2.1 AB, BC, CD and DA are the distributors. The current through a distributor is not constant because tapping are taken at various place along its length. While designing a distributor, voltage drop along its length is the main consideration since the statutory limit of voltage variation is ± 6% of the rated value at the consumer's terminals [3].

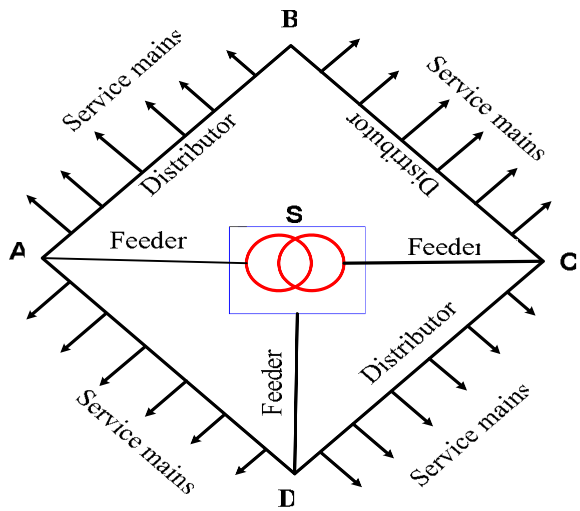


Figure 2.1 Single line diagram of a typical low tension distribution system

(iii) Service Mains. A service main is generally a small cable which connects the distributor to the consumer's terminal.

2.3 Classification of Distribution Systems

A distribution system may be classified on the basis of:

(i) Nature of Current: According to nature of current, distribution system can be classified as

(a) Direct current (D.C) distribution system (b) Alternating current (A.C) distribution system.

Now-a-days a.c system is universally adopted for distribution of electric power as it is simpler and more economical than direct current method.

(ii) Types of Construction: According to type of construction, distribution system is classified as

(a) Overhead system (b) underground system

The overhead system is generally employed for distribution as it is 5 to 10 times cheaper than the equivalent underground system [3]. In general, the underground system is used at places where overhead construction is impracticable or prohibited by the local low.

(iii) Scheme of operation: According to scheme of operation, distribution system may be classified as: (a) radial delivery network (b) ring main system (c) interconnected system. Each scheme has its own advantages and disadvantages.

2.3.1 Alternating current distribution

Nowadays electrical energy is generated, transmitted and distributed in the form of alternating current. One important reason for the wide spread use of alternating current in preference to direct current is the fact that alternating voltage can be conveniently changed in magnitude by means of a transformer. The a.c distribution system is classified in to (i) primary distribution system and (ii) secondary distribution system, which are described briefly hereunder.

(i) Primary Distribution System: Electrical power from the generating station is transmitted at high voltage to the substation located in or near the city. The transmission system voltage is stepped down to lower levels by distribution substation transformers. The primary distribution system, which is the concern of this thesis, that portion of the power network between the distribution substation and the utilization transformers. It is that part of a.c distribution system which operates at voltages somewhat higher than general utilization and handles large blocks of electrical energy than the average low voltage consumer uses. The voltage used for primary distribution depends up on the amount power to be conveyed and the distance of the substation required to be fed. The most commonly used distribution voltages are usually in the range of 4 to 34.5 KV. The primary distribution voltages used by Ethiopian Electric power corporation

(EEPCO) are 33 and 15 KV. Due to economic considerations, primary distribution is carried out by 3-phase, 3-wire system.

(ii) Secondary Distribution System: it is that part of a.c distribution system which includes the range of voltages at which the ultimate consumer utilises the electrical energy delivered to them. The secondary distribution employs 380/220V, 3-phase, 4-wire system. The primary distribution circuit delivers power to various substations, called distribution substations. The substations are situated near the consumer's localities and contain step down transformers. At each distribution substation, the voltage is stepped down to 380/220V and power is delivered by 3- phase, 4-wire a.c. system.

2.3.2 Overhead distribution system

The distribution system can be overhead or underground. Overhead lines are generally mounted on wooden, concrete or steel poles which are arranged to carry distribution transformers in addition to the conductors. The underground system uses conduit, cables and manholes under the surface of streets and sideways. The choice between overhead and underground system depends up on a number of widely differing factors. The advantages of overhead and underground are given in table 2.1

Table 2.1 Overhead vs. underground: Advantage of each

Overhead	Underground
Cost: significantly less cost, especially initial cost	Aesthetics: much less visual clutter
Flexibility	Safety: less chance for public contact
Reliability: shorter outage durations because of faster fault finding and faster repair	Less voltage drop because reactance is lower due to closer spacing of conductors
Loading: overhead distribution conductor has higher current capacity	Low maintenance cost because of less chance to fault

2.3.3 Connection schemes of distribution system

(i) Radial System: In this system, separate feeders radiate from a single substation and feed the distributors at one end only. Figure 2.2(a) shows a single line diagram of a radial system for DC system. Distribution where a feeder OC supplies a distributor AB at point A. Obviously, the distributors are at one point only i.e. point A in this case. Figure 2.2(b) shows a single line diagram of radial system for AC distribution. The radial system is employed only when power is generated at low voltage and the substation is located at the centre of load.

This is the simplest distribution circuit and has the lowest initial cost. However, it suffers from the following drawbacks

(a) The end of the distributors nearest to the feeder point will be heavily loaded

(b) The customers are dependent on a single feeder and single distributor. Therefore, any fault on the feeder distributor cuts off supply to the customers who are on the side of the fault away from the substation

(c) The customers at the distant end of the distribution would be subjected to serious voltage fluctuations when the load on the distributor changed.

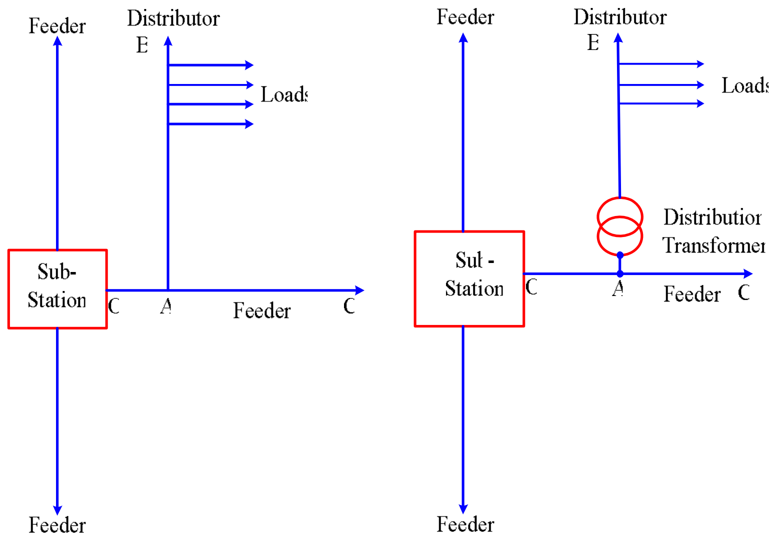


Figure 2.2 Radial distribution systems

(ii) Ring system: In this system, the primaries of distribution transformers form a loop. The loop circuit starts from the substation bus-bars, makes a loop through the area to be served, and returns to the substation. Figure 2.3 shows the single line diagram of ring main system for a.c distribution where substation supplies to the closed feeder LMNOPQRS. The distributors are tapped from different points M, O, and Q of the feeder through distribution transformers.

The ring main system has the following advantages:

(a) There are less voltage fluctuations at consumers terminals

(b) The system is very reliable as each distributor is fed via two feeders. In the event of fault on any section of feeder, the continuity of supply is maintained. For example, suppose that fault occurs at any point F of section SLM of the feeder. Then section SLM of the feeder can be isolated for repairs and at the same time continuity of supply is maintained to all the customers via the feeder SRQPONM.

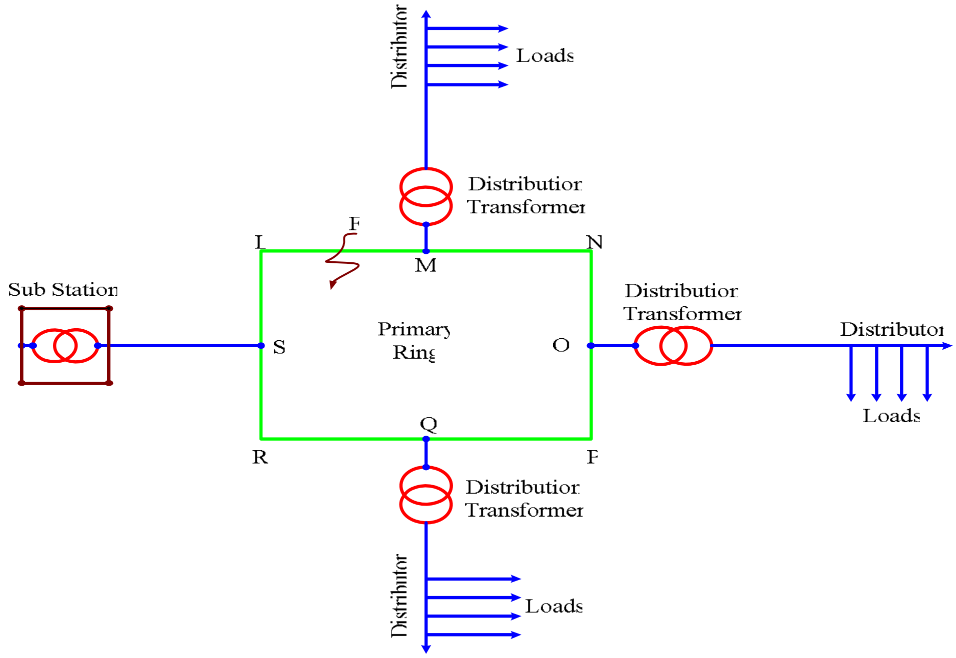


Figure 2.3 Ring main systems

(iii) Inter Connected System: When the feeder ring is energized by two or more than two generating stations or substations, it is called inter connected system. Figure 2.4 shows a single line diagram of inter connected system where the closed feeder ring ABCD is supplied by two substations S1 and S2 at point D and C respectively. Distributors are connected to points O, P, Q and R of the feeder ring through distribution transformers.

The inter-connected system has the following advantages:

(a) It increases the service reliability

(b) Any area fed from one generating station during peak load hours can be fed from the other generating station. This reduces reserve power capacity and increase efficiency of the system

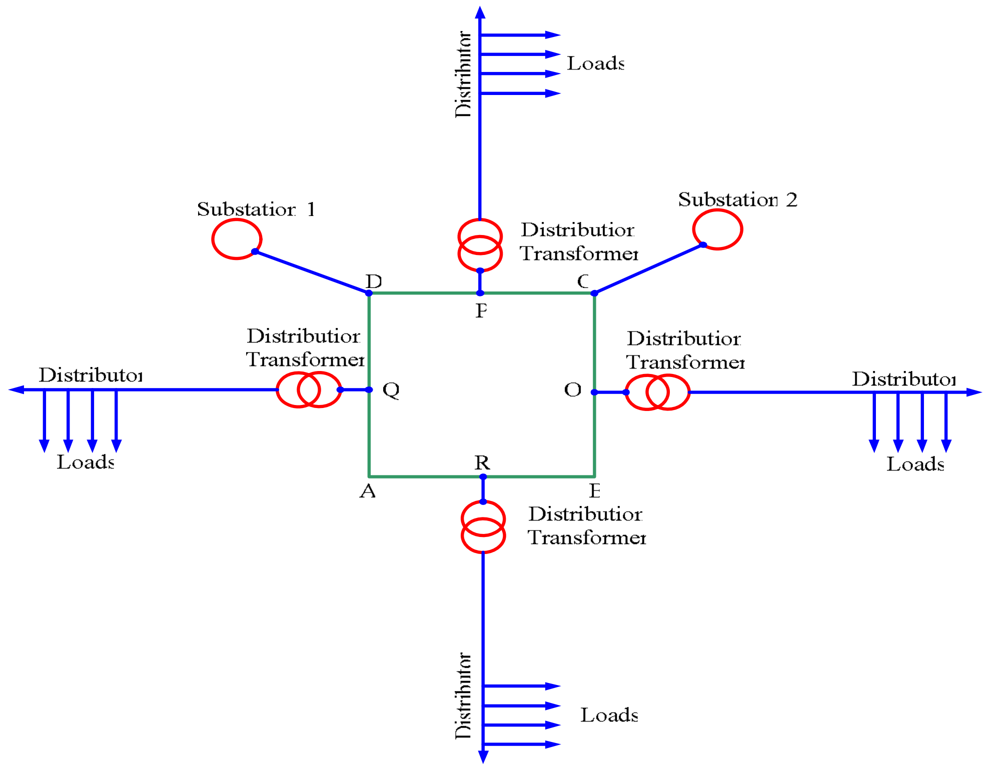


Figure 2.4 Inter connected system

2.4 Series Impedances of the Overhead lines for Distribution System

Overhead lines have resistance and reactance that impedes the flow of current. These impedancevalues are necessary for voltage drop, power flow, short circuit, and line loss calculations.

Depending up on the degree of accuracy required, impedances can be calculated using Carson – Clem's equations where no assumptions are made, or the impedances can be determined from tables where a wide variety of assumptions are made.

The series impedance of a single phase, two phases (V-phase), or three phase distribution line consists of the resistance of the conductors and the self and mutual inductive reactance resulting from the magnetic fields surrounding the conductors. The inductive reactance (self and mutual) components of the impedance are a function of the total magnetic fields surrounding a conductor.

The series impedance per phase of a transposed three phase line of conductors *a, b, c,* is given by:

$$z_i = r_i + jx_i \tag{2.1}$$

$$z_i = r_i + j0.06283 ln \frac{D_{eq}}{GMR_I} \ \Omega/km \tag{2.2}$$

Where r_i=conductor resistance in Ω/km obtained from data table.

x_i = conductor inductive reactance in Ω/km

z_i = conductor series impedance per phase

$D_{eq} = \sqrt[3]{D_{ab}.D_{bc}.D_{ca}}$ meters

GMR_i -geometric mean radius of conductor i in meter

D_{eq} – Geometric mean distance of the conductors

For a solid, round, nonmagnetic conductors, the relationship between the actual radius and the GMR is: $\frac{GMR}{r} = e^{\frac{-1}{4}}$

For stranded conductors, the GMR is given by:

$$GMR = k.r \tag{2.3}$$

Where k = the GMR factor which is given in table 2.2

r = actual conductor radius

Table 2.2 GMR factor[4]

Strands	1	3	7	19	37	61
GMR factor, k	0.7788	0.6778	0.7256	0.7577	0.7678	0.7722

When distribution systems consists of single phase, two phase and untransposed three phase lines serving unbalanced loads, it is necessary to retain the identity of the self and mutual impedance terms of the conductors, and take into account the ground return path for the unbalanced currents. The inductive reactance will be assumed to be at a frequency of 50Hz, and the length of the conductor will be assumed to be one kilometer, with these assumptions the self and mutual impedances are given by:

$$z_{ii} = r_i + j0.062832 \ln \frac{1}{GMR_i} \; \Omega/\text{km} \tag{2.4}$$

$$z_{ij} = j0.062832 \ln \frac{1}{D_{ij}} \; \Omega/\text{km} \tag{2.5}$$

Where

z_{ii} = self impedance of conductor i in Ω/km

z_{ij} = mutual impedance between conductors i and j in Ω/km

r_i = resistance of conductor i in Ω/km

D_{ij} = distance between conductors i and j in meter

2.4.1 Carson-Clem's equations

Carson's equations permit to consider the influence of the earth resistance and of the currents that flow through it.

Since a distribution feeder is inherently unbalanced, every analysis that would show some precision should not make any assumptions regarding conductor sizes, the spacing between conductors, and transposition. In order to face this problem of accuracy, in his paper in 1926 Carson employed the image theory to develop equations that calculate the self-impedance with earth return and mutual impedances with common earth return for an arbitrary number of overhead conductors.

The image theory states that every conductor at a given distance above ground has an image conductor the same distance below ground.

Carson assumed the earth is an infinite, uniform solid with a flat uniform upper surface and a constant resistivity. Any end effects introduced at the neutral grounding points are not large at power frequencies, and were therefore neglected [5]. These equations can also be applied to underground cables.

