# THE EFFECT OF SEEDED THERMAL LIGHT IN THE CAVITY OF A NON-DEGENERATE PARAMETRIC OSCILLATOR COUPLED TO VACUUM RESERVOIR


**By**

**Sisay Sime Hailu**


A Thesis Submitted to
the Department of Applied physics
School of Applied Natural Science

Presented in Partial Fulfilment of the Requirement for the Degree of Master's of Science in Physics

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This is to certify that the thesis prepared by **Sisay Sime**:- **The effect of seeding thermal light in a cavity of non degenerate parametric oscillator coupled to vacuum reservoir** and submitted for partial fulfilment of the requirement for the degree of master in physics complies with regulation of university meets the accepted standard with respect to originality and ability.

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# DECLARATION

I hereby declare that this thesis is my original work and has not been presented for a degree in any other universities, and all sources of material used for this thesis have been duly acknowledged.

Name: Sisay Sime

Signature: ____________________

This M.sc thesis has been submitted for examination with my approval as thesis advisor supervisor.

Name: Dr. Tewodiros Yirgashewa

Signature: ____________________

date of submission:

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# Contents

# Abstract

*We studied the quantification of entanglement, quadrature squeezing and statistical properties of light produced by a non degenerate parametric oscillator coupled to vacuum reservoir in which the cavity is initially seeded by thermal light. We obtained the c-number Langevin equations associated with the normal ordering employing the pertinent master equation. We used the solutions of the resulting c-number Langevin equations and correlation properties of the noise forces to calculate quadrature variances, mean photon number, variance of the photon number and entanglement of the produced light. We have found that the effect of seeding thermal light is to increase the mean photon number, but degrades the two mode squeezing and entanglement.*

# 1

# INTRODUCTION

## 1.1 Background

Light has played a special role in our attempts to understand nature both classically and quantum mechanically. Classically light field consists of waves with well defined amplitude and phase such is not the case when we treat the field quantum mechanically. Squeezing and entanglement are quantum properties of light which do not have a classical analog. Recent development of technology greatly improved our ability to control individual quantum system. This brings quantum technology beyond academic research to control the level of concrete industrial programs.

Quantum optics is the subject that deals with optical phenomena that can only be explained by treating light as a stream of photons rather than as electromagnetic waves. In principle, the subject is as old as quantum theory it self, but in practice, it is a relatively new one, and has really only come to the fore during the last quarter of the twentieth century. The quantum properties of light are largely determined by the state of the light mode and the well known quantum states are;the number state, the coherent state, the chaotic state and the squeezed state.

Squeezing is one of the interesting non classical features of light that has been attracting attention and studied by many authors [1-9]. Squeezing light can be generated by quantum optical processes such as parametric oscillation [1-8], second harmonic generation [1,8,9] and four-wave mixing [8,9].

A parametric oscillator has been considered as an important source of squeezed light. It is one of the most interesting and well characterized optical device in quantum optics. In this device a pump photon interacts with a non linear crystal inside a cavity and is down converted in two highly correlated photons. If these photons have the same frequency and the device is called a degenerate parametric oscillator, otherwise it is called a non degenerate parametric oscillator.

Another interesting non-classical features of light is entanglement, which was studied by many authors [12-18]. Quantum entanglement is a physical phenomenon which is created usually by direct interaction between sub-atomic particles such as photons, electrons and molecules and then separated. Before interaction each particle is described by its own quantum state. The two-mode squeezing state itself is an entangled state which entitles the idler mode and signal mode in the frequency domain [18]. Recently, it has been shown that the non degenerate parametric oscillator can generate macroscopic entangled state. It is believed that the key in gradient of quantum information is entanglement which has been recognized as the essential resource for quantum teleportation,quantum decoding,quantum computation and quantum cryptography [19].

In this M.sc thesis, we study the squeezing and entanglement properties of light produced by a non degenerate parametric oscillator whose cavity initially seeded by thermal light coupled to vacuum reservoir.

## 1.2 Objectives

### 1.2.1 General objective of the study

The general objectives of this thesis is to study the effect of seeding thermal light in the cavity of a non degenerate parametric oscillator coupled to ordinary vacuum reservoir.

### 1.2.2 Specific objectives of the study

The specific objectives of this thesis are:

- To construct the Hamiltonian
- To determine the master equation
- To show c-number Langevin equations and their solutions
- To calculate the quadrature variances
- To calculate the mean and the variance of photon number
- To calculate the quantum entanglement

# 2

# REVIEW LITERATURE

Squeezed state are non-classical states characterized by a reduction of quantum fluctuations(noise) in one quadrature component below the vacuum level(standard), or below the achievable in a coherent state [20,21] at the expense of increased fluctuations in the other component such that the product of these fluctuations still obeys the uncertainty relation [8,22].

It was Takahashi [24] who, in 1965 first pointed out that a degenerate parametric amplifier enhances the noise in one quadrature component and alternates it in the other quadrature. This prediction has been confirmed by several authors for degenerate and non degenerate parametric amplifiers and oscillators. Operating below threshold, the parametric amplifiers is a source of squeezed states. In the initial experiments carried out to observe squeezing, a noise reduction of 4-17% relative to observe the standard quantum limit [25]. Squeezed light has potential applications in low-noise communication, weak signal detection and optical amplification measurements [10,11]. Over the years, several authors have analyzed the quantum properties of squeezed light generated by various quantum optical systems [1-10].

T.Yirgashewa [26] has studied squeezing and statistical properties of the light produced by a non degenerate parametric oscillator with the cavity modes driven by coherent light and coupled to a two-mode squeezed vacuum reservoir. He has found that the two-mode squeezed vacuum increases the degree of squeezing. He also found that for $\omega$=0 the noise in the plus quadrature is completely suppressed. However, the two mode driving light has no effect on the squeezing properties of the cavity modes. The two mode deriving light and the two mode squeezed vacuum increases the mean and the variance of the photon number.

M.Bekele has studied the squeezing and entanglement properties of the light produced by non-degenerate three-level laser with a strong driving coherent light and squeezed vacuum reservoir. He has found that the squeezing and entanglement of the cavity modes will be higher for large values of linear gain coefficient and the degree of entanglement is directly proportional to the degree of squeezing of the two- mode light but the degree of entanglement for the out put modes will disappear for large values of linear gain coefficient. Despite of this, the two mode squeezed vacuum reservoir considerably increases the degree of squeezing as well as entanglement in the cavity, but the deriving coherent light does not have any effect on the squeezing and entanglement. Moreover, the mean photon number increases considerably due to the driving coherent light and the squeezed vacuum reservoir.

A.Abebe has studied the squeezing and entanglement of light produced by a non degenerate three-Level laser whose cavity contains two degenerate parametric amplifiers coupled to squeezed vacuum reservoir. He has found that the light produced by the system under consideration is in squeezed and entangled state. He also found that the degree of squeezing and entanglement for the system under consideration increase with the amplitude of the parametric amplifiers and with the parameter of squeezed vacuum reservoir. Moreover, the squeezing of the cavity modes is greater than that of out put modes.

Research in quantum entanglement was initiated by a 1935 paper, by Albert Einstein, Boris Podo'sky and Nathan Rosen come to the conclusion that some quantum effects travel faster than light, which is contradiction to the theory of relativity. The presented is called the EPR paradox. In response to the EPR paradox, Irish Physicist John Stewart Bell performed a thought experiment showing that at least one of the quantum mechanics must be false [29]. Bell introduced inequalities that satisfy the assumptions of local realism and then showed that for certain quantum states they are violated. Experimental violation of Bell's inequalities was confirmed repeatedly by some quantum systems [30,31]. EPR paradox has become the basis to define the term entanglement of states, which is a type of correlation between particles that have no classical counter part [32].

S. Tesfa [33] has studied that the role of seeding thermal light in the cavity of a two-photon correlated emission laser coupled to vacuum reservoir. He has found that while designing the experimental set up, one should not worry the remnant radiation in the cavity, since its effect would be significantly nominal at large time scale due to emission absorption process. Hence this reasoning acclaims that the degree of two mode squeezing is independent of the intensity of the thermal seeded light at large time scale. In connection with these absorption mechanisms, it is natural to observe that the effect of the thermal light in the cavity is sucked from the cavity in a level that it would not affect the non classical effect of the radiation. As long as the radiation in the cavity is resonant with the radiative level of the atoms, it is not difficult to infer that the out comes remains the same irrespective of the type of the seed. In short his study indicates that the effect of the radiation in the cavity at the beginning would be efficiently washed out by the lasing process.

# 3

# METHODOLOGY

## 3.1 Description of the involved systems

### 3.1.1 Thermal light

It is a well established fact that the radiation generated by conventional sources of light can be taken as the best example of chaotic light. For instance, the light produced by a laser machine operating below threshold exhibits chaotic nature. For the sake of convenience, let us consider the thermal light in a cavity maintained at equilibrium with the walls of the same cavity at temperature T.Entrophy is defined interims of density operator as

$$s = -k_B Tr(\hat{\rho} ln \hat{\rho}), \tag{3.1}$$

where $k_B$ is the Boltzmann constant and Tr stands for trace operation and $\hat{\rho}$ is the density operator for thermal light. Using the fact that at equilibrium entropy should be maximum and $Tr(\hat{\rho}) = 1$, the density operator for the thermal(chaotic) light is related to the temperature by

$$\hat{\rho} = \frac{e^{-\hat{H}/kT}}{Tr(e^{-\hat{H}/k_B T})}, \tag{3.2}$$

where $\hat{H} = w\hat{a}^\dagger \hat{a}$ is the Hamiltonian of the radiation field when the zero point energy is shifted by $\frac{w}{2}$ and $\hbar = 1$ for convenience. The density operator can be described in the number state applying the completeness relation for number state $\hat{I} = \sum_{n=0}^{\infty} |n\rangle\langle n|$ and assuming that the number state is orthonormal $\langle n|m\rangle = \delta_{nm}$ as

$$\hat{\rho} = \sum_{n=0}^{\infty} \frac{\overline{n}^n}{(1+\overline{n})^{1+n}} |n\rangle\langle n|, \tag{3.3}$$

where $\overline{n}$ is the mean photon number for the chaotic light. Since the chaotic light is not expected to exhibit correlation, the density operator for two mode light can be treated as separable and is takes the form

$$\hat{\rho}_{ab} = \sum_{n_a, n_b}^{\infty} \frac{\overline{n}_a^{n_a} \overline{n}_b^{n_b} |n_a, n_b\rangle\langle n_a, n_b|}{(1+\overline{n}_a)^{1+n_a}(1+\overline{n}_b)^{1+n_b}}, \tag{3.4}$$

where $\overline{n}_a$ and $\overline{n}_b$ are the mean photo numbers of the respective modes. On the other hand, the number state in general can be describable in terms of the vacuum state as

$$|n\rangle = \frac{(\hat{a}^\dagger)^n}{\sqrt{n}}|0\rangle. \tag{3.5}$$

Which can also be expressed in terms of the coherent state using the power series expansion of the form $e^{\alpha\hat{a}^\dagger} = \sum_{n=0} \frac{(\alpha\hat{a}^\dagger)^n}{n!}$ as

$$|\alpha\rangle = e^{\frac{-\alpha^*\alpha}{2}} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}}|n\rangle. \tag{3.6}$$

Now applying the density operator Eq. (3.4) along with the relation with coherent state Eq. (3.6) and the definition of the mean value $\langle O\rangle = Tr(\hat{O}\hat{\rho})$ for any operator $\hat{O}$, it is possible to verify for two-mode chaotic light that

$$\langle\alpha^*\alpha\rangle = \overline{n}_a, \tag{3.7}$$

$$\langle\beta^*\beta\rangle = \overline{n}_b, \tag{3.8}$$

$$\langle\alpha\beta\rangle = \langle\alpha^2\rangle = \langle\beta^2\rangle = \langle\alpha\rangle = \langle\beta\rangle = 0. \tag{3.9}$$

It is worth noting that if it is found necessary the mean photon number of each state can be related to the frequency of the mode and temperature of the wall by

$$\overline{n}_i = \frac{1}{e^{w_i/k_B T} - 1}, \tag{3.10}$$

with i= a, b.

