TWO LEVEL ATOM COUPLED TO SQUEEZED VACUUM INSIDE A COHERENTLY DRIVEN CAVITY

By

Kemaw Aragaw Tilahun

A Thesis Submitted to

the Department of Applied Physics

School of Applied Natural Science

Presented In Partial Fulfilment of the Requirement for the Degree of

Master's of science in Physics

Office of Graduate Studies

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Examination Committee Approval Sheet

This is to certify that the thesis prepared by **Kemaw Aragaw** :- **Two-Level Atom Coupled to Squeezed Vacuum Inside a Coherently Driven Cavity** and submitted for partial fulfilment of the requirement for the degree of master in physics complies with the regulation of university meets the accepted standard with respect to originality and ability.

Approved by the Examination Committee

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DECLARATION

I hereby declare that this Msc Thesis is my original work and has not been presented for a degree in any other universities, and that all sources of material used for the thesis have been duly acknowledged.

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Abstract

The squeezing and statistical properties of the light produced by a two level atom coupled to squeezed vacuum inside a coherently driven cavity is studied applying c-number Langevin equations. Employing the solutions of these equations, we have determined the atomic inversion, the quadrature variance of the cavity radiation, the photon statistic of the cavity mode and the second order correlation function. If so turned out that the quadrature variance of the cavity mode exhibit squeezing and the squeezing occurs in the plus quadrature of the cavity mode. Further more, the degree of squeezing and mean photon number of the cavity mode increases with the presence of the squeezed vacuum and as the amplitude of the driving mode increases. But the degree of squeezing decrease when κ *increase. And also the atomic inversion increase as the atomic decay constant increase. Moreover, we have found that the cavity mode radiation exhibits photon antibouncching and poissonian photon statistics.*

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1

Introduction

1.1 Background

Quantum physics had developed in the first half of the twentieth century, by understanding how photons and matter interacted and inter-related[1]. In 1953, the maser which emits coherent microwaves and later in 1960 the laser which emits coherent light were discovered. Light played a special role in our attempt to understand nature both classically and quantum mechanically. Classically, light consists of waves with a well-defined amplitude and phase such as not case when we treat the field quantum mechanically[2]. Recent development of technology greatly improved our ability to control individual quantum system. This brings quantum technology beyond academic research to control the level of concrete individual program.

Quantum optics is a field of study that deals with the interaction of radiation with matter. It also deals mainly with the quantum properties of the light generated by the various optical systems such as lasers and with the effect of dynamics of atoms. The quantum properties of light largely determined by the state of the light mode and the well quantum states are coherent, chaotic, number and squeezed[1,3]. The interaction of radiation with atom is one of the central problem in quantum optics. This kind of interaction is involved in various physical process of interest such as resonance fluorescence and laser dynamics.

Squeezing is one of the non-classically feature of light that has attached a great deal of interest. It has a potential application in low noise optical communication and weak signal detection. If the band width of the squeezed vacuum is the large enough, it can be treated as reservoir. However, the squeezed vacuum contrary to the ordinary vacuum, caries some phase information and the behavior[4,5 and 6].

An atom with only two energy eigen values is described by a two dimensional state space spanned by a two energy eigen state $|e\rangle$ and $|g\rangle$. The two state constitute a complete orthonormal system. The corresponding energy eigen values are E_2 and E_1.The excited state in the two level atom systems has a finite time due to spontaneous emission back to the ground state. When an electron is an excited energy state, it must eventually decay to lower level, giving off a photo of radiation[7]. This event is called spontaneous emission and the photon is emitted in a random direction and random phase. The average time taken for the electron to decay is called the time constant for spontaneous emission.

Spontaneous emission of an exited two-level atom results as an effect of its interaction with a continuum of vacuum modes that play a role of reservoir to the atom. The spontaneous emission of rate is usually supposed to be an inherent property of the atom, but in rates depend on the mode structure of the atomic environment[5,6]. An atom is driven on resonance by strong monochromatic laser beam, the structure of atomic level changes dramatically.

In this Msc thesis we study the squeezing and statistical properties of the light produced by a two level atom coupled to squeezed vacuum inside a coherently driven cavity.

1.2 Objectives

1.2.1 General Objective

The general objective of this thesis is to study the atomic dynamics of the two-level atom coupled to squeezed vacuum inside a coherently driven cavity.

1.2.2 Specific Objectives

The specific objectives of this study are:

- To construct the Hamiltonian of the system.
- To dirive the master equation of the system.
- To find the c-number Langevin equations of the system.
- To calculate the quadrature variances of the system.
- To calculate mean and variance of photon number of the system.

2

Review of Literature

The statistical and squeezing properties of light by two-level laser has been investigated by several authors[1, 2, 3, 4, 5 and 6]. B.Betros [2] has studied squeezing and statistical properties of light produced by two-level atom coupled to a squeezed vacuum reservoir. He was found that the light produced by the system under consideration is in a squeezed state for the case if the atom are initially prepared in the lower level and the squeezed occurred in the plus quadrature. His result shows that squeezed vacuum reservoir increase the mean photon number.

T.Mulatu[4] has studied the squeezing and statistical properties of the light produced by the two-level laser in which the cavity mode atom are coupled to squeezed vacuum reservoir. He had calculated the quadrature variance of the cavity mode and atomic variables. His result shows that the cavity mode and atomic variable exhibit squeezing. And the squeezed vacuum increase the degree of squeezing and mean photon number of the cavity mode. But the degree of squeezing decrease when the cavity mode and cavity atom decay constant.

The dynamics of two level atom interacting with the single mode radiation have been investigated by several authors[7-15]. F. kassahun has analyzed the dynamics of two level atom coupled to squeezed vacuum reservoir. He has carried out his analysis after driving the master equation for a two level atom directly coupled to a squeezed vacuum reservoir. The result indicates that the atom oscillates back and forth b/n the upper and lower[5].

A.Messikh has realized the response of a two-level atoms when injected to the cavity will interacted the system[11]. He has analyzed that a light source that exhibits unique properties such as directionality, monochromaticity and coherence which distinguish the laser from the other source of light and has found a wide variety of applications in the modern technology[12].

The generation and detection of squeezed light increasing attention is being given to the study of interaction of squeezed light with a two-level atom[7-13]. Gardiner[13] studied a single two-level atom embedded in a broadband squeezed vacuum and showed that the two quadratures of polarizations decays at two distinct decay rates.

Parkins and Gardiner considered a single two-level atom in a cavity when the squeezed light is incident upon one of the out put mirrors. In recent year a great deal of attention has been focused on the study of the interaction of the one atom or of a small collection of atoms with a single mode of resonant cavity[13]. The boundary conditions associated with the cavity may leads to substantial modification of the radiate process relative to this encountered in free space emission and absorption. The presence of cavity mode enhances the coupling between the atoms and electromagnetic field. In particular, if the coupling of the collective atomic polarization to the intra-cavity field is sufficiently strong compared with the relevant dissipative process, an oscillatory regime is encountered in which the field radiated by the atom excites the cavity mode, which subsequently re-excites the atomic polarization and so on.

M.Eleccion and H.Svelto Analyzed that a laser must be produced emissions that is lost trough absorption. They seek here to analyze the quantum properties of a light emitted by two-level laser atoms available in a cavity injected to the lower level at a constant rate γ_a thus taking in to account the interaction of the two-level atoms with a cavity mode and the damping of the cavity mode by a squeezed vacuum reservoir. They obtain the quadrature variance and the photon statistics for the light emitted by the atoms. This analyze is carried by putting the noise operators associated with the squeezed vacuum reservoir and with out considering the interaction of the two level atoms with the squeezed vacuum reservoir.[14].

Jin and Xiao[15] have considered N two-level atom placed inside a parametric oscillator in a good cavity limit. They have found that under strong interaction limit, the presence of the two-level atom inside a parametric oscillator increases the amount of intra cavity squeezed from its maximum value of 50% to maximum value of 75% percent.

F. Kassahun [16] has drived a generalized Fokker- Planck equation without using the system size expansion for a single two-level atom placed inside a coherently driven cavity and the atom is coupled to a broad band squeezed vacuum . His results include atomic and cavity detuning and also atomic and cavity decays. Using this Fokker-Planck equation he has studied atomic inversion, fluorescent spectrum and second order intensity correlation function of the transmitted light and fluorescent light in the bad cavity limit.

3

Methodology

In this chapter we consider a two-level atom coupled to a squeezed vacuum inside a coherently driven cavity. We first constract the Hamiltonian of the system under consideration, using this Hamiltonian we drive the master equation. And with the aid of this master equations, we employ c-number Langevin equations and calculation relations of the noise forces are obtained. we then determine the solutions of the resulting c-number Langevin equations.

3.1 Master Equation

In this section we wish to drive the master equation representing a two level atom coupled to squeezed vacuum inside a coherently driven cavity. We drive the equation of evolution for density operator representing the interaction of the atom with a coherently driven cavity, the atom with the reservoir,the cavity with the reservoir and the driven with the cavity in the system separately. Finally, using the resulting master equation of the system under the consideration.

The interaction of two-level atom with a squeezed vacuum as well as the reservoir and the squeezed vacuum with reservoir is described by the Hamiltonian

$$\hat{H} = \hat{H}_{AC} + \hat{H}_{AR} + \hat{H}_{CR} + \hat{H}_{DC}, \tag{3.1}$$

where

$$\hat{H}_{AC} = ig(\hat{\sigma}_{+}\hat{a} - \hat{a}^{\dagger}\hat{\sigma}_{-}), \tag{3.2}$$

represents the interaction of a two-level atom with the cavity.

$$\hat{H}_{AR} = i\lambda(\sigma_{+}\hat{a}_{in} - \hat{a}^{\dagger}_{in}\sigma_{-}), \tag{3.3}$$

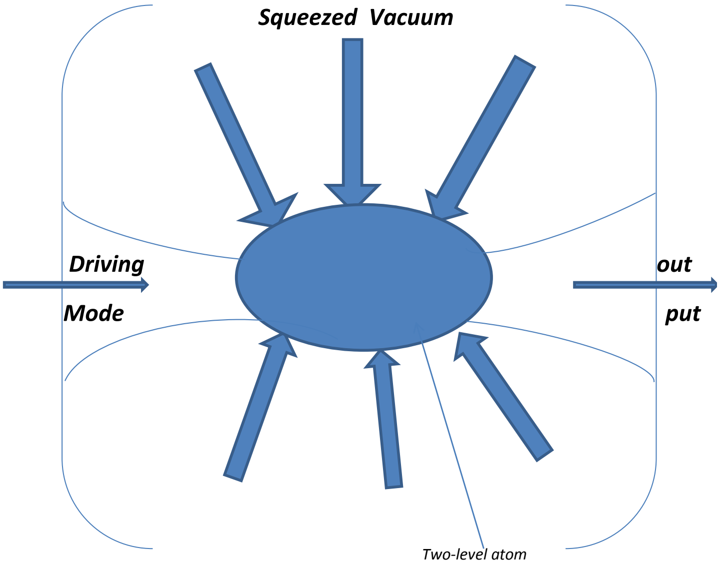

Fig. 3.1: Schematic representation of two level atom coupled to squeezed vacuum inside a coherently driven cavity.

represents the interaction of a two-level atom with reservoir.

$$\hat{H}_{CR} = i\eta(\hat{a}\hat{a}^{\dagger}_{in} - \hat{a}^{\dagger}\hat{a}_{in}), \tag{3.4}$$

represents the interaction of the cavity with reservoir.

$$\hat{H}_{DC} = -i\epsilon(\hat{a}^{\dagger} - \hat{a}), \tag{3.5}$$

represents the interaction of driven mode with the cavity mode.
These are the Hamiltonian with the pump mode treated classically, where g, λ, k, η being the appropriate coupling constant and ϵ, considered to be real and constant is proportional to the amplitude the driving mode, $\hat{a}(\hat{a}^{\dagger})$ is the annihilation(creation) operator for the cavity mode,$\hat{a}_{in}(\hat{a}^{\dagger}_{in})$ is the annihilation(creation)operator for the reservoir and $\hat{\sigma}_{+}(\hat{\sigma}_{-})$ is the rising(lowering) atomic operator satisfying the following commutation relation.

$$[\hat{a}, \hat{a}^{\dagger}] = [\hat{a}_{in}, \hat{a}^{\dagger}_{in}] = 1, \tag{3.6}$$

$$[\hat{\sigma}_{+}, \hat{\sigma}_{-}] = \hat{\sigma}_{z}, \tag{3.7}$$

$$[\hat{\sigma}_{\pm}, \hat{\sigma}_{z}] = \mp\hat{\sigma}_{\pm}, \tag{3.8}$$

where σ_z is atomic inversion.