The modified Carson's equations will be used to compute the primitive self and mutual impedances of overhead lines which are given by (the derivations are given in appendix D):

$$z_{ii} = r_i + 0.049348 + j0.062832\left(\ln\frac{1}{GMR_i} + 8.025174\right)\Omega/_{km} \tag{2.6}$$

$$z_{ij} = 0.049348 + j0.062832\left(\ln\frac{1}{D_{ij}} + 8.025174\right)\Omega/_{km} \tag{2.7}$$

Equations 2.6 and 2.7 are used to compute the elements of an n conductor by n conductor primitive impedance matrix. . The primitive impedance matrix for a three phase line with m neutral will be of the form:

$$[Z_{ptimitive}] = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} & Z_{an1} & Z_{an2} & Z_{anm} \\ Z_{ba} & Z_{bb} & Z_{bc} & Z_{bn1} & Z_{bn2} & Z_{bnm} \\ Z_{ca} & Z_{cb} & Z_{cc} & Z_{cn1} & Z_{cn2} & Z_{cnm} \\ Z_{n1a} & Z_{n1b} & Z_{n1c} & Z_{n1n1} & Z_{n1n2} & Z_{n1nm} \\ Z_{n2a} & Z_{n2b} & Z_{n2c} & Z_{n2n1} & Z_{n2n2} & Z_{n2nm} \\ Z_{nma} & Z_{nmb} & Z_{nmc} & Z_{nmn1} & Z_{nmn2} & Z_{nmnm} \end{bmatrix} \tag{2.8}$$

An overhead four wire grounded Wye distribution line segment will result in a *4 x 4* matrix (due to three-phase wires and ground wire). As the Carson's paper suggests, the return current due to unbalanced loads and non linear characteristics of the electrical equipment can be modeled by a conducting plane located at a certain depth below the earth surface. The line scheme of the three-phase wires with neutral grounded is hereunder.

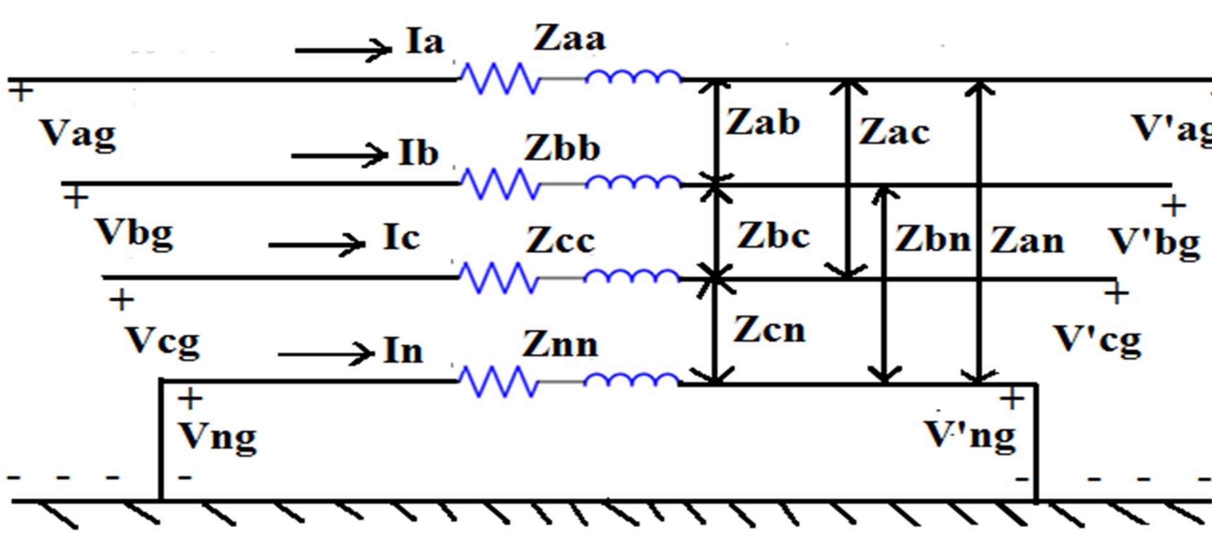


Figure 2.5 Representative three-phase line scheme with neutral wire grounded**.**

Thus, the primitive impedance matrix for a three phase line with a neutral grounded line segment will be of the form:

$$[\hat{z}_{primitive}] = \begin{bmatrix} \hat{z}_{aa} & \hat{z}_{ab} & \hat{z}_{ac} & \hat{z}_{an} \\ \hat{z}_{ba} & \hat{z}_{bb} & \hat{z}_{bc} & \hat{z}_{bn} \\ \hat{z}_{ca} & \hat{z}_{cb} & \hat{z}_{cc} & \hat{z}_{cn} \\ \hat{z}_{na} & \hat{z}_{nb} & \hat{z}_{nc} & \hat{z}_{nn} \end{bmatrix} \tag{2.9}$$

For most applications this primitive impedance matrix needs to be reduced. One standard method of reduction is the Kron reduction. The assumption is made that the line has a multigrounded neutral. The Kron reduction method applies Kirchhoff's voltage law to the circuit.

By KVL for the figure 2.5

$$\begin{bmatrix} V_{abc} \\ V_{ng} \end{bmatrix} = \begin{bmatrix} V'_{abc} \\ V'_{ng} \end{bmatrix} + \begin{bmatrix} Z_{ij} & Z_{in} \\ Z_{nj} & Z_{nn} \end{bmatrix} * \begin{bmatrix} I_{abc} \\ I_n \end{bmatrix} \tag{2.10}$$

Because the neutral is grounded, the voltages V_{ng} and V'_{ng} are equal to zero. It means that the equation written in equation (2.11) can now be re-written as

$$\begin{bmatrix} V_{abc} \\ 0 \end{bmatrix} = \begin{bmatrix} V'_{abc} \\ 0 \end{bmatrix} + \begin{bmatrix} Z_{ij} & Z_{in} \\ Z_{nj} & Z_{nn} \end{bmatrix} * \begin{bmatrix} I_{abc} \\ I_n \end{bmatrix} \tag{2.11}$$

$$[V_{abc}] = [V'_{abc}] + [Z_{ij}] * [I_{abc}] + [Z_{in}] * [I_n] \tag{2.12}$$

$$[0] = [Z_{nj}] * [I_{abc}] + [Z_{nn}] * [I_n] \tag{2.13}$$

Solve equation (2.13) for $[I_n]$, gives

$$[I_n] = -[Z^{-1}{}_{nn}][Z_{nj}][I_{abc}] \tag{2.14}$$

Substitute $[I_n]$ in equation (2.12) and solve for $[Z_{abc}]$, gives

$$[Z_{abc}] = [Z_{ij}] - [Z_{in}] * [Z_{nn}]^{-1} * [Z_{nj}] \tag{2.15}$$

The phase impedance matrix is given by [6]:

$$[z_{abc}] = [\widehat{z_{ij}}] - [\widehat{z_{in}}][\widehat{z_{nn}}]^{-1}[\widehat{z_{nj}}] \tag{2.16}$$

Equation 2.16 is the final form of the kron reduction techniques. The final phase impedance matrix becomes:

$$[z_{abc}] = \begin{bmatrix} z_{aa} & z_{ab} & z_{ac} \\ z_{ba} & z_{bb} & z_{bc} \\ z_{ca} & z_{cb} & z_{cc} \end{bmatrix} \Omega/_{km} \tag{2.17}$$

The phase impedance matrix for a three wire delta line is determined by the application of Carson's equations without the kron reduction step.

2.4.2 Sequence impedances

Many times the analysis of a feeder will use only the positive and zero sequence impedances for the line segment.

If a line is assumed to be transposed, the phase impedance matrix is modified so the three diagonal terms are equal and all of the off-diagonal terms are equal. When this is done the self and mutual impedances are defined as:

$$z_s = \frac{1}{3}(z_{aa} + z_{bb} + z_{cc})\ \Omega/_{km} \tag{2.18}$$

$$z_m = \frac{1}{3}(z_{ab} + z_{bc} + z_{ca})\ \Omega/_{km} \tag{2.19}$$

The sequence impedances can be determined directly as:

$$z_{00} = z_s + 2z_m\ \ \Omega/_{km} \tag{2.20}$$

$$z_{11} = z_{22} = z_s - z_m\ \Omega/_{km} \tag{2.21}$$

Where z_{00} and z_{11} are zero and positive sequence impedances respectively

2.5 Requirements of a Distribution System

It is mandatory to maintain the supply of electrical power within the requirements of many types of consumers. Following are the necessary requirements of a good distribution system:

(i) Proper Voltage: One important requirement of a distribution system is that voltage variations at consumer's terminals should be as low as possible. The changes in voltage are generally caused due to the variation of load on the system. Low voltage causes loss of revenue, inefficient lighting and possible burning out of motors. High voltage causes lamps to burn out permanently and may cause failure of other appliances. Therefore, a good distribution system should ensure that the voltage variations at consumer's terminals are within permissible limits. The statutory limit of voltage variations is $\pm 6\%$ of the rated value at the consumer's terminals. Thus, if the declared voltage is 220 V, then the highest voltage of the consumer should not exceed 233 V while the lowest voltage of the consumer should not be less than 207 V.

(ii) Availability of Power Demand: Power must be available to the consumers in any amount that they may require from time to time. For example, motors may be started or shut down, lights may be turned on or off, without advance warning to the electric supply company. As electrical energy can not be stored, therefore, the distribution system must be capable of supplying load demands of the consumers. This necessitates that operating staff must continuously study load patterns to predict in advance those major load changes that follow the known schedules.

(iii) Reliability: Modern industry is almost dependent on electric power for its operation. Homes and office buildings are lighted, heated, cooled and ventilated by electric power. This call for reliable service. Unfortunately, electric power, like everything else that is manmade, can never be absolutely reliable. However, the reliability can be improved to a considerable extent by

(a) inter connected system (b) reliable automatic control system (c) providing additional reserve facilities.

2.6 Distribution System Loss Reduction Technique

The methods for loss reduction are numerous. They include Demand Side Management (DSM) programs, distribution transformer load management, reconfiguration, reconductoring, voltage up-rating, and shunt capacitor installation.

2.6.1 Demand side management

Some utilities have set up demand side management policies to recommend customers to defer the use of home appliances such as backing and cooking stoves, cloth washers and dryers to off-peak hours. The Ethiopian electric power corporation put a notice on electricity sale bill that there is high power demand between 9 to 11 A.M and 7 to 10 PM local time and recommends power demanding home electric appliances be used other than this times. To further increase the incentive of using electricity at off-peak times, some utilities have been established time-of-use (TOU) rates with less expensive energy charges during off-peak hours. TOU rates have been mandatory in California for all customers above 500 kW since 1978, as a state wide policy response to the energy crises of 1973. These rates are mandatory in several U.S states but the size threshold varies by state. (J. Terry Cousins, Using time off use (TOU) tariffs in industrial, commercial and residential applications effectively)

The effectiveness of a DSM program depends on the participation of the customers. Using certain household appliances at off-peak times may not be convenient for some customers. In some instances, the encouragement of using electricity at non-peak times has even caused new peaks to occur. Furthermore, this form of power peak-shaving will have less effect in a deregulated environment like Ethiopia as the producers of electricity are likely to only have a KWh rate and no KW rate for the demand. From an economic savings perspective, there is less motivation for the promotion of DSM programs.

2.6.2 Distribution transformer load management

Distribution transformer load management involves balancing the load between phases and resizing over and under-utilized transformers to reduce losses. Ideally, phase currents and voltages of all distribution transformers would need to be monitored in order to practice distribution transformer load management properly. However, for Ethiopian utility, such measurements are only available at the substation level. To monitor every pole and pad mounted transformer downstream from the substation would prove to be costly. At the very least, distribution transformer load management can be practiced at the planning stages of a distribution system by sizing transformers for maximum efficiency.

2.6.3 Reconfiguration

Network reconfiguration refers to the closing and opening of switches in a power distribution system in order to alter the network topology, and thus the flow of power from the substation to the consumers. Many distribution systems have a radial structure and are equipped with sectionalizing switches for fault isolation or scheduled maintenance of a section. These switches can also be used to transfer loads between unevenly loaded feeders. During normal operating conditions, networks reconfigured for two purposes: (i) to minimize the system real power losses in the network; and (ii) to relive the overload in the feeders. Loss reduction by reconfiguration is highly system dependent.

2.6.4 Reconductoring

It is obvious that the use of feeder conductors with larger cross-sectional areas has associated lower I^2R losses. The more expensive larger sized cables and the installation costs are likely to forbid reconductoring as a feasible means for loss reduction. Moreover, feeder upgrades with heavier conductors can only be economical justified for older networks that are operating close to their design capacity.

2.6.5 Voltage Up-rating

By increasing the primary distribution systems voltage, the same amount of power can be delivered at lower currents, thus lowering losses. Voltage up-rating like reconductoring has required much capital expenditure (small benefit/cost ratio).

2.6.6 Shunt Capacitor installation

The installation of shunt capacitors in primary distribution systems reduces losses by supplying reactive currents to oppose the out-of-phase component of the feeder currents required by inductive loads. It is this reduction or elimination of the reactive component of the currents that reduces the I^2R losses.

2.6.7 Comparison of loss reduction techniques

Given that the majority of the losses are found at the distribution level, many utilities can achieve significant savings from implementation of one or a combination of the loss reduction methods. Benefit/cost ratios can be established to determine which loss reduction measures are the most suited for application. In European study, benefit/cost ratios of various loss reduction schemes have been established and obtained as 2 to 8, 0.6 to 7, 1.5 to 3, up to 13, and 1 to 15 for capacitor placement, reconductoring, voltage uprating, reconfiguration and distribution transformer load management respectively [7]. Capacitor placement, reconfiguration and distribution transformer load management rank among the highest with benefit/cost ratios. Reconfiguration and distribution transformer load management may have greater benefits in Europe where the systems are more heavily loaded. Capacitor placement remains as one of the most popular methods of loss reduction since it does not required as much capital expenditure as reconductoring or voltage uprating. Furthermore, distribution system circuits always have an inductive component and proper capacitor placement will always result in some loss reduction. In addition, from an operation point of view, installation of fixed capacitor does not require any significant upgrading of Supervisory Control and Data Accusation (SCADA) system equipment; whereas the implementation of a new reconfiguration scheme or transformer loads management system would require a substantial increase of additional SCADA capabilities of monitoring and control. Thus, capacitor placement is, in most cases, an ideal distribution system loss reduction measures a utility can choose to improvement [7].

2.7 Overview of Reactive Power

Reactive power is a subject of great concern for the operation of alternating current (AC) power systems [8]. It has always been challenge to obtain the balance between a minimum amount of reactive power flow (to maximize capacity for active power flow) and sufficient amount of reactive power flow to maintain a proper system voltage profile. Even though reactive power is not widely understood outside of the power engineering community, it remains one of the most important aspects of AC power system operation. Those involved with maintaining and operating power systems must constantly be concerned with the balance between reactive power supply and demand as much as with active power supply and demand. The reliable and economic use of electric power depends on an availability of sources for leading and lagging reactive power that can be appropriately dispatched to the system.

2.8 Reactive Power Compensation

Reactive power compensation is defined as the management of reactive power to improve the performance of ac power system.

The earliest distribution systems did not use any form of reactive power compensation. Any reactive requirements by the components of the system or by the loads served were supplied by the synchronous generator. This led to very inefficient utilization of the system so utility companies developed rate structures that penalized loads of low power factor. The first uses of shunt capacitors on power systems was in the 1920s, at large industrial plants where reductions in electric charges justified providing local reactive power compensation.

Before capacitors, synchronous motors were (and continue to be) employed in industrial plants on processes requiring large amounts of constant mechanical power.

The synchronous motors could be controlled to provide some amount of reactive supply, often enough to compensate for the reactive consumption of induction machines. Synchronous condensers were used, beginning in the 1930s, on transmission and sub-transmission systems to provide a variable source of reactive power.

In the 1930s, the advantages of series capacitors became apparent and began to find applications in distribution systems and industrial installation [9].

In general, shunt capacitors have been commonly used to provide reactive power compensation in distribution systems. They are provided to minimize power and energy losses, maintain best voltage regulations for load buses and improve system security. The amount of compensation provided is very much linked to the placement of capacitors in the distribution system which essentially includes determination of location, size, number and type of capacitors to be placed in the system [10]. A large variety of research work has been done on capacitor placement problem in the past. All the approaches differ from each other by the way of their problem formulation and the problem solution method employed. Ng. H (1999) classifies the optimum capacitor allocation methods in to: (1) analytical methods, (2) numerical programming methods, (3) heuristics methods, and (4) Artificial intelligent (AI) based methods. Each method has its own advantages and drawbacks. The choice of which method to use is dependent on: the problem to be solved, the complexity of the problem, the accuracy of results required, and the practicality of implementation. Hamouda. A et al. (2007) have used analytical method of the reactive energy compensation on a radial distribution lines. In addition they applied the forward and backward sweep iteration methods for the determination of the voltage magnitude. Subrahanyam. J (2005) has proposed the novel method to determine the best locations for capacitor placement in unbalanced radial distribution networks and simple Genetic Algorithm (GA) is used to find the optimal sizing of capacitor bank. In this paper the energy cost, capacitor installation cost and purchase costs are included in the formulated objective functions. Ellithy. K et al. (2008) have used genetic algorithms techniques for the determination of the location and sizes of the capacitor to be placed in distribution networks in an optimal manner to reduce the energy losses and peak power losses of the networks. A combination of fuzz-GA method to resolve the capacitor placement problem is used by Hsiao.Y et al. (2004). The problem formulation considers three distinct objective functions related to minimize the total cost for energy loss and capacitor to be installed, as well as decreasing the deviation of bus voltage and improving the margin loading of feeder. Seifi.A (2008) has used fuzzy reasoning for sitting of capacitors in radial distribution feeders, Dynamic programming (DP) for sizing and Genetic Algorithm (GA) for finding shape of membership functions which are used in fuzzy reasoning stage.