### 3.1.2 The interaction between the cavity mode and the ordinary vacuum reservoir

We seek to study the effect of seeding thermal light in the cavity of a non degenerate parametric oscillation coupled to vacuum reservoir via a single -port mirror. In order to study the dynamics of the cavity radiation of the combined system, it is quite necessary to find the corresponding equations of evaluation. To this end, the contribution of the initial thermal light in the cavity and the two mode vacuum reservoir to wards the master equation can be determined. In general a system coupled to a reservoir can be determined by the Hamiltonian:

$$\hat{H} = \hat{H}_S + \hat{H}_{SR}, \tag{3.11}$$

where $\hat{H}_{SR}$ describes the interaction between the cavity mode and the reservoir and $\hat{H}_S$ is the Hamiltonian of the system describing the interaction in the cavity. Suppose $\hat{X}_{SR}$ is the density operator for the system and the reservoir. Then we can write the equation of evolution of this density operator as:

$$\frac{d\hat{X}_{SR}(t)}{dt} = -i[\hat{H}_S(t) + \hat{H}_{SR}(t), \hat{X}_{SR}(t)]. \tag{3.12}$$

In order to obtain the time evolution of the system alone, which we are interested in ,we trace $\hat{X}_{SR}(t)$ over the reservoir variables, that is, the density operator for the system $\hat{\rho}(t) = Tr_R(\hat{X}_{SR}(t))$. In view of this,Eq. (3.12)takes the form

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}_S(t), \hat{\rho}(t)] - iTr_R[\hat{H}_{SR}(t), \hat{X}_{SR}(t)]. \tag{3.13}$$

Furthermore, a formal solution of Eq. (3.12)can be written as

$$\hat{X}_{SR}(t) = \hat{X}_{SR}(0) - i\int_0^t [\hat{H}_S(t') + \hat{H}_{SR}(t'), \hat{X}_{SR}(t')]dt'. \tag{3.14}$$

To proceed further, we need to introduce a certain approximation scheme. To this end, assuming here is a weak interaction between the system and reservoir, we can write approximately valid relation $\hat{X}_{SR}(t') = \hat{\rho}(t')\hat{R}$, where $\hat{R}$ is the density operator for the reservoir assumed to be constant in time. Using this approximation,Eq. (3.14)becomes

$$\hat{X}_{SR}(t) = \hat{\rho}(0)\hat{R} - i\int_0^t [\hat{H}_S(t') + \hat{H}_{SR}(t'), \hat{\rho}(t')\hat{R}]dt'. \tag{3.15}$$

Inserting this result into Eq. (3.13), we get

$$\begin{aligned}\frac{d\hat{\rho}(t)}{dt} &= -i[\hat{H}_s, \hat{\rho}(t)] - i[\langle\hat{H}_{SR}(t)\rangle_R, \hat{\rho}(0)] \\ &\quad - \int_0^t [\langle\hat{H}_{SR}(t)\rangle_R, [\hat{H}_s(t'), \hat{\rho}(t')]]dt' \\ &\quad - \int_0^t Tr_R[\hat{H}_{SR}(t), [\hat{H}_{SR}(t'), \hat{\rho}(t')\hat{R}]]dt'.\end{aligned} \tag{3.16}$$

It should be noted that $\hat{\rho}(0)$ represents the density operator at unit time and the density that describes the chaotic light in the cavity.

Furthermore, the interaction of a two mode cavity radiation with a two mode reservoir can be described in the interaction picture by the Hamiltonian

$$\begin{aligned}\hat{H}_{SR}(t) &= i\sum \lambda_u(\hat{a}^\dagger\hat{a}_u e^{i(w_a-w_u)t} - \hat{a}\hat{a}_u^\dagger e^{-i(w_a-w_u)t}) \\ &\quad + i\sum \lambda_v(\hat{b}^\dagger\hat{b}_v e^{i(w_b-w_v)t} - \hat{b}\hat{b}_v^\dagger e^{-i(w_b-w_v)t}),\end{aligned} \tag{3.17}$$

where $w_o = \frac{w_a+w_b}{2}$,with $w_a$ and $w_b$ stands for the frequencies and $(\hat{a}, \hat{b})$ are the annihilation operators for the cavity modes. In addition, $(\hat{a}_u, \hat{b}_v)$ are the annihilation operators for the reservoir modes having frequencies $w_u$ and $w_v$ respectively. $\lambda_u$ and $\lambda_v$ are coupling constants between the cavity modes and reservoir modes. Two mode vacuum reservoir operators satisfy the following properties:

$$\langle \hat{a}_u \rangle_R = \langle \hat{b}_v \rangle_R = 0. \tag{3.18}$$

$$\langle \hat{a}_u^\dagger \hat{a}_v \rangle_R = \langle \hat{b}_u^\dagger \hat{b}_v \rangle_R = 0. \tag{3.19}$$

$$\langle \hat{a}_u \hat{a}_v^\dagger \rangle_R = \langle \hat{b}_u \hat{b}_v^\dagger \rangle_R = \delta_{uv}. \tag{3.20}$$

$$\langle \hat{a}_u \hat{a}_v \rangle_R = \langle \hat{a}_u^\dagger \hat{a}_v^\dagger \rangle_R = 0. \tag{3.21}$$

$$\langle \hat{b}_u \hat{b}_v \rangle_R = \langle \hat{b}_u^\dagger \hat{b}_v^\dagger \rangle_R = 0. \tag{3.22}$$

$$\langle \hat{a}_u \hat{b}_v \rangle_R = \langle \hat{a}_u^\dagger \hat{b}_v^\dagger \rangle_R = 0. \tag{3.23}$$

$$\langle \hat{a}_u \hat{b}_v^\dagger \rangle_R = \langle \hat{a}_u^\dagger \hat{b}_v \rangle_R = 0. \tag{3.24}$$

On account of Eq. (3.18), we easily see that $\langle \hat{H}_{SR} \rangle_R = 0$. Hence in view of this result,Eq. (3.16) reduces to

$$\begin{aligned}\frac{d\rho(t)}{dt} &= -i[\hat{H}_S(t)\hat{\rho}(t)] - \int_0^t Tr_R(\hat{H}_{SR}(t)\hat{H}_{SR}(t')\hat{R}\hat{\rho}(t'))dt' \\ &- \int_0^t \hat{\rho}(t')Tr_R(\hat{R}\hat{H}_{SR}(t')\hat{H}_{SR}(t))dt' \\ &+ \int_0^t Tr_R(\hat{H}_{SR}(t)\hat{\rho}(t')\hat{R}\hat{H}_{SR}(t'))dt' \\ &+ \int_0^t Tr_R(\hat{H}_{SR}(t)\hat{R}\hat{\rho}(t')\hat{H}_{SR}(t'))dt'.\end{aligned} \tag{3.25}$$

Applying the Hamiltonian,Eq. (3.17) and Eqs. (3.19)-(3.24), we readily obtain

$$-Tr_R(\hat{H}_{SR}(t)\hat{H}_{SR}(t')\hat{R} = [p_1\hat{a}\hat{a}^\dagger + p_2\hat{b}\hat{b}^\dagger], \tag{3.26}$$

where

$$p_1 = -\sum_u \lambda_u^2 e^{-i(w_a - w_u)(t-t')}. \tag{3.27}$$

$$p_2 = -\sum_v \lambda_v^2 e^{-i(w_b - w_a)(t-t')}. \tag{3.28}$$

Assuming the frequencies of the reservoir modes to be closely spaced, the summation can be changed to integral over frequency. Then applying Markov's approximation [34] to the resulting integral we find the following result:

$$= \sum_u \lambda_u^2 e^{\pm i(w_a - w_u)(t-t')} = \kappa_j \delta(t-t'), \tag{3.29}$$

in which $\kappa_j$=$2\pi g(w_j)\lambda^2(w_j)(j = a, b)$ is defined to be the damping constant for cavity mode $j$ with $g(w_j)$ being the density operator for the reservoir modes which assumed to be at resonance with the cavity mode j. In our analysis, we assume that $\kappa_a$=$\kappa_b$=$\kappa$ for convenience. Therefore, on account of these results, Eqs. (3.27) and (3.28) take the form

$$p_1 = p_2 = -\kappa\delta(t-t'). \tag{3.30}$$

Substitution of Eq. (3.30) in to (3.26) yields

$$-iTr_R(\hat{H}_{SR}(t)\hat{H}_{SR}(t')\hat{R} = -\kappa\delta(\hat{a}\hat{a}^\dagger + \hat{b}\hat{b}^\dagger)(t-t'). \tag{3.31}$$

It then follows that

$$-\int_0^t Tr_R(\hat{H}_{SR}(t)\hat{H}_{SR}(t')\hat{R}\hat{\rho}(t')dt' = -\frac{\kappa}{2}(\hat{a}\hat{a}^\dagger\hat{\rho} + \hat{b}\hat{b}^\dagger\hat{\rho}). \tag{3.32}$$

In a similar fashion,one can easily verify that

$$-\int_0^t \hat{\rho}(t')Tr_R(\hat{R}\hat{H}_{SR}(t')\hat{H}_{SR}(t)dt' = -\frac{\kappa}{2}(\hat{\rho}\hat{a}\hat{a}^\dagger + \hat{\rho}\hat{b}\hat{b}^\dagger). \tag{3.33}$$

Furthermore, applying Eq. (3.17), we readily get

$$Tr_R(\hat{H}_{SR}(t')\hat{\rho}(t')\hat{R}\hat{H}_{SR}(t)) = -[p_1\hat{a}^\dagger\hat{\rho}(t')\hat{a} + p_2\hat{b}^\dagger\hat{\rho}(t')\hat{b}]. \tag{3.34}$$

With the aid of Eq. (3.30), we find

$$Tr_R(\hat{H}_{SR}(t')\hat{\rho}(t')\hat{R}\hat{H}_{SR}(t)) = \frac{\kappa}{2}(\hat{a}\hat{\rho}\hat{a}^\dagger + \hat{b}\hat{\rho}\hat{b}^\dagger). \tag{3.35}$$

It can also be established in a similar way that

$$Tr_R(\hat{H}_{SR}(t')\hat{R}\hat{\rho}(t')\hat{H}_{SR}(t)) = \frac{\kappa}{2}(\hat{a}\hat{\rho}\hat{a}^\dagger + \hat{b}\hat{\rho}\hat{b}^\dagger). \tag{3.36}$$

Upon substituting Eqs. (3.32),(3.33),(3.35)and (3.36) in to Eq. (3.25), we obtain the master equation for cavity modes coupled to a two-mode vacuum reservoir to be

$$\frac{d\hat{\rho}(t)}{dt} = -i[\hat{H}_S(t), \hat{\rho}(t)] + \frac{\kappa}{2}(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}) + \frac{\kappa}{2}(2\hat{b}\hat{\rho}\hat{b}^\dagger - \hat{b}^\dagger\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^\dagger\hat{b}). \tag{3.37}$$

This entails that the thermal light in the cavity does not contribute to the master equation.

### 3.1.3 The down conversion process

We seek to derive the master equation for a non-degenerate parametric oscillator coupled to vacuum reservoir via a single port mirror.

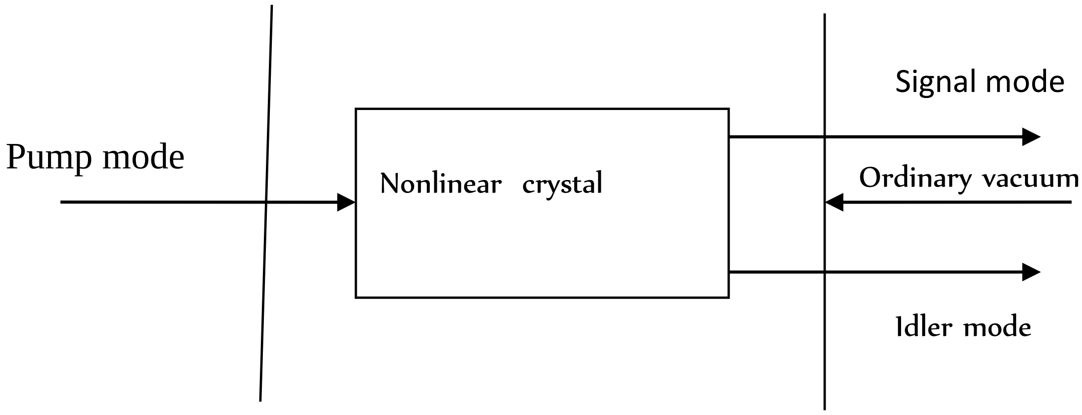


*Fig. 3.1:* A schematic representation of non degenerate parametric oscillators coupled to ordinary vacuum .

In a non degenerate parametric oscillator a pump photon of frequency $w$ is down converted into a highly correlated signal and idler photons of frequencies $w_a$ and $w_b$ such that $w = w_a + w_b$. The process of the parametric interaction can be described by the Hamiltonian

$$\hat{H} = i\lambda(\hat{a}\hat{b}\hat{c}^\dagger - \hat{a}^\dagger\hat{b}^\dagger\hat{c}), \tag{3.38}$$

where a(b) is the annihilation operator for the signal (idler)mode, $\hat{c}$ is the annihilation operator for the pump mode and $\lambda$ is the coupling constant with with the pump mode treated classically, the Hamiltonian can be written as

$$\hat{H} = i\varepsilon(\hat{a}\hat{b} - \hat{a}^\dagger\hat{b}^\dagger), \tag{3.39}$$

where $\varepsilon$, considered to be real and constant, is proportional to the the amplitude of the pump mode.

With the aid of Eqs. (3.37) and (3.39), one can write

$$\begin{aligned}\frac{d\hat{\rho}}{dt} &= \varepsilon(\hat{a}\hat{b}\hat{\rho} - \hat{\rho}\hat{a}\hat{b} + \hat{\rho}\hat{a}^{\dagger}\hat{b}^{\dagger} - \hat{a}^{\dagger}\hat{b}^{\dagger}\hat{\rho}) + \frac{k}{2}(2\hat{a}\hat{\rho}\hat{a}^{\dagger} - \hat{a}^{\dagger}\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^{\dagger}\hat{a}) \\ &+ \frac{k}{2}(2\hat{b}\hat{\rho}\hat{b}^{\dagger} - \hat{b}^{\dagger}\hat{b}\hat{\rho} - \hat{\rho}\hat{b}^{\dagger}\hat{b}).\end{aligned} \tag{3.40}$$

Therefore, Eq. (3.40) represents the master equation of a non-degenerate parametric oscillator coupled to vacuum reservoir. In which the cavity damping constant $k$ is assumed to be the same for both the signal and idler modes.