3.1.1 The master equation for the interaction of the two-level atom with reservoir

We consider a system (a two-level atom) denoted by 'A' interacting with reservoir denoted by the 'R'. The combined density operator is denoted by $\hat{\chi}(t)$. The equation of evolution for $\hat{\chi}(t)$ is given by

$$\frac{d\hat{\chi}(t)}{dt} = -i[\hat{H}_{AR}, \hat{\chi}(t)]. \tag{3.9}$$

The reduced operator for the system $\hat{\rho}(t)$ is obtained by taking a trace over of the reservoir variables. i.e

$$\hat{\rho}(t) = Tr_R\hat{\chi}(t). \tag{3.10}$$

In view of Eq.(3.10), we see that

$$\frac{d\hat{\rho}(t)}{dt} = -iTr_R[\hat{H}_{AR}, \hat{\chi}(t)]. \tag{3.11}$$

On the other hand, $\frac{d\hat{\chi}((t)}{dt}$ an be expressed as

$$\frac{d\hat{\chi}(t)}{dt} = \lim_{\chi\to 0}\frac{\hat{\chi}(t) - \hat{\chi}(t-h)}{h}. \tag{3.12}$$

On the base of this equation, can be written as

$$\hat{\chi}(t) = \hat{\chi}(t-h) + h\frac{d\hat{\chi}(t)}{dt}, \tag{3.13}$$

with $h \to 0$. Substitute Eq. (3.13) into Eq. (3.11), we have

$$\frac{d\hat{\rho}(t)}{dt} = -iTr_R[\hat{H}_{AR}, \hat{\chi}(t-h)] + ihTr_R[\hat{H}_{AR}, \hat{\chi}(t)]]. \tag{3.14}$$

In order to obtain mathematically manageable equation for evaluation of the reduced density operator, we need to replace $\hat{\chi}(t)$ by same approximately valid expression by assuming that the density operator 'R' for the reservoir is remain the same in time. Taking into this assumption, the total density operator can be written as

$$\hat{\chi}(t) = \hat{\rho}(t)R. \tag{3.15}$$

in view of this Eq. (3.14) takes the form

$$\frac{d\hat{\rho}(t)}{dt} = -i[\langle\hat{H}_{AR}\rangle_R, \hat{\rho}(t)] - 2ihTr_R(\hat{H}_{AR}\hat{\rho}(t)\hat{H}_{AR}) + ih\langle\hat{H}^2_{AR}\rangle_R\hat{\rho}(t) - ih\hat{\rho}(t)\langle\hat{H}^2_{AR}\rangle_R. \tag{3.16}$$

With the aid of Eq. (3.3) the expectation value of the vacuum reservoir is

$$\langle\hat{H}_{AR}\rangle = 0. \tag{3.17}$$

One can easily show that

$$[\langle \hat{H}_{AR} \rangle, \hat{\rho}(t-h)] = 0. \tag{3.18}$$

Furthermore, applying Eq. (3.3) and the cyclic operator, we can get

$$hTr_R(\hat{H}_{AR}\hat{\rho}(t)\hat{H}_{AR}) = \frac{\gamma}{2}(N+1)\hat{\sigma}_+\hat{\rho}(t)\hat{\sigma}_- + \frac{\gamma}{2}N\hat{\sigma}_-\hat{\rho}(t)\hat{\sigma}_+ + \frac{\gamma}{2}M(\hat{\sigma}_+\hat{\rho}(t)\hat{\sigma}_+ + \hat{\sigma}_-\hat{\rho}(t)\hat{\sigma}_-) \tag{3.19}$$

and

$$h\langle \hat{H}_{AR}^2 \rangle = \frac{\gamma}{2}((N+1)\hat{\sigma}_+\hat{\sigma}_- + N\hat{\sigma}_-\hat{\sigma}_+), \tag{3.20}$$

where

$$\gamma = 2h\lambda^2, \tag{3.21}$$

$$\langle \hat{a}_{in}^\dagger \hat{a}_{in} \rangle = N, \tag{3.22}$$

$$M = -\langle -\hat{a}_{in}^2 \rangle = -\langle -\hat{a}_{in}^{\dagger 2} \rangle = 0. \tag{3.23}$$

Using Eqs. (3.18), (3.19), (3.20) and (3.21) into (3.16) we can obtain

$$\begin{aligned} \frac{d\hat{\rho}}{dt} &= \frac{\gamma}{2}(N+1)(2\hat{\sigma}_-\hat{\rho}\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-) \\ &+ \frac{\gamma}{2}N(2\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+\hat{\rho} - \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+) \quad , \\ &+ \gamma M(\hat{\sigma}_+\hat{\rho}\hat{\sigma}_+ + \hat{\sigma}_-\hat{\rho}\hat{\sigma}_-) \end{aligned} \tag{3.24}$$

where $N = sinh^2 r$ and $M = sinhrcoshr$ Eq.(3.24) represents the reduced density operator for the two-level atom with the ordinary reservoir.

3.1.2 The master equation for the interaction of the cavity with reservoir

We consider the system (squeezed vacuum)denoted by $'S'$ interacting with reservoir which is denoted by R. Denoting total density operator for squeezed vacuum coupled to a vacuum reservoir by $\hat{\chi}(t)$, the time evolution of this density operator is given by

$$\frac{d\hat{\chi}(t)}{dt} = -i[\hat{H}_{CR}(t), \hat{\chi}(t)], \tag{3.25}$$

where $\hat{H}_{CR}(t)$, the Hamiltonian for the interaction of the squeezed vacuum with the reservoir. Taking into account the relation is

$$\hat{\rho}(t) = Tr_R(\hat{\chi}(t)), \tag{3.26}$$

we see that the reduced operator density evolves in time is to be

$$\frac{d\hat{\rho}(t)}{dt} = -iTr_R[\hat{H}_{CR}(t), \hat{\chi}(t)]. \tag{3.27}$$

In which Tr_R,indicates that the trace is taken over the reservoir variables only. On the other hand, a formal solution of Eq.(3.27) can be put in the form

$$\hat{\chi}(t) = \hat{\chi}(0) - i\int_0^t [\hat{H}_{CR}(t')\hat{\chi}(t')]dt'. \tag{3.28}$$

Applying the born approximation

$$\hat{\chi}(t') = \hat{\rho}(t')R, \tag{3.29}$$

$$\hat{\chi}(0) = \hat{\rho}(0)R. \tag{3.30}$$

We can write Eq.(3.28) as

$$\hat{\chi}(t) = \hat{\rho}(0)R - i\int_0^t [\hat{H}_{CR}(t'), \hat{\rho}(t')R]dt'. \tag{3.31}$$

Now substituting Eq. (3.24) into Eq. (3.20), there follows

$$\frac{d\hat{\rho}(t)}{dt} = -i[\langle\hat{H}_{CR}(t)\rangle_R, \hat{\rho}(0)] - \int_0^t Tr_R[\hat{H}_{CR}(t'), \hat{\rho}(t)R]dt'. \tag{3.32}$$

We consider the reservoir to be composed of a large sub modes. The interaction of a squeezed vacuum with a reservoir can be described by the Hamiltonian

$$\hat{H}_{CR} = i\hbar\Sigma\lambda_\kappa(\hat{a}^\dagger\hat{b}_k e^{-i(\omega_0-\omega_\kappa)t} - \hat{a}\hat{b}_k e^{-i(\omega_0-\omega_\kappa)t}), \tag{3.33}$$

where λ_κ, coupling constant for squeezed and the reservoir sub vacuums. $\hat{a}_{in}$ is the annihilation operator for the reservoir sub vacuum. we then see that

$$\langle\hat{H}_{CR}(t)\rangle_R = i\hbar\Sigma_\kappa\lambda_\kappa(\hat{a}^\dagger\langle\hat{b}_k\rangle e^{i(\omega_0\omega_\kappa)t} - \hat{a}\langle\hat{b}_k\rangle e^{-i(\omega_0-\omega_\kappa)t}). \tag{3.34}$$

Taking into account the property of vacuum reservoir operators,

$$\langle\hat{b}_k\rangle_R = \langle\hat{b}_k\rangle = 0. \tag{3.35}$$

We easily see that

$$\langle\hat{H}_{CR}(t)\rangle_R = 0. \tag{3.36}$$

In view of Eq.(3.36) and $\hat{H}_{CR}(t') = \hat{H}_{CR}(t')\hat{H}_{CR}(t)\hat{\rho}(t')R\hat{H}_{CR}(t') = \hat{H}_{CR}(t')\hat{\rho}(t')R\hat{H}_{CR}(t')$ Eq. (3.32) reduced to

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} = 2\int_0^t Tr_R(\hat{H}_{CR}(t)\hat{\rho}(t')R\hat{H}_{CR}(t'))dt' - \int_0^t \langle\hat{H}_{CR}(t)\hat{H}_{CR}(t')\rangle_R\hat{\rho}(t')dt' \\ - \int_0^t \hat{\rho}(t)\langle\hat{H}_{CR}(t)\hat{H}_{CR}(t')\rangle_R dt'.\end{aligned} \tag{3.37}$$

With the aid of Eq.(3.33), we can write

$$-\langle \hat{H}_{CR}(t)\hat{H}_{CR}(t')\rangle_R = I_1\hat{a}^\dagger\hat{a} + I_2\hat{a}\hat{a}^\dagger + I_3\hat{a}^2 + I_4\hat{a}^{\dagger 2} \tag{3.38}$$

and

$$Tr_R(H_{CR}(t')\hat{\rho}(t')\hat{H}_{CR}(t)) = I_1\hat{a}\hat{\rho}\hat{a}^\dagger + I_2\hat{a}^\dagger\hat{\rho}\hat{a} + I_3\hat{a}^\dagger\hat{\rho}\hat{a}^\dagger + I_4\hat{a}\hat{\rho}\hat{a}, \tag{3.39}$$

where

$$I_1 = -\sum_{k,k'} \lambda_k\lambda_{k'}\langle\hat{b}_k\hat{b}_k\rangle e^{i(\omega_0-\omega_k)t - i(\omega_0-\omega_{k'})t'}, \tag{3.40}$$

$$I_2 = -\sum_{k,k'} \lambda_k\lambda_{k'}\langle\hat{b}_k\hat{b}_k\rangle e^{-i(\omega_0-\omega_k)t + i(\omega_0-\omega_{k'})t'}, \tag{3.41}$$

$$I_3 = \sum_{k,k'} \lambda_k\lambda_{k'}\langle\hat{b}_k\hat{k}_k\rangle e^{-i(\omega_0-\omega_k)t + i(\omega_0-\omega_{k'})t'}, \tag{3.42}$$

$$I_4 = \sum_{k,k'} \lambda_k\lambda_{k'}\langle\hat{b}_k\hat{b}_k\rangle e^{i(\omega_0-\omega_k)t + i(\omega_0-\omega_{k'})t'}. \tag{3.43}$$

We note that for vacuum reservoir we have,

$$\langle\hat{b}_k\hat{b}_k\rangle = N\delta_{kk'} = 0, \tag{3.44}$$

$$\langle\hat{b}_k\hat{b}_k\rangle = (N+1)\delta_{kk'} = \hat{\delta}_{kk'}, \tag{3.45}$$

$$\langle\hat{b}_k\hat{b}_k\rangle = \langle\hat{b}_k\hat{b}_k\rangle = 0, \tag{3.46}$$

with

$$N = \sinh^2 r = 0, \tag{3.47}$$

$$M = \sinh r \cosh r = 0. \tag{3.48}$$

On account of Eq. (3.45) we see that

$$I_1 = -\sum_k \lambda_k^2 e^{i(\omega_0-\omega_k)t}. \tag{3.49}$$

Assuming the frequencies of the reservoir sub modes to be closed spaced the summation over k can be converted into an integration over ω'. One can then write

$$I_1 = -\int_0^\infty g(\omega)\lambda^2(\omega)e^{\pm i(\omega-\omega_0)(t-t')}d\omega', \tag{3.50}$$

where $g(\omega)$ is the density state of the vacuum reservoir . We expect ω to vary little around ω_0. In view of this, we can replace $g(\omega)$ and $\lambda^2(\omega)$ by $g(\omega_0)$and $\lambda^2(\omega_0)$ and extend the lower limit of integration to ∞, so that

$$I_1 = g(\omega_0)\lambda^2(\omega)\int_{-\infty}^{\infty} e^{\pm i(\omega_0-\omega)(t-t')}d\omega. \tag{3.51}$$

Carrying out the integration upon setting $\omega' = \omega_0 - \omega$,we get

$$I_1 = -\kappa_A\delta(t-t'), \tag{3.52}$$

where

$$\kappa = 2\pi g(\omega_0)\lambda^2(\omega_0), \tag{3.53}$$

is the vacuum decay constant. Following the similar procedure we easily verify that

$$I_2 = -\kappa N\delta(t-t') = 0, \tag{3.54}$$

$$I_3 = I_4 = -\kappa M\delta(t-t') = 0, \tag{3.55}$$

where $N = M = 0$, for vacuum reservoir.