2.9 Installation of Capacitor bank to Distribution feeder

Shunt capacitors are connected in parallel with the primary distribution feeder of a system. Distribution capacitors can be pole mounted or pad mounted. Example of pole mounted is shown in figure 2.6 below.

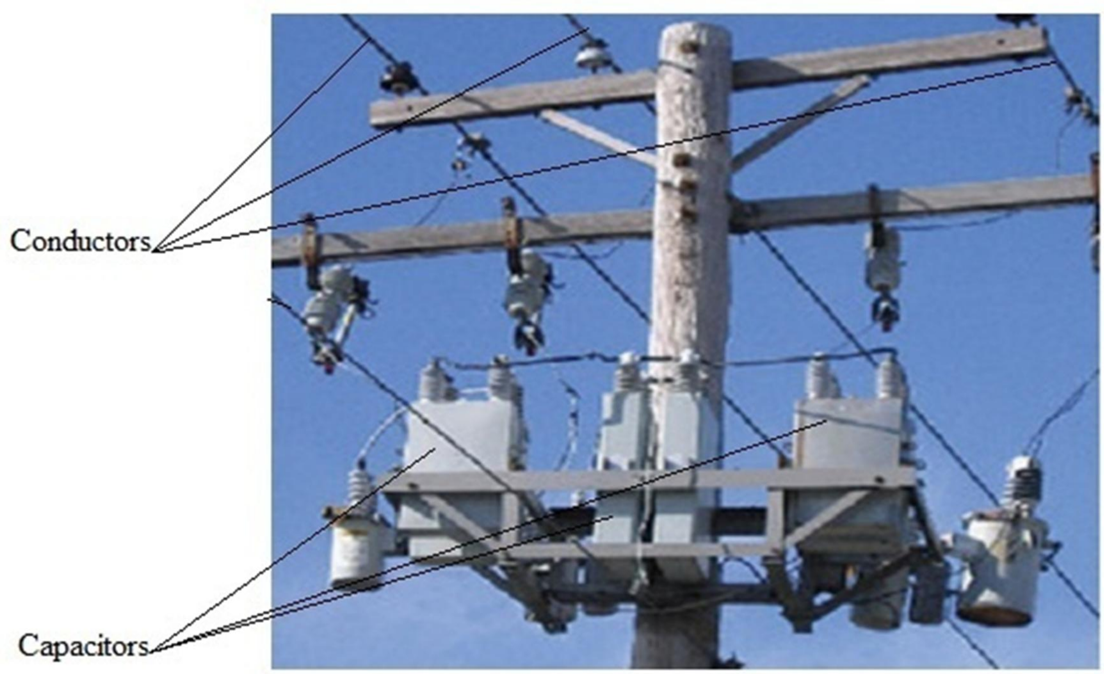


Figure 2.6 Pole mounted medium voltage capacitor

Pole mounted capacitor are the least expensive way for providing the reactive power from the point of view of installation.

Pad mounted capacitors are used where the power distribution circuit is placed underground. The disadvantage of this kind of capacitors can be the big size and aesthetics.

These shunt capacitors supply the out-of-phase component of the current required by inductive loads. Typically, for distribution loads, the current lags the voltage. Thus, a shunt capacitor draws leading currents which counteracts against the lagging component of the current at the point of installation. As a result, the power factor of the circuit is improved; the voltage rise occurs at the point of the capacitor installation, and more importantly, the reactive component of the current reduced to lower the I^2R losses. When reactive power is supplied by generation, ratings of generators, transformers, transmission and distribution lines must be increased accordingly. However, when shunt capacitors are installed in distribution system, the reactive power demand from generation is decreased, thus releasing savings in the capacity for generators, transformers, and distribution lines.

The installation of shunt capacitors in primary distribution network has the following economical benefits.

1. Released generation capacity
2. Released transmission capacity
3. Released substation capacity

In generally shunt capacitor decrease KVA loading on the source generator and networks relive an overload condition or make capacity available for additional load growth.

4. Additional advantages in distribution system:
   - Reduced energy (copper) losses due to the reduction of current
   - Reduced voltage drop and consequently improved voltage regulation
   - Released capacity of feeder and associated apparatus

- Postponement or elimination of capital expenditure due to system improvement and /or expansion
- Increase power factor

Voltage drop can be reduced by the application of shunt capacitor. A correctly selected and located shunt capacitor assures that the voltage at the load will be within the allowable limit at the heavy load condition. However at light loading, the same capacitor will increase the voltage to above the allowable limit. The way to avoid this is to use switched capacitor banks. The capacitors are switched in during heavy load condition and switched out during light load condition.

3. DATA COLLECTION AND ANALYSIS

To obtain the benefits from installation of shunt capacitors, it is necessary to select the appropriate place and size of capacitors for maximum benefits. For this work data are collected from EEPCO, the substation data sheet and measurements are taken from the feeder lines and the maximum real and reactive load demand at each distribution transformer using fluke power quality analyzer. From the substation the feeder which delivers power to Maichew town and some rural customers is considered for this work because as it incorporates loads such as industrial, commercial, and domestic. Besides to this, the feeder has a maximum demand at the substation. The average power factor, as it is recorded, is 0.75 at the substation.

3.1 Thirty Four-Bus Radial Distribution Feeder

The single line diagram, the load data and feeder line parameters are taken from the thirty four-bus radial distribution system.

In addition to the radial configuration, thirty four-bus feeder consists of a substation connected to node 0 which is taken as the reference node, 33 branches and thirty four nodes of which 24 nodes are connected to loads through distribution transformers, the remaining 9 are common coupling nodes. A single line diagram of this feeder is shown in figure 3.1 10.5463 kilometres long all aluminum conductor (viz. AAC-95 and AAC -50) overhead lines are used to distribute medium voltage (15 KV) power from substation to the distribution transformers. Both types of conductors have the same conductor arrangements as shown in figure 3.2.

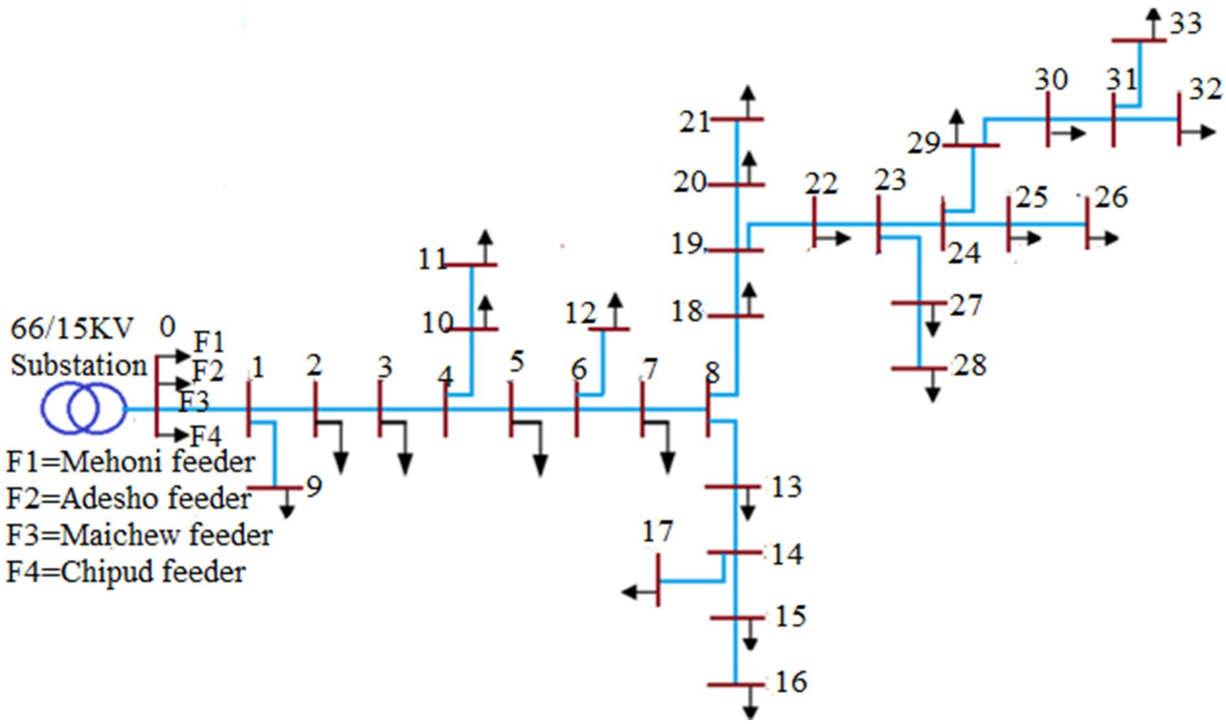


Figure 3.1 Single line diagram of thirty four-bus distribution system

The overhead conductors' parameters are given in table 3.1 where the resistance of the conductors is obtained from the standard overhead conductor data sheet and the GMRs are calculated using equation 2.3 and table 2.2.

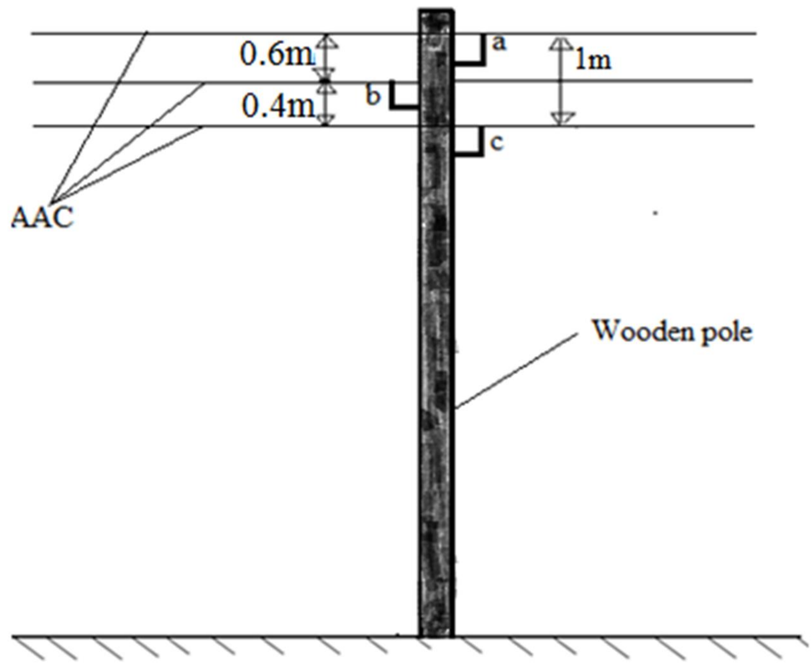


Figure 3.2 Primary distribution three-phase lines configurations

Table 3.1 Over head MV conductor data

Conductor type	Nominal area (mm^2)	Actual area (mm^2)	Stranding and wire diameter	Overall diameter (mm)	Actual diameter (mm)	GMR (mm)	Resistance (Ω/km)
AAC	50	49.48	7/3.00	9.0	7.9377	2.880	0.5786
AAC	95	93.27	19/2.5	12.5	10.8975	4.129	0.3084

Using equations 2.22, figures 3.2 and table 3.1 the branch positive sequence impedances are calculated (detailed calculation are given in appendix B) and the final results are tabulated in table 3.2

The maximum transformer loads demand (real and reactive demand) are measured using Fluke power quality analyzer, to evaluate the maximum losses, for each load node at the secondary of distribution transformers and the data are shown in the last two columns of table 3.2. The distribution transformers are assumed to be hundred percent efficient.

Table 3.2 Thirty four-bus overhead conductor line and load data

Branch No.	Sending node	Receiving node	Conductor type	Length (km)	R (Ω)	X(Ω)	Receiving end load	
							Real power (kw)	Reactive power (Kvar)
1	0	1	AAC-95	1.5000	0.462648	0.487177	0	0
2	1	2	AAC-95	0.0600	0.018506	0.019271	32	18
3	2	3	AAC-95	0.0750	0.023132	0.240885	135	113.6
4	3	4	AAC-95	0.3300	0.101783	0.105989	0	0
5	4	5	AAC-95	0.0750	0.023132	0.024089	74.6	64.3
6	5	6	AAC-95	0.2400	0.074024	0.077083	0	0
7	6	7	AAC-50	0.3000	0.173580	0.103145	153	146
8	7	8	AAC-50	0.3000	0.173580	0.103145	0	0
9	1	9	AAC-50	0.3375	0.195278	0.116038	54	44.6
10	4	10	AAC-50	0.3488	0.201787	0.119906	63	49.3
11	10	11	AAC-50	0.0900	0.052074	0.0309434	42	32.8
12	6	12	AAC-50	0.1125	0.065093	0.0386792	74	53.7
13	8	13	AAC-50	0.7500	0.433950	0.2578613	86	76.4
14	13	14	AAC-50	0.2250	0.130185	0.0773584	0	0
15	14	15	AAC-50	0.6000	0.347160	0.206289	68	53
16	15	16	AAC-50	1.2500	0.72325	0.4297688	140	136
17	14	17	AAC-50	0.1875	0.108488	0.064465	56	43.7
18	8	18	AAC-50	0.1125	0.065093	0.038680	150	13.8
19	18	19	AAC-50	0.5250	0.303765	0.180503	0	0
20	19	20	AAC-50	0.5250	0.303765	0.180503	54	48
21	20	21	AAC-50	0.3450	0.199617	0.118616	138	121
22	19	22	AAC-50	0.0750	0.043395	0.025786	140	127

Branch No.	Sending node	Receiving node	Conductor type	Length (km)	R (Ω)	X(Ω)	Receiving end load	
							Real power (kw)	Reactive power (Kvar)
23	22	23	AAC-50	0.1650	0.095469	0.056729	0	0
24	23	24	AAC-50	0.2400	0.138864	0.082516	0	0
25	24	25	AAC-50	0.1875	0.108488	0.064465	42	18
26	25	26	AAC-50	0.3000	0.173570	0.103145	137	119
27	23	27	AAC-50	0.2850	0.164901	0.097987	60	43
28	27	28	AAC-50	0.0525	0.030377	0.0180503	38	17
29	24	29	AAC-50	0.1650	0.095469	0.056729	63	47
30	29	30	AAC-50	0.1500	0.086790	0.0515723	43	25
31	30	31	AAC-50	0.1125	0.065093	0.038679	0	0
32	31	32	AAC-50	0.4500	0.260370	0.154717	88	84.6
33	31	33	AAC-50	0.0750	0.043395	0.025786	112	98

3.2 Distribution Feeder Analysis

3.2.1 Power flow analysis

The load-flow analysis in a power system has paramount importance because it reveals the electrical performance and power-flows of the system operating under steady state and it provides the real and reactive power losses and voltages at different nodes of the system.

Power flow analysis is used to ensure that electrical power transfer from generators to consumers through the grid system is stable, reliable and economic. In a three phase ac power system active and reactive power flows from the generating station to the load through different networks buses and branches. The flow of active and reactive power is called power flow or load flow.

Power flow studies provide as systematic mathematical approach for determination of various bus voltages, their phase angles, active and reactive power flows through different branches, generators and loads under steady state condition.

In a distribution system there is typically one voltage specified bus, the substation bus, which is connected radially to the load buses. The solution to the power flow problem provides information on the voltage magnitudes and angles at each bus, the real and reactive power supplied or absorbed at the substation, the real and reactive power flows in each line section, and the system losses. In the capacitor placement problem the load flow study is used to calculate the overall power loss, the voltage magnitude and angle at each bus and the branch reactive current flow. The results are used for the determination of location and size of capacitors in order to optimize the losses which are caused by the flow of reactive currents. In the distribution analysis the following models are used for the components encountered in this thesis.

3.2.2 Modeling of distribution system components

The individual components of a distribution system are modeled by their mathematical equivalents to perform the power flow analysis. Modeling of distribution system components such as line sections, loads and shunt components are treated hereunder.

3.2.2.1 Representation of the lines

The modeling of distribution lines is a very crucial point in the analysis of distribution feeder. Generally, the length of most distribution line sections is short (<80km). In addition, the voltage level of distribution systems is lower than that of transmission systems and hence the π model is not used because line shunt capacitance is found to be negligible. Instead, the short line model is used to represent the line sections for distribution systems whereby the distribution line sections are modeled as series impedances ($Z = rl + jxl$).