### 3.1.4 C-number Langevien equation

The dynamics of a cavity mode coupled to the reservoir can be also described using the c-number Langevin equation. In this section we seek to obtain the c-number Langevin equation of Eq. (3.40) to determine the equations of the evaluation for the expectation values of normally ordered cavity modes operators. The time evolution of the expectation value of an operator $\hat{A}$ in the Schrodinger picture can be expressed as [1]

$$\frac{d}{dt}\langle\hat{A}\rangle = Tr(\frac{d\hat{\rho}}{dt}\hat{A}). \tag{3.41}$$

Now taking into account Eq. (3.40) along with Eq. (3.41),one can write

$$\begin{aligned}\frac{d}{dt}\langle\hat{a}\rangle &= \varepsilon Tr(\hat{a}\hat{b}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}\hat{b}\hat{a} + \hat{\rho}\hat{a}^{\dagger}\hat{b}^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{b}^{\dagger}\hat{\rho}\hat{a}) \\ &+ \frac{k}{2}(Tr2\hat{a}\hat{\rho}\hat{a}^{\dagger}\hat{a} - \hat{a}^{\dagger}\hat{a}\hat{\rho}\hat{a} - \hat{\rho}\hat{a}^{\dagger}\hat{a}\hat{a}) \\ &+ \frac{k}{2}(Tr2\hat{b}\hat{\rho}\hat{b}^{\dagger}\hat{a} - \hat{b}^{\dagger}\hat{b}\hat{\rho}\hat{a} - \hat{\rho}\hat{b}^{\dagger}\hat{b}\hat{a}).\end{aligned} \tag{3.42}$$

Applying the cyclic property of the trace operation together with the commutation relations

$$[\hat{a}, \hat{a}^{\dagger}] = 1, \tag{3.43}$$

$$[\hat{a}, \hat{b}] = [\hat{a}^{\dagger}, \hat{b}^{\dagger}] = [\hat{a}, \hat{b}^{\dagger}] = 0, \tag{3.44}$$

and

$$[\hat{a}^{\dagger}, \hat{a}^{2}] = -2\hat{a}, \tag{3.45}$$

we readily find

$$\frac{d}{dt}\langle\hat{a}(t)\rangle = -\frac{k}{2}\langle\hat{a}(t)\rangle - \varepsilon\langle\hat{b}^{\dagger}(t)\rangle. \tag{3.46}$$

It can also be shown in a similar manner that

$$\frac{d}{dt}\langle\hat{b}(t)\rangle = -\frac{k}{2}\langle\hat{b}(t)\rangle - \varepsilon\langle\hat{a}^{\dagger}(t)\rangle, \tag{3.47}$$

$$\frac{d}{dt}\langle\hat{a}(t)\hat{b}(t)\rangle = -k\langle\hat{a}(t)\hat{b}(t)\rangle - \varepsilon\langle\hat{a}^{\dagger}(t)\hat{a}(t)\rangle - \varepsilon\langle\hat{b}^{\dagger}(t)\hat{b}(t)\rangle - \varepsilon, \tag{3.48}$$

$$\frac{d}{dt}\langle\hat{a}(t)\hat{a}(t)\rangle = -2\varepsilon\langle\hat{a}(t)\hat{b}^{\dagger}(t)\rangle - k\langle\hat{a}(t)\hat{a}(t)\rangle, \tag{3.49}$$

$$\frac{d}{dt}\langle\hat{a}^{\dagger}(t)\hat{a}(t)\rangle = -k\langle\hat{a}^{\dagger}(t)\hat{a}(t)\rangle - \varepsilon(\langle\hat{a}(t)\hat{b}(t)\rangle + \langle\hat{a}^{\dagger}(t)\hat{b}^{\dagger}(t)\rangle). \tag{3.50}$$

The c- number equations corresponding to Eqs. $(3.46)$,$(3.47)$,$(3.48)$,$(3.49)$ and $(3.50)$ became

$$\frac{d}{dt}\langle\alpha(t)\rangle = -\frac{k}{2}\langle\alpha(t)\rangle - \varepsilon\langle\beta^{*}(t)\rangle, \tag{3.51}$$

$$\frac{d}{dt}\langle\beta(t)\rangle = -\frac{k}{2}\langle\beta(t)\rangle - \varepsilon\langle\alpha^{*}(t)\rangle, \tag{3.52}$$

$$\frac{d}{dt}\langle\alpha(t)\beta(t)\rangle = -k\langle\alpha(t)\beta(t)\rangle - \varepsilon\langle\alpha^{*}(t)\alpha(t)\rangle - \varepsilon\langle\beta^{*}(t)\beta(t)\rangle - \varepsilon, \tag{3.53}$$

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = -2\varepsilon\langle\alpha(t)\beta^{*}(t)\rangle - k\langle\alpha^{2}(t)\rangle, \tag{3.54}$$

$$\frac{d}{dt}\langle\alpha^{*}(t)\alpha(t)\rangle = -k\langle\alpha^{*}(t)\alpha(t)\rangle - \varepsilon(\langle\alpha(t)\beta(t)\rangle + \langle\alpha^{*}(t)\beta^{*}(t)\rangle). \tag{3.55}$$

On the basis of Eqs. $(3.51)$ and $(3.52)$,one can write

$$\frac{d}{dt}\alpha(t) = -\frac{k}{2}\alpha(t) - \varepsilon\beta^{*}(t) + f_{\alpha}(t) \tag{3.56}$$

and

$$\frac{d}{dt}\beta(t) = -\frac{k}{2}\beta(t) - \varepsilon\alpha^{*}(t) + f_{\beta}(t), \tag{3.57}$$

where $f_{\alpha}(t)$ and $f_{\beta}(t)$ are noise forces. We observe that Eq. $(3.51)$ and the expectation value of Eq. $(3.56)$ as well as Eq. $(3.52)$ and the expectation value of Eq. $(3.57)$ will have the same form if

$$\langle f_{\alpha}(t)\rangle = \langle f_{\beta}(t)\rangle = 0. \tag{3.58}$$

Using Eqs. $(3.56)$ and $(3.57)$ together with the relation

$$\frac{d}{dt}\langle\alpha(t)\beta(t)\rangle = \langle\alpha(t)\beta(t)\rangle + \langle\alpha(t)\beta(t)\rangle, \tag{3.59}$$

one can write

$$\frac{d}{dt}\langle\alpha(t)\beta(t)\rangle = -k\langle\alpha(t)\beta(t)\rangle - \varepsilon\langle\beta^*(t)\beta(t)\rangle - \varepsilon\langle\alpha^*(t)\alpha(t)\rangle \\ + \langle\beta(t)f_\alpha(t)\rangle + \langle\alpha(t)f_\beta(t)\rangle. \tag{3.60}$$

Comparison of Eqs. (3.53) and (3.60) indicates that

$$\langle\alpha(t)f_\beta(t)\rangle + \langle\beta(t)f_\alpha(t)\rangle = -\varepsilon. \tag{3.61}$$

The formal solution of Eqs .(3.51) and (3.52) can be written as

$$\alpha(t) = \alpha(0)e^{\frac{-(kt)}{2}} + \int_0^t e^{-k\frac{t-t'}{2}}[f_\alpha(t') - \varepsilon\beta^*(t')]dt', \tag{3.62}$$

$$\beta(t) = \beta(0)e^{\frac{-(kt)}{2}} + \int_0^t e^{-k\frac{t-t'}{2}}[f_\beta(t') - \varepsilon\alpha^*(t')]dt'. \tag{3.63}$$

Assuming the noise force $f_\beta(t)$ at time t can not affect the system variables at earlier times,we see that

$$\langle\alpha(0)f_\beta(t)\rangle = \langle\alpha(0)\rangle\langle f_\beta(t)\rangle = 0, \tag{3.64}$$

$$\langle\beta(t')^* f_\beta(t)\rangle = \langle\beta(t')^*\rangle\langle f_\beta(t)\rangle = 0. \tag{3.65}$$

Thus on account of Eqs. (3.58),(3.62),(3.64) and (3.65),we obtain

$$\langle\alpha(t)f_\beta(t)\rangle = \int_0^t e^{-k\frac{t-t'}{2}}\langle f_\beta(t)f_\alpha(t')\rangle dt'. \tag{3.66}$$

It can also be verified in a similar fashion that

$$\langle\beta(t)f_\alpha(t)\rangle = \int_0^t e^{-k\frac{t-t'}{2}}\langle f_\alpha(t)f_\beta(t')\rangle dt'. \tag{3.67}$$

Now in view of Eqs. (3.61),(3.51) and (3.52) and the assumption that

$$\langle\alpha(t)f_\beta(t')\rangle = \langle f_\beta(t)f_\alpha(t')\rangle, \tag{3.68}$$

we have

$$\int_0^t e^{-k\frac{t-t'}{2}}\langle f_\beta(t)f_\alpha(t')\rangle dt' = \int_0^t e^{-k\frac{t-t'}{2}}\langle f_\alpha(t)f_\beta(t')\rangle dt' = \frac{-\varepsilon}{2}. \tag{3.69}$$

Now on account of [1]

$$\int_0^t e^{-a\frac{t-t'}{2}}\langle f(t)g(t')\rangle dt' = D, \tag{3.70}$$

we assert that

$$\langle f(t)g(t')\rangle = 2D\delta(t-t'), \tag{3.71}$$

where a is a constant and D is a constant or some function of the time t. We then see that

$$\langle f_\alpha(t) f_\beta(t')\rangle = \langle f_\beta(t) f_\alpha(t')\rangle = -\varepsilon\delta(t-t'). \tag{3.72}$$

Furthermore Eq. $(3.56)$ along with the relation

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = 2\langle\alpha(t)\alpha(t)\rangle. \tag{3.73}$$

$$\frac{d}{dt}\langle\alpha(t)\alpha(t)\rangle = -k\langle\alpha^2(t)\rangle - 2\varepsilon\langle\alpha(t)\beta^*(t)\rangle + 2\langle\alpha(t) f_\alpha(t)\rangle. \tag{3.74}$$

Comparison of Eqs. $(3.54)$ and $(3.74)$ shows that

$$\langle\alpha(t) f_\alpha(t)\rangle = 0. \tag{3.75}$$

Applying Eq. $(3.62)$ together with $(3.75)$, we find

$$\int_0^t e^{-k\frac{t-t'}{2}}\langle f_\alpha(t) f_\alpha(t')\rangle dt'. \tag{3.76}$$

From which follows

$$\langle f_\alpha(t) f_\alpha(t')\rangle = 0. \tag{3.77}$$

It can also be established in a similar manner that

$$\langle f_\beta(t) f_\beta(t')\rangle = \langle f^*_\alpha(t) f_\beta(t')\rangle = 0. \tag{3.78}$$

Moreover employing Eq. $(3.56)$ and its complex conjugate along with the relation

$$\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = \langle\alpha^*(t)\alpha(t)\rangle + \langle\alpha^*(t)\alpha(t)\rangle, \tag{3.79}$$

we find

$$\begin{aligned}\frac{d}{dt}\langle\alpha^*(t)\alpha(t)\rangle = -k\langle\alpha^*(t)\alpha(t)\rangle - \varepsilon(\langle\beta(t)\alpha(t)\rangle + \langle\alpha^*(t)\beta^*(t)\rangle)\\ + \langle\alpha(t) f^*_\alpha(t)\rangle + \langle\alpha^*(t) f_\alpha(t)\rangle.\end{aligned} \tag{3.80}$$

Comparison of Eqs. (3.55) and (3.80) indicates that

$$\langle\alpha(t) f^*_\alpha(t)\rangle + \langle\alpha^*(t) f_\alpha(t)\rangle = 0. \tag{3.81}$$

With the aid of Eq. (3.62) and its complex conjugate, one can write

$$\langle\alpha(t) f^*_\alpha(t)\rangle = \int_o^t e^{-k(t-t')/2}\langle f^*_\alpha(t) f_\alpha(t')\rangle dt', \tag{3.82}$$

$$\langle\alpha^*(t) f_\alpha(t)\rangle = \int_o^t e^{-k(t-t')/2}\langle f_\alpha(t) f^*_\alpha(t')\rangle dt'. \tag{3.83}$$

In view of Eqs. (3.81),(3.82) and (3.83) and the assumption that

$$\langle f_\alpha^*(t) f_\alpha(t')\rangle = \langle f_\alpha(t) f_\alpha^*(t')\rangle, \tag{3.84}$$

we get

$$\int_0^t e^{-k(t-t')/2}\langle f_\alpha^*(t) f_\alpha(t')\rangle dt' = \int_0^t e^{-k(t-t')/2}\langle f_\alpha(t) f_\alpha^*(t')\rangle dt' = 0 \tag{3.85}$$

Hence on account of Eqs. (3.70) and (3.71),we note that

$$\langle f_\alpha^*(t) f_\alpha(t')\rangle = \langle f_\alpha(t) f_\alpha^*(t')\rangle = 0. \tag{3.86}$$

It can also be established in a similar procedure that

$$\langle f_\beta^*(t) f_\beta(t')\rangle = \langle f_\beta(t) f_\beta^*(t')\rangle = 0. \tag{3.87}$$

Eqs. (3.58),(3.72),(3.77),(3.78),(3.86) and (3.87) describe the correlation properties of the noise forces $f_\alpha(t)$ and $f_\beta(t)$ associated with the normal ordering.