Up on substituting the Eq. (3.53), (3.54), (3.55) into Eq. (3.38)and (3.39) we have

$$-\langle\hat{H}_{CR}(t)\hat{H}_{CR}(t')\rangle = -\kappa\hat{a}^\dagger\hat{a} \tag{3.56}$$

and

$$Tr_R(\hat{H}_{CR}(t')\hat{\rho}(t')R\hat{H}_{CR}(t) = -\kappa\hat{a}\hat{\rho}\hat{a}^\dagger. \tag{3.57}$$

It then follows that

$$\int_0^{\infty}\langle\hat{H}_{CR}(t)\hat{H}_{CR}(t')\rangle R\hat{\rho}(t')dt' = \frac{\kappa}{2}\hat{a}^\dagger\hat{a}\hat{\rho}(t), \tag{3.58}$$

$$\int_0^{\infty}\hat{\rho}(t')\langle\hat{H}_{CR}(t)\hat{H}_{CR}(t')\rangle R\hat{\rho}(t')dt' = \frac{\kappa}{2}\hat{\rho}(t)\hat{a}^\dagger\hat{a}, \tag{3.59}$$

$$\int_0^{\infty} Tr_R(\hat{H}_{CR}(t')\hat{\rho}(t')R\hat{H}_{CR}(t))dt' = -\frac{\kappa}{2}\hat{a}\hat{\rho}(t)\hat{a}^\dagger. \tag{3.60}$$

Combination of Eq. (3.37), (3.58), (3.59) and (3.60) leads to

$$\frac{d\hat{\rho}}{dt} = \frac{\kappa}{2}(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}). \tag{3.61}$$

Eq. (3.61)represents the equation of reduced density operator for the cavity with the vacuum reservoir.

3.1.3 The master equation for the interaction of the driving light with the cavity

The in the action of the driving coherent light interaction with the cavity mode can be described by the Hamiltonian

$$\hat{H}_{DC} = -i\epsilon(\hat{a}^\dagger - \hat{a}), \tag{3.62}$$

where ϵ, is considered to be real and constant, which is proportion to the amplitude of driven mode. And $\hat{a}$ is the annihilation operator of the atom. The time evolution of the density operator of Eq. (3.62) is

$$\frac{d\hat{\rho}(t)}{dt} = \epsilon(\hat{a}^\dagger\hat{\rho} - \hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger + \hat{\rho}\hat{a}). \tag{3.63}$$

3.1.4 The master equation for the interaction of the two-level atom with cavity

The interaction of the two level atom with squeezed vacuum can be described at resonant by the hamiltonian

$$\hat{H}_{AC} = ig(\hat{\sigma}_+\hat{a} - \hat{a}^\dagger\hat{\sigma}_-), \tag{3.64}$$

where g is the coupling constant b/n the atom and the cavity.
The equation of evolution of this density operator corresponding to the hamiltonian Eq. (3.64)

$$\frac{d\hat{\rho}(t)}{dt} = g(\hat{\sigma}_+\hat{a}\hat{\rho} - \hat{a}^\dagger\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{a} + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_-). \tag{3.65}$$

Finally, with the aid of Eq. (3.24), (3.61), (3.62) and (3.65), we write

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= g(\hat{\sigma}_+\hat{a}\hat{\rho} - \hat{a}^\dagger\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{a} + \hat{\rho}\hat{a}^\dagger\hat{\sigma}_-) + \epsilon(\hat{a}^\dagger\hat{\rho} - \hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger + \hat{\rho}\hat{a})\\ &+ \frac{\gamma}{2}N(2\hat{\sigma}_+\rho\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+\hat{\rho} - \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+) + \frac{\gamma}{2}(N+1)(2\hat{\sigma}_-\rho\hat{\sigma}_+ - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-)\\ &+ \gamma M(\hat{\sigma}_+\hat{\rho}\hat{\sigma}_+ + \hat{\sigma}_-\hat{\rho}\hat{\sigma}_-) + \frac{\kappa}{2}(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}).\end{aligned} \tag{3.66}$$

This equation represents the master equation of a two-level atom coupled to squeezed vacuum in side a coherently driven cavity mode and the cavity made coupled to ordinary vacuum.

3.2 C-number Langevein Equation

In this section we seek to obtain the c-number Langevin equation for the cavity mode employing the master equation in Eq. (4.66). In view of the relation

$$\frac{d}{d}\langle\hat{A}(t)\rangle = Tr(\frac{d}{dt}\hat{\rho}(t)\hat{A}), \tag{3.67}$$

one can write

$$\begin{aligned}\frac{d\langle\hat{a}\rangle}{dt} &= gTr(\hat{\sigma}_+\hat{a}\hat{\rho}\hat{a} - \hat{a}^\dagger\hat{\sigma}_-\hat{\rho}\hat{a} - \hat{\rho}\hat{\sigma}_+\hat{\rho}\hat{a} - \hat{\rho}\hat{\sigma}_-\hat{a}\hat{a} + \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+\hat{a}) + \epsilon Tr(\hat{a}^\dagger\hat{\rho}\hat{a} - \hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^\dagger\hat{a} + \hat{\rho}\hat{a}^2)\\ &+ \frac{\gamma}{2}NTr(2\hat{\sigma}_+\rho\hat{\sigma}_-\hat{a} - \hat{\sigma}_-\hat{\sigma}_+\hat{\rho}\hat{a} - \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+\hat{a}) + \frac{\gamma}{2}(N+1)Tr(2\hat{\sigma}_-\rho\hat{\sigma}_+\hat{a} - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho}\hat{a} - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-\hat{a})\\ &+ \gamma MTr(\hat{\sigma}_+\hat{\rho}\hat{\sigma}_+\hat{a} + \hat{\sigma}_-\hat{\rho}\hat{\sigma}_-\hat{a}) + \frac{\kappa}{2}Tr(2\hat{a}\hat{\rho}\hat{a}^\dagger\hat{a} - \hat{a}^\dagger\hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^\dagger\hat{a}\hat{a}).\end{aligned} \tag{3.68}$$

Applying the cyclic property of the trace trace operator, we get

$$\frac{d\langle\hat{a}\rangle}{dt} = -\frac{\kappa}{2}\langle\hat{a}\rangle - g\langle\hat{\sigma}_-\rangle + \epsilon. \tag{3.69}$$

Following the same procedure, it can easily be verified that

$$\frac{d\langle\hat{a}^2\rangle}{dt} = -\kappa\langle\hat{a}^2\rangle - 2g\langle\hat{a}\hat{\sigma}_-\rangle + 2\epsilon\langle\hat{a}\rangle, \tag{3.70}$$

$$\frac{d\langle\hat{a}^\dagger\hat{a}\rangle}{dt} = \kappa\langle\hat{a}^\dagger\hat{a}\rangle - g(\langle\hat{\sigma}_+\hat{a}\rangle + \langle\hat{\sigma}_-\hat{a}^\dagger\rangle) + \epsilon(\langle\hat{a}\rangle + \langle\hat{a}^\dagger\rangle). \tag{3.71}$$

On account of Eq. (3.66), we see that

$$\begin{aligned}\frac{d\langle\hat{\sigma}_-\rangle}{dt} &= gTr(\hat{\rho}_+\hat{a}\hat{\rho}\hat{\sigma}_- - \hat{a}^\dagger\hat{\sigma}_-\hat{\rho}\hat{\sigma}_- - \hat{\rho}\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- - \hat{\rho}\hat{\sigma}_- + \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+\hat{\sigma}_-) + \epsilon Tr(\hat{a}^\dagger\hat{\rho}\hat{\sigma}_- - \hat{a}\hat{\rho}\hat{\sigma}_- - \hat{\rho}\hat{a}^\dagger\hat{\sigma}_- + \hat{\rho}\hat{a}\hat{\sigma}_-)\\ &+ \frac{\gamma}{2}NTr(2\hat{\sigma}_+\rho\hat{\sigma}_-\hat{\sigma}_- - \hat{\sigma}_-\hat{\sigma}_+\hat{\rho}\hat{\sigma}_- - \hat{\rho}\hat{\sigma}_-\hat{\sigma}_+\hat{\sigma}_-) + \frac{\gamma}{2}(N+1)Tr(2\hat{\sigma}_-\rho\hat{\sigma}_+\hat{\sigma}_- - \hat{\sigma}_+\hat{\sigma}_-\hat{\rho}\hat{\sigma}_- - \hat{\rho}\hat{\sigma}_+\hat{\sigma}_-\hat{\sigma}_-)\\ &+ \gamma Mtr(\hat{\sigma}_+\hat{\rho}\hat{\sigma}_+\hat{\sigma}_- + \hat{\sigma}_-\hat{\rho}\hat{\sigma}_-\hat{\sigma}_-) + \frac{\kappa}{2}Tr(2\hat{a}\hat{\rho}\hat{a}^\dagger\hat{\sigma}_- - \hat{a}^\dagger\hat{a}\hat{\rho}\hat{\sigma}_- - \hat{\rho}\hat{a}^\dagger\hat{a}\hat{\sigma}_-).\end{aligned} \tag{3.72}$$

With the aid of cyclic property of the trace operation, the commutation relations of the boson and atomic operators, we find

$$\frac{d\langle\hat{\sigma}_-\rangle}{dt} = -\frac{\gamma}{2}(1+2N)\langle\hat{\sigma}_-\rangle + \gamma M\langle\hat{\sigma}_+\rangle - g\langle\hat{\sigma}_z\hat{a}\rangle. \tag{3.73}$$

Following the same procedure, it can be easily obtained that

$$\frac{d\hat{\sigma}_{ee}}{dt} = -\gamma(1+2N)\langle\hat{\sigma}_{ee}\rangle + g(\langle\hat{\sigma}_+\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_-\rangle) + \epsilon(\langle\hat{\sigma}_{ee}\hat{a}^\dagger\rangle - \langle\hat{\sigma}_{ee}\hat{a}\rangle - \hat{a}^\dagger\hat{\sigma}_{ee} + \hat{a}\hat{\sigma}_{ee}) + \gamma N, \tag{3.74}$$

$$\frac{d\hat{\sigma}_{ff}}{dt} = -\gamma(1+2N)\langle\hat{\sigma}_{ee}\rangle + g(\langle\hat{\sigma}_+\hat{a}\rangle + \langle\hat{a}^\dagger\hat{\sigma}_-\rangle) + \epsilon(\langle\hat{\sigma}_{ee}\hat{a}^\dagger\rangle - \langle\hat{\sigma}_{ee}\hat{a}\rangle - \langle\hat{a}^\dagger\hat{\sigma}_{ee}\rangle + \langle\hat{a}\hat{\sigma}_{ee}\rangle) + \gamma N, \tag{3.75}$$

where $\hat{\sigma}_{ee} = \hat{\sigma}_+\hat{\sigma}_-$ and $\hat{\sigma}_{ff} = \hat{\sigma}_-\hat{\sigma}_+$

$$\begin{aligned}\frac{d}{dt}\langle\sigma_-\hat{a}\rangle &= -\frac{1}{2}(k + \gamma(1+2N)\langle\hat{\sigma_-}\hat{a}\rangle + \gamma M\langle\hat{\sigma_+}\hat{a}\rangle - g\langle\hat{\sigma_z}\hat{a}^2\rangle\\ &- \epsilon(\langle\hat{\sigma_-}\hat{a}\hat{a}^\dagger\rangle - \langle\hat{\sigma_-}\hat{a}^2\rangle - \langle\hat{a}^\dagger\hat{\sigma_-}\hat{a}\rangle + \langle\hat{a}\hat{\sigma_-}\hat{a}\rangle)),\end{aligned} \tag{3.76}$$

$$\frac{d}{dt}\langle\sigma_{+}\hat{a}\rangle = -\frac{1}{2}(k+\gamma(1+2N)\langle\hat{\sigma_{+}}\hat{a}\rangle + \gamma M\langle\hat{\sigma_{-}}\hat{a}\rangle - g\langle\hat{\sigma_{z}}\hat{a}^{\dagger}\hat{a}\rangle - g\langle\hat{\sigma_{ee}}\rangle \\ - \epsilon(\langle\hat{\sigma_{+}}\hat{a}\hat{a}^{\dagger}\rangle - \langle\hat{\sigma_{+}}\hat{a}^{2}\rangle - \langle\hat{a}^{\dagger}\hat{\sigma_{+}}\hat{a}\rangle + \langle\hat{a}\hat{\sigma_{+}}\hat{a}\rangle)). \tag{3.77}$$