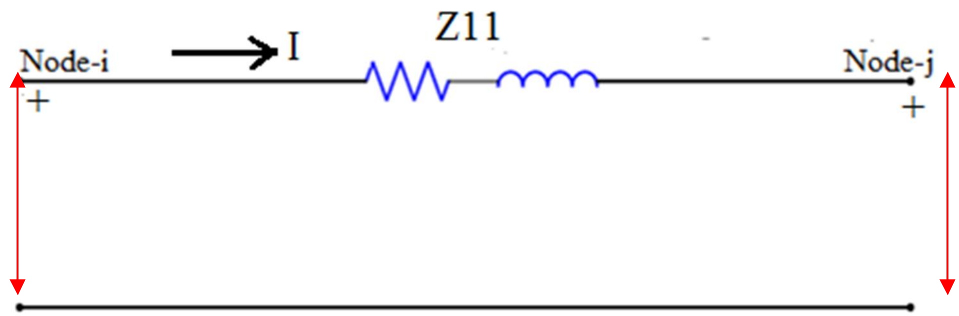


Figure 3.3 Radial feeder single phase model

$$V_{ig} = V_{jg} + Z_{11}I \tag{3.1}$$

Where Z_{11} - the positive sequence impedance

3.2.2.2 Representation of the loads

The loads on a distribution system are typically specified by the complex power consumed. The specified load will be the three phase maximum diversified demand. This demand can be specified as KVA and power factor, KW and power factor, or KW and Kvar. In this thesis the demand are specified as KW and Kvar from measurements. The voltage specified will always be the voltage at the low-voltage terminals of the distribution substation.

The loads on a distribution feeder modeled as constant real and reactive power (constant PQ). The load models developed are to be used in the iterative process of a power flow program where the load voltages are initially assumed. All models are initially defined by a complex total power and an assumed line-to-line voltage. In this model the line-to-line voltages will change during each iteration until convergence is achieved.

Shunt capacitor banks are used in distribution system to help in voltage regulation and to provide reactive power support. The capacitor banks are represented by their equivalent injected currents.

3.2.2.3 Substation

Substations in power flow analysis is modeled as infinite sources as a reference bus, where voltage magnitude can be controlled using regulators and/or taps on the transformers.

3.2.3 Power Flow Solution for Radial Distribution System

The common conventional load flow procedures like Newton Raphson and Gauss Seidel methods have less accuracy and take much time to reach convergence and hence become inefficient in the analysis of distribution systems, due to the special features of distribution networks, that is, radial structure, high R/X ratio, unbalanced loads, untransposed lines, single-phase and two-phase laterals. These characteristic features make the distribution systems power flow computation different and somewhat difficult to analyze as compared to the transmission systems when the conventional power flow algorithms are employed [2]. Due to its computational efficiency and robust convergence characteristic, forward/backward sweep based algorithms have gained the most popularity for distribution systems load flow analysis. In the present study, network topology based forward/backward sweep algorithms have been used for load flow analysis. This is a factor causes the distributions to be ill-conditioned for conventional power flow methods [11].

The Backward/Forward sweep based method (BFS), the ladder iterative network based method, which employs Kirchhoff's current and voltage laws, is found to be more robust in solving the power flow issues of RDS. This methodology finds out the node currents and corresponding branch currents in backward sweep mode from leaf (terminal) nodes towards the source node and magnitude of voltage at every node are calculated in the forward sweep from the source end towards the leaf nodes. For the six bus radial distribution system shown in Fig.3.4, the current in each branch and voltage at every node is calculated by using BFS based load flow technique. In backward sweep mode, the load current injection of every node is calculated using Eq. (3.2).

$$I_i = \left(\frac{S_i}{V_i}\right)^* \text{ where i 2, 3, …, n} \tag{3.2}$$

Where Si* is the conjugate of complex power of ith node, Vi* is the conjugate of ith node voltage and n is the total number of nodes.

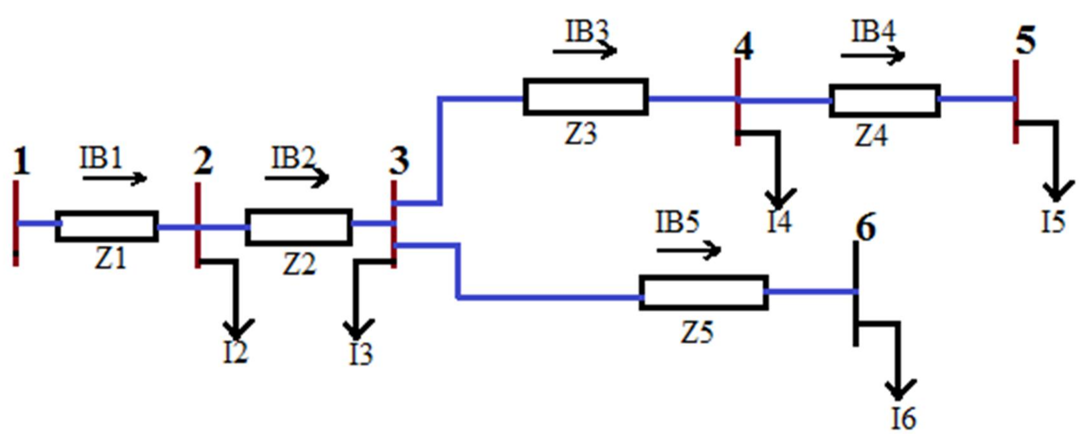


Figure 3.4 Six bus radial distribution network

3.2.3.1 Formulation of BIBC matrix

After the calculation of node currents, the branch currents are obtained by adding the current of up- stream node as,

$IB_5 = I_6$

$IB_4 = I_5$

$IB_3 = I_4 + I_5$

$IB_2 = I_6 + I_5 + I_4 + I_3$

$IB_1 = I_6 + I_5 + I_4 + I_3 + I_2$

Where I_2, I_3….I_6 are the equivalent injected load currents at their respective nodes. The Bus injection to Branch current (BIBC) matrix of the six bus RDS system is a connectivity matrix relating the bus currents and the branch currents and it can be written in matrix format as,

$$\begin{bmatrix} IB_1 \\ IB_2 \\ IB_3 \\ IB_4 \\ IB_5 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} \tag{3.3}$$

The above equation can hence forth be written in a précised manner as,

$$[BC] = [BIBC][I] \tag{3.4}$$

The BIBC matrix is an upper triangular matrix and contains values of 0 and +1 only.

3.2.3.2 Formation of BCBV (Branch current to Bus voltage) matrix

BCBV is a connectivity matrix for branch current to bus voltage constituted for calculation of the node voltages in the forward sweep mode. The node voltages are calculated starting from the root node and ending up with the leaf node. The BCBV matrix can be formulated as shown in Eq. (3.5),

$$[BCBV] = [BIBC]^T[ZD] \tag{3.5}$$

Where T stands for transpose of the BIBC Matrix and [ZD] is the impedance matrix with the impedance values arranged in diagonal format as per their branch numbering as in Eq. (3.6)

$$[\mathrm{ZD}] = \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 \\ 0 & Z_2 & 0 & 0 & 0 \\ 0 & 0 & Z_3 & 0 & 0 \\ 0 & 0 & 0 & Z_4 & 0 \\ 0 & 0 & 0 & 0 & Z_5 \end{bmatrix} \tag{3.6}$$

Where Z1, Z2, Z3, Z4, Z5 are the impedance values of the respective branches in the six bus distribution system. The impedance values are arranged in an orderly manner based on the branch numbering of the RDS. The BCBV matrix can be obtained by multiplying the transposed BIBC matrix and impedance matrix as shown,

$$[\mathrm{BCBV}] = \begin{bmatrix} Z1 & 0 & 0 & 0 & 0 \\ Z1 & Z2 & 0 & 0 & 0 \\ Z1 & Z2 & Z3 & 0 & 0 \\ Z1 & Z2 & Z3 & Z4 & 0 \\ Z1 & Z2 & 0 & 0 & Z5 \end{bmatrix} \tag{3.7}$$

The node voltages of the above six bus distribution system can be estimated by using BCBV matrix and branch current matrix [IB] as in the Eq. (3.8).

$$\begin{bmatrix} V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} = \begin{bmatrix} V_1 \\ V_1 \\ V_1 \\ V_1 \\ V_1 \end{bmatrix} - \begin{bmatrix} Z_1 & 0 & 0 & 0 & 0 \\ Z_1 & Z_2 & 0 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & 0 & 0 \\ Z_1 & Z_2 & Z_3 & Z_4 & 0 \\ Z_1 & Z_2 & 0 & 0 & Z_5 \end{bmatrix} \begin{bmatrix} IB_1 \\ IB_2 \\ IB_3 \\ IB_4 \\ IB_5 \end{bmatrix} \tag{3.8}$$

Where, V1 is the sub-station bus voltage assumed to be 1 p.u. A flat start is assumed in the first iteration of backward sweep with an assumption of all node voltages as 1 p.u and the bus current and branch currents are computed in backward sweep mode from end nodes. In forward sweep mode, starting from source node towards the leaf nodes the bus voltages are updated with branch currents computed in the preceding backward sweep iteration. The iterative process of above said backward/forward sweep stops when certain convergence criterion is met. Once after the convergence of the solution is met, the estimated branch current flowing in the system and respective branch impedances are considered for the calculation of the real and reactive power loss in the system.

Generally, based on the ladder iterative load flow, the backward sweep will determine a computed source voltage V_0. For the first iteration of the load flow, it is assumed that all the end node voltages have a voltage of the reference node voltage (Vs) at the source node. The currents drawn by each of the loads are then calculated by:

$$I_i = \left(\frac{S_i}{V_i} \right)^* \tag{3.9}$$

The currents flowing through each section of the feeder are calculated by summing the load currents downstream of the section. The node voltages can be then calculated by:

$$V_i = V_{i+1} + Z_{i+1}.I_{i+1} \tag{3.10}$$

The newly calculated voltages are used to recalculate the load currents using equation (3.9). The first iteration will produce a source node voltage that is not equal to the specified source voltage Vs. The forward sweep commences by using the specified source voltage and line currents from the backward sweep. Kirchhoff's voltage law also used in the forward sweep to compute the voltage at node n+1, which is given by equation 3.11.

$$V_{i+1} = V_i - Z_{i+1}.I_{i+1} \tag{3.11}$$

This procedure repeated for each line segment until a new voltage is determined at the last node. Using the voltage at the last node, a second backward sweep is started that will lead a new computed voltage at the source. The forward and backward sweep process is continued until the difference between the computed and the specified voltage at the source is within a given tolerance.

The power losses of a line segment can be computed as the difference of the input power to a line segment minus the output power of the line segment.

The total loss in the feeder is obtained by:

$$S_{Loss} = \sum_{i=0}^{n-1} \left(v_i - v_j \right) . I_{ij}^{*} \tag{3.12}$$

Where

vi and vj are the line voltages of bus i and j respectively

j = the bus connected to bus i through branch $i\ j$

The ladder iterative power flow solution methods is summarized in the flow chart in figure 3.5

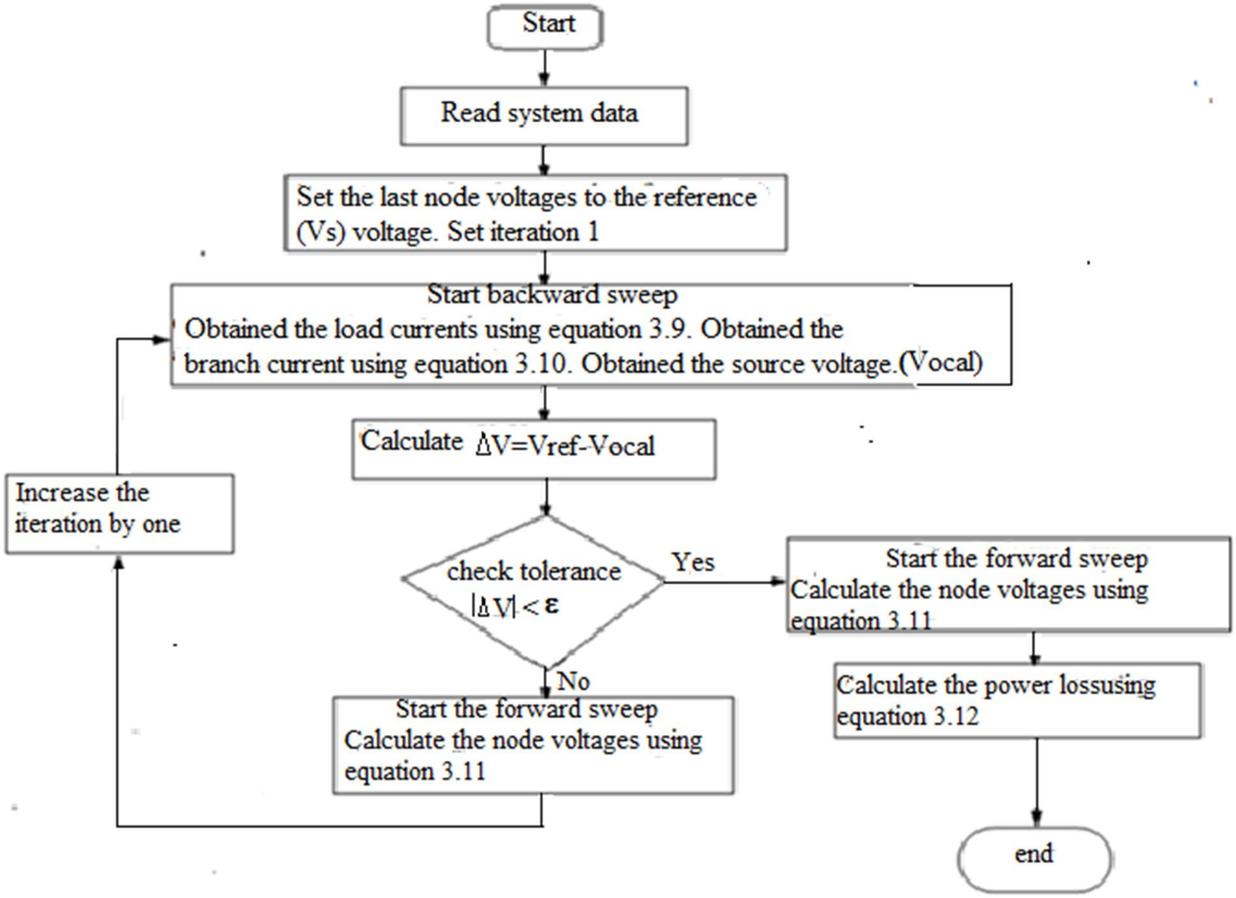


Figure 3.5 Flow chart for power flow solution using ladder iterative method

The total I^2R loss (P_{TL}) in a distribution system having n number of branches is given by

$$P_{TL} = \sum_{i=1}^{n} {I_i}^2 R_i \tag{3.13}$$

Here Ii and Ri are the branch current magnitude and resistance, respectively, of the ith branch. The branch current can be obtained from the load flow study. The branch current has two components; active Ia and reactive component Ir. The total loss associated with the active and reactive components of branch currents can be written as

$$P_{TL} = P_{La} + P_{Lr} \tag{3.14}$$

$$P_{TL} = \sum_{i=1}^{n} I_{ai}{}^{2} R_i + \sum_{i=1}^{n} I_{ri}{}^{2} R_i \quad (3.15)$$

Note that for a given configuration of single source radial networks, the loss P_{La} associated with the active component of branch currents cannot be minimized because all active power must be supplied by the source at the root bus. However, the reactive power which depends on inductive characteristic loads can be minimized by placing shunt capacitors. This is because the reactive component of branch current is reduced. This paper presents a method that minimizes the loss due to the reactive component of the branch current by optimally placing the capacitors and thereby reduces the total loss in the distribution system. A two stage methodology is applied here. In the first stage optimum location of the capacitors are determined and in the second stage the sizes of the capacitors are determined for maximum real power loss reduction.

Similarly, the total reactive power loss in the system is determined as:

$$Q_{TL} = \sum_{i=1}^{n} I_{ai}{}^{2} X_i + \sum_{i=1}^{n} I_{ri}{}^{2} X_i \quad (3.16)$$

4. OPTIMAL CAPACITOR PLACEMENT ON A RADIAL DISTRIBUTION SYSTEM

4.1 Introduction

The general problem for capacitor placement is to determine the optimal number, location, sizes and types for capacitors to be installed on a distribution feeder to maximize loss savings subjected to operating constraints.