In order to obtain the solutions of Eqs. (3.56) and (3.57), we introduce a new variable defined by

$$z_\pm = \alpha(t) \pm \beta^*(t). \tag{3.88}$$

Applying Eq. (3.56) along with the complex conjugate of Eq. (3.57) we obtain,

$$\frac{dz_\pm}{dt} = -\frac{1}{2}\lambda_\pm z_\pm + f_\alpha \pm f_\beta^*, \tag{3.89}$$

where

$$\lambda_\pm = k \pm 2\varepsilon. \tag{3.90}$$

According to Eq. (3.89) together with (3.90),the equation of evolution for $z_-(t)$ has no defined solution for $k < 2\varepsilon$. We then identify $k = 2\varepsilon$ as the threshold condition. Thus for $2\varepsilon < k$ the solution of Eq. (3.68) can be put in the form

$$z_\pm(t) = z_\pm(0)e^{(-\lambda\pm t)/2} + \int_0^t e^{-\lambda\pm(t-t')/2}(f_\alpha(t') \pm f_\beta^*(t'))dt'. \tag{3.91}$$

It then follows that

$$\alpha(t) = E_+(t)\alpha(0) + E_-(t)\beta^*(0) + F_+(t) + F_-(t), \tag{3.92}$$

$$\beta(t) = E_+(t)\beta(0) + E_-(t)\alpha^*(0) + F_+^*(t) - F_-^*(t), \tag{3.93}$$

where

$$E_{\pm}(t) = \frac{1}{2}(e^{-\lambda_{+}t/2} \pm e^{-\lambda_{-}t/2}), \tag{3.94}$$

$$F_{\pm}(t) = \frac{1}{2}\int_{0}^{t} e^{-\lambda\pm(t-t')/2}[f_{\alpha}(t') \pm f^{*}_{\beta}(t)]dt'. \tag{3.95}$$

It is worth noting that $\alpha(0)$ and $\beta(0)$ along with their corresponding complex conjugate represents the thermal light. It is clearly shown in Eqs. (3.92)and (3.93) that the thermal light entered in to the evolution of the system as initial condition as anticipated. But if the source of the chaotic light remains in the cavity through out the lasing operation, the effect of the thermal light should also be included in the correlations of the noise.

Employing the resulting solution of the c-number Langevin equation along with the correlation relations of the noise forces, we calculate the quadrature variance of cavity modes, mean photon number, variance of photon number and entanglement.

# 4

# RESULT AND DISCUSSION

## 4.1 Quadrature fluctuations

We next proceed to determine the quadrature variances for the cavity radiation produced by a non degenerate parametric oscillator coupled to a vacuum reservoir.

### 4.1.1 Quadrature variance of cavity mode

We seek to describe the two light modes represented by $\hat{a}$ and $\hat{b}$ by the operator

$$\hat{c} = \frac{1}{\sqrt{2}}(\hat{a} + \hat{b}). \tag{4.1}$$

Particularly, the squeezing properties of the cavity radiation can be studied applying the corresponding quadrature operators defined by

$$c_+ = \hat{c}^\dagger + \hat{c}, \tag{4.2}$$

and

$$c_- = i(\hat{c}^\dagger - \hat{c}). \tag{4.3}$$

Using the commutation relation

$$[\hat{a},\ \hat{a}^\dagger] = [\hat{b},\ \hat{b}^\dagger] = 1, \tag{4.4}$$

it can easily be shown that

$$[\hat{c}, \hat{c}^\dagger] = 1. \tag{4.5}$$

With the aid of Eqs. (4.2),(4.3) and (4.5), one can readily verify that

$$[c_+, c_-] = 2i. \tag{4.6}$$

A two-mode light is said to be in a squeezed state if either $\triangle c_{+} < 1$ and $\triangle c_{-} > 1$ or $\triangle c_{+} > 1$ and $\triangle c_{-} < 1$ such that $\triangle c_{+} \triangle c_{-} \geq 1$. The variance of the quadrature operators are defined by

$$\triangle \hat{c_{\pm}}^{2}(t) = \langle \hat{c_{\pm}}^{2}(t) \rangle - \langle \hat{c_{\pm}}(t) \rangle^{2}. \tag{4.7}$$

Applying Eqs. (4.2),(4.3) and (4.7) in the normal order as

$$\begin{aligned} \triangle \hat{c_{\pm}}^{2}(t) = 1 \pm \langle c^{+2}(t) + c^{2}(t) \pm 2c^{+}(t)c(t) \rangle \\ \mp \langle c^{+}(t) \pm c(t) \rangle^{2}. \end{aligned} \tag{4.8}$$

This can be expressed in terms of c-number variables associated with the normal ordering as

$$\begin{aligned} \triangle \hat{c_{\pm}}^{2}(t) = 1 \pm \langle \gamma^{*2}(t) + \gamma^{2}(t) \pm 2\gamma^{*}(t)\gamma(t) \rangle \\ \mp \langle \gamma^{*}(t) \pm \gamma(t) \rangle^{2}, \end{aligned} \tag{4.9}$$

where $\gamma(t)$ is the c-number variables corresponding to the operator $\hat{c}(t)$. Introducing a new variable defined by

$$\gamma_{\pm}(t) = \gamma^{*}(t) \pm \gamma(t). \tag{4.10}$$

Eq. (4.9)can be rewritten as

$$\triangle \hat{c_{\pm}}^{2}(t) = 1 \pm (\langle \gamma_{\pm}^{2}(t) \rangle - \langle \gamma_{\pm}(t) \rangle^{2}) \tag{4.11}$$

or

$$\triangle \hat{c_{\pm}}^{2}(t) = 1 \pm \langle \gamma_{\pm}(t), \gamma_{\pm}(t) \rangle. \tag{4.12}$$

The c-number variable equation corresponding to Eq. (4.1) is expressible as

$$\gamma(t) = \frac{1}{\sqrt{2}}(\alpha(t) + \beta(t)). \tag{4.13}$$

We note that

$$\frac{d}{dt}\gamma(t) = \frac{1}{\sqrt{2}}(\frac{d}{dt}\alpha(t) + \frac{d}{dt}\beta(t)). \tag{4.14}$$

On account of Eqs. (4.41)and (4.42), we obtain

$$\frac{d}{dt}\gamma(t) = \frac{-k}{2}\gamma(t) - \varepsilon\gamma^{*}(t) + \frac{1}{\sqrt{2}}(F_{\alpha}(t) + F_{\beta}(t)). \tag{4.15}$$

With the help of Eq. (4.10), one can easily gets

$$\frac{d}{dt}\gamma_{\pm}(t) = \frac{d}{dt}\gamma^{*}(t) \pm \frac{d}{dt}\gamma(t). \tag{4.16}$$

Employing Eq. (4.15) and its complex conjugate along with Eq. (4.16), we find,

$$\begin{aligned} \frac{d}{dt}\gamma_{\pm}(t) = -\frac{1}{2}\lambda_{\pm}\gamma_{\pm} + \frac{1}{\sqrt{2}}[(F^{*}_{\alpha}(t) \\ \pm F_{\alpha}(t)) + (F^{*}_{\beta}(t) \pm F_{\beta}(t))], \end{aligned} \tag{4.17}$$

where

$$\lambda_{\pm} = k \pm 2\varepsilon. \tag{4.18}$$

The equation of evolution for $\gamma_{-}(t)$ has no solution for $k < 2\varepsilon$. Then $k = 2\varepsilon$ is identified as the threshold condition. We thus realize that our analysis will be confined to a non degenerate parametric oscillator operating below threshold. The solution of Eq. (4.17) for $2\varepsilon < k$ can be written as

$$\begin{aligned} \gamma_{\pm}(t) = \gamma_{\pm}(0)e^{-(\lambda\pm t)/2} + \int_{0}^{t} e^{-\lambda\pm(t-t')/2} \\ \frac{1}{\sqrt{2}}[(F^{*}_{\alpha}(t') \pm F_{\alpha}(t')) + (F^{*}_{\beta}(t') \pm F_{\beta}(t'))]dt'. \end{aligned} \tag{4.19}$$

Now in view of Eq. (3.58),we obtain

$$\langle\gamma_{\pm}(t)\rangle = \langle\gamma_{\pm}(0)\rangle e^{-\lambda\pm t/2}. \tag{4.20}$$

Furthermore, applying the relation

$$\frac{d}{dt}(\gamma^{2}_{\pm}(t)) = 2\gamma_{\pm}(t)\frac{d}{dt}\gamma_{\pm}(t), \tag{4.21}$$

together with Eq. (4.17) we find

$$\begin{aligned} \frac{d}{dt}(\gamma^{2}_{\pm}(t)) = -\lambda\gamma^{2}_{\pm}(t) + \sqrt{2}\gamma_{\pm}(t)F^{*}_{\alpha}(t) \\ \pm \sqrt{2}\gamma_{\pm}(t)F_{\alpha}(t) + \sqrt{2}\gamma_{\pm}(t)F^{*}_{\beta}(t) \\ \pm \sqrt{2}\gamma_{\pm}(t)F_{\beta}(t), \end{aligned} \tag{4.22}$$

so that multiplying Eq.(4.19)by $F^{*}(t)$, we get

$$\begin{aligned} \langle\gamma_{\pm}(t)F^{*}(t)\rangle = \langle\gamma_{\pm}(0)F^{*}(t)\rangle e^{-\lambda\pm t/2} \\ + \frac{1}{\sqrt{2}}\int_{0}^{t} e^{-\lambda\pm(t-t')/2}\langle F^{*}_{\alpha}(t')F^{*}(t) \pm F_{\alpha}(t')F^{*}(t)\rangle dt' \\ + \frac{1}{\sqrt{2}}\int_{0}^{t} e^{-\lambda\pm(t-t')/2}\langle F^{*}_{\beta}(t')F^{*}(t) \pm F_{\beta}(t')F^{*}(t)\rangle dt'. \end{aligned} \tag{4.23}$$

Taking into account Eqs. (3.43),(3.57),(3.62)and(3.63) and the fact that a noise force at time t can not affect the system variables at the earlier times. We readily obtain

$$\langle\gamma_{\pm}(t)F^{*}(t)\rangle = \frac{-\varepsilon}{\sqrt{2}}\int_{0}^{t} e^{-\lambda\pm(t-t')/2}\delta(t-t')dt'. \tag{4.24}$$

Now on the basis of the relation

$$D\int_0^t e^{-a(t-t')/2}\delta(t-t')dt' = \tfrac{D}{2},$$

we arrive at

$$\langle\gamma_{\pm}(t)F^{*}(t)\rangle = \frac{-1}{2\sqrt{2}}(\varepsilon). \tag{4.25}$$

Multiplying Eq. (4.19), by $F_{\alpha}(t)$

$$\langle\gamma_{\pm}(t)F_{\alpha}(t)\rangle = \mp\frac{1}{2\sqrt{2}}(\varepsilon), \tag{4.26}$$

$$\langle\gamma_{\pm}(t)F^{*}_{\beta}(t)\rangle = -\frac{1}{2\sqrt{2}}(\varepsilon), \tag{4.27}$$

and

$$\langle\gamma_{\pm}(t)F_{\beta}(t)\rangle = \mp\frac{1}{2\sqrt{2}}(\varepsilon). \tag{4.28}$$

On account of Eqs. (4.25),(4.26),(4.27)and(4.28) one can write Eq. (4.22)as

$$\frac{d}{dt}\langle\gamma^2_{\pm}(t)\rangle = -\lambda\gamma^2_{\pm}(t) - 2\varepsilon. \tag{4.29}$$

Up on substituting Eqs. (4.20)in to (4.29), we find

$$\frac{d}{dt}\langle\gamma^2_{\pm}(t)\rangle = -\lambda\langle\gamma^2_{\pm}(t)\rangle - 2\varepsilon. \tag{4.30}$$

The solution of Eq. (4.30) can be written as

$$\langle\gamma^2_{\pm}(t)\rangle = \langle\gamma_{\pm}(0)\rangle e^{-\lambda\pm t} + (-2\varepsilon)\int_0^t e^{-\lambda\pm(t-t')/2}dt'. \tag{4.31}$$

carrying out the integration, we obtain

$$\langle\gamma^2_{\pm}(t)\rangle = \langle\gamma^2_{\pm}(0)\rangle e^{-\lambda\pm t} - \frac{2\varepsilon}{\lambda\pm}(1-e^{-\lambda\pm t}). \tag{4.32}$$

On the other hand, using Eq. (4.20) one can write

$$\langle\gamma_{\pm}(t)\rangle^2 = \langle\gamma_{\pm}(0)\rangle^2 e^{-\lambda\pm t}. \tag{4.33}$$

Hence substitution of Eqs. (4.32)and (4.33) in to (4.11)results in

$$\triangle c^2_{+}(t) = 1 + \langle\triangle c^2_{+}(0)\rangle e^{-\lambda_{+}t} - \frac{2\varepsilon}{\lambda_{+}}(1-e^{-\lambda_{+}t}), \tag{4.34}$$

$$\triangle c^2_{-}(t) = 1 - \langle\triangle c^2_{-}(0)\rangle e^{-\lambda_{-}t} + \frac{2\varepsilon}{\lambda_{-}}(1-e^{-\lambda_{-}t}), \tag{4.35}$$

where

$$\triangle c^2_{\pm}(0) = 1 + (\langle\gamma^2_{\pm}(0)\rangle - \langle\gamma_{\pm}(0)\rangle^2). \tag{4.36}$$

Taking cavity mode to be initially in a chaotic state, we find

$$\triangle c_+^2(0) = \overline{n}_a + \overline{n}_b + 1, \tag{4.37}$$

up on substituting Eqs. (4.37) in to (4.34) results in

$$\triangle c_+^2(t) = 1 + (\overline{n}_a + \overline{n}_b + 1)e^{-\lambda_+ t} - \frac{2\varepsilon}{\lambda_+}(1 - e^{-\lambda_+ t}). \tag{4.38}$$