We note that Eq. (3.69-3.71) and (3.73-3.77) are put in the normal order. The c-number equations corresponding to the above equations are obtained by converting $a \rightarrow \alpha$, $\sigma_{-} \rightarrow m$, $\sigma_{+}\sigma_{-} \rightarrow N_{e}$, $\sigma_{-}\sigma_{+} \rightarrow N_{f}$

$$\frac{d}{dt}\langle\alpha\rangle = \frac{-\kappa}{2}\langle\alpha\rangle - g\langle m\rangle + \epsilon, \tag{3.78}$$

$$\frac{d}{dt}\langle\alpha^{2}\rangle = -\kappa\langle\alpha^{2}\rangle - 2g\langle\alpha m\rangle + 2\epsilon\langle\alpha\rangle, \tag{3.79}$$

$$\frac{d}{dt}\langle\alpha^{*}\alpha\rangle = \kappa\langle\alpha^{*}\alpha\rangle - g(\langle m^{*}\alpha\rangle - \langle m\alpha^{*}\rangle) + \epsilon(\langle\alpha\rangle + \langle\alpha^{*}\rangle), \tag{3.80}$$

$$\frac{d}{dt}\langle m\rangle = -\frac{\gamma}{2}(1+2N)\langle m\rangle + \gamma M\langle m^{*}\rangle - g\langle(N_{e}-N_{f})\hat{a}\rangle, \tag{3.81}$$

$$\frac{d}{dt}\langle N_{e}\rangle = -\gamma(1+2N)\langle N_{e}\rangle + g(\langle m^{*}\alpha\rangle + \langle\alpha^{*}m\rangle) + \gamma N, \tag{3.82}$$

$$\frac{d}{dt}\langle N_{f}\rangle = -\gamma(1+2N)\langle N_{f}\rangle - g(\langle m^{*}\alpha\rangle + \langle\alpha^{*}m\rangle) + \gamma N, \tag{3.83}$$

$$\frac{d}{dt}\langle m\alpha\rangle = -\frac{1}{2}(\kappa+(1+2N)\gamma)\langle m\alpha\rangle + \gamma M\langle m^{*}\alpha\rangle - g\langle(N_{e}-N_{f})\alpha^{2}\rangle, \tag{3.84}$$

$$\frac{d}{dt}\langle m^{*}\alpha\rangle = -\frac{1}{2}(\kappa+(1+2N)\gamma)\langle m^{*}\alpha\rangle + \gamma M\langle m\alpha\rangle - g\langle(N_{e}-N_{f})\alpha^{*}\alpha\rangle - g\langle N_{e}\rangle. \tag{3.85}$$

On the basis of Eq. (3.69), we can write

$$\frac{d}{dt}\alpha = -\frac{\kappa}{2}\alpha(t) - gm(t) + f_{\alpha}(t) + \varepsilon, \tag{3.86}$$

where $f_{\alpha}(t)$ is a noise force whose correlation properties remain to be determined. we observe that Eq.(3.68) and the expectation value of Eq.(3.86) will have the same form if

$$\langle f_{\alpha}(t)\rangle = 0. \tag{3.87}$$

Moreover, applying the relation $\frac{d}{dt}\langle\alpha^{2}\rangle = 2\langle\alpha\frac{d\alpha}{dt}\rangle$ along with Eq.(3.86) we have

$$\frac{d}{dt}\langle\alpha^{2}\rangle = -\kappa\langle\alpha^{2}\rangle - 2g\langle\alpha m\rangle + 2\epsilon\langle\alpha\rangle + \langle\alpha(t)f_{\alpha}(t)\rangle + \langle f_{\alpha}(t)\alpha(t)\rangle. \tag{3.88}$$

Comparison of this with Eq. (3.79) indicates that

$$\langle \alpha(t) f_\alpha(t)\rangle + \langle f_\alpha(t)\alpha(t)\rangle = 0. \tag{3.89}$$

The formula solution of Eq. (3.86)can be written as

$$\alpha(t) = \alpha(0)e^{-\frac{\kappa t}{2}} + \int_0^t dt' e^{-\frac{\kappa(t-t')}{2}}(-gm(t') + \epsilon + f_\alpha(t')). \tag{3.90}$$

It then follows that

$$\begin{aligned}\langle f_\alpha(t)\alpha(t)\rangle + \langle \alpha(t) f_\alpha(t)\rangle = 2[\langle \alpha(0) f_\alpha(t)\rangle + \langle f_\alpha(t)\alpha(0)\rangle]e^{-\frac{\kappa t}{2}} - \int_0^t e^{-k(t-t')/2}dt' \\ [\langle m\alpha(t)\rangle + 2\varepsilon\langle\alpha(t)\rangle + \langle \alpha(t) f_\alpha(t')\rangle + \langle f_\alpha(t')\alpha(t)\rangle].\end{aligned} \tag{3.91}$$

Multiply Eq. (3.90) by $f_\alpha(t)$,we get

$$\langle f_\alpha(t)\alpha(t)\rangle = \langle f_\alpha(t)\alpha(0)\rangle e^{-\frac{\kappa t}{2}} + \int_0^t dt' e^{-\frac{\kappa(t-t')}{2}}(-g\langle f_\alpha(t) m(t')\rangle + \epsilon\langle f_\alpha(t)\rangle + \langle f_\alpha(t) f_\alpha(t')\rangle .) \tag{3.92}$$

Since a noise force at a certain time should not affect the atomic and light mode variables at an earlier time, one can write

$$\langle f_\alpha(t)\alpha(t)\rangle = \langle f_\alpha(t)\rangle\langle\alpha(t)\rangle = 0 \tag{3.93}$$

and

$$\langle f_\alpha(t) m(t')\rangle = \langle f_\alpha(t)\rangle\langle m(t)\rangle = 0. \tag{3.94}$$

In view of these results along with Eq. (3.87),Eq. (3.92) reduced to

$$\langle f_\alpha(t)\alpha(t)\rangle = \int_0^t dt' e^{-\frac{\kappa(t-t')}{2}}\langle f_\alpha(t) f_\alpha(t')\rangle. \tag{3.95}$$

We also note that

$$\langle \alpha(t) f_\alpha(t)\rangle = \int_0^t dt' e^{-\frac{\kappa(t-t')}{2}}\langle f_\alpha(t') f_\alpha(t)\rangle. \tag{3.96}$$

Taking into Eq.(3.89) along with Eq.(3.95) and (3.96), we get

$$\int_0^t dt' e^{-\frac{\kappa(t-t')}{2}}\langle f_\alpha(t) f_\alpha(t')\rangle + \int_0^t dt' e^{-\frac{\kappa(t-t')}{2}}\langle f_\alpha(t') f_\alpha(t)\rangle) = 0 \tag{3.97}$$

and assuming that

$$\langle f_\alpha(t) f_\alpha(t')\rangle = \langle f_\alpha(t') f_\alpha(t)\rangle. \tag{3.98}$$

We have

$$2\int_0^t dt' e^{-\frac{\kappa(t-t')}{2}}\langle f_\alpha(t') f_\alpha(t)\rangle = 0. \tag{3.99}$$

We finally get

$$\langle f_\alpha(t') f_\alpha(t)\rangle = 0. \tag{3.100}$$

Furthermore, employing the relation

$$\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = \langle\frac{d\alpha^*(t)}{dt}\alpha(t)\rangle + \langle\alpha^*(t)\frac{d\alpha(t)}{dt}\rangle, \tag{3.101}$$

along Eq.(3.86) and its complex conjugate, we have

$$\begin{aligned}\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle &= \kappa\langle\alpha^*(t)\alpha(t)\rangle - g(\langle m^*(t)\alpha(t)\rangle - \langle m(t)\alpha^*(t)\rangle) \\ &\quad + \epsilon(\langle\alpha\rangle + \langle\alpha^*(t)\rangle) + \langle\alpha^*(t)f_\alpha(t) + \langle f^*_\alpha(t)\alpha(t)\rangle.\end{aligned} \tag{3.102}$$

Comparison of Eq. (3.80) and (3.102) , shows that

$$\langle\alpha^*(t)f_\alpha(t)\rangle + \langle f^*_\alpha(t)\alpha(t)\rangle = 0. \tag{3.103}$$

Applied Eq. (3.86) allowing with its adjoint and the fact that the noise force at a certain time does not affect the cavity mode variables at earlier, we find

$$\int_0^t e^{-\frac{\kappa(t-t')}{2}}(\langle f^*(t)f(t)\rangle + \langle f^*(t)f(t)\rangle)dt' = 0. \tag{3.104}$$

Assuming that $\langle f^*(t)f(t')\rangle = \langle f^*(t)f(t')\rangle$, we see that

$$\int_0^t e^{-\frac{\kappa(t-t')}{2}}\langle f^*(t')f(t)\rangle = 0. \tag{3.105}$$

In view of the property of Dirac delta function

$$\int_0^t f(t')\delta(t-t')dt' = \frac{1}{2}f_\alpha, \tag{3.106}$$

Eq.(3.105) can be written as

$$\int_0^t e^{-\frac{\kappa}{2}(t-t')}\langle f^*(t)f(t')\rangle dt' = 0 \tag{3.107}$$

It then follows that

$$\langle f^*(t)f(t')\rangle = 0. \tag{3.108}$$

Follows a similar procedure , we can also establish the following properties

$$\langle f_m(t)\rangle = 0,\ \langle f_m(t')f_\alpha(t)\rangle = \langle f^*_m(t)f_\alpha(t')\rangle = 0, \tag{3.109}$$

$$\langle f_m(t)f^*_m(t')\rangle = \langle f^*_m(t)f_m(t')\rangle = \gamma N\delta(t-t'), \tag{3.110}$$

$$\langle f_m(t')f_m(t)\rangle = \langle f^*_m(t)f^*_m(t')\rangle = 0, \tag{3.111}$$

where f_m is the noise force of the cavity atom. The results described by Eqs.(3.87),(3.100),(3.108),(3.109),(3.11 (3.111) represent the correlation properties of the noise forces $f_\alpha(t)$ and $f_m(t)$associated with normal ordering.