Power loss in the radial distribution system is comparatively higher than the transmission system because of its higher R/X ratio and untransposed lengthy lines with unbalanced loads. The presence of higher resistance in the distribution network give rise to more power loss in the feeder lines and its sub laterals and it makes the distribution system less efficient for supplying quality power to the tail end consumers. Portion of these losses are caused by the reactive current flowing in the network. There is a need for improving the efficiency in the system by reducing the power loss in different possible ways. One such way of reducing the power loss is by installing shunt capacitors in the system at required locations to inject reactive power partially. Installation of shunt capacitor enhances the feeder capacity, reduces power loss, and improves voltage regulation of the RDS to the desired level. The objective of reducing power loss within the bounded constraints can be achieved by optimally sizing the capacitor banks and placing them in optimal locations. If the place and sizing is not appropriate it may result in over compensation during light load condition.

Generally, distribution system is fed at only one point and the structure of the network is mainly radial. For such a system all active power demands and losses must be supplied by the source at root bus. However, addition of shunt capacitors can generate the reactive power and therefore it is not necessary to supply all reactive power demands and losses by the source. Thus, there is a provision to minimize the loss associated with the reactive power flow through the branches.

The purpose of placing capacitors along distribution feeders is to lower the total power loss and bring the bus voltage within specified limits while minimizing the total power loss. The total I^2R power loss (p_{Tloss}) in a distribution system having n number of branches is given by [5]:

$$P_{Tloss} = \sum_{j=1}^{n} I_j^2 R_j \tag{4.1}$$

Where R_j and I_j are the resistance and the line current magnitude respectively of the j^{th} branch, which is obtained from the load flow solution. The branch currents have two components, namely, (1) those due to the in-phase or active component of current and (2) those due to the out-of-phase or reactive component of current.

$$I_j = I_{aj} + jI_{rj} \tag{4.2}$$

The I^2R total loss is the summation of the losses associated with the active and reactive components of the branch currents. The I^2R loss associated with the active and reactive components of branch currents can be written as:

$$P_{TLoss} = \sum_{j=1}^{n} \left(I_{aj}^2 R_j + I^2{}_{rj}.R_j \right) \tag{4.3}$$

$$p_{La} = \sum_{j=1}^{n} I_{aj}^2 R_j \quad and \quad P_{Lr} = \sum_{j=1}^{n} I_{rj}^2 R_j \tag{4.4}$$

Shunt capacitors supply capacitive reactive power to the system at the point where they are connected, mainly to counteract the out-of-phase component of current required by an inductive load.

For a single source radial network, the active current component I_a depends only on the circuit load. The losses P_{La} due to the in-phase or active component of branch currents are not significantly affected by the application of shunt capacitors, because all active power must be supplied by the source at the root bus. However, the loss P_{Lr} associated with the reactive component of branch currents can be minimized by supplying part of the reactive power demand locally [2].

4.2 Optimum locations for capacitors

To identify the candidate location of shunt capacitors to be placed to accomplish the proposed objective function of loss minimization, the computation of differentiating the total power loss saving with respect to the capacitor sizes of all nodes of the RDS is necessary.

The method used here is first formulating the node-to-branch incidence matrix (G) of a given configuration. Using the node-to-branch incidence matrix, the sequence of node to be compensated is identified by repetitive applications of loss minimization technique. Once the sequence of nodes to be compensated is identified, the corresponding optimal size at the compensated nodes can be determined simultaneously by minimizing the loss saving equation with respect to the capacitor Kvar size.

Node after branch determination**:** to determine the nodes after each branch of the feeder (G), first the branch-to-node incidence matrix IM is constructed. In IM the rows indicate the identification numbers of branch and the columns the identification numbers of nodes. The generic elements IM (i, j) are assumed to have the values significations of which are given below:

$$IM = (i, j) = \begin{cases} +1, & \text{if } j \text{ is the receiving end of branch } i \\ -1, & \text{if } j \text{ is the sending end of branch } i \\ 0, & \text{otherwise} \end{cases} \tag{4.5}$$

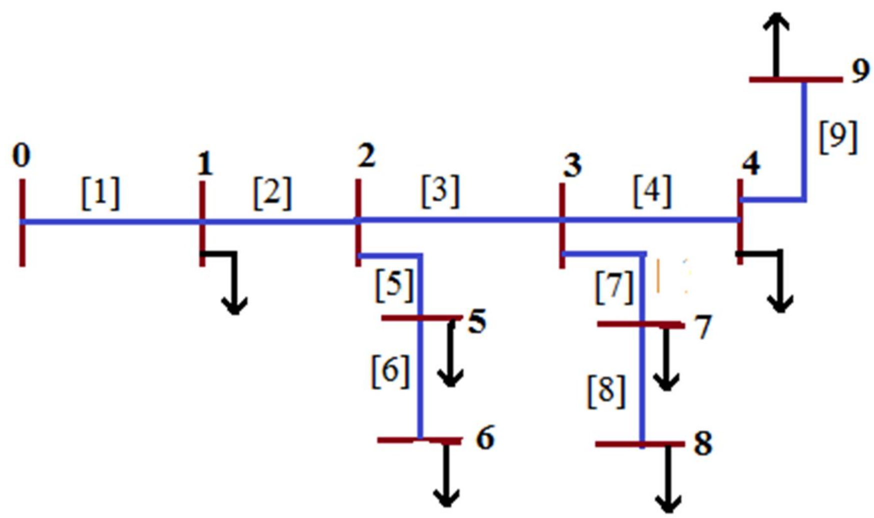


Figure 4.1 Nine node radial distribution feeder

For the feeder example of figure 5.1, the branch-to-node incidence matrix is:

$$IM = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

From the branch-to-node incidence matrix, the node-to-branch incidence matrix G is deduced by simply inversion of IM. In the node-to-branch incidence matrix, the nodes identifiers are shown on rows and the column numbers identify the branches numbers. For the feeder of figure 4.1, the node-to-branch incidence matrix G is:

$$G = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Considering single source radial distribution system with n branches, let a capacitor Q_{ci} be placed at bus i. The capacitor draws a reactive current I_c and for radial network it changes only the reactive component of the current of branches between the source and the capacitor unit buses. The current of other branches is unaffected by the capacitors. Thus the new reactive branch current $I_r^{'}$ due to the addition of capacitor at node i is given by:

$$I_{rj}^{'} = I_{rj} + G_{ij}\frac{Q_{ci}}{|v_i|} \tag{4.6}$$

Here G_{ij} is the i^{th} row and j^{th} column element of matrix G and I_{rj} is the reactive current of the j^{th} branch in the original system obtained from the load flow solutions.

The loss $P_{Lr}^{'}$ associated with the reactive component of branch currents when the capacitor is connected in the system can be written as:

$$P_{Lr}^{'} = \sum_{j=1}^{n}\left(I_{rj} + G_{ij}\frac{Q_{ci}}{|v_i|}\right)^2 R_j \tag{4.7}$$

The loss saving ΔP_{LSi} which is obtained due to the capacitor Q_{ci} at bus i is:

$$\Delta P_{LSi} = P_{Lr} - P_{Lr}^{'} = \sum_{j=1}^{n} I_{rj}^{2} R_j - \sum_{j=1}^{n} \left(I_{rj} + G_{ij} \frac{Q_{ci}}{|v_i|} \right)^2 R_j$$
$$\Delta P_{LSi} = -\sum_{j=1}^{n} \left(2 I_{rj} G_{ij} \frac{Q_{ci}}{|v_i|} + G_{ij} \left(\frac{Q_{ci}}{|v_i|} \right)^2 \right) R_j \tag{4.8}$$

The capacitor size that provides the maximum loss saving can be obtained from differentiating the loss saving of equation 4.8 with respect to the capacitor size and equating to zero:

$$\frac{\partial \Delta P_{LSi}}{\partial Q_{ci}} = -2 \sum_{j=1}^{n} \left(G_{ij} I_{rj} + G_{ij} \frac{Q_{ci}}{|v_i|} \right) R_j = 0 \tag{4.9}$$

Solving the above equation for Q_{ci}, we will know the amount of reactive power that the capacitor has to be produce (the capacitor size) at various locations.

$$Q_{ci} = -|v_i| \frac{\sum_{j=1}^{n} G_{ij} A_j}{\sum_{j=1}^{n} G_{ij} R_j} \tag{4.10}$$

Where $A_j = I_{rj} . R_j$, $j = 1,2,\ldots n$

The above process helps to identify the capacitor bus that resulted highest loss savings. When the candidate bus is identified and compensated, the above technique also be used to identify the next and subsequent nodes.

4.3 Optimum capacitor values

Once a suitable sequence of capacitor location is found using singly located capacitors at each bus, the next step is to find the global optimal size of capacitors of the system. The capacitors obtained in equation 4.10 is a local optimum values, but not a global optimum. From the node-to-branch incidence matrix, identify and select the whole row of the compensated buses identifiers (i), and take the transposed (G^c).

$$G^c = [G(i,:)]^T \tag{4.11}$$

Where $i = 1,2,...,k$, k - the number of capacitor buses

The new reactive component of branch currents after the buses are compensated by the capacitors is given by

$$I_{rj}^{''} = I_{rj} + \sum_{i=1}^{k}\left(G_{ji}^{c} \frac{Q_{ci}}{|v_i|} \right) \tag{4.12}$$

The loss $P_{Lr}^{''}$ associated with the new reactive current is

$$P_{Lr}^{''} = \sum_{j=1}^{n}\left(I_{rj} + \sum_{i=1}^{k} G_{ji}^{c} \frac{Q_{ci}}{|v_i|} \right)^2 R_j \tag{4.13}$$

The loss saving due to the capacitor placement is the difference between equation 4.4 and 4.13

$$\Delta P_{LS}^{'} = P_{Lr} - P_{Lr}^{''}$$
$$\Delta P_{LS}^{'} = -\sum_{j=1}^{n}\left(2I_{rj}\sum_{i=1}^{k} G_{ji}^{c} \frac{Q_{ci}}{|v_i|} + \left(\sum_{i=1}^{k} G_{ji}^{c} \frac{Q_{ci}}{|v_i|} \right)^2 \right) R_j \tag{4.14}$$

Maximizing the loss saving of equation 4.14 is given by

$$\frac{\partial \Delta P_{LS}^{'}}{\partial Q_{ck}} = 0 \tag{4.15}$$

Solving Q_{ck} from equation 4.15 (the derivation is given in appendix C), is given by a set of linear algebraic equations as

$$[C][Q_{ck}] = [D] \tag{4.16}$$

The global optimum capacitors size for the maximum loss saving (in Kvar) can be obtained

$$[Q_{copt}] = [C]^{-1}[D] \tag{4.17}$$

Where C is a KxK square matrix and D is a K-dimensional vector. The elements of C and D are given by

$$C_{mk} = \frac{1}{|v_k|} \sum_{j=1}^{n} \left(G_{jm}^{c} . G_{jk}^{c} \right) R_j \tag{4.18}$$

$$D_k = -\sum_{j=1}^{n} G_{jk}^{c} I_{rj} R_j \tag{4.19}$$

4.4 Types of shunt capacitors

Fixed capacitors are suitable for steady loads which do not have a large variance. For loads that swing widely and have a shorter lasting peak, switched capacitors are more appropriate.

One way of determining whether a load is considered as a steady or has a short lasting peak is by examining its load and loss factor.

By definition, load factor is [12]

$$F_{LD} = \frac{AverageLoad}{PeakLoad} \tag{4.20}$$

Thus, a load factor approaching the value 1 indicates that the average load is very close to the peak and the load is steady. However, a low load factor indicates the opposite, of a load that has more variation with time. Similarly, the loss factor is defined as [12]

$$F_{Ls} = \frac{AveragePowerLoss}{PeakPowerLoss} \tag{4.21}$$

The loss factor only applies to the I^2R loss of the system. Therefore, a loss factor approaching to the value 1 indicates that the load currents are quit steady; where as smaller loss factor indicates more varying load currents and:

- If the load is steady, the value of the loss factor approaching the value of the load factor, $F_{Ls} \to F_{LD}$
- If the load has a very short lasting peak, the value of the loss factor approaches to the square of the load factor, $F_{Ls} \to F_{LD}^2$

Therefore, in general the value of the loss factor is

$$F_{LD}^2 < F_{Ls} < F_{LD} \qquad (4.22)$$

Using above relationship the approximate formula for loss factor is given by [10]

$$F_{Ls} = 0.3F_{LD} + 0.7F_{LD}^2 \qquad (4.23)$$

Using equation 4.23, several rules can be established to determine where a suitable capacitor location requires a fixed or switched capacitor. The rules are:

- If the load factor is approaching a value of 1 or is considered high, the loss factor is also high indicating that the load is steady, and then only a fixed capacitor is required.
- If the load factor is low, the loss factor is even lower indicating that the load varies widely with a shorter peak, and a much higher losses occur during the peak, then a switched capacitor is desired

Further investigating of the load factor also aid in determining the sizes of the switched capacitor required for the nodes suited for switched capacitors. The optimum level of reactive compensation is best when the value of the per-unit total reactive compensation is close to the loss factor [7].

Thus, using this correlation, and given the above equation, the following rules can be established to determined if the node is suitable for switched capacitor and also the amount of compensation level that should be switched on during peak period.

- If load factor, is high, then use only a fixed capacitor of the size, 100% of the compensation level found
- If load factor, is medium, then use a fixed capacitor of the size, $(0.3F_{LD} + 0.7F_{LD}^2)$ x100% of the compensation level found and a switched capacitor for the remainder of the compensation level.
- If load factor, is low, then use a fixed capacitor of the size F_{LD}^2x100% of the compensation level found and a switched capacitor for the remainder of the compensation level.

The decision values for low, medium and high is given in [13] as less than 0.35, between 0.35 and 0.65 and greater than 0.65 respectively.

4.5 Algorithm

The load flow program calculates the branch currents, the node voltage magnitude and phase angle, and the real and reactive power loss before compensation. The reactive currents and the node voltages are used to obtain the maximum loss savings and the corresponding capacitor sizes. These maximum loss saving determine the node most suited for capacitor installation. By compensating the selected bus with the corresponding capacitors the process repeats until a significant loss saving is achieved. Once the nodes are identified, the optimal size of capacitors to be placed at the chosen nodes is determined using the loss minimization techniques for multiple capacitor locations. Using the Load factors of the compensated node the need and size of the switched capacitors are determined. Finally the load flow program runs to calculate the power loss reduction, the node voltage and branch currents after compensation Figure 4.2 illustrates the steps followed for the study of capacitor placement for loss reduction.

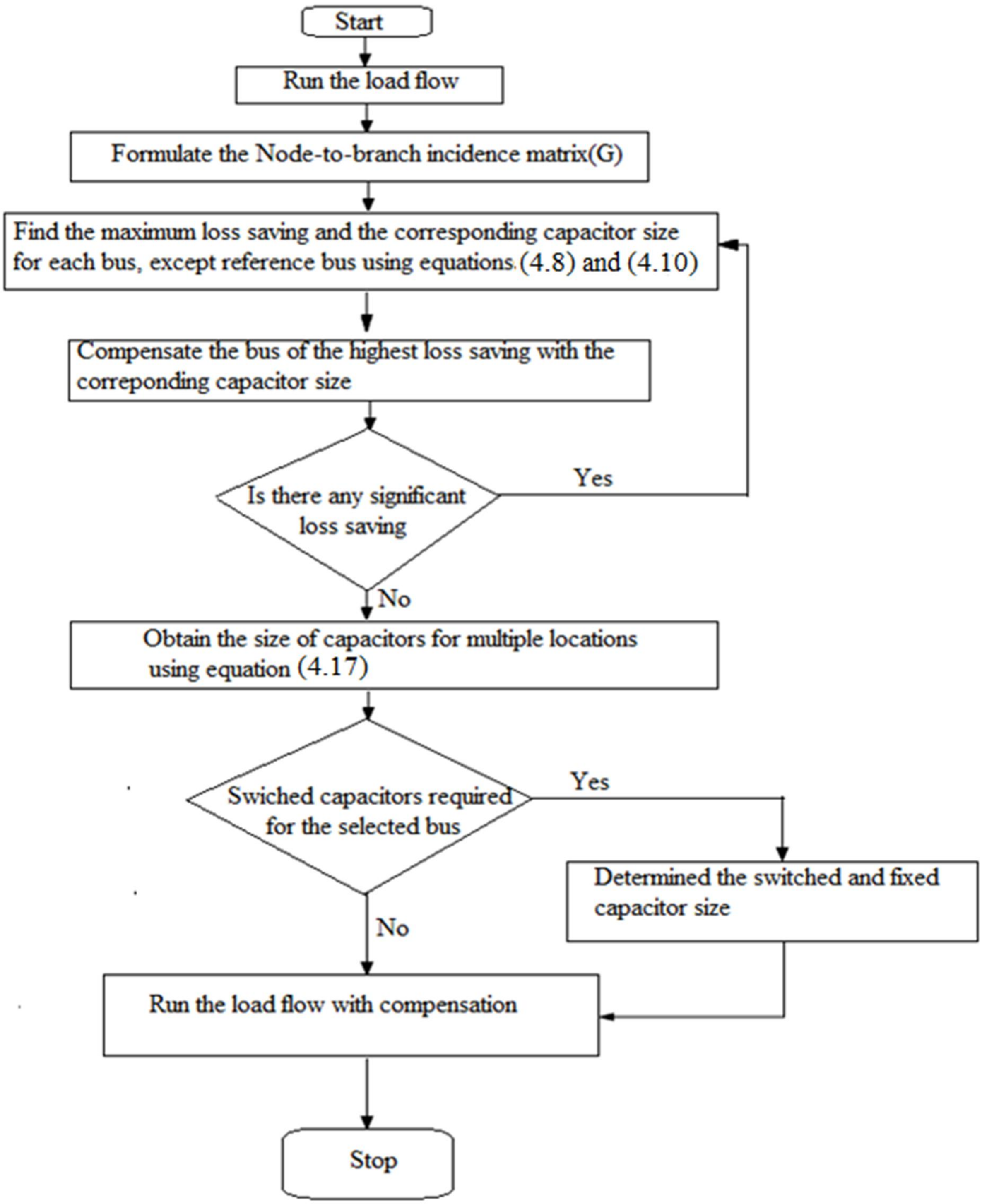


Figure 4.2 Flow chart

5. DISCUSSIONS OF THE RESULTS

Before solving the capacitor placement, it is important to find the node voltages and branch currents at each node of the distribution system. The branch currents are used to calculate the real and reactive power loss at each branch and the node voltages are used to find the size of capacitors needed to be installed. The algorithm given in section 4.5 is used on thirty four-bus radial feeder of figure 3.1. The power flow method given in chapter three is implemented on this feeder using MATLAB which is power full software used to analyze complex analysis.