Following the same procedure

$$\triangle c_-^2(0) = \overline{n}_a + \overline{n}_b + 1, \tag{4.39}$$

$$\triangle c_-^2(t) = 1 + (\overline{n}_a + \overline{n}_b + 1)e^{-\lambda_- t} + \frac{2\varepsilon}{\lambda_-}(1 - e^{-\lambda_- t}). \tag{4.40}$$

At steady state we have

$$(\triangle c_+^2)_{ss} = 1 - \frac{2\varepsilon}{\lambda_+}, \tag{4.41}$$

$$(\triangle c_-^2)_{ss} = 1 + \frac{2\varepsilon}{\lambda_-}, \tag{4.42}$$

where ss stands for steady state. We easily notice that the squeezing occurs in the plus quadrature. It is also clear that the best squeezing in the non degenerate parametric oscillator is achieved at threshold $(k = 2\varepsilon)$

$$(\triangle c_+^2)_{ss} = \frac{1}{2}, \tag{4.43}$$

$$(\triangle c_-^2)_{ss} = \rightarrow \infty. \tag{4.44}$$

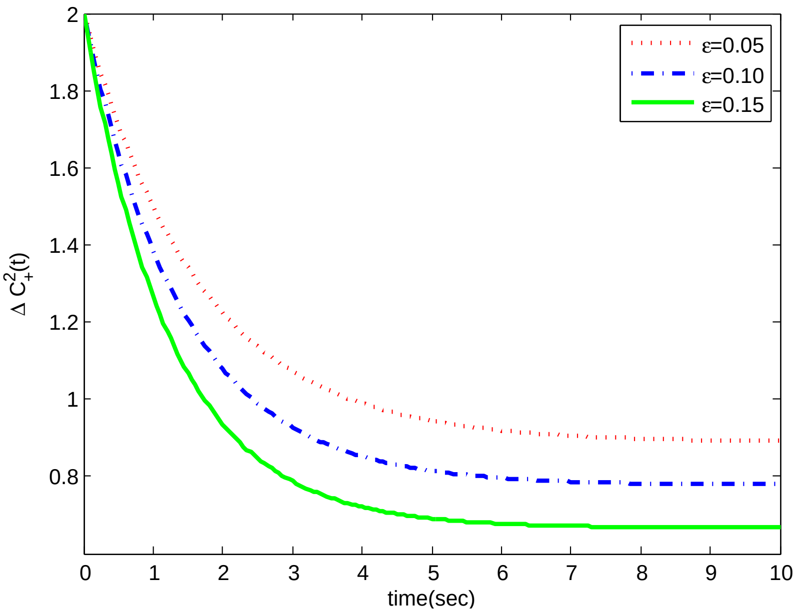


*Fig. 4.1:* Plots of the plus quadrature variance$(\Delta C_+^2)$of cavity radiation versus time for $\kappa = 0.5$,$\overline{n}_a = \overline{n}_b = 0$ and different values of $\epsilon$ ,for $\epsilon = 0.15$(solid curve), $\epsilon = 0.10$ (dashed curve) and $\epsilon = 0.05$ (doted curve).

The plots in Fig.4.1 indicate that as the intensity of the pump mode increases the quadrature variance is decreases. This implies that the intensity of a pump mode increases the degree of squeezing.

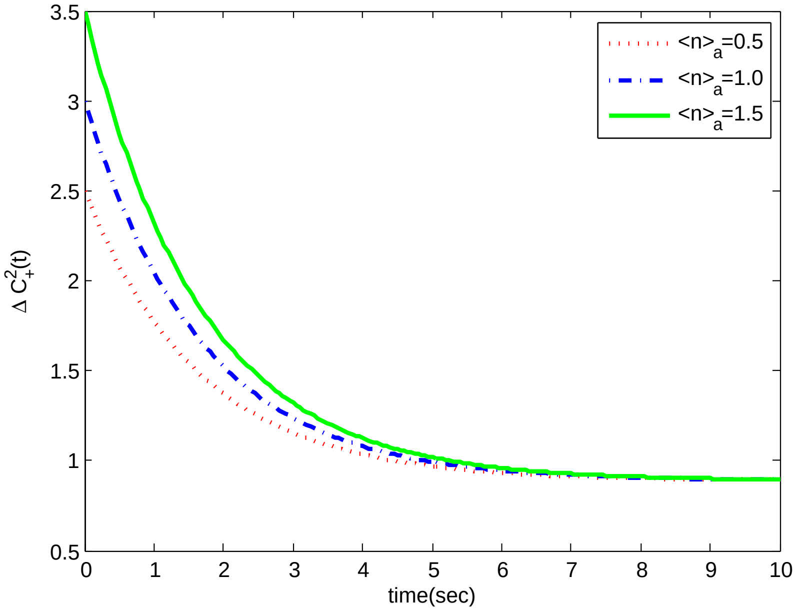


*Fig. 4.2:* Plots of the plus quadrature variance($\Delta C_{+}^{2}$)of cavity radiation versus time for $\kappa = 0.5$,$\overline{n}_b = 0$,$\epsilon = 0.05$ and different values of $\overline{n}_a$ ,for $\overline{n}_a = 1.5$(solid curve), $\overline{n}_a = 1.0$ (dashed curve) and $\overline{n}_a = 0.5$ (doted curve).

If one critically observe Fig.4.2, one may see that the degree of two-mode squeezing is almost the same after t=6. This entails that the effect of the seed is relatively short lived.

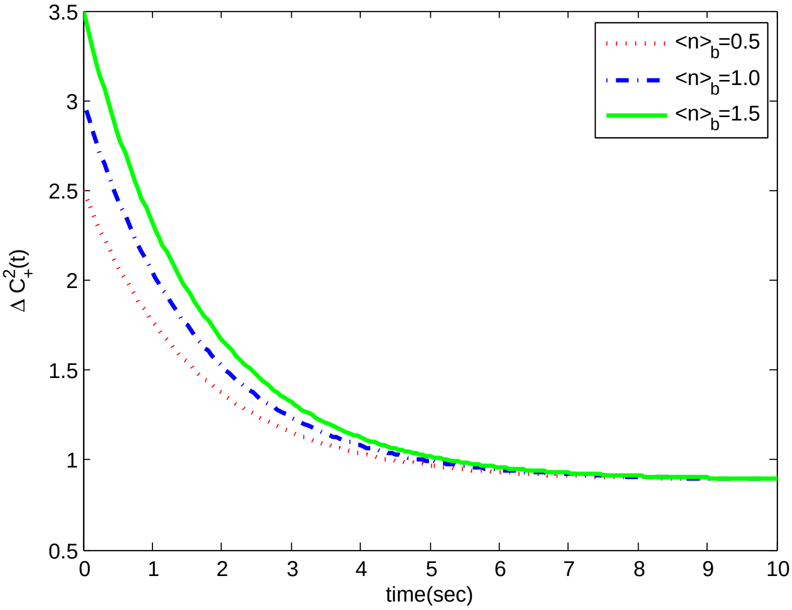


*Fig. 4.3:* Plots of the plus quadrature variance($\Delta C_+^2$)of cavity radiation versus time for $\kappa = 0.5$,$\overline{n}_a = 0$,$\epsilon = 0.05$ and different values of $\overline{n}_b$ ,for $\overline{n}_b = 1.5$(solid curve), $\overline{n}_b = 1.0$ (dashed curve) and $\overline{n}_b = 0.5$ (doted curve).

If one critically observe Fig.4.3, one may see that the degree of two-mode squeezing is almost the same after t=6. This entails that the effect of the seed is relatively short lived.

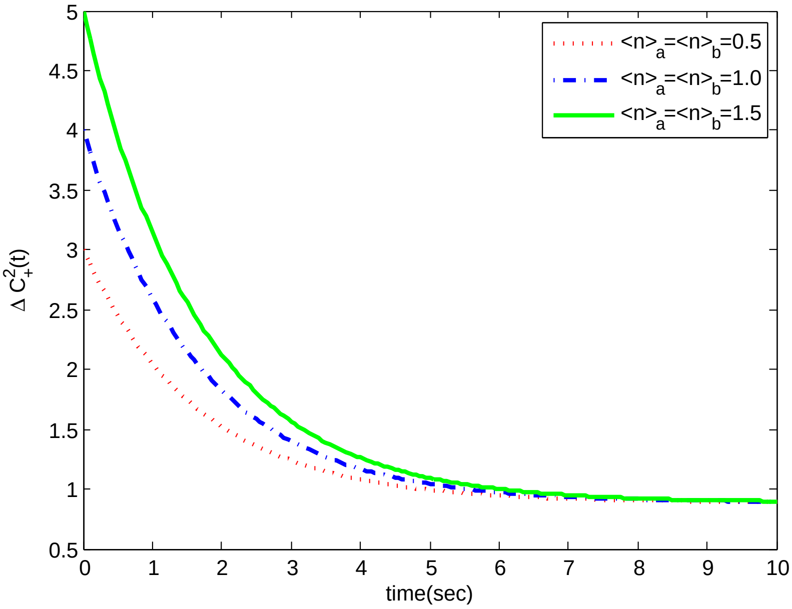


*Fig. 4.4:* Plots of the plus quadrature variance($\Delta C_{+}^{2}$)of cavity radiation versus time for $\kappa = 0.5$,$\epsilon = 0.05$ and different values of $\overline{n}_a = \overline{n}_b$ ,for $\overline{n}_a = \overline{n}_b = 1.5$(solid curve), $\overline{n}_a = \overline{n}_b = 1.0$ (dashed curve) and $\overline{n}_a = \overline{n}_b = 0.5$ (doted curve).

It is clearly presented in Figs.4.2 and 4.3 that the degree of two-mode squeezing is significantly impaired by chaotic light. The more intense the seeding chaotic light the more the squeezing is damaged. If one critically observes Figs.4.2,4.3 and 4.4 ,one may see that the degree of two-mode squeezing is almost the same for all cases after t=6. This entails that the effect of the seed is relatively short lived.

It is not difficult to realize that from Figs.4.2,4.3 and 4.4 and Eq.(4.41) the degree of squeezing at large time scale is independent of the mean photon number of the chaotic light. This is mainly because as time progress there are more photons that pass through the cavity in a way absorb the available radiation. Since the number of chaotic light in the radiation can not increases, the absorption mechanism can diminished it in time. At this juncture, it would be appropriate noting that the photons are continuously seeded with constant rate and left the cavity. In connection to this what most probably happen is a photon after absorbing a chaotic light emits a light which is not necessarily has the same nature as it absorbs. Since this process successively repeated, the chaotic light in the cavity is converted to a correlated light or lost all the way which leads to diminishing effect of the seed on the properties of the cavity radiation.

## 4.2 Photon statistics

### 4.2.1 Mean photon number

For the system under consideration the number of photons in the cavity can be represented by the number operator.

$$\hat{n} = \hat{n}_a + \hat{n}_b. \tag{4.45}$$

where $\hat{n}_a = \hat{a}^\dagger \hat{a}$ and $\hat{n}_b = \hat{b}^\dagger \hat{b}$ are the photon number operators for mode a and b. The mean photon number for the cavity mode can also be expressed in terms of c-number variables associated with the normal ordering as

$$\langle \hat{n} \rangle = \langle \alpha^*(t)\alpha(t) \rangle + \langle \beta^*(t)\beta(t) \rangle. \tag{4.46}$$

Thus on account of Eqs. (3.92) and (3.93), and assuming the cavity modes to be initially in a two mode thermal light, one can readily verify that

$$\langle \alpha(t) \rangle = \langle \beta(t) \rangle = 0. \tag{4.47}$$

Taking Eq. (3.92) in to account one can write

$$\begin{aligned} \langle \alpha^*(t)\alpha(t) \rangle &= E_+(t)\langle \alpha^*(t)\alpha(0) \rangle + E_-(t)\langle \alpha^*(t)\beta^*(0) \rangle \\ &+ \langle \alpha^*(t)F_+(t) \rangle + \langle \alpha^*(t)F_-(t) \rangle. \end{aligned} \tag{4.48}$$

In the same sprit, using the complex conjugate of Eq. (3.92), one readily sees that

$$\begin{aligned} \langle \alpha^*(t)\alpha(0) \rangle &= E_+(t)\langle \alpha^*(0)\alpha(0) \rangle + E_-(t)\langle \alpha(0)\beta(0) \rangle \\ &+ \langle \alpha(0)F_+^*(t) \rangle + \langle \alpha(0)F_-^*(t) \rangle. \end{aligned} \tag{4.49}$$

On the basis of the fact that the noise force at time t does not affect the cavity mode variables at earlier times and taking the cavity modes to be initially in a chaotic state one gets

$$\langle \alpha^*(t)\alpha(0) \rangle = E_+\overline{n}_a. \tag{4.50}$$

It is also possible to verify in a similar manner that

$$\langle \alpha^*(t)\beta^*(0) \rangle = E_-\overline{n}_b, \tag{4.51}$$

$$\langle \alpha^*(t)F_+(t) \rangle = \langle F_+^*(t)F_+(t) \rangle + \langle F_-^*(t)F_+(t) \rangle, \tag{4.52}$$

$$\langle \alpha^*(t)F_-(t) \rangle = \langle F_+^*(t)F_-(t) \rangle + \langle F_-^*(t)F_-(t) \rangle. \tag{4.53}$$