3.2.1 Solution of c-number Langevein equation

Here we seek to obtain the solution of the differential equation

$$\frac{d}{dt}\alpha(t) = -\frac{\kappa}{2}\alpha(t) - gm(t) + f_\alpha(t) + \varepsilon, \tag{3.112}$$

$$\frac{d}{dt}m(t) = -\frac{\gamma}{2}(1+2N)m(t) + \gamma M m^*(t) - g(2N_e - 1)\alpha(t) + f_m(t). \tag{3.113}$$

We note that Eq.(3.113) is not linear differential equation. In order to make solvable and linear, we set that $-gN_e\alpha$ is Zero. Then we see that

$$\frac{dm(t)}{dt} = -\frac{\gamma}{2}(1+2N)m(t) + \gamma M m^*(t) + f_m + g\alpha(t). \tag{3.114}$$

Introducing new variables defined by

$$\alpha_\pm(t) = \alpha^*(t) \pm \alpha(t), \tag{3.115}$$

$$m_\pm = m^*(t) \pm m(t), \tag{3.116}$$

and using Eq.(3.112) and (3.114) along with their complex conjugate, we can write

$$\frac{d}{dt}\alpha_\pm(t) = -\frac{\kappa}{2}\alpha_\pm(t) - gm_\pm(t) + f_{\alpha\pm}(t) + \varepsilon, \tag{3.117}$$

$$\frac{d}{dt}m_\pm(t) = -\frac{\gamma}{2}(1+2N\mp 2M)m_\pm(t) - g\alpha_\pm(t) + f_{m\pm}(t). \tag{3.118}$$

Or these equation can be written in matrix as

$$\frac{d}{dt}Y_\pm(t) = -\frac{1}{2}Z_\pm Y_\pm + F_\pm(t), \tag{3.119}$$

where

$$Y_\pm(+) = \begin{pmatrix} \alpha_\pm(t) \\ m_\pm(t) \end{pmatrix}, \tag{3.120}$$

$$Z_\pm(t) = \begin{pmatrix} \kappa & 2g \\ 2g & -\gamma e^{\mp 2r} \end{pmatrix}, \tag{3.121}$$

$$F_\pm(t) = \begin{pmatrix} f_{\alpha\pm}(t) + \varepsilon \pm \varepsilon \\ f_{m\pm}(t) \end{pmatrix}. \tag{3.122}$$

Introducing a unitary matrix defined by

$$U = \begin{pmatrix} u_{11} & u_{12} \\ u_{21} & u_{22} \end{pmatrix}, \tag{3.123}$$

with $U_1 = \begin{pmatrix} u_{11} \\ u_{21} \end{pmatrix}$;and $U_2 = \begin{pmatrix} u_{12} \\ u_{22} \end{pmatrix}$ being the eigenvectors of matrix, $Z_\pm$ we write

$$\frac{d}{dt}Y_\pm(t) = -\frac{1}{2}UU^{-1}Z_\pm UU^{-1}Y_\pm(t) + F_\pm(t). \tag{3.124}$$

Multiplying both sides on the left by U^{-1} we see that

$$\frac{d}{dt}(U^{-1}Y_\pm(t) = -\frac{1}{2}R_\pm(U^{-1}Y_\pm(t) + U^{-1}F_\pm(t), \tag{3.125}$$

where

$$R_\pm = U^{-1}Z_\pm U = \begin{pmatrix} \lambda_{1\pm} & 0 \\ 0 & \lambda_{2\pm} \end{pmatrix}. \tag{3.126}$$

In which $\lambda_{1\pm}$ and$\lambda_{2\pm}$are the eigenvalues of matrix $Z_\pm$. We note that Eq.(3.125)can have a well defined solution if and only if both $\lambda_{1\pm}$ and $\lambda_{2\pm}$ are greater than zero. Hence the solution of Eq.(3.125)can be written as

$$Y_\pm(t) = Ue^{-R_\pm\frac{t}{2}}U^{-1}Y_\pm(0) + \int_0^t Ue^{-R_\pm\frac{(t-t')}{2}}U^{-1}F_\pm(t)dt'. \tag{3.127}$$

We next proceed to find eigenvalues and eigenvectors of the matrix $Z_\pm$. Using Eq.(3.121), the eigenvalue equation

$$Z_\pm Ui = \lambda_\pm Ui, \tag{3.128}$$

we find the characteristic equation equation

$$\lambda_\pm^2 - \lambda_\pm(\kappa + \gamma e^{\mp 2r}) + \gamma e^{\mp 2r} - 4g = 0, \tag{3.129}$$

and the roots of this quadratic equation are

$$\lambda_{1\pm} = \frac{1}{2}\Big(\ \kappa + \gamma e^{\mp 2r} + G\ \Big), \tag{3.130}$$

$$\lambda_{2\pm} = \frac{1}{2}\Big(\ \kappa + \gamma e^{\mp 2r} - G\ \Big), \tag{3.131}$$

where

$$G = \sqrt{(\kappa \pm \gamma e^{\mp 2r})^2 + 16g^2}. \tag{3.132}$$

We note that Eq.(3.125) will have a well defined solution for $\kappa+\gamma e^{\mp 2r} > G$. Hence we can consider $\kappa+\gamma e^{\mp 2r}=G$ as threshold with Eq.(3.121), we can find the equation corresponding to $\lambda_{1\pm}$

$$B_{1\pm}u_{11}-2gu_{21}=0, \tag{3.133}$$

in which

$$B_{1\pm}=\frac{1}{2}\Big(\ \gamma e^{\mp 2r}+G-\kappa\ \Big). \tag{3.134}$$

Taking into account the normalization condition

$$u_{11}^2+u_{21}^2=1, \tag{3.135}$$

we get

$$u_{11}=\frac{2g}{\sqrt{B_{1\pm}^2+4g^2}}, \tag{3.136}$$

$$u_{21}=\frac{B_{1\pm}}{\sqrt{B_{1\pm}^2+4g^2}}. \tag{3.137}$$

Similarly, the elements of eigenvector corresponding to $\lambda_{2\pm}$ are to be

$$u_{12}=\frac{2g}{\sqrt{B_{2\pm}^2+4g^2}}, \tag{3.138}$$

$$u_{22}=\frac{B_{2\pm}}{\sqrt{B_{2\pm}^2+4g^2}}, \tag{3.139}$$

where

$$B_{2\pm}=\frac{1}{2}\Big(\ \gamma e^{\mp 2r}-(\kappa+G)\ \Big). \tag{3.140}$$

Putting Eq.(3.136), (3.137), (3.138) and (3.139) in to Eq.(3.123) we see that

$$U=\begin{pmatrix} \frac{2g}{\sqrt{B_{1\pm}^2+4g^2}} & \frac{2g}{\sqrt{B_{2\pm}^2+4g^2}} \\ \frac{B_{1\pm}}{\sqrt{B_{1\pm}^2+4g^2}} & \frac{B_{2\pm}}{\sqrt{B_{2\pm}^2+4g^2}} \end{pmatrix}. \tag{3.141}$$

Eq.(4.141) is 2×2 matrix.Then its inverse is

$$U^{-1}=-\frac{1}{4gG}\begin{pmatrix} B_{2\pm}\sqrt{B_{1\pm}^2+4g^2} & -g\sqrt{B_{1\pm}^2+4g^2} \\ -B_{1\pm}\sqrt{B_{1\pm}^2+4g^2} & 2g\sqrt{B_{2\pm}^2+4g^2}, \end{pmatrix}, \tag{3.142}$$

since Eq.(3.127) is a diagonal matrix. We observe that

$$e^{-R_{\pm}\frac{t}{2}}=\begin{pmatrix} e^{-\frac{1}{2}\lambda_{1\pm}t} & 0 \\ 0 & e^{-\frac{1}{2}\lambda_{2\pm}t} \end{pmatrix}. \tag{3.143}$$

$$e^{-R_\pm \frac{t-t'}{2}} = \left(\begin{array}{cc} e^{-\frac{1}{2}\lambda_{1\pm}(t-t')} & 0 \\ 0 & e^{-\frac{1}{2}\lambda_{2\pm}(t-t')} \end{array} \right), \tag{3.144}$$

From which follows

$$Ue^{-R_\pm \frac{t}{2}} U^{-1} = \left(\begin{array}{cc} p_{1\pm}(t) & q_{1\pm}(t) \\ q_{2\pm}(t) & p_{2\pm}(t), \end{array} \right), \tag{3.145}$$

$$Ue^{-R_\pm \frac{t-t'}{2}} U^{-1} = \left(\begin{array}{cc} p_{1\pm}(t-t') & q_{1\pm}(t-t') \\ q_{2\pm}(t-t') & p_{2\pm}(t-t') \end{array} \right), \tag{3.146}$$

where

$$p_{1\pm}(t) = \frac{B_{1\pm}}{G} e^{-\frac{1}{2}\lambda_{2\pm}t} - \frac{B_{2\pm}}{G} e^{-\frac{1}{2}\lambda_{1\pm}t}, \tag{3.147}$$

$$p_{2\pm}(t) = \frac{B_{1\pm}}{G} e^{-\frac{1}{2}\lambda_{1\pm}t} - \frac{B_{2\pm}}{G} e^{-\frac{1}{2}\lambda_{2\pm}t}, \tag{3.148}$$

$$q_{1\pm}(t) = \frac{2g}{G} e^{-\frac{1}{2}\lambda_{1\pm}t} - \frac{2g}{G} e^{-\frac{1}{2}\lambda_{2\pm}t}, \tag{3.149}$$

$$q_{2\pm}(t) = \frac{2g}{G} e^{-\frac{1}{2}\lambda_{2\pm}t} - \frac{2g}{G} 4ge^{-\frac{1}{2}\lambda_{1\pm}t}, \tag{3.150}$$

$$p_{1\pm}(t-t') = \frac{B_{1\pm}}{G} e^{-\frac{1}{2}\lambda_{2\pm}(t-t')} - \frac{B_{2\pm}}{G} e^{-\frac{1}{2}\lambda_{1\pm}(t-t')}, \tag{3.151}$$

$$p_{2\pm}(t-t') = \frac{B_{1\pm}}{G} e^{-\frac{1}{2}\lambda_{1\pm}(t-t')} - \frac{B_{2\pm}}{G} e^{-\frac{1}{2}\lambda_{2\pm}(t-t')}, \tag{3.152}$$

$$q_{1\pm}(t-t') = \frac{2g}{G} e^{-\frac{1}{2}\lambda_{1\pm}(t-t')} - \frac{2g}{G} e^{-\frac{1}{2}\lambda_{2\pm}(t-t')}, \tag{3.153}$$

$$q_{2\pm}(t-t') = \frac{2g}{G} 4ge^{-\frac{1}{2}\lambda_{2\pm}(t-t')} - \frac{2g}{G} 4ge^{-\frac{1}{2}\lambda_{1\pm}(t-t')}. \tag{3.154}$$

With the aid of Eq.(3.120),(3.128),(3.142),(3.143),(3.144)and(3.145), we finally obtain

$$\alpha_\pm(t) = p_{1\pm}(t)\alpha_\pm(0) + q_{1\pm}(t)\alpha_\pm(0) + V_{1\pm}(t), \tag{3.155}$$

$$\sigma_{m\pm}(t) = p_{2\pm}(t)m_\pm(0) + q_{2\pm}(t)\alpha_\pm(0) + V_{2\pm}(t), \tag{3.156}$$

where

$$V_{1\pm}(t) = \int_0^t \Big(\; p_{1\pm}(t-t')(f_{\alpha\pm}(t') + \varepsilon \pm \varepsilon) + q_{1\pm}(t-t')f_{m\pm}(t') \; \Big) dt', \tag{3.157}$$

$$V_{2\pm}(t) = \int_0^t \Big(\; p_{2\pm}(t-t')f_{m\pm}(t') + q_{2\pm}(t-t')(f_{\alpha\pm}(t') + \varepsilon \pm \varepsilon) \; \Big) dt'. \tag{3.158}$$

4

Result and Discussion

4.1 Atomic Inversion

With the aid of Eq.(3.82) and (3.83) one can write

$$\frac{d}{dt}\langle\sigma_z\rangle = \gamma(1+2N)\langle\sigma_z\rangle + 4g\langle\alpha^* m\rangle. \tag{4.1}$$

At steady state, we have

$$\langle\sigma_z\rangle_{ss} = \frac{4g(\langle\alpha^* m\rangle_{ss})}{\gamma(1+2N)}, \tag{4.2}$$

where ss stands for steady state. Now employing Eq.(3.109),(3.110),(3.155) and (3.156), we can write

$$\langle\alpha^* m\rangle = \frac{1}{2}\langle\alpha_+ m_+\rangle - \langle\alpha_- m_-\rangle + \langle\alpha_- m_+\rangle - \langle\alpha_+ m_-\rangle. \tag{4.3}$$