5.1 Analysis of Thirty Four-Bus Radial Distribution Feeder models

The single line diagram, the line and load data of 15 KV, 34-bus system which are given in figure 3.1 and table 3.2 respectively are consider hereunder.

The power flow analysis given in chapter three is used to find the node voltages and the total real and reactive power losses before compensation and the results are given in table 5.1

Table 5.1 Power flow solution of a 34 bus system before compensation

Bus No.	Voltage magnitude (Kv)	Angle(degrees)	Branch Active power loss (Kw)	Branch Reactive power loss (Kvar)
0	15.000000	000000	0	0
1	14.884009	-0.061592	14.167	15.801
2	14.879010	-0.065463	0.55916	0.70535
3	14.874109	-0.069337	0.54963	0.69714
4	14.851015	-0.081019	2.5234	2.7918
5	14.846016	-0.084905	0.48217	0.61509
6	14.814011	-0.069620	3.9453	2.6075
7	14.787007	-0.054246	3.2392	2.0524

8	14.763004	-0.042690	2.5507	1.6773
9	14.883009	-0.061596	0.0036246	0.0030002
10	14.849015	-0.081030	0.0141254	0.0110704
11	14.848015	-0.081035	0.0028251	0.0022124
12	14.813011	-0.069622	0.0049901	0.0036299
13	14.747002	-0.031082	0.442710	0.26449
14	14.743002	-0.031094	0.0716364	0.0631933
15	14.736001	-0.023329	0.124460	0.061604
16	14.725003	-0.011673	0.132230	0.073084
17	14.740002	-0.027210	0.0143527	0.00530
18	14.756004	-0.042712	0.506150	0.42638
19	14.728001	-0.023342	1.998100	1.1328
20	14.722001	-0.019459	0.0904424	0.0603323
21	14.719001	-0.015571	0.0370776	0.0175325
22	14.725001	-0.023346	0.147350	0.11799
23	14.719001	-0.019463	0.268400	0.14460
24	14.712001	-0.015578	0.257350	0.15342
25	14.710001	-0.015580	0.024326	0.0186294
26	14.708000	-0.011687	0.0267092	0.0068719
27	14.718001	-0.019465	0.0066553	0.0040778
28	14.717009	-0.019466	0.0025808	0.0011556
29	14.709000	-0.015581	0.0623922	0.0519324
30	14.707000	-0.011687	0.0471494	0.0117149

31	14.706000	-0.011688	0.0135945	0.0124161
32	14.703000	-0.007794	0.0236982	0.0112752
33	14.705000	-0.011689	0.0076125	0.0066637
Total power loss			32.332 Kw	29.607 Kvar

5.1.1 Results based on Capacitor Placement

The thirty four-buses radial distribution feeder has a total load of (2042.6+j1592.8) KVA and the total losses associated with the active and reactive power without capacitors are 32.332Kw and 29.607Kvar respectively.

The approximate loss saving due to the supply of reactive component of current using shunt capacitor at each node and the respective capacitor size is obtained by using equations 4.8 and 4.10 respectively. The results are computed with MATLAB as given in table 5.2

Table 5.2 Loss saving due to capacitor placement at each node of thirty four-bus feeder

Bus No.	Loss savings (Kw)	Capacitor size required (Kvar)	Bus No.	Loss savings (Kw)	Capacitor size required (Kvar)
1	5.4685	1618.2	18	10.5590	1363.4
2	5.6752	1615.9	19	10.9590	1242.2
3	5.9275	1612.6	20	9.6852	1066.9
4	6.8712	1581.3	21	8.9750	977.35
5	7.0590	1572.5	22	10.941	1223.8
6	8.4833	1502.7	23	10.7840	1179.5
7	9.6427	1453.5	24	10.515	1118.8
8	10.4890	1396.8	25	10.0680	1063.4

Bus No.	Loss savings (Kw)	Capacitor size required (Kvar)	Bus No.	Loss savings (Kw)	Capacitor size required (Kvar)
9	3.9355	1151.1	26	9.4235	985.101
10	5.3350	1206.7	27	9.9179	1079.3
11	5.0295	1135.5	28	9.7558	1062.0
12	7.9071	1396.6	29	10.2310	1075.5
13	8.9628	1101.8	30	9.9589	1037.7
14	8.5773	1036.4	31	9.7558	1010.7
15	7.6792	894.75	32	8.8475	906.81
16	6.3180	698.69	33	9.5887	991.8
17	8.1154	977.98	The maximum loss saving is obtained at node 19		

From table 5.2 the maximum loss savings is 10.9590 Kw, which is obtained by compensating node 19 with 1242.2 Kvar capacitors. Therefore the first candidate bus to be compensated is bus number 19.

The same process and equations 4.6, 4.8 and 4.10 are used to find the next bus to be compensated after compensated bus 19 and the results are depicted in table 5.3.

Table 5.3 Loss saving and capacitor size obtained after node 19 is compensated

Bus No.	Loss savings (Kw)	Capacitor size required (Kvar)	Bus No.	Loss savings (Kw)	Capacitor size required (Kvar)
1	0.2748577	362.78	18	0.0802167	118.84
2	0.2831520	360.95	19[1]	----	----
3	0.2921874	358.03	20	0.0074808	29.650
4	0.2968383	328.67	21	0.0148242	39.721
5	0.2928516	320.29	22	0.0018388	15.866
6	0.2408151	253.18	23	0.0128080	40.649
7	0.1941303	206.23	24	0.0382101	67.443
8	0.1235874	151.62	25	0.0453318	71.352
9	0.2138827	268.34	26	0.0550224	75.281
10	0.2613450	267.08	27	0.0152900	42.378
11	0.2494278	252.88	28	0.0152417	41.964
12	0.2307420	238.57	29	0.0521273	76.768
13	0.2782846	194.15	30	0.0628493	82.434
14	0.3099421	197.01	31	0.0699594	85.585
15	0.3670333	195.61	32	0.0785740	85.457
16	0.4199633	180.14	33	0.0718561	85.834
17	0.2998581	187.99	The maximum loss saving is obtained by compensating node 16 with 180.14 Kvar capacitor		

From the table 5.3, after placing 1242.2Kvar at bus 19, it was found the next capacitor size of 180.14 Kvar at bus 16 which results a further loss saving of 0.4199633 Kw. Therefore the optimum location and sequence of nodes to be compensated for the maximum loss savings are 19 and 16.

The capacitors size obtained using equation 4.10 is local maximum values. Therefore it is necessary to find the global maximum values. Using equations 4.17 the global optimum capacitor sizes are found as 1041.4 Kvar and 264 Kvar for buses 19 and 16 respectively. When the capacitor sizes are rounded to the nearby standard sizes, it becomes 1050 Kvar and 300 Kvar for nodes 19 and 16 respectively.

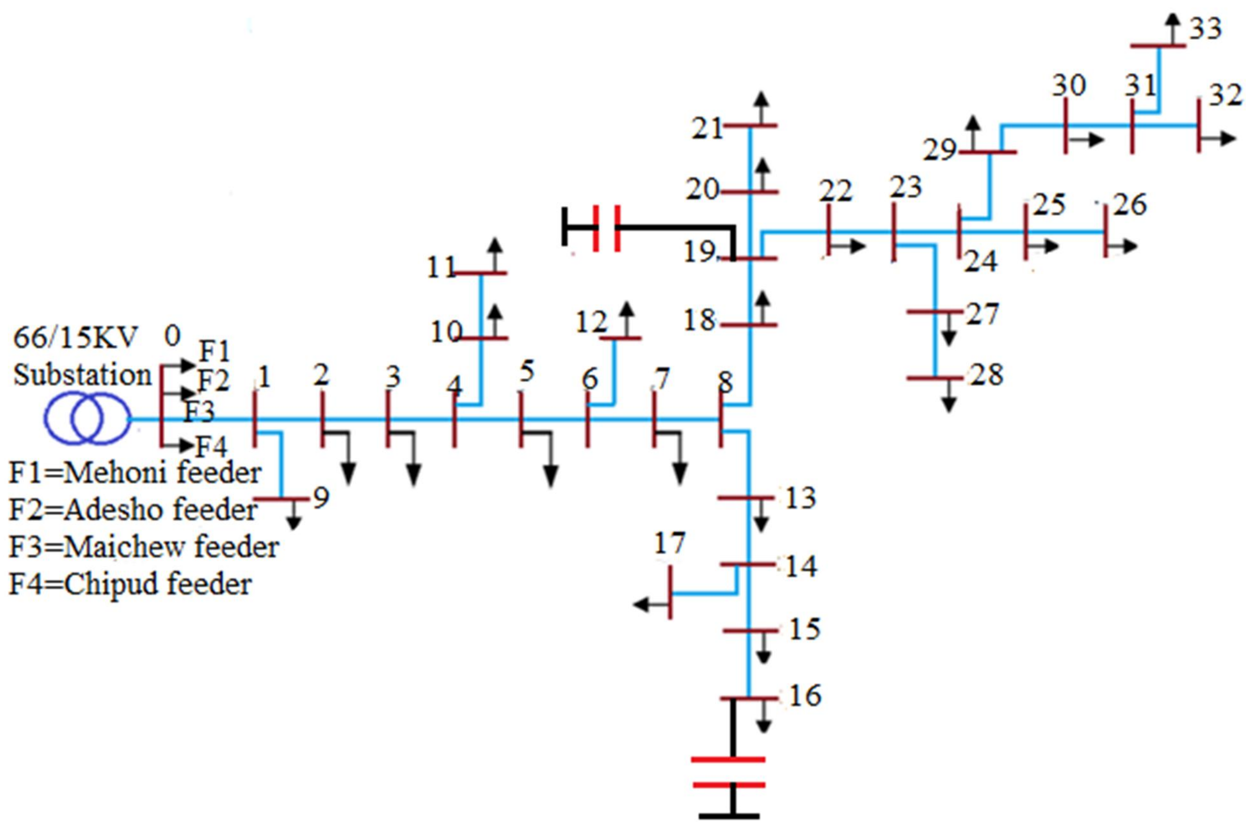


Figure 5.1 Thirty four-bus radial distribution feeder after compensation

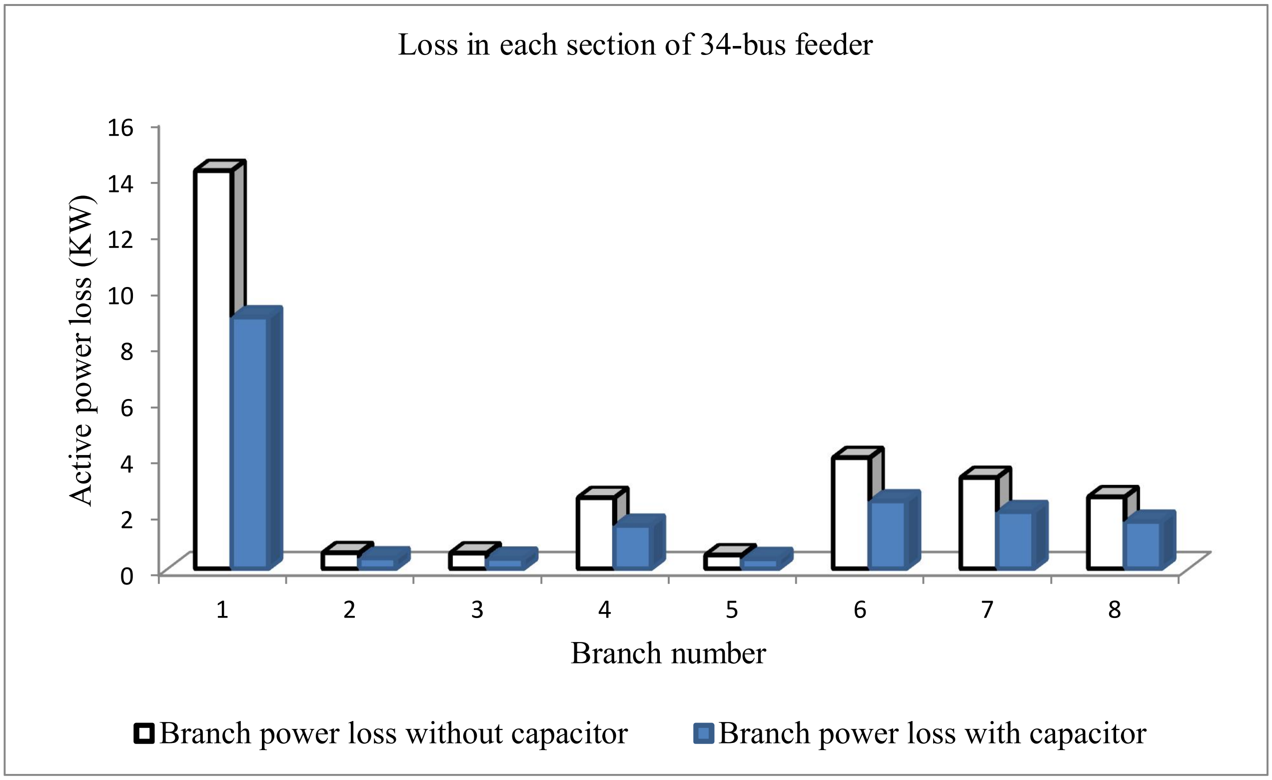
Load flow is run after the identified buses are compensated with the selected capacitor sizes and the results are tabulated in table 5.4

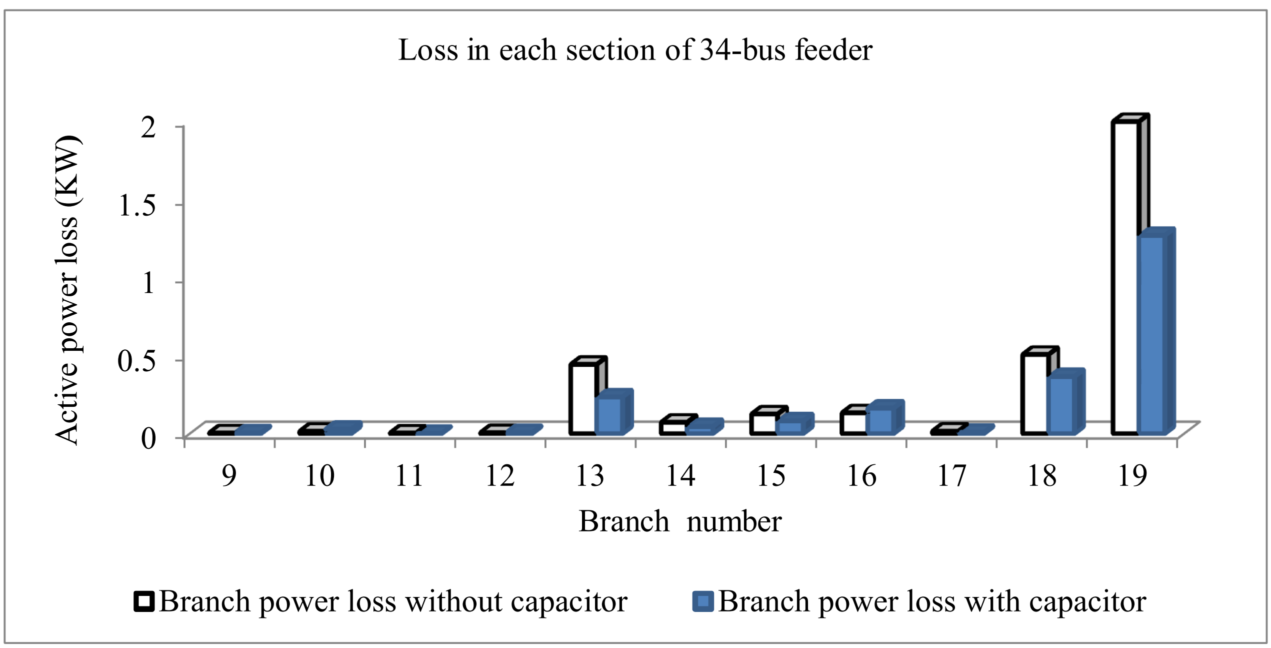
Table 5.4 Load flow solutions for 34 bus feeder after compensation