Hence on substituting Eqs. (4.50),(4.51),(4.52) and(4.53) in to Eq. (4.48)

$$\begin{aligned}\langle\alpha^*(t)\alpha(t)\rangle = \langle F_+^*(t)F_+(t)\rangle + \langle F_-^*(t)F_-(t)\rangle + \langle F_+^*(t)F_-(t)\rangle \\ \langle F_-^*(t)F_+(t)\rangle + E_+^2\overline{n}_a + E_-^2\overline{n}_b.\end{aligned} \tag{4.54}$$

Next we evaluate the correlation functions in Eq. (4.54). To this end, on the basis of Eq. (3.95), one has

$$\begin{aligned}\langle F_-^*(t)F_-(t)\rangle = \frac{1}{4}\int_0^t & e^{-\lambda_-(2t-t'-t'')/2}[\langle F_\alpha^*(t')F_\alpha(t'')\rangle \\ & + \langle F_\beta(t')F_\beta^*(t'')\rangle - \langle F_\alpha^*(t')F_\beta^*(t'')\rangle \\ & - \langle F_\beta(t')F_\alpha(t'')\rangle]dt'dt'',\end{aligned} \tag{4.55}$$

So that on account of Eq. (3.72) and (3.78), we find

$$\langle F_-^*(t)F_-(t)\rangle = \frac{\varepsilon}{2}\int_0^t e^{-\lambda_-(2t-t'-t'')/2}\delta(t'-t'')dt'dt''. \tag{4.56}$$

Up on carrying out the integration, we obtain

$$\langle F_-^*(t)F_-(t)\rangle = \frac{\varepsilon}{2\lambda_-}(1-e^{-\lambda_- t}). \tag{4.57}$$

Furthermore, it can also be established in a similar manner that

$$\langle F_+^*(t)F_+(t)\rangle = \frac{-\varepsilon}{2\lambda_+}(1-e^{-\lambda_+ t}). \tag{4.58}$$

Moreover, using Eq. (3.95), we can write

$$\begin{aligned}\langle F_+^*(t)F_-(t)\rangle = \frac{1}{4}\int_0^t & e^{-[(\lambda_++\lambda_-)t-\lambda_+t'-\lambda_-t'']/2}[F_\alpha^*(t')F_\alpha(t'') \\ & - F_\alpha^*(t')F_\beta^*(t'') + F_\beta(t')F_\alpha(t'') \\ & - F_\beta(t')F_\beta^*(t'')]dt'dt''.\end{aligned} \tag{4.59}$$

With the aid of Eqs. (3.72)and (3.78), we get

$$\langle F_+^*(t)F_-(t)\rangle = 0. \tag{4.60}$$

We also note that

$$\langle F_-^*(t)F_+(t)\rangle = 0. \tag{4.61}$$

Hence substitution of (4.57),(4.58),(4.60)and (4.61) in to (4.54)results in

$$\begin{aligned}\langle\alpha^*(t)\alpha(t)\rangle = & \frac{\varepsilon}{2\lambda_-}(1-e^{-\lambda_- t}) - \frac{\varepsilon}{2\lambda_+}(1-e^{-\lambda_+ t}) \\ & + E_+^2\overline{n}_a + E_-^2\overline{n}_b,\end{aligned} \tag{4.62}$$

with the aid of Eq. (3.94) we get

$$\begin{aligned}\langle\alpha^*(t)\alpha(t)\rangle &= \frac{\varepsilon}{2\lambda_-}(1-e^{-\lambda_- t}) - \frac{\varepsilon}{2\lambda_+}(1-e^{-\lambda_+ t})\\ &+ \frac{\overline{n}_a}{4}[e^{-\lambda_+ t} + e^{-\lambda_- t} + 2e^{-(\lambda_+ + \lambda_-)t/2}]\\ &+ \frac{\overline{n}_b}{4}[e^{-\lambda_+ t} + e^{-\lambda_- t} - 2e^{-(\lambda_+ + \lambda_-)t/2}].\end{aligned} \tag{4.63}$$

In addition, it can be verified following a similar procedure that

$$\begin{aligned}\langle\beta^*(t)\beta(t)\rangle &= \frac{\varepsilon}{2\lambda_-}(1-e^{-\lambda_- t}) - \frac{\varepsilon}{2\lambda_+}(1-e^{-\lambda_+ t})\\ &+ \frac{\overline{n}_b}{4}[e^{-\lambda_+ t} + e^{-\lambda_- t} + 2e^{-(\lambda_+ + \lambda_-)t/2}]\\ &+ \frac{\overline{n}_a}{4}[e^{-\lambda_+ t} + e^{-\lambda_- t} - 2e^{-(\lambda_+ + \lambda_-)t/2}].\end{aligned} \tag{4.64}$$

$$\begin{aligned}\langle\alpha(t)\beta(t)\rangle &= \frac{\overline{n}_a}{4}[e^{-\lambda_+ t} - e^{-\lambda_- t}] + \frac{\overline{n}_b}{4}[e^{-\lambda_+ t} - e^{-\lambda_- t}]\\ &- \frac{\epsilon}{2\lambda_+}(1-e^{-\lambda_+ t}) - \frac{\epsilon}{2\lambda_-}(1-e^{-\lambda_- t}).\end{aligned} \tag{4.65}$$

Therefore, on combining Eqs. (4.63) and(4.64), we obtain

$$\begin{aligned}\langle\hat{n}\rangle &= \frac{\varepsilon}{\lambda_-}(1-e^{-\lambda_- t}) - \frac{\varepsilon}{\lambda_+}(1-e^{-\lambda_+ t})\\ &+ \frac{\overline{n}_a}{2}(e^{-\lambda_+ t} + e^{-\lambda_- t}) + \frac{\overline{n}_b}{2}(e^{-\lambda_+ t} + e^{-\lambda_- t}).\end{aligned} \tag{4.66}$$

At steady state, we have

$$\langle\hat{n}\rangle = \frac{4\varepsilon^2}{\lambda_+\lambda_-}. \tag{4.67}$$

This represents the mean number of the signal-idler photons produced by the parametric oscillator.

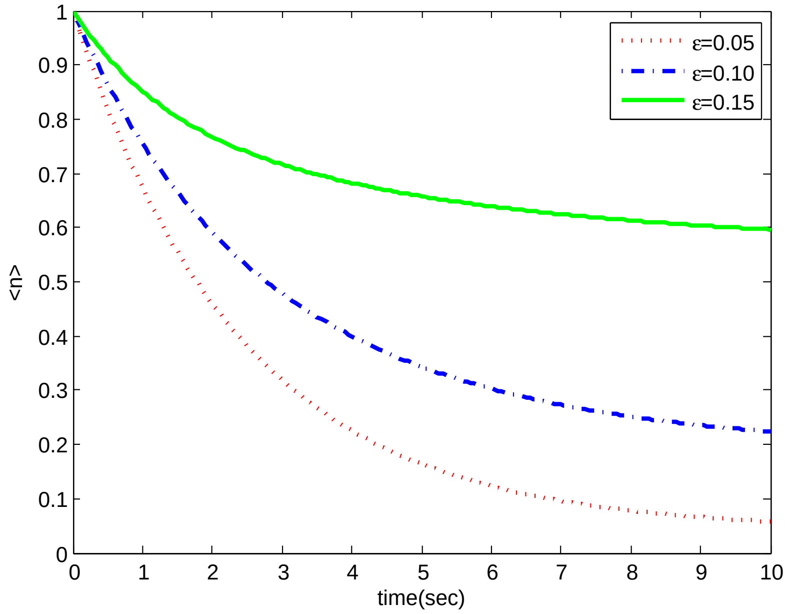


*Fig. 4.5:* Plots of the mean photon number of cavity radiation versus time for $\kappa = 0.5$,$\overline{n}_a = \overline{n}_b = 0$ and different values of $\epsilon$ ,for $\epsilon = 0.15$(solid curve), $\epsilon = 0.10$ (dashed curve) and $\epsilon = 0.05$ (doted curve).

The plots in Fig.4.5 indicate that as the intensity of the pump mode decreases the mean photon is also decreases.

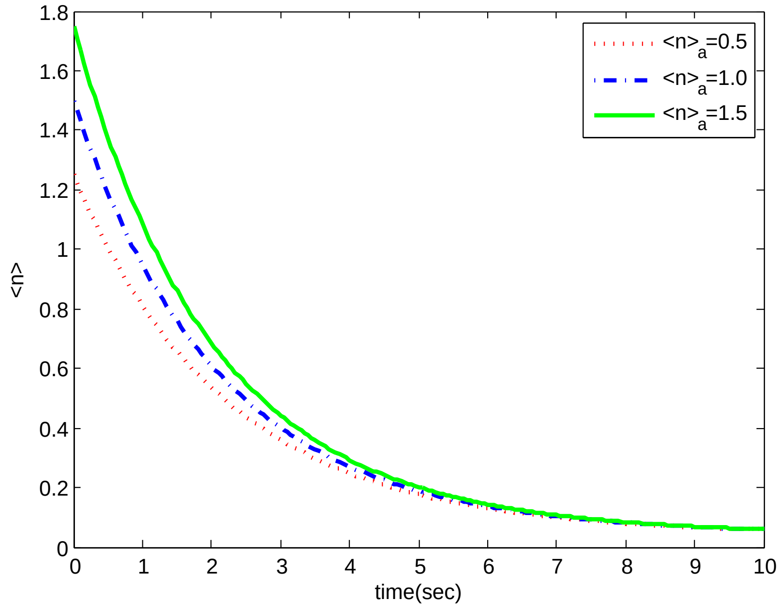


*Fig. 4.6:* Plots of the mean photon number of cavity radiation versus time for $\kappa = 0.5$,$\overline{n}_b = 0$,$\epsilon = 0.05$ and different values of $\overline{n}_b$ ,for $\overline{n}_a = 1.5$(solid curve), $\overline{n}_a = 1.0$ (dashed curve) and $\overline{n}_a = 0.5$ (doted curve).

It is clearly shown in Fig.4.6, that the mean photon number decreases with time.

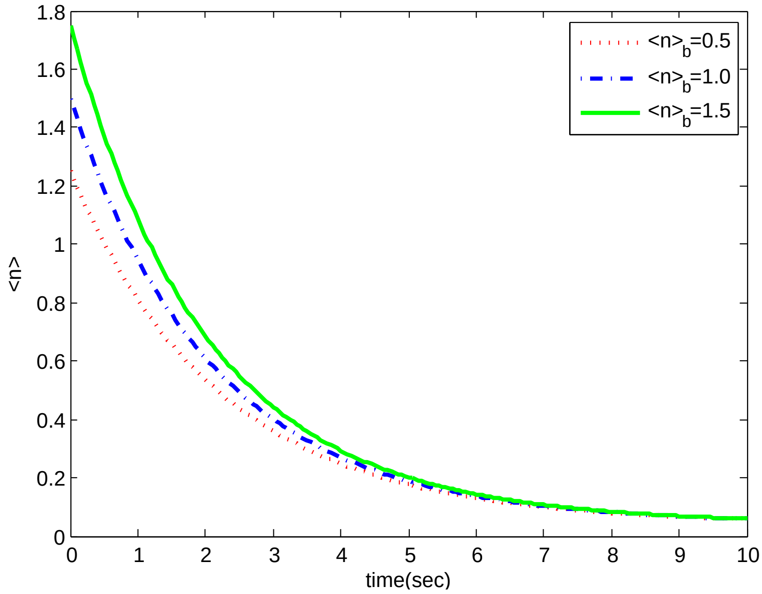


*Fig. 4.7:* Plots of the mean photon number of cavity radiation versus time for $\kappa = 0.5$,$\overline{n}_a = 0$,$\epsilon = 0.05$ and different values of $\overline{n}_b$ ,for $\overline{n}_b = 1.5$(solid curve), $\overline{n}_b = 1.0$ (dashed curve) and $\overline{n}_b = 0.5$ (doted curve).

It is clearly shown in Fig.4.7, that the mean photon number decreases with time.

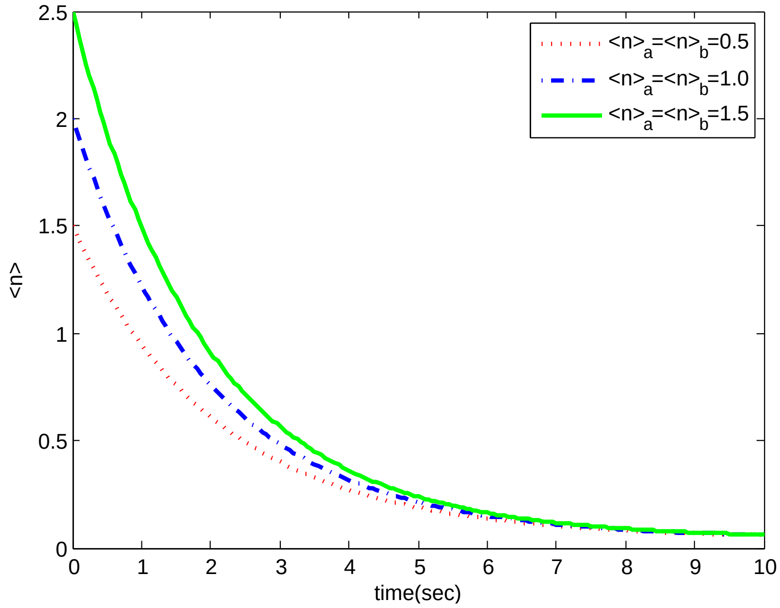


*Fig. 4.8:* Plots of the mean photon number of cavity radiation versus time for $\kappa = 0.5$,$\epsilon = 0.05$ and different values of $\overline{n}_a = \overline{n}_b$ ,for $\overline{n}_a = \overline{n}_b = 1.5$(solid curve), $\overline{n}_a = \overline{n}_b = 1.0$ (dashed curve) and $\overline{n}_a = \overline{n}_b = 0.5$ (doted curve).