Applying the Eq.(3.155) and (3.156) along with the assumption of the cavity is initially vacuum and the fact the noise force at certain time does not affect the cavity mode variables at earlier time, we see that

$$\langle\alpha^* m\rangle = \frac{1}{2}\Big(\ \langle V_{1+}V_{2+}\rangle - \langle V_{1-}V_{2-}\rangle\ \Big). \tag{4.4}$$

Using Eq.(3.157) and (3.158) we get

$$\begin{aligned}\langle\alpha^* m\rangle = \int_0^t dt'[2\varepsilon p_{1+}(t-t')q_{2+}(t-t') + p_{2-}(t-t')q_{1-}(t-t')\gamma N \\ - p_{2+}(t-t')q_{1+}(t-t')\gamma N].\end{aligned} \tag{4.5}$$

With Eq.(3.151),(3.152),(3.153)and (3.154), we can get

$$\int dt' p_{1+}(t-t')q_{2+}(t-t') = \int_0^t dt' 2\frac{B_{2+}B_{1+}}{G^2}e^{-\frac{1}{2}(\lambda_{1+}+\lambda_{2+})(t-t')} - \frac{B_{2+}B_{1+}}{G^2}e^{-\lambda_{1+}(t-t')} - \frac{B_{2+}B_{1+}}{G^2}e^{-\lambda_{2+}(t-t')}, \tag{4.6}$$

$$\int dt' q_{1+}(t-t')p_{2+}(t-t') = \int_0^t dt' \frac{B_{1+}^2}{G^2} e^{-\lambda_{1+}(t-t')} - 2\frac{B_{2+}B_{1+}}{G^2} e^{-\frac{1}{2}(\lambda_{1+}+\lambda_{2+})(t-t')} + \frac{B_{2+}^2}{G^2} e^{-\lambda_{2+}(t-t')}, \tag{4.7}$$

$$\int dt' q_{1-}(t-t')p_{2-}(t-t) = \int_0^t dt' \frac{B_{1-}^2}{G^2} e^{-\lambda_{1-}(t-t')} - 2\frac{B_{2-}B_{1-}}{G^2} e^{-\frac{1}{2}(\lambda_{1-}+\lambda_{2-})(t-t')} + \frac{B_{2-}^2}{G^2} e^{-\lambda_{2-}(t-t')}. \tag{4.8}$$

Substitute Eqs.(4.6-4.8) into Eq.(4.5)and integrate,yields at steady state

$$\begin{aligned} \langle \alpha^* m \rangle = & \frac{2g}{G_2^2}[\varepsilon[\frac{2\gamma e^{-4r} - 2ke^{-2r}}{2\gamma e^{-2r}(1+k) + k^2 - G_2^2}] + \gamma N[\frac{4kG_2^2 + k^2}{\gamma e^{2r}(\gamma e^{2r} + 2k) + k^2 + G_2^2}] \\ & + \varepsilon + \gamma N] - \frac{2g}{G_1^2}[\frac{\gamma e^{-2r} - \kappa}{2(\kappa + \gamma e^{-2r})} - \frac{4\kappa G_1^2 + k^2}{\gamma e^{-2r}(\gamma e^{2r} + 2k) + k^2 + G_1^2}]\gamma N, \end{aligned} \tag{4.9}$$

where

$$G_1 = \sqrt{((k + \gamma e^{-2r})^2 + 16g^2)} \tag{4.10}$$

$$G_2 = \sqrt{((k + \gamma e^{2r})^2 + 16g^2)} \tag{4.11}$$

On the account Eq.(4.10) and (4.11) Eq.(4.9) yields,

$$\begin{aligned} \langle \alpha^* m \rangle_{ss} = & \frac{2g}{(k+\gamma e^{-2r})^2 + 16g^2}[\varepsilon(\frac{2\gamma e^{-2r}(\gamma e^{-2r} - k)}{\gamma e^{-2r}(\gamma e^{-2r} + 2) + 2k^2}) + \gamma N(\frac{4k((k+\gamma e^{2r})^2 + 16g^2)^{\frac{1}{2}} + k^2}{2k^2 + 2\gamma^2 e^{4r}}) \\ & + \frac{\varepsilon + \gamma N}{1+2N}] - \frac{2gN}{(k+\gamma e^{-2r})^2 + 16g^2}[\frac{e^{-2r} - k}{2(\gamma e^{-2r} + k)} - \frac{((k+\gamma e^{-2r})^2 + 16g^2)^{\frac{1}{2}} + k^2}{2k^2 + 2\gamma^2 e^{-4r}}]. \end{aligned} \tag{4.12}$$

On the account of this result, Eq.(4.2) becomes

$$\begin{aligned} \langle \sigma_z \rangle_{ss} = & \frac{4g^2}{(1+2N)(k+\gamma e^{-2r})^2 + 16g^2}[[\frac{\varepsilon}{\gamma}(\frac{2\gamma e^{-2r}(\gamma e^{-2r} - k)}{\gamma e^{-2r}(\gamma e^{-2r} + 2) + 2k^2}) \\ & + (\frac{4k((k+\gamma e^{2r})^2 + 16g^2)^{\frac{1}{2}} + k^2}{2k^2 + 2\gamma^2 e^{4r}})N + \varepsilon + \gamma N] \\ & - [\frac{\gamma e^{-2r} - k}{2(\gamma e^{-2r} + k)} - \frac{((k+\gamma e^{-2r})^2 + 16g^2)^{\frac{1}{2}} + k^2}{2k^2 + 2\gamma^2 e^{-4r}}]N], \end{aligned} \tag{4.13}$$

is the population (atomic) inversion.

The plots atomic inversion vs coupling constant in Fig.(4.1) show that the population inversion increase with the atomic decay constant.

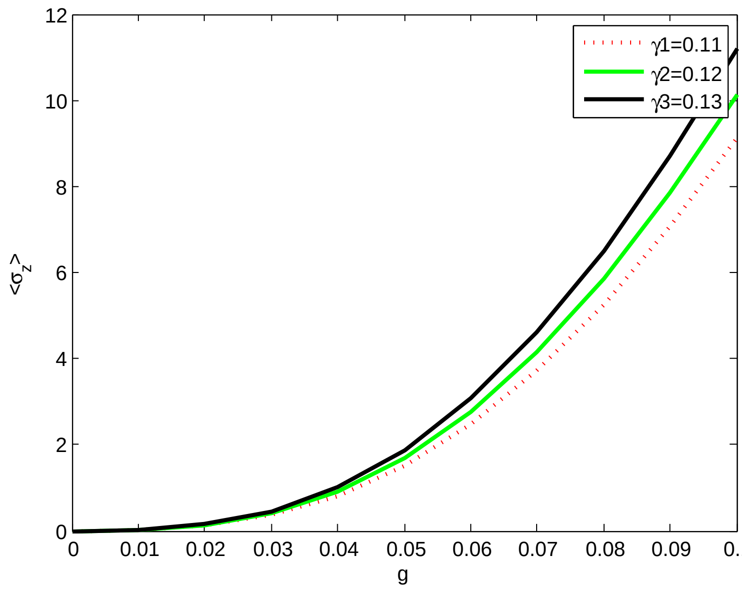

Fig. 4.1: Plots of the population inversion [Eq.(4.31)] versus g for $\varepsilon = 0.8$,$r = 0.5$,$\kappa = 0.8$ and different values of γ ,for $\gamma = 0.11$(doted curve), $\gamma = 0.12$ (dashed curve) and $\gamma = 0.13$ (solid curve).

4.2 Quadrature fluctuation of the cavity mode

The quadrature for the cavity mode represented by $a\pm$ are defined.

$$\hat{a}_+ = \hat{a}^\dagger + \hat{a} \tag{4.14}$$

$$\hat{a}_- = i(\hat{a}^\dagger - \hat{a}) \tag{4.15}$$

and Hermitian and satisfy the commutation relation $[\hat{a}_+, \hat{a}_-] = 2i$, The quadrature variances defined by

$$\Delta\hat{a}_\pm^2 = \langle\hat{a}_\pm^2(t)\rangle - \langle\hat{a}_\pm(t)\rangle^2, \tag{4.16}$$

Can be expressed in terms of c-number variables associated with the normal ordering as,

$$\Delta\hat{a}_\pm^2 = 1 \pm \langle\alpha_\pm(t), \alpha_\pm(t)\rangle \tag{4.17}$$

where $\alpha_\pm^2$ is given by Eq. (3.115). Assuming that cavity mode to be initially in vacuum and taking into account Eq. (3.155) along with Eq.(3.87) one easily shows that

$$\langle\alpha_\pm(t)\rangle = 0. \tag{4.18}$$

On account of this, Eq.(4.35) becomes,

$$\Delta\hat{a}_\pm^2 = 1 \pm \langle\alpha_\pm^2(t)\rangle. \tag{4.19}$$

Furthermore, applying this result and Eq.(3.118) along with the assumption that the cavity is initially vacuum and the fact the noise force at certain time does not affect the cavity mode variables at earlier time.We see that,

$$\langle \alpha_{\pm}^2(t)\rangle = \langle V_{1\pm}(t)V_{1\pm}(t)\rangle. \tag{4.20}$$

Taking into account Eqs.(3.157), (4.12) and (4.13) we have

$$\langle \alpha_{\pm}^2(t)\rangle = \pm 2\int_0^t dt' \left[\ p_{1\pm}^2(t-t')(\varepsilon \pm \varepsilon) + q_{1\pm}^2(t-t')\gamma N\ \right]. \tag{4.21}$$

Applying Eqs.(3.151) and (3.153) and carry out the integration, we find

$$\begin{aligned}\langle \alpha_{\pm}^2(t)\rangle = {} & \pm 2(-\frac{B_{1\pm}^2}{G^2\lambda_{2\pm}}(1-e^{-\lambda_{2\pm}t}) - \frac{B_{2\pm}^2}{G^2\lambda_{1\pm}}(1-e^{-\lambda_{1\pm}t}) \\ & + \frac{4B_{1\pm}B_{2\pm}}{G^2(\lambda_{2\pm}+\lambda_{1\pm})}(1-e^{-\frac{1}{2}(\lambda_{2\pm}+\lambda_{1\pm})t}))(\varepsilon \pm \varepsilon) \\ & + (-\frac{B_{2\pm}^2B_{1\pm}^2}{4g^2G^2\lambda_{2\pm}}(1-e^{\lambda_{2\pm}t}) - \frac{B_{2\pm}^2B_{1\pm}^2}{4g^2G^2\lambda_{1\pm}}(1-e^{\lambda_{1\pm}t}) \\ & + \frac{B_{2\pm}^2B_{1\pm}^2}{g^2G^2(\lambda_{2\pm}+\lambda_{1\pm})}(1-e^{-\frac{1}{2}(\lambda_{2\pm}+\lambda_{2\pm})t}))\gamma N\end{aligned}. \tag{4.22}$$

Substituting Eq.(4.22) into Eq(4.19), we have

$$\begin{aligned}\Delta\hat{a}_{\pm}^2(t) = {} & 1 + 2(-\frac{B_{1\pm}^2}{G^2\lambda_{2\pm}}(1-e^{-\lambda_{2\pm}t}) - \frac{B_{2\pm}^2}{G^2\lambda_{1\pm}}(1-e^{-\lambda_{1\pm}t}) \\ & + \frac{4B_{1\pm}B_{2\pm}}{G^2(\lambda_{2\pm}+\lambda_{1\pm})}(1-e^{-\frac{1}{2}(\lambda_{2\pm}+\lambda_{1\pm})t}))(\varepsilon \pm \varepsilon) \\ & + (-\frac{B_{2\pm}^2B_{1\pm}^2}{4g^2G^2\lambda_{2\pm}}(1-e^{\lambda_{2\pm}t}) - \frac{B_{2\pm}^2B_{1\pm}^2}{4g^2G^2\lambda_{1\pm}}(1-e^{\lambda_{1\pm}t}) \\ & + \frac{B_{2\pm}^2B_{1\pm}^2}{g^2G^2(\lambda_{2\pm}+\lambda_{1\pm})}(1-e^{-\frac{1}{2}(\lambda_{2\pm}+\lambda_{2\pm})t}))\gamma N\end{aligned}, \tag{4.23}$$

so that at steady state, this equation can be written in the form

$$\begin{aligned}\Delta\hat{a}_{\pm}^2(t) = {} & 1 + 2(\frac{\gamma e^{\mp 2r}-k}{\gamma e^{\mp 2r}+k} - \frac{\gamma e^{\mp 2r}-k+G}{\gamma e^{\mp 2r}+k-G} + \frac{\gamma e^{\mp 2r}+k-G}{\gamma e^{\mp 2r}+k+G})(\frac{\varepsilon \pm \varepsilon}{G}) \\ & + 2(\frac{(\gamma e^{\mp 2r}-k)^2}{\gamma e^{\mp 2r}+k} - \frac{(\gamma e^{\mp 2r}-k)^2}{\gamma e^{\mp 2r}+k+G} - \frac{(\gamma e^{\mp 2r}-k)^2}{\gamma e^{\mp 2r}+k-G})\frac{\gamma N}{2}.\end{aligned} \tag{4.24}$$