Bus No.	Voltage magnitude (Kv)	Angle (degrees)	Bus No.	Voltage magnitude (Kv)	Angle (degrees)
0	15.000000	000000	17	14.839535	-0.486495
1	14.928117	-0.226449	18	14.848509	-0.474624
2	14.925125	-0.234172	19	14.833661	-0.540765
3	14.922137	-0.245738	20	14.827652	-0.537120
4	14.909199	-0.295911	21	14.824642	-0.533364
5	14.906209	-0.303658	22	14.830651	-0.537011
6	14.885291	-0.357974	23	14.824652	-0.537229
7	14.867378	-0.408505	24	14.818643	-0.533580
8	14.852477	-0.459066	25	14.816643	-0.533652
9	14.927117	-0.226464	26	14.813634	-0.529893
10	14.906194	-0.292127	27	14.823652	-0.537265
11	14.906194	-0.292127	28	14.823652	-0.537265
12	14.884291	-0.357998	29	14.815633	-0.529821
13	14.842526	-0.482536	30	14.812634	-0.529928
14	14.840543	-0.490322	31	14.811634	-0.529964
15	14.837587	-0.509729	32	14.809624	-0.526167
16	14.834699	-0.556177	33	14.811634	-0.529964
Total active power loss (Kw)			20.871		
Total reactive power loss (Kvar)			17.585		

From table 5.4 the total active and reactive power losses after bus 19 and 16 are compensated found to be 20.871 KW and 17.585 Kvar and before compensation were 32.332 Kw and 29.607

Kvar respectively. Therefore 11.461 Kw active powers and 12.022 Kvar reactive powers are saved by compensating node 19 and 16 with the total capacitor size of 1350 Kvar. The losses in each branch is reduced after loss optimization, and it is clearly shown in figure 5.2





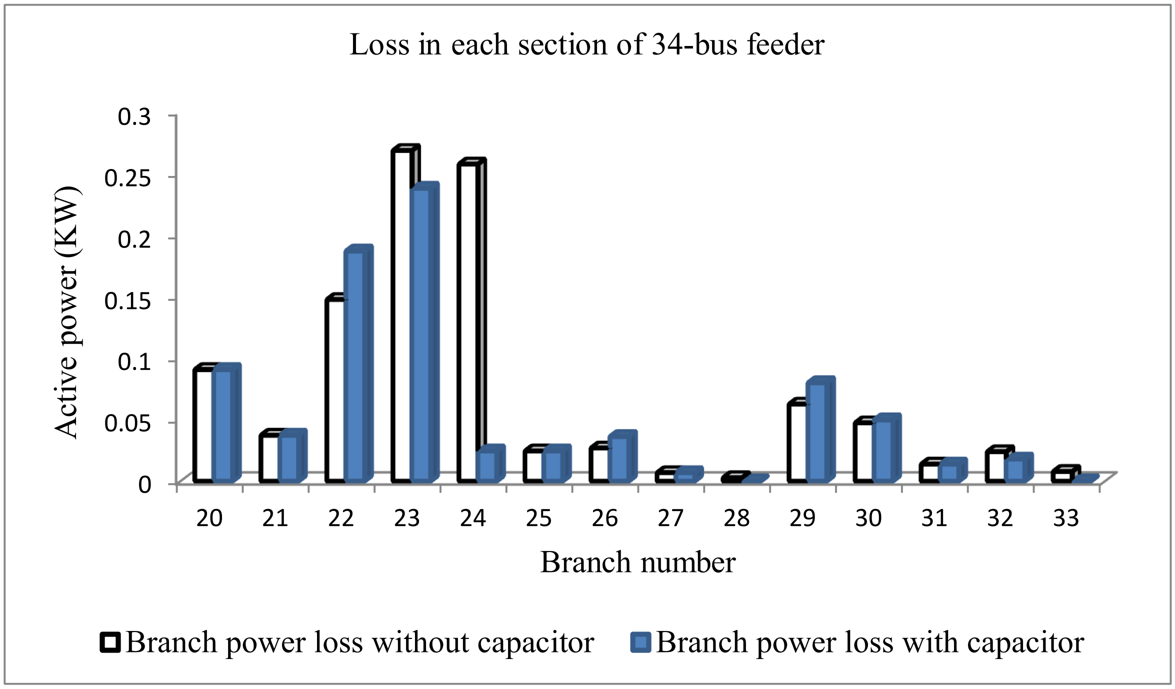


Figure 5.2 Branch power losses before and after compensation

From table 5.4, it is observed that the voltage profile is improved after the selected node is compensated. The minimum node voltage which was 0.98020009 Pu before compensation becomes 0.987308298 Pu after compensation.

5.1.2 Determining switched capacitors

The load factor of bus 16 is 0.63 for thirty four-bus system, therefore these nodes quantify to have both switched and fixed capacitors. Using section 4.4 the switched capacitor sizes are 150 Kvar and the remaining 150 Kvar are fixed values.

5.2 Explanation on Savings and Economic Benefits of the Results

It is expected that the use of electricity generated by hydro power and diesel generators system in both urban and rural areas would improve quality life of the households; through improving social services in healthy, education and water, agro-industry and other income generating activities. But due to economic and technical challenges, EEPCO does not fulfill the demand of electric power. Hence, to fulfill the gap, a considerable effort should be done in parallel for additional power generation and to minimize the losses in distribution of powers for further uses both in the utility and consumer sides.

In section 5.1 the analysis and results on the thirty four-bus feeder was given. A considerable percentage improvement in electric loss reduction is observed. The detailed explanation in Kwh savings and the socio-economic benefits that will be obtained based on the results in section 5.1 are given in this section.

From table 5.1 and 5.4 the total active and reactive power losses before compensation were found 32.332 Kw and 29.607 Kvar and after compensation it becomes 20.871 Kw and 17.585 Kvar respectively under the peak load condition. It is clear that 11.461 Kw active powers and 12.022 Kvar reactive powers are saved after loss optimization. With reference to the given total demand as much as 1.639 % of the total demand is lost in the distribution feeder, where as this value is reduced to 1.020 % after loss optimization.

The annual total electric energy saving due to optimization of primary distribution feeder loss taking quarter of the 24 hours is about 25.09959 Mwh. With an average load factor of the feeder 0.6 the actual energy that can be commercialized is 15.059754 Mwh. It is clear that it is a significant number. With an average monthly consumption of a residence be 150 Kwh, more than 8 households can be electrified. Financially about 5,662.50 birr per annul will be collected assuming average billing of 0.376 Birr/Kwh. Assuming fixed and switched capacitor investment cost is 63.7 Birr/Kvar[14], then the cost of the capacitor will be 85,995 birr. The pay back period for the capacitor cost will be about 15 years. Here due to the safe use of the system components (distribution transformers, feeder lines, protective devices, relays, etc.) is increased extra cost will not be invested.

5.3 Thirty four-Bus Feeder Voltage Profile

Capacitor placement in the primary distribution feeder, besides optimization of feeder loss will also improve the voltage regulation of the system. It is usually convenient to discus distribution feeder voltage regulation in terms of voltage profiles of the feeder, because the voltages are everywhere different on the feeder. A profile simply indicates the feeder voltage magnitude versus location on the feeder. One of the factors that determine the quality of power supply is maintaining the sender voltage and the receiver voltage as closer as possible. In order to achieve this, practically different compensating devices are placed along the distribution system. Among them is capacitor placement.

Considering tables 5.1 and 5.4 the voltages at each node decreases as one move from the reference node to the last node, due to the voltage drop in the line. The capacitor produces a voltage rise because of its leading current flowing through the inductive reactance of the feeder. This capacitor improves the voltage level at each node which is shown in figure 5.4 below.

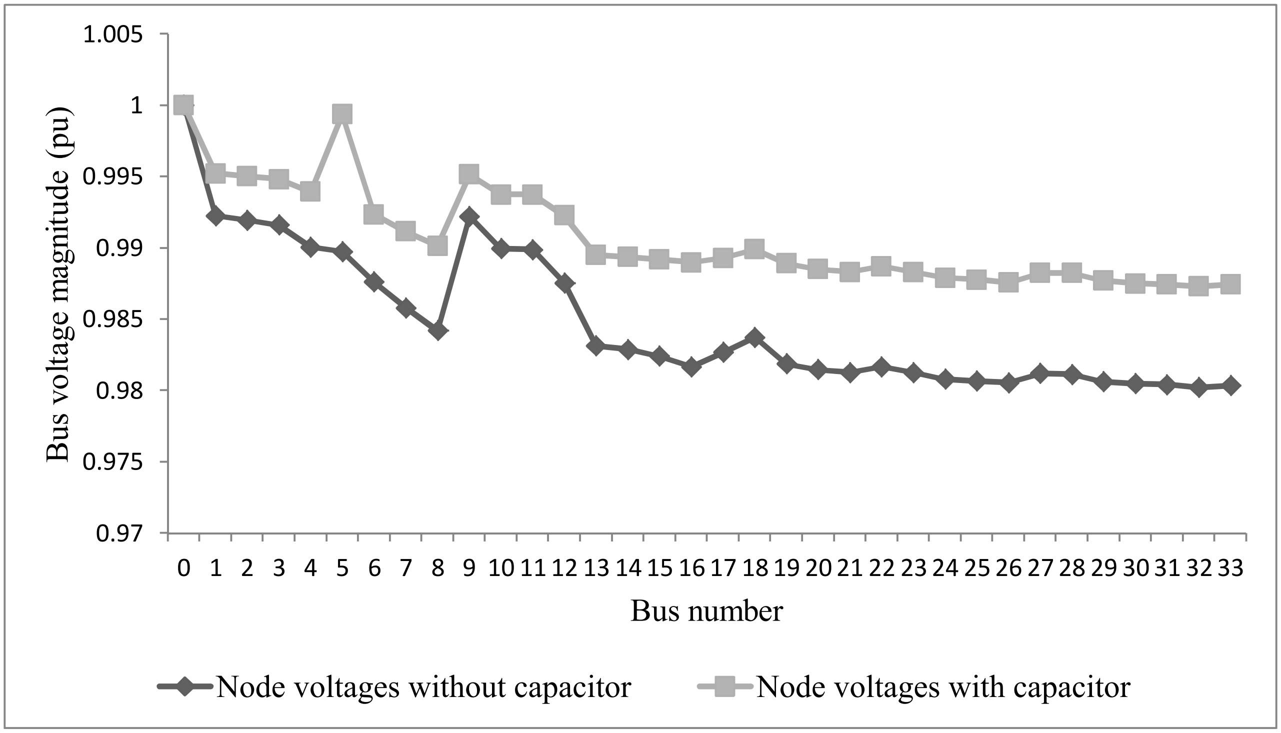


Figure 5.3 Voltage profile of 34 bus feeder

From figure 5.4, it is clearly observed that the voltage profile is improved after the selected node is compensated. The minimum node voltage after compensation is 0.987308 Pu which was 0.980200 Pu before compensation. Therefore its voltage regulation improved from 2.02% to 1.29 %. Therefore it is concluded that due to capacitor placement the voltage profile and hence the voltage regulation of the system is improved.

5.4 Summary of the Result

Increased demand for power is pushing power distribution networks to their upper limits and beyond that. The more power flows through a distribution network, the more voltage drop becomes excessive and power quality seriously declines. In addition, reactive power losses cause vast amount of energy to be wasted unnecessarily when power is distributed to end users. The simplest and cost effective solution is to install shunt capacitor banks at selected points along the distribution lines.

Optimum capacitor placement in a distribution system, which essentially includes the optimal location and size of capacitors, is required to minimize the losses associated with the reactive component of branch currents. The location of capacitors is identified through optimizing the losses, which caused by the reactive component of branch currents obtained from the load flow solutions, by placing a capacitor at each node. The optimal capacitor size at each selected location is determined through optimizing of the loss saving equations with respect to the capacitor sizes for multiple nodes identified for capacitor placement. The fixed and switched capacitor sizes are determined based on the load factor.

Thirty four-bus distribution system of the Maichew town is considered for optimal capacitor placement to minimize the distribution feeder loss that caused by the reactive component of branch currents, in turn to improve the system performance. The result shows that the reduction of 35.447854% of the total loss for thirty four bus system. In addition to voltage regulation improvement, the total feeder capacity is released by 554.09128KVA. Detailed explanation on capacitor size required, saving of electrical energy, and the socio-economic benefits of optimizing the losses in the medium voltage distribution networks through optimum capacitor placement in a distribution lines are given.

The results of the previous sections can be summarized shortly as follows in the following tables

Table 5.5 Thirty four-bus system

Description	Without compensation	With compensation
Total capacitor required (Kvar)	0	1350
Total line current (A)	175.909529	139.334236
Released line current (A)	---	36.575293
Total reactive power demand (Kvar)	1622.407	260.385
Total released reactive power (Kvar)	….	1362.022
Minimum voltage (Pu)	0.980200	0.987308
Voltage regulation (%)	2.02	1.29
Improvement of voltage regulation (%)	….	0.73
Total loss (Kw)	32.332	20.871
Total loss reduction (%)	…	35.447854
Total demand (Kw)	2074.932	2063.471
Total released demand (Kw)	…	11.461
Total feeder capacity (KVA)	2633.926109	2079.834829
Total released feeder capacity (KVA)	…	554.09128

6. CONCLUSION AND RECOMMENDATIONS

6.1 Conclusion

Minimization of distribution feeder loss due to optimum reactive power compensation in the distribution networks is successfully achieved through identifying the optimal location and size of capacitors. For identifying the sequence of nodes to be compensated repetitive loss minimization technique using single capacitors at each bus is used. For the global capacitor size determination the loss minimization techniques for multiple capacitors are used. This method is used in thirty four-bus radial distribution feeder, this results a significant loss reduction and voltage regulation improvement.

Reduction of reactive current flow by installing shunt capacitors in the primary distribution system releases the generation, substation capacities and reduces the transmission and distribution line losses as a result efficiency of the system improved. Minimization of the distribution line losses which is caused by reactive current flow saved a large amount of electric energy annually. The saved energy has a great impact socially and economically both for the utility and the consumers while the saved energy is supplied to the consumers. Besides satisfying the consumer's need the utility returns the capacitors cost and collect extra money in a long run by supplying the saved energy to new consumers where as the consumer could have infrastructure facilities and continuity of power supply.

Minimization of losses in distribution system and electrification of new areas with saved energy reduces the chance of using diesel generators hence contributes in reduction of global warming which is caused by emission of carbon dioxide to the atmosphere from diesel generators.

Regarding the voltage profile, loss minimization using shunt capacitors reduces the current flows in each section of the feeder and hence the voltage drop in the section reduced. Therefore the voltage profile is improved after the feeder line is compensated and the quality of the supply is improved.

6.2 Recommendation

In the present energy crises implementing energy loss reduction techniques has a paramount effect in the development of the country socially and economically. Capacitor for loss minimizations has a relatively low cost and shorter return period than other methods, and it is advisable to implement this method in conjunction with other natural compensation methods such as phase balancing to satisfy the increased load demand. Distribution feeders having short length had lower line losses despite of higher reactive current flow. Therefore it is recommended to compensate the high reactive power flow by the consumers rather than the utilities.

In this thesis, optimization of unconstrained loss saving equation is carried out to obtain the optimal location and size of capacitors, but for further study optimization of loss can be done using one of artificial intelligent methods like genetic algorithms considering constraint parameters such as bus voltage and capacitor cost.

For longer distribution feeders, though the loads are balanced at substation, unbalance will be created due to the difference in mutual coupling between the phase lines. Therefore, other researchers can extend this work for further study by considering unbalanced three phase system.