It is clearly shown in Figs.4.6,4.7 and 4.8 that the mean photon number decreases with time. If one critically observes Figs.4.6,4.7 and 4.8 one may see that the mean photon number is almost the same for all cases after t=7.

### 4.2.2 The variance of the photon number

The variance of photon number can be expressed as

$$\triangle n^2 = \langle n^2 \rangle - \langle n \rangle^2. \tag{4.68}$$

Applying Eq. (4.45) the variance can be put in the form

$$\triangle n^2 = \Delta n_a^2 + \Delta n_b^2 + 2\langle \hat{n}_a \hat{n}_b \rangle - 2\langle \hat{n}_a \rangle \langle \hat{n}_b \rangle. \tag{4.69}$$

On the other hand, the photon number variance for mode a can be written in the normal order as

$$\Delta n_a^2 = \langle \hat{a}^{\dagger 2} \hat{a}^2 \rangle + \langle \hat{a}^\dagger \hat{a} \rangle - \langle \hat{a}^\dagger \hat{a} \rangle^2. \tag{4.70}$$

This can also be expressed interims of c-number variables associated with the normal ordering as

$$\Delta n_a^2 = \langle \alpha^{*2} \alpha^2 \rangle + \langle \alpha^* \alpha \rangle - \langle \alpha^* \alpha \rangle^2. \tag{4.71}$$

Following a similar procedure the photon number variance for mode b can also be written as

$$\Delta n_b^2 = \langle \beta^{*2}\beta^2 \rangle + \langle \beta^*\beta \rangle - \langle \beta^*\beta \rangle^2. \tag{4.72}$$

In addition,one can also write

$$\langle \hat{n}_a \hat{n}_b \rangle = \langle \alpha^*\alpha\beta^*\beta \rangle. \tag{4.73}$$

Hence substitution of Eqs. (4.70),(4.72) and (4.73) in to (4.69) yields

$$\begin{aligned}\Delta n^2 &= \langle \alpha^{*2}\alpha^2 \rangle + \langle \beta^{*2}\beta^2 \rangle + \langle \alpha^*\alpha \rangle + \langle \beta^*\beta \rangle \\ &- (\langle \alpha^*\alpha \rangle + \langle \beta^*\beta \rangle)^2 + 2\langle \alpha^*\alpha\beta^*\beta \rangle.\end{aligned} \tag{4.74}$$

In view of Eq. (4.46),we see that

$$\Delta n^2 = \langle \hat{n} \rangle - \langle \hat{n} \rangle^2 + \langle \alpha^{*2}\alpha^2 \rangle + \langle \beta^{*2}\beta^2 \rangle + 2\langle \alpha^*\alpha\beta^*\beta \rangle. \tag{4.75}$$

Furthermore Eqs. (3.92) and (3.93) can be re written as

$$\alpha(t) = E_+(t)\alpha(0) + E_-(t)\beta^*(0) + \eta_\alpha(t), \tag{4.76}$$

$$\beta(t) = E_+(t)\beta(0) + E_-(t)\alpha^*(0) + \eta_\beta(t), \tag{4.77}$$

where $E_\pm(t)$ is defined by Eq. (3.94)

$$\begin{aligned}\eta_\alpha(t) = \frac{1}{2}[&\int_0^t e^{-\lambda_+(t-t')/2}(F_\alpha(t') + F_\beta^*(t'))dt' \\ &+ \int_0^t e^{-\lambda_-(t-t')/2}(F_\alpha(t') - F_\beta^*(t'))dt'],\end{aligned} \tag{4.78}$$

And

$$\begin{aligned}\eta_\alpha(t) = \frac{1}{2}[&\int_0^t e^{-\lambda_+(t-t')/2}(F_\alpha^*(t') + F_\beta(t'))dt' \\ &- \int_0^t e^{-\lambda_-(t-t')/2}(F_\alpha^*(t') - F_\beta(t'))dt'].\end{aligned} \tag{4.79}$$

Employing Eq. (4.76) and its complex conjugate along with the assumption that the cavity radiation is initially in two mode thermal state we get

$$\begin{aligned}\langle \alpha^{*2}\alpha^2 \rangle &= 4(E_+^2\overline{n}_\alpha + E_-^2(\overline{n}_\beta))\langle \eta_\alpha^*\eta_\alpha \rangle \\ &+ 2(E_+\alpha^* + E_-\beta)\langle \eta_\alpha^*\eta_\alpha^2 \rangle \\ &+ 2(E_+\alpha + E_-\beta^*)\langle \eta_\alpha^{*2}\eta_\alpha \rangle + \langle \eta_\alpha^{*2}\eta_\alpha^2 \rangle.\end{aligned} \tag{4.80}$$

We recall that Gaussian variables with vanishing means satisfy the relation [1]

$$\langle ABCD \rangle = \langle AB \rangle\langle CD \rangle + \langle AC \rangle\langle BD \rangle + \langle AD \rangle\langle BC \rangle. \tag{4.81}$$

Next we need to show that $\eta_\alpha(t)$is Gaussian variable. One can rewrite Eq. (4.78)as

$$\eta_\alpha = \eta_+ + \eta_-, \tag{4.82}$$

in which

$$\eta_+ = \frac{1}{2}\int_0^t e^{-\lambda_+(t-t')/2}(F_\alpha(t') + F^*_\beta(t'))dt' \tag{4.83}$$

And

$$\eta_- = \frac{1}{2}\int_0^t e^{-\lambda_-(t-t')/2}(F_\alpha(t') - F^*_\beta(t'))dt'. \tag{4.84}$$

Employing Eqs. (4.83) and (4.84), the time evolution for the expectation values of $\eta_+$ and $\eta_-$ can be expressed as

$$\frac{d}{dt}\langle\eta_+\rangle = \frac{-\lambda_+}{2}\langle\eta_+\rangle \tag{4.85}$$

And

$$\frac{d}{dt}\langle\eta_-\rangle = \frac{-\lambda_-}{2}\langle\eta_-\rangle. \tag{4.86}$$

Inspection of Eqs. (4.85) and (4.86) indicates that $\eta_+$ and $\eta_-$ are Gaussian variables. Thus in view of Eq. (4.82) we see that $\eta_\alpha$ is also Gaussian variable. In addition ,using Eq. (4.78), one easily gets

$$\langle\eta_\alpha(t)\rangle = 0. \tag{4.87}$$

Hence on the basis of the relation (4.81), we find

$$\langle\eta_\alpha\rangle = \langle\eta^*_\alpha\rangle = \langle\eta_\alpha\eta^{*2}_\alpha\rangle = \langle\eta^*_\alpha\eta^2_\alpha\rangle = 0, \tag{4.88}$$

$$\langle\eta^2_\alpha\eta^{*2}_\alpha\rangle = \langle\eta^{*2}_\alpha\rangle\langle\eta^2_\alpha\rangle + 2\langle\eta^*_\alpha\eta_\alpha\rangle^2, \tag{4.89}$$

so that application of these results in (4.80) leads to

$$\begin{aligned}\langle\alpha^{*2}\alpha^2\rangle = 4(E^2_+\overline{n}_\alpha + E^2_-(\overline{n}_\beta))\langle\eta^*_\alpha\eta_\alpha\rangle \\ + \langle\eta^{*2}_\alpha\rangle\langle\eta^2_\alpha\rangle + 2\langle\eta^*_\alpha\eta_\alpha\rangle^2.\end{aligned} \tag{4.90}$$

Now with the help of Eq. (4.80), we can write

$$\begin{aligned}\langle\eta^2_\alpha\rangle = \frac{1}{4}[\int_0^t e^{-\lambda_+(2t-t'-t'')/2}\langle(F_\alpha(t') + F^*_\beta(t'))(F_\alpha(t'') + F^*_\beta(t''))dt'dt''\rangle \\ + \int_0^t e^{-\lambda_-(2t-t'-t'')/2}\langle(F_\alpha(t') - F^*_\beta(t'))(F_\alpha(t'') - F^*_\beta(t''))dt'dt''\rangle \\ + \int_0^t e^{[-(\lambda_++\lambda_-)t-\lambda_+t'-\lambda_-t'']/2}\langle(F_\alpha(t') + F^*_\beta(t'))(F_\alpha(t'') \\ - F^*_\beta(t''))dt'dt''\rangle \int_0^t e^{[-(\lambda_++\lambda_-)t-\lambda_-t'-\lambda_+t'']/2}\langle(F_\alpha(t') - F^*_\beta(t')) \\ (F_\alpha(t'') - F^*_\beta(t''))dt'dt''\rangle].\end{aligned} \tag{4.91}$$

Using Eqs. (3.77) and (3.78) we get

$\langle F_\alpha(t)F_\alpha(t')\rangle = 0, \langle F^*_\alpha(t)F_\beta(t')\rangle = 0$

$$\langle \eta_\alpha^2\rangle = 0. \tag{4.92}$$

$$\langle \eta_\alpha^{*2}\rangle = 0. \tag{4.93}$$

Furthermore, applying Eq. (4.78) and its complex conjugate

$$\langle \eta^*_\alpha \eta_\alpha\rangle = \frac{1}{4}[\langle \tau_1 + \tau_2 + \tau_3 + \tau_4\rangle], \tag{4.94}$$

with the aid of Eqs (3.72) and (3.78), we get

$$\langle \tau_1\rangle = 2\epsilon \int_0^t e^{-\lambda_+(2t-t'-2t'')/2}\delta(t-t'')dt''dt', \tag{4.95}$$

$$\langle \tau_2\rangle = 2\epsilon \int_0^t e^{-\lambda_-(2t-t'-2t'')/2}\delta(t-t'')dt''dt', \tag{4.96}$$

$$\langle \tau_3\rangle = \langle \tau_4\rangle. \tag{4.97}$$

Up on carrying out the integration Eqs. (4.95) and (4.96) turns over in to

$$\langle \tau_1\rangle = \frac{-2\epsilon}{\lambda_+}(1-e^{-\lambda_+ t}), \tag{4.98}$$

$$\langle \tau_2\rangle = \frac{-2\epsilon}{\lambda_-}(1-e^{-\lambda_- t}). \tag{4.99}$$

Hence substitution of Eqs. (4.98) and (4.99) into Eq. (4.94) results in

$$\langle \eta^*_\alpha \eta_\alpha\rangle = \frac{1}{2}[H-G], \tag{4.100}$$

in which

$$G = \frac{\epsilon}{\lambda_+}(1-e^{-\lambda_+ t}). \tag{4.101}$$

$$H = \frac{\epsilon}{\lambda_-}(1-e^{-\lambda_- t}). \tag{4.102}$$

Therefore, on account of Eqs. (4.92),(4.93) and (4.100) Eq. (4.90) takes the form

$$\langle \alpha^{*2}\alpha^2\rangle = 2[E_+^2\overline{n}_\alpha + E_-^2\overline{n}_\beta](H-G) + \frac{1}{2}(H-G)^2. \tag{4.103}$$

One can also establish in a similar manner that

$$\langle \beta^{*2}\beta^2\rangle = 2[E_+^2\overline{n}_\beta + E_-^2\overline{n}_\alpha](H-G) + \frac{1}{2}(H-G)^2. \tag{4.104}$$

On the other hand, employing Eqs. (4.76) and (4.77) along with the assumption that the cavity radiation is initially in a two mode thermal state, we can write

$$\begin{aligned}\alpha^*\alpha\beta^*\beta = {} & E_+^4\alpha^*\alpha\beta^*\beta + E_+^2E_-^2\alpha\alpha^*\alpha^*\alpha + E_+^2\alpha^*\alpha\eta_\beta^*\eta_\beta + E_+^2E_-^2\beta\beta^*\beta^*\beta \\ & + E_-^4\beta\beta^*\alpha\alpha^* + E_-^2\beta\beta^*\eta_\beta^*\eta_\beta + E_+^2\alpha^*\beta^*\eta_\alpha\eta_\beta + E_+E_-\alpha^*\alpha\eta_\alpha\eta_\beta \\ & + E_+\alpha^*\eta_\alpha\eta_\beta^* + E_+^2\alpha\beta\eta_\alpha^*\eta_\beta^* + E_+E_-\alpha\alpha^*\eta_\alpha^*\eta_\beta^* + E_+\alpha\eta_\alpha^*\eta_\beta^*\eta_\beta \\ & + E_-E_+\beta\beta^*\eta_\alpha\eta_\beta + E_-^2\beta\alpha\eta_\alpha\eta_\beta + E_-\beta\eta_\alpha\eta_\beta^*\eta_\beta + E_+E_-\beta^*\beta\eta_\alpha^*\eta\eta_\beta^* \\ & + E_-^2\beta^*\alpha^*\eta_\alpha^*\eta_\beta^* + \beta^*\eta_\alpha^*\eta_\beta^*\eta_\beta + E_+^2\beta^*\beta\eta_\alpha^*\eta_\alpha + E_-^2\alpha\alpha^*\eta_\alpha^*\eta_\alpha \\ & + E_+\beta^*\eta_\alpha^*\eta_\alpha\eta_\beta + E_+\beta\eta_\alpha^*\eta_\alpha\eta_\beta^* + E_-\alpha\eta_\alpha^*\eta_\alpha\eta_\beta + E_-\alpha^*\eta_\alpha^*\eta_\alpha\eta_\beta^* \\ & + \eta_\alpha^*\eta_\alpha\eta_\beta^*\eta_\beta.\end{aligned} \tag{4.105}$$