Substituting Eq. (3.132) into Eq. (4.43), we have

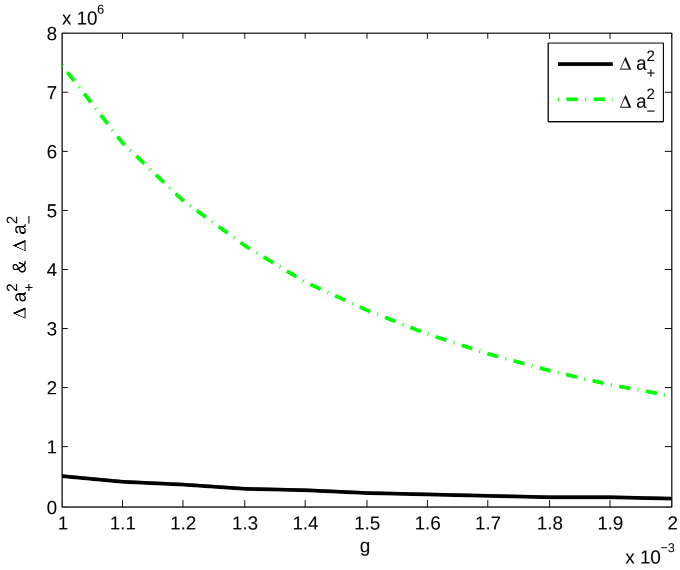

Fig. 4.2: Plots of the quadrature variances [Eq.(4.44)] versus g, for $\varepsilon = 0.5$,$r = 0.5$, $\kappa = 0.8$,$\gamma = 0.8$, for Δa_{+} (solid line) and Δa_{-} (dash doted curve)

$$\Delta\hat{a}^2_{\pm}(t) = 1 + 2\left(\frac{\gamma e^{\mp 2r} - k}{\gamma e^{\mp 2r} + k} - \frac{\gamma e^{\mp 2r} - k + \sqrt{(k+\gamma e^{\mp 2r})^2 + 16g^2}}{\gamma e^{\mp 2r} + k - \sqrt{(k+\gamma e^{\mp 2r})^2 + 16g^2}} + \frac{\gamma e^{\mp 2r} + k - \sqrt{(k+\gamma e^{\mp 2r})^2 + 16g^2}}{\gamma e^{\mp 2r} + k + \sqrt{(k+\gamma e^{\mp 2r})^2 + 16g^2}} \right) \left(\frac{\varepsilon \pm \varepsilon}{\sqrt{(k + \gamma e^{\mp 2r})^2 + 16g^2}} \right)$$
$$+ 2\left(\frac{(\gamma e^{\mp 2r} - k)^2}{\gamma e^{\mp 2r} + k} - \frac{(\gamma e^{\mp 2r} - k)^2}{\gamma e^{\mp 2r} + k + \sqrt{(k+\gamma e^{\mp 2r})^2 + 16g^2}} - \frac{(\gamma e^{\mp 2r} - k)^2}{\gamma e^{\mp 2r} + k - \sqrt{(k+\gamma e^{\mp 2r})^2 + 16g^2}} \right) \frac{\gamma N}{2}, \tag{4.25}$$

This equation represents the quadrature variances of the light mode in the cavity for the two-level atom coupled to squeezed vacuum inside a coherently driven cavity at steady state. The plots in the figure show that the light mode produced by the system under consideration is squeezed state and the squeezing occurs in the plus quadrature.

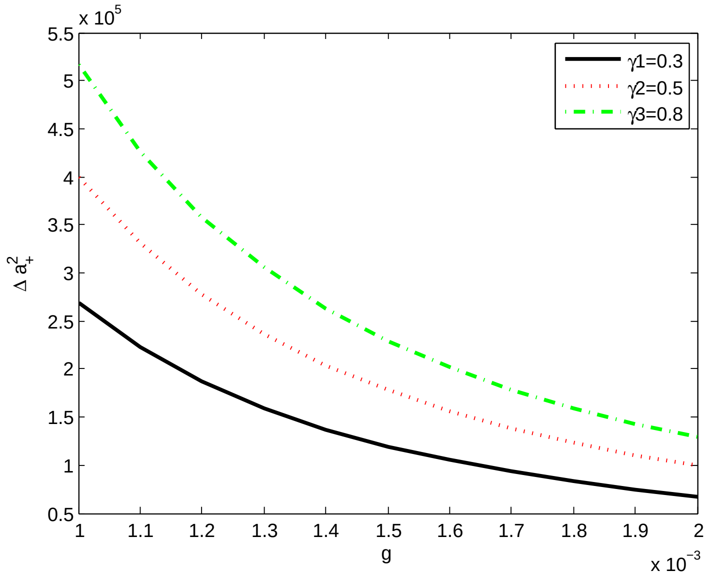

Fig. 4.3: Plots of the quadrature variances Δa_+[Eq.(4.25)] versus g for $\kappa = 0.8$, $r = 0.5$, $\varepsilon = 0.5$ and for different values of γ ,for $\gamma 1 = 0.3$(solid curve), $\gamma 2 = 0.5$ (doted curve) and $\gamma 3 = 0.8$ (dash doted curve).

We observe from Fig.(4.3) that the degree of squeezing increases as the atomic decay constant decrease.

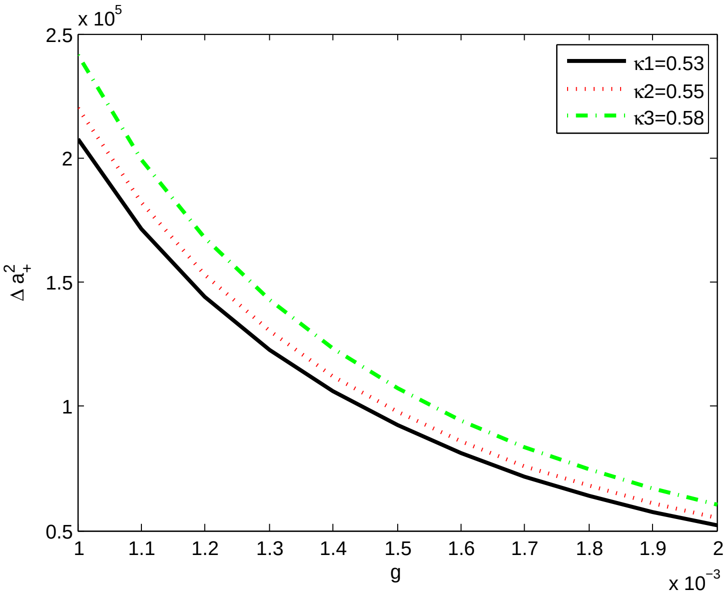

Fig. 4.4: Plots of the quadrature variances Δa_+[Eq.(4.25)] versus g for $\varepsilon = 0.5$, $\gamma = 0.8$, $r = 0.8$ and for different values of κ, for $\kappa 1 = 0.53$(solid curve), $\kappa 2 = 0.55$ (doted curve) and $\kappa 3 = 0.58$ (dashed doted curve).

We observe from the plots in fig.(4.4) that the degree of squeezing increases as the cavity mode decay constant decrease.

4.3 Second order correlation function

The second order correlation can be expressed interms of the atomic operator as,

$$g^{(2)}(t') = \frac{\langle \hat{\sigma}_+(t)\hat{\sigma}_+(t+t')\hat{\sigma}_-(t+t')\hat{\sigma}_-(t)\rangle}{\langle \hat{\sigma}_+(t)\hat{\sigma}_-(t)\rangle^2}. \tag{4.26}$$

We recall that

$$\langle \hat{\sigma}_+(t+t')\hat{\sigma}_-(t+t')\rangle = \frac{(\langle \hat{\sigma}_z(t+t')\rangle + 1)}{2}. \tag{4.27}$$

Further, more the formal solution of Eq.(4.1) can be written as

$$\langle \hat{\sigma}_z(t+t')\rangle = \langle \hat{\sigma}_z(t)\rangle e^{-\gamma(1+2N)t'} + \frac{4g\langle \alpha^* m\rangle}{\gamma(1+2N)}(1 - e^{-\gamma(1+2N)t'}). \tag{4.28}$$

From which follows

$$\frac{(\langle \hat{\sigma}_z(t+t')\rangle + 1)}{2} = \frac{1}{2}(\langle \hat{\sigma}_z(t)\rangle + 1)e^{-\gamma(1+2N)t'} + \frac{\gamma(1+2N) + 4g\langle \alpha^* m\rangle}{2\gamma(1+2N)}(1 - e^{-\gamma(1+2N)t'}). \tag{4.29}$$

In view of Eq.(4.26), we see that

$$\langle \hat{\sigma}_+(t+t')\hat{\sigma}_-(t+t')\rangle = \langle \hat{\sigma}_+(t)\hat{\sigma}_-(t)\rangle e^{-\gamma(1+2N)t'} + \frac{\gamma(1+2N) + 4g\langle \alpha^* m\rangle}{2\gamma(1+2N)}(1 - e^{-\gamma(1+2N)t'}). \tag{4.30}$$

On applying of the quantum regression theorem, the steady state solution of Eq.(4.46)takes the form

$$\langle\hat{\sigma}_z\rangle_{ss} = \frac{4g\langle\alpha^*\rangle}{\gamma(1+2N)}. \tag{4.31}$$

From which follows

$$\frac{\langle\sigma_z(t)\rangle + 1}{2} = \frac{4g\langle\alpha^* m\rangle + \gamma(1+2N)}{2\gamma(1+2N)}. \tag{4.32}$$

Thus, on the account of Eq. (4.27) we get

$$\langle\sigma_+(t)\sigma_-(t)\rangle = \frac{4g\langle\alpha^* m\rangle + \gamma(1+2N)}{2\gamma(1+2N)}. \tag{4.33}$$

The second order correlation function then be written as,

$$g^{(2)}(t') = \frac{\frac{4g\langle\alpha^* m\rangle+\gamma(1+2N)}{2\gamma(1+2N)}}{\langle\sigma_+(t)\sigma_-(t)\rangle}\left(\ 1 - e^{-\gamma(1+2N)t'}\ \right), \tag{4.34}$$

$$g^{(2)}(t') = 1 - e^{-\gamma(1+2N)t'}. \tag{4.35}$$

For $t' = 0$,

$$g^{(2)}(0) = 0. \tag{4.36}$$

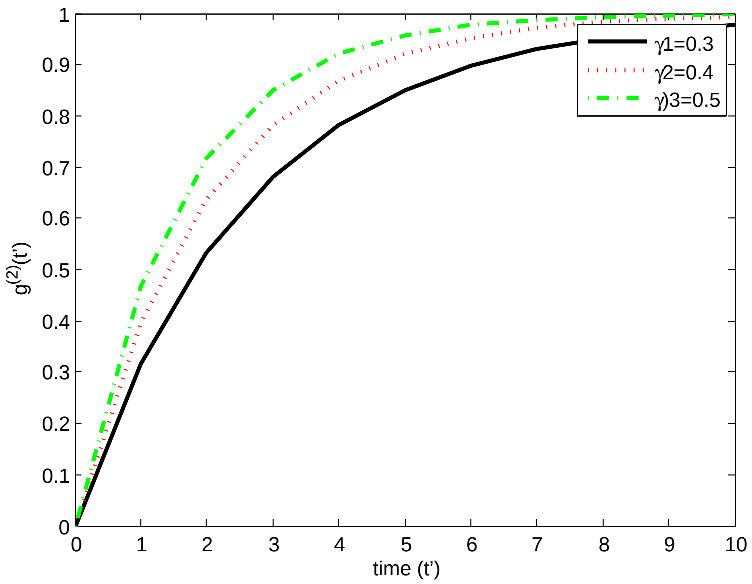

Fig. 4.5: Plots of the second order correlation function [Eq.(4.35)] versus t' for $r = 0.3$ and different values of γ, for $\gamma = 0.3$(dashed curve), $\gamma = 0.4$ (doted curve) and $\gamma = 0.5$ (dash doted curve).