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APPENDIX A: DAILY DEMAND PROFILE AT SUBSTATION

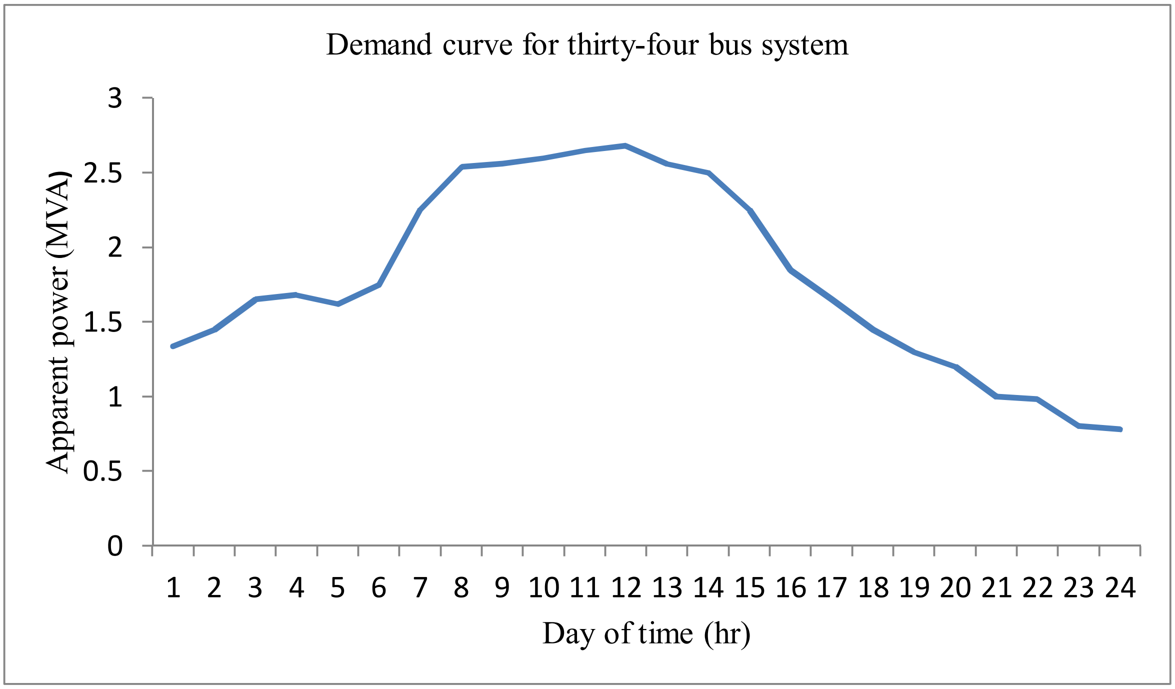

Figure A.1 Demand curve of thirty four- bus system

APPENDIX B: OVERHEAD LINE IMPEDANCE CALCULATIONS

Using the data in figure 3.2, table 3.1 and applying equations (2.6) and (2.7), the impedances are obtained as:

1. AAC-95

Using equation (2.6) the self impedances for phase conductors are

$$z_{aa} = r_a + 0.049348 + j0.062832\left(\ln\frac{1}{GMR_a} + 8.025174\right)\Omega/km$$

$$= 0.3084 + 0.049348 + j0.062832\left(\ln\frac{1}{0.004129} + 8.025174\right)\Omega/km$$

$$= (0.357748 + j0.849168)\Omega/km$$

$$z_{aa} = z_{bb} = z_{cc}$$

Applying equation 2.7 for the mutual impedance between phases a and b, phase a and c, phase b and c.

$$z_{ab} = 0.049348 + j0.062832(\ln\frac{1}{D_{ab}} + 8.025174)\Omega/km$$

$$= 0.049348 + j0.062832(\ln\frac{1}{0.6} + 8.025174)\Omega/km$$

$$= (0.049348 + j0.536334)\Omega/km$$

$$z_{ac} = 0.049348 + j0.062832(\ln\frac{1}{D_{ac}} + 8.025174)\Omega/km$$

$$= 0.049348 + j0.062832(\ln\frac{1}{1} + 8.025174)\Omega/km$$

$$= (0.049348 + j0.504238)\Omega/km$$

$$z_{bc} = 0.049348 + j0.062832(\ln\frac{1}{D_{bc}} + 8.025174)\Omega/km$$

$$= 0.049348 + j0.062832(\ln\frac{1}{0.4} + 8.025174)\Omega/km$$

$$= (0.049348 + j0.561810)\Omega/km$$

The positive sequence impedance is obtained using equations (2.19) to (2.22)

Applying equation (2.19) the self impedance is

$$z_s = \frac{1}{3}(z_{aa} + z_{bb} + z_{cc})\Omega/km$$

$$= (0.357748 + j0.849168)\Omega/km$$

Using equation (2.20) the mutual impedance is

$$z_m = \frac{1}{3}(z_{ab} + z_{bc} + z_{ca})\Omega/km$$

$$= \frac{1}{3}(0.148044 + 1.60238)\Omega/km$$

$$= (0.049348 + j0.534127)\Omega/km$$

The positive sequence impedance z_{11} of AAC-95 is obtained using equation (2.22) as:

$$z_{11} = z_s - z_m \ \Omega/km$$

$$= (0.3084 + j0.315041)\Omega/km$$

The line segments positive sequence impedances are obtained by multiplying the impedance per kilometre by its length and the result is depicted in table 3.2.

2. AAC-50

Like AAC-95 the process and equations are followed to obtain the section impedances of the feeder.

$$z_{aa} = r_a + 0.049348 + j0.062832\left(\ln\frac{1}{GMR_a} + 8.025174\right)\Omega/km$$

$$= 0.5786 + 0.049348 + j0.062832\left(\ln\frac{1}{0.0028\ 8\ 0} + 8.025174\right)\Omega/km$$

$$= (0.627948 + j0.871803)\Omega/km$$

$$z_{aa} = z_{bb} = z_{cc}$$

The mutual impedance between phases a and b, phase a and c, phase b and c.

$$z_{ab} = 0.049348 + j0.062832(\ln\frac{1}{D_{ab}} + 8.025174)\Omega/km$$

$$= 0.049348 + j0.062832(\ln\frac{1}{0.6} + 8.025174)\Omega/km$$

$$= (0.049348 + j0.536334)\Omega/km$$

$$z_{ac} = 0.049348 + j0.062832(\ln\frac{1}{D_{ac}} + 8.025174)\Omega/km$$

$$= 0.049348 + j0.062832(\ln\frac{1}{1} + 8.025174)\Omega/km$$

$$= (0.049348 + j0.504238)\Omega/km$$

$$z_{bc} = 0.049348 + j0.062832(\ln\frac{1}{D_{bc}} + 8.025174)\Omega/km$$

$$= 0.049348 + j0.062832(\ln\frac{1}{0.4} + 8.025174)\Omega/km$$

$$= (0.049348 + j0.561810)\Omega/km$$

The positive sequence impedance is obtained using equations (2.19) to (2.22)

$$z_s = \frac{1}{3}(z_{aa} + z_{bb} + z_{cc})\Omega/km$$

$$= (0.627948 + j0.871803)\Omega/km$$

$$z_m = \frac{1}{3}(z_{ab} + z_{bc} + z_{ca})\Omega/km$$

$$= \frac{1}{3}(0.148044 + 1.583964)\Omega/km$$

$$= (0.049348 + j0.534127)\Omega/km$$

The positive sequence impedance z_{11} of AAC-50 is obtained using equation (2.22) as:

$$z_{11} = z_s - z_m \; \Omega/km$$

$$= (0.5786 + j0.337676)\Omega/km$$

The line segments positive sequence impedances are obtained by multiplying the impedance per kilometre by its length as tabulated in table 3.2.

Since the line is short, the shunt admittance of overhead lines is neglected.

APPENDIX C: DERIVATION OF OPTIMAL CAPACITOR SIZE FOR MULTIPLE CAPACITOR LOCATIONS

The total active power loss of n branch feeder before compensation is

$$P_{Lr} = \sum_{j=1}^{n} I_{rj}^{2}.R_j$$

The new reactive component of branch currents after the buses are compensated is given by:

$$I_{rj}^{''} = I_{rj} + \sum_{i=1}^{k}\left(G_{ji}^{c}\frac{Q_{ci}}{v_i}\right)$$

The total power loss after compensation is:

$$P_{Lr}^{''} = \sum_{j=1}^{n}\left(I_{rj}^{''}\right)^{2}.R_j$$

The loss saving due to the capacitor placement is the difference of the loss before compensation and the loss after compensation which is given as:

$$\Delta p_{LS}^{'} = \sum_{j=1}^{n}(I_{rj}^{2}.R_j - \left(I_{rj}^{''}\right)^{2}.R_j)$$

$$= \sum_{j=1}^{n}\left(I_{rj}^{2} - \left(I_{rj}^{''}\right)^{2}\right).R_j$$

$$= -\sum_{j=1}^{n}\left(2I_{rj}.\sum_{i=1}^{k}G_{ji}^{c}\frac{Q_{ci}}{|v_i|} + \left(\sum_{i=1}^{k}G_{ji}^{c}\frac{Q_{ci}}{|v_i|}\right)^{2}\right).R_j$$

The optimal capacitor currents for maximum loss saving can be obtained by differentiating the loss saving equation with respect to the capacitor size and equating to zero as:

$$\frac{\partial \Delta p_{LS}^{'}}{\partial Q_{ck}} = -\sum_{j=1}^{n}\left(2I_{rj}.G_{jk}^{c} + 2\sum_{i=1}^{k} G_{ji}^{c}\frac{Q_{ci}}{|v_i|}.G_{jk}^{c}\right).R_j = 0$$

$$\Rightarrow -\sum_{j=1}^{n} G_{jk}^{c} I_{rj} R_j = \sum_{j=1}^{n} G_{jk}^{c} R_j \sum_{i=1}^{k} G_{ji}^{c}\frac{Q_{ci}}{|v_i|}$$

$$\Rightarrow \left[\frac{1}{|v_i|}\sum_{j=1}^{n}\left(G_{jk}^{c}.G_{ji}^{c}\right)R_j\right]\left[Q_{copt}\right] = -\sum_{j=1}^{n} G_{jk}^{c} I_{rj} R_j$$

$$[C]\left[Q_{copt}\right] = [D]$$

$$\left[Q_{copt}\right] = [C]^{-1}[D]$$

where:

$$C_{mk} = \frac{1}{|v_k|}\sum_{j=1}^{n}\left(G_{jm}^{c}.G_{jk}^{c}\right)R_j$$

$$D_k = -\sum_{j=1}^{n} G_{jk}^{c} I_{rj} R_j$$

Where k is the number of capacitor bus

Q_{copt} = k dimensional vector consisting of capacitor size in Kvar

G^c = a matrix of nxk dimension

v_i = k dimensional vector consists of voltage magnitude of capacitor buses

APPENDIX D: DERIVATION OF MODIFIED CARSON'S EQUATION

Carson-Clem's equations

In his paper, Carson assumes the earth is an infinite, uniform solid with a flat uniform upper surface and a constant resistivity. Any end effects introduced at the neutral grounding points are not large at power frequency, and are therefore neglected.

Carson made use of conductor images; that is every conductor at a given distance above ground has an image conductor the same distance below ground.

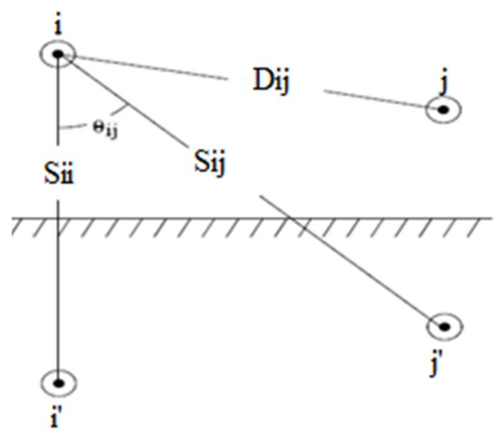

Figure D.1 conductors and images

Self impedance of conductor i :

$$z_{ii} = r_i + 4wP_{ii}G + j\,(x_i + 2wG\ln\frac{S_{ii}}{RD_i} + 4wQ_{ii}G)\ \Omega / mile \qquad (d.1)$$

Mutual impedance between i and j :

$$z_{ij} = 4wP_{ij}G + j\,(2wG\ln\frac{S_{ij}}{D_{ij}} + 4wQ_{ij}G)\,\Omega/mile \qquad \text{(d.2)}$$

Where r_i =resistance of conductor i in $\Omega/mile$

w = 2πf= system angular frequency in radian per second,

f= system frequeny = 50Hz

$G = 0.1609344x10^{-3}\Omega/mile$ $= 10^{-4}\Omega$/km

S_{ij} =Distance between conductor i and j in feet

RD_i = radius of conductor i in feet

Q_{ij} =Angle between a pair of lines drawn from conductor i to its own image and to the image of conductor j

$$x_i = 2\omega G\ln\frac{RD_i}{GMR_i}\quad \Omega/mile \qquad \text{(d.3)}$$

$$p_{ij} = \frac{\pi}{8} - \frac{1}{3\sqrt{2}}K_{ij}\cos(\theta_{ij}) + \frac{K_{ij}^2}{16}\cos(2\theta_{ij}).(0.6728 + \ln\frac{2}{K_{ij}}) \qquad \text{(d.4)}$$

$$Q_{ij} = -0.0386 + \frac{1}{2}\ln\frac{2}{k_{ij}} + \frac{1}{3\sqrt{2}}k_{ij}\cos(\theta_{ij}) \qquad \text{(d.5)}$$

Modified Carson-Clem's equations

Since the use of Carson's original equations was troublesome, some approximations were made in driving the modified Carson's equations. These approximations involve the terms associated with P_{ij} and Q_{ij}, by using only the first term of the variable P_{ij} and the first two terms of Q_{ij}. So the two variables were defined as constants correction factors.

$$P_{ij} = \pi/8 \tag{d.7}$$

$$Q_{ij} = -0.03860 + \frac{1}{2}\ln\frac{2}{K_{ij}} \tag{d.8}$$

Substituting equation (d.6) in to equation (d.8)

$$\begin{aligned} Q_{ij} &= 3.8393 - \frac{1}{2}\ln S_{ij} + \frac{1}{4}\ln\frac{\rho}{f} \\ or \quad 2Q_{ij} &= 7.6786 - \ln S_{ij} + \frac{1}{2}\ln\frac{\rho}{f} \end{aligned} \tag{d.9}$$

Substituting equation (d.3) in to equation (d.1)

$$z_{ii} = r_i + 4\omega P_{ii}G + j\left(2\omega G \ln\frac{RD_i}{GMR_i} + 2\omega G \ln\frac{S_{ii}}{RD_i} + 4\omega Q_{ii}G\right)\Omega/\text{mile}$$

$$= r_i + 4\omega P_{ii}G + j2\omega G(\ln\frac{S_{ii}}{GMR_i} + 2Q_{ii}) \tag{d.10}$$

Substituting for $2Q_{ii}$

$$z_{ii} = r_i + 4\omega P_{ii}G\left(\ln\frac{1}{GMR_i} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f}\right)\Omega/\text{mile}$$

$$
\begin{aligned}
&p_{ii} = \frac{\pi}{8} \\
&G = 10^{-4} \quad \Omega / km \\
&\omega = 2\pi f, \quad f = 50\ HZ \\
&\rho = 100\ \Omega - m \\
&r_i \quad is\ resistance\ in \quad \Omega / km
\end{aligned}
\qquad \text{(d.11)}
$$

Substituting equation (d.11) in to the last equation of (d.10) we obtained:

$$z_{ii} = r_i + 0.049348 + j0.062832\left(\ln\frac{1}{GMR_i} + 8.025174\right)\Omega/\text{km} \qquad \text{(d.12)}$$

Simplifying equation (d.2), we obtained:

$$
\begin{aligned}
z_{ij} &= 4wP_{ij}G + j\,2wG\,(\ln\frac{S_{ij}}{D_{ij}} + 2Q_{ij})\ \Omega / mile \\
&= 4\omega p_{ij}G + j2\omega G(\ln\frac{1}{D_{ij}} + 7.6786 + \frac{1}{2}\ln\frac{\rho}{f})
\end{aligned}
\qquad \text{(d.13)}
$$

Substituting the values in equation (d.11), equation (d.13) becomes

$$z_{ij} = 0.049348 + j0.062832(\ln\frac{1}{D_{ij}} + 8.025174)\Omega/\text{km} \qquad \text{(d.14)}$$

Equations (d.12) and (d.14) are final modified Carson's equations in Ω / km

Where $GMR_i \quad and \quad D_{ij}$ are given in meters