From Fig. 4.5, we observe that $g^{(2)}(0) = 0$ for $t' = 0$ and $g^{(2)}(t') > 0$ for $t' > 0$. Thus we see that for $t' > 0$, $g^{(2)}(t') > g^{(2)}(0)$. The light thus exhibits the phenomena of photon anti bounching, for a relative small value of t' the second order correlation function is less than unity.

4.4 Photon Statistics

In this section we seek to calculate the mean and variance of the photon number of the cavity mode produced by the two-level atom coupled to squeezed vacuum inside a coherently driven cavity coupled to ordinary vacuum reservoir.

4.4.1 Mean photon number

We here wish to calculate the mean photon number of the cavity mode. The mean photon number defined by

$$\overline{n} = \langle \hat{a}^{\dagger}, \hat{a} \rangle, \tag{4.37}$$

can be written in terms of c-number variable associated to the normal order as,

$$\overline{n} = \langle \alpha^{*}, \alpha \rangle. \tag{4.38}$$

Or employing Eq. (3.115), (3.155), we can rewrite as

$$\overline{n} = \frac{1}{4}(\langle \alpha_{+}^{2}(t) \rangle - \langle \alpha_{-}^{2}(t) \rangle). \tag{4.39}$$

In view of Eq. (4.22), we obtain

$$\begin{aligned}\overline{n} = &\Big(-\tfrac{B_{1+}^2}{\lambda_{2+}}(1-e^{-\lambda_{2+}t}) - \tfrac{B_{2+}^2}{\lambda_{1+}}(1-e^{-\lambda_{1+}t}) + \tfrac{4B_{1\pm}B_{2+}}{\lambda_{2+}+\lambda_{1+}}(1-e^{-\frac{1}{2}(\lambda_{2+}+\lambda_{1+})t}) \Big)(\frac{\varepsilon}{G^2}) \\ &+ \Big(-\tfrac{B_{2+}^2 B_{1+}^2}{\lambda_{2+}}(1-e^{\lambda_{2+}t}) - \tfrac{4B_{2+}^2 B_{1+}^2}{\lambda_{1+}}(1-e^{\lambda_{1+}t}) + \tfrac{B_{2+}^2 B_{1+}^2}{\lambda_{2+}+\lambda_{1+}}(1-e^{-\frac{1}{2}(\lambda_{2+}+\lambda_{2+})t}) \Big)\frac{\gamma N}{2G^2}) \\ &+ \Big(-\tfrac{B_{2-}^2 B_{1-}^2}{\lambda_{2-}}(1-e^{\lambda_{2-}t}) - \tfrac{B_{2-}^2 B_{1-}^2}{\lambda_{1-}}(1-e^{\lambda_{1-}t}) + \tfrac{4B_{2-}^2 B_{1-}^2}{\lambda_{2-}+\lambda_{1-}}(1-e^{-\frac{1}{2}(\lambda_{2-}+\lambda_{2-})t}) \Big)\frac{\gamma N}{2G^2}).\end{aligned} \tag{4.40}$$

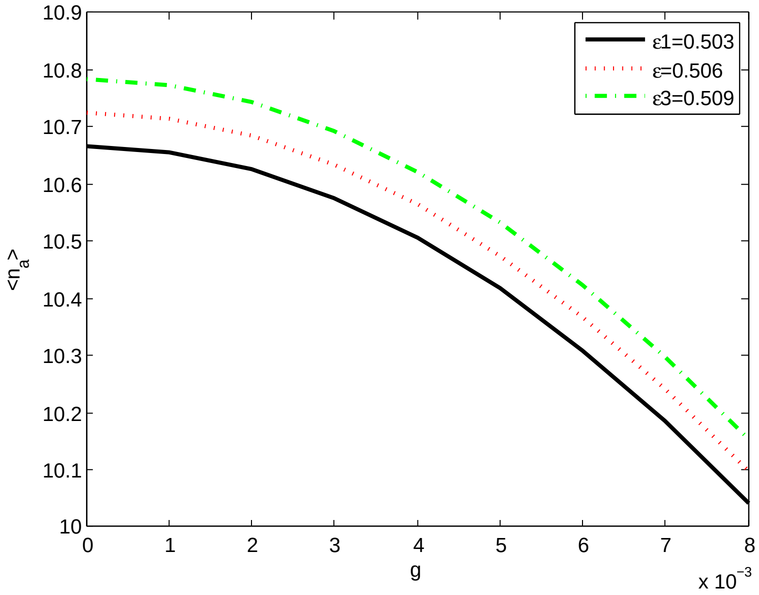

Fig. 4.6: Plots of the the mean photon number [Eq.(4.42)] versus g for $\kappa = 0.8$, $\gamma = 0.8$, $r = 0.5$, and different values of ε ,for $\varepsilon 1 = 0.503$(solid curve), $\varepsilon 2 = 0.506$ (doted curve) and $\varepsilon 3 = 0.509$ (dash curve).

The plots in the figure show that the mean photon number increase as the driving mode amplitude increase.

Hence at steady state, Eq. (4.40) takes the form,

$$\begin{aligned}\overline{n} = &\Big(-\frac{B_{1+}^2}{\lambda_{2+}} - \frac{B_{2+}^2}{\lambda_{1+}} + \frac{4B_{1\pm}B_{2+}}{\lambda_{2+}+\lambda_{1+}} \Big)(\frac{\varepsilon}{G^2}) \\ &+ \Big(-\frac{B_{2+}^2B_{1+}^2}{\lambda_{2+}} - \frac{4B_{2+}^2B_{1+}^2}{\lambda_{1+}} + \frac{B_{2+}^2B_{1+}^2}{\lambda_{2+}+\lambda_{1+}} \Big)\frac{\gamma N}{2G^2} \\ &+ \Big(-\frac{B_{2-}^2B_{1-}^2}{\lambda_{2-}} - \frac{B_{2-}^2B_{1-}^2}{\lambda_{1-}} + \frac{4B_{2-}^2B_{1-}^2}{\lambda_{2-}+\lambda_{1-}} \Big)\frac{\gamma N}{2G^2}.\end{aligned} \tag{4.41}$$

Substitute Eqs. (3.130), (3.131), (3.132), (3.134) and (3.140) into (4.41), then we find

$$\begin{aligned}\overline{n} = &\Big(\frac{\gamma e^{-2r}-k}{\gamma e^{-2r}+k} - \frac{\gamma e^{-2r}-k+((k+\gamma e^{-2r})^2+16g^2)^{\frac{1}{2}}}{\gamma e^{-2r}+k-((k+\gamma e^{-2r})^2+16g^2)^{\frac{1}{2}}} + \frac{\gamma e^{-2r}+k-((k+\gamma e^{-2r})^2+16g^2)^{\frac{1}{2}}}{\gamma e^{-2r}+k+((k+\gamma e^{-2r})^2+16g^2)^{\frac{1}{2}}} \Big)(\frac{\varepsilon}{((k+\gamma e^{-2r})^2+16g^2)^{\frac{1}{2}}}) \\ &+ \Big(\frac{(\gamma e^{-2r}-k)^2}{\gamma e^{-2r}+k} - \frac{(\gamma e^{-2r}-k)^2}{\gamma e^{-2r}+k+((k+\gamma e^{-2r})^2+16g^2)^{\frac{1}{2}}} - \frac{(\gamma e^{-2r}-k)^2}{\gamma e^{-2r}+k-((k+\gamma e^{-2r})^2+16g^2)^{\frac{1}{2}}} \Big)\frac{\gamma N}{4} \\ &+ \Big(\frac{(\gamma e^{+2r}-k)^2}{\gamma e^{+2r}+k} - \frac{(\gamma e^{+2r}-k)^2}{\gamma e^{+2r}+k+((k+\gamma e^{+2r})^2+16g^2)^{\frac{1}{2}}} - \frac{(\gamma e^{+2r}-k)^2}{\gamma e^{+2r}+k-((k+\gamma e^{+2r})^2+16g^2)^{\frac{1}{2}}} \Big)\frac{\gamma N}{4}.\end{aligned} \tag{4.42}$$

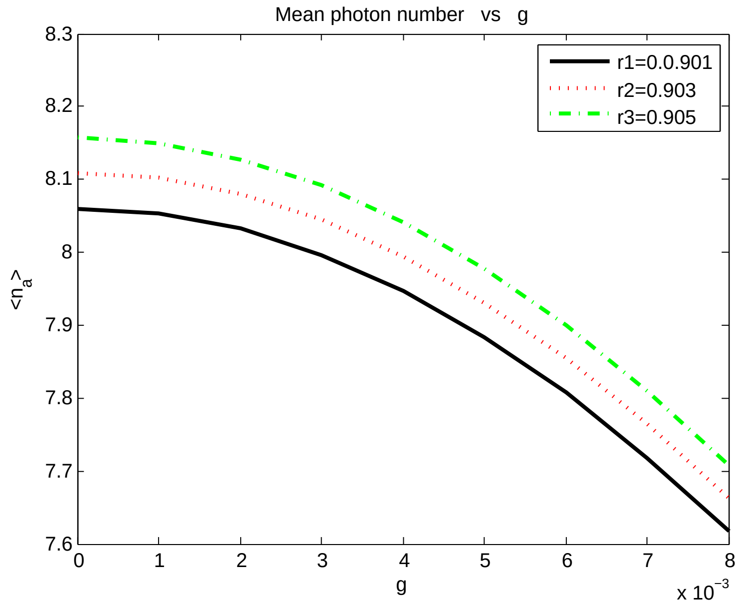

Fig. 4.7: Plots of the the mean photon number [Eq.(4.42)] versus g for $\varepsilon = 0.5$, $\kappa = 0.8$, $\gamma = 0.5$ and different values of r ,for $r = 0.901$(solid curve), $r = 0.903$ (doted curve) and $r3 = 0.905$ (dashed curve).

From the plots we observe, that the mean photon number increase as the the squeezing parameter increase.

4.4.2 The variance of the photon number

we next proceed calculate the variance of the the photon for the two level atom coupled to a squeezed vacuum inside a coherently driven cavity. The variance of the photon defined by

$$\Delta \overline{n}^2 = \langle (\hat{a}^\dagger \hat{a})^2 \rangle - \langle (\hat{a}^\dagger \hat{a}) \rangle^2 \tag{4.43}$$

Can be expressed interims of c-number variable associated with normal ordering as

$$\Delta \overline{n}^2 = \langle \alpha^{*2} \alpha^2 \rangle + \overline{n} - \overline{n}^2 \tag{4.44}$$

Recalling that

$$\langle \alpha^{*2} \alpha^2 \rangle = \overline{n}^2 \tag{4.45}$$

Substituting Eq.(4.45) into Eq.(4.44), we obtain

$$\Delta \overline{n}^2 = \overline{n} \tag{4.46}$$

One can easily see from this results that

$$(\Delta \overline{n})^2 = \overline{n}, \tag{4.47}$$

Hence the radiation in the cavity has poissonian statistics.

5

Conclusion

In this thesis we have studied the squeezing and statistical properties of a two level atom coupled to squeezed vacuum inside a coherently driven cavity. Using the pertinent master equation, we have obtained the c-number Langevien equation associated with the normal ordering. Applying the solutions of the resulting c-number Langevien equations and correlation properties of the noise force, we have calculated the atomic inversion, the quadrature variance of the cavity mode, the mean and the variance of the photon number and the second order correlation function. The result showed that the quadrature variance of the cavity mode exhibit squeezing and the squeezing occurs in the plus quadrature of cavity mode. Further more, mean photon number of the cavity mode increases with the presence of the squeezed vacuum and the degree of squeezing increases as the amplitude of the driving mode increase. But the degree of squeezing decrease when κ increase. And also the atomic inversion increase as the atomic decay constant increase. Moreover, we have found that the cavity mode radiation exhibits photon antibounching and poissonian photon statistics.

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