We recall that $\eta_\alpha(t)$ and $\eta_\beta(t)$ are gaussian variables with vanishing means. Thus on the basis of the relation Eq. (4.81), we get

$$\langle \eta_\alpha\eta_\beta\eta_\beta^*\rangle = \langle \eta_\alpha^*\eta_\beta\eta_\beta^*\rangle = \langle \eta_\alpha\eta_\alpha^*\eta_\beta^*\rangle = \langle \eta_\alpha\eta_\alpha^*\eta_\beta\rangle = 0, \tag{4.106}$$

$$\langle \eta_\alpha^*\eta_\alpha\eta_\beta^*\eta_\beta\rangle = \langle \eta_\alpha^*\eta_\alpha\rangle\langle \eta_\beta^*\eta_\beta\rangle + \langle \eta_\alpha^*\eta_\beta^*\rangle\langle \eta_\alpha\eta_\beta\rangle + \langle \eta_\alpha^*\eta_\beta\rangle\langle \eta_\alpha\eta_\beta^*\rangle, \tag{4.107}$$

so that applying these results together with the fact that $\eta_\alpha(t)$ and $\eta_\beta(t)$ have zero means ,Eq. (4.105) reduces to

$$\begin{aligned}\langle \alpha^*\alpha\beta^*\beta\rangle = {} & (E_+^4 + E_-^4)\overline{n}_a\overline{n}_b + E_+^2E_-^2(\overline{n}_a^2 + \overline{n}_b^2) + (E_+^2 + E_-^2)(\overline{n}_a + \overline{n}_b)\langle \eta_\alpha^*\eta_\alpha\rangle \\ & + 2(E_+^2 + E_-^2)\alpha\beta\eta_\alpha\eta_\beta + 2E_+E_-(\overline{n}_a + \overline{n}_b)\eta_\alpha\eta_\beta + \\ & + \langle \eta_\alpha^*\eta_\alpha\rangle\langle \eta_\beta^*\eta_\beta\rangle + \langle \eta_\alpha^*\eta_\beta^*\rangle\langle \eta_\alpha\eta_\beta\rangle + \langle \eta_\alpha^*\eta_\beta\rangle\langle \eta_\alpha\eta_\beta^*.\end{aligned} \tag{4.108}$$

Now with the help of Eqs. (4.78) and (4.79),one can write

$$\begin{aligned}\langle \eta_\alpha\eta_\beta^*\rangle = {} & \frac{1}{4}[\int_0^t e^{-\lambda_+(2t-t'-t'')/2}\langle (F_\alpha(t') + F_\beta^*(t'))(F_\alpha(t'') + F_\beta^*(t''))\rangle dt'dt'' \\ & - \int_0^t e^{-\lambda_-(2t-t'-t'')/2}\langle (F_\alpha(t') - F_\beta^*(t'))(F_\alpha(t'') - F_\beta^*(t''))\rangle dt'dt'' \\ & - \int_0^t e^{-[(\lambda_+ + \lambda_-)t - \lambda_+t' - \lambda_-t'']/2}\langle (F_\alpha(t') + F_\beta^*(t'))(F_\alpha(t'') - F_\beta^*(t''))\rangle dt'dt'' \\ & - \int_0^t e^{-[(\lambda_+ + \lambda_-)t - \lambda_-t' - \lambda_+t'']/2}\langle (F_\alpha(t') - F_\beta^*(t'))(F_\alpha(t'') + F_\beta^*(t''))\rangle dt'dt''].\end{aligned} \tag{4.109}$$

On account of Eqs. (3.77) and (3.78), we obtain

$$\langle \eta_\alpha \eta_\beta^* \rangle = 0. \tag{4.110}$$

We also note that

$$\langle \eta_\alpha^* \eta_\beta \rangle = 0. \tag{4.111}$$

Furthermore, using Eqs. (4.78) and (4.79), we can write

$$\begin{aligned}\langle \eta_\alpha \eta_\beta \rangle = \frac{1}{4}[ &\int_0^t e^{-\lambda_+(2t-t'-t'')/2} \langle (F_\alpha(t') + F_\beta^*(t'))(F_\alpha^*(t'') + F_\beta(t''))\rangle dt'dt'' \\ &- \int_0^t e^{-\lambda_-(2t-t'-t'')/2} \langle (F_\alpha(t') - F_\beta^*(t'))(F_\alpha^*(t'') - F_\beta(t''))\rangle dt'dt'' \\ &- \int_0^t e^{-[(\lambda_+ + \lambda_-)t - \lambda_+ t' - \lambda_- t'']/2} \langle (F_\alpha(t') + F_\beta^*(t'))(F_\alpha^*(t'') - F_\beta^( t''))\rangle dt'dt'' \\ &+ \int_0^t e^{-[(\lambda_+ + \lambda_-)t - \lambda_- t' - \lambda_+ t'']/2} \langle (F_\alpha(t') - F_\beta^*(t'))(F_\alpha^*(t'') + F_\beta(t''))\rangle dt'dt''].\end{aligned} \tag{4.112}$$

Applying Eqs. (3.77) and (3.78)

$$\langle \eta_\alpha \eta_\beta \rangle = \frac{-\epsilon}{2}(\frac{1 - e^{-\lambda_+ t}}{\lambda_+}) - \frac{\epsilon}{2}(\frac{1 - e^{-\lambda_- t}}{\lambda_-}). \tag{4.113}$$

In view of Eqs. (4.101) and (4.102) Eq. (4.113) can be written as

$$, \langle \eta_\alpha \eta_\beta \rangle = \frac{-1}{2}(H + G) \tag{4.114}$$

where G and H are defined by Eqs. (4.101) and (4.102) we also see that

$$\langle \eta_\alpha^* \eta_\beta^* \rangle = \frac{-1}{2}(H + G). \tag{4.115}$$

Hence combination of Eqs. (4.100),(4.110),(4.111),(4.114) and (4.115) with Eq. (4.108) results in

$$\begin{aligned}\langle \alpha^* \alpha \beta^* \beta \rangle = &(E_+^4 + E_-^4)\overline{n}_a \overline{n}_b + E_+^2 E_-^2 (\overline{n}_a^2 + \overline{n}_b^2) + \frac{1}{2}(\overline{n}_a + \overline{n}_b)[(E_+^2 + E_-^2)(H - G)] \\ &- [(E_+^2 + E_-^2)(\alpha(t)\beta(t))(H + G)] - [E_+ E_- (\overline{n}_a t \overline{n}_b)(H + G)] + \frac{1}{2}(H^2 + G^2).\end{aligned} \tag{4.116}$$

Moreover, with the aid of Eqs. (4.66),(3.94) ,(4.101) and (4.102) we find

$$\begin{aligned}\langle \overline{n} \rangle^2 = &H^2 + G^2 - 2HG + 2H\overline{n}_a(E_+^2 + E_-^2) + 2H\overline{n}_a(E_+^2 + E_-^2) \\ &- 2G\overline{n}_a(E_+^2 + E_-^2) - 2G\overline{n}_b(E_+^2 + E_-^2) + \overline{n}_a^2(E_+^2 + E_-^2)^2 \\ &+ \overline{n}_b^2(E_+^2 + E_-^2)^2 + 2\overline{n}_a\overline{n}_b(E_+^2 + E_- 2)^2.\end{aligned} \tag{4.117}$$

Finally on account of Eqs. (4.103),(4.104),(4.116) and (4.117) Eq. (4.75)can be put in the form

$$\begin{aligned}\triangle n^2 = \langle\hat{n}\rangle - [(E_+^2\overline{n}_a + E_-^2\overline{n}_b)^2 + (\overline{n}_a + \overline{n}_b)(E_+^2 + E_-^2)(H - G)\\ - 2(E_+^2 + E_-^2)\alpha(t)\beta(t)(H + G) - 2E_+E_-(H + G) + H^2 + G^2.\end{aligned} \tag{4.118}$$

With the help of Eqs. (3.94),(4.101) and (4.102) the variance takes at steady state the form

$$\triangle n_{ss}^2 = \langle\hat{n}\rangle_{ss} + H^2 + G^2. \tag{4.119}$$

We note that $\triangle n_{ss}^2 > \langle\hat{n}\rangle_{ss}$ and hence the cavity radiations has supper poissonian statistics.

$$\triangle n_{ss}^2 = \frac{\varepsilon}{\lambda_-}(1 + \frac{\varepsilon}{\lambda_-}) + \frac{\varepsilon}{\lambda_+}(\frac{\varepsilon}{\lambda_+} - 1). \tag{4.120}$$

This represents the variance of the photon number of the signal idler modes produced by parametric oscillator.

## 4.3 Quantification of entanglement

Based on different physical context and mathematical considerations several sufficient inseparability criteria for composite state have been proposed [12-17] for continuous variables. One of these alternatives is the criterion associated with the logarithmic negativity. For a Gaussian density operator the logarithmic negativity [12-15] is completely defined by the symplectic spectrum of the partial transpose of the covariance matrix. It is given by

$$EN = max[0, -\log_2 V_-], \tag{4.121}$$

where is $V_-$ the smallest eigen value. up on solving the eigen value equations for sympletic spectrum of the covariance matrix of the partially transposed density operator, the smallest eigen value is found to have the form [16-19]

$$V_- = [\frac{\Delta - \sqrt{\Delta^2 - 4det\Omega}}{2}]^{\frac{1}{2}}, \tag{4.122}$$

with $\Delta = detA + detB - 2detC_{(AB)}$, in which A and B are the covariance matrices that represent each modes separately while $C_{AB}$ in the intermodal correlations and $(\Omega\ )$ is the covariance matrix to be defined later [16,20]. The expression of the two mode covariance matrix $(\Omega)$ interims of the three 2*2 A,B and $C_{AB}$ can be expressed as the following

$$\Omega = \begin{pmatrix} A & C_{AB} \\ C_{AB}^T & B \end{pmatrix}, \tag{4.123}$$

the covariance matrix $\Omega$ of elements $\Omega_{ij} = \frac{1}{2}\langle \hat{x}_i\hat{x}_j + \hat{x}_j\hat{x}_i\rangle - \langle \hat{x}_i\rangle\langle \hat{x}_j\rangle$. It is essential to note that the involved quadrature operators can be defined as $\hat{x}_1 = \hat{a} + \hat{a}^\dagger, \hat{x}_2 = i[\hat{a}^\dagger - \hat{a}], \hat{x}_3 = \hat{x}_1 = \hat{b} + \hat{b}^\dagger and \hat{x}_4 = i[\hat{b}^\dagger - \hat{b}]$. It is worth noting that $C_+ = \frac{1}{\sqrt{2}}(\hat{x}_1 + \hat{x}_3)$ and $C_= \frac{1}{\sqrt{2}}(\hat{x}_2 + \hat{x}_4)$. With this introduction,the extended covariance matrix turns out to have the form.

$$\Omega = \begin{pmatrix} q & 0 & c & 0 \\ 0 & q & 0 & -c \\ c & 0 & r & 0 \\ 0 & -c & 0 & r \end{pmatrix}, \tag{4.124}$$

where $q = 2\langle \hat{a}^\dagger(t)\hat{a}(t)\rangle + 1$ , $r = 2\langle \hat{b}^\dagger(t)\hat{b}(t)\rangle + 1$ and $c = \langle \hat{a}(t)\hat{b}(t)\rangle + \langle \hat{a}^\dagger(t)\hat{b}(t)^\dagger\rangle$ which can be expressed interims of c-number variables associated with the normal ordering as $q = 2\langle \alpha^*(t)\alpha(t)\rangle + 1,$ $r = 2\langle \beta^*(t)\beta(t)\rangle + 1$ and $c = \langle \alpha(t)\beta(t) + \langle \alpha^*(t)\beta^*(t)\rangle.$

The two-mode gaussian composite state is said to be entangled when the logarithmic negativity is positive, that is if $E_N = -\log_2 V_-$ which implies that $\log_2 V_-$ should be negative. This, on other hand,holds true provided that

$$V_- < 1. \tag{4.125}$$

Stands the condition of detecting entanglement.
In order to quantify entanglement with the aid Eq. (4.125) the involved correlations should be calculated from equations of evolutions. Following straight for ward algebra, it is possible to show at steady state $(V_-) < 1.$
Furthermore, making use of Eqs. (4.123),(4.124) and the accompanying definitions q, r and c, one can readily see that

$$detA = 4\langle \alpha^*(t)\alpha(t)\rangle^2 + 4\langle \alpha^*(t)\alpha(t)\rangle + 1, \tag{4.126}$$

$$detB = 4\langle \beta^*(t)\beta(t)\rangle^2 + 4\langle \beta^*(t)\beta(t)\rangle + 1, \tag{4.127}$$

$$detC_{AB} = -4\langle \alpha(t)\beta(t)\rangle^2, \tag{4.128}$$

$$det\Omega = 16[(\frac{1}{4} + \frac{\langle \alpha^*\alpha\rangle + \langle \beta^*\beta\rangle}{2} + \langle \alpha^*\alpha\rangle\langle \beta^*\beta\rangle)^2 - (\langle \alpha\beta\rangle^2)^2]. \tag{4.129}$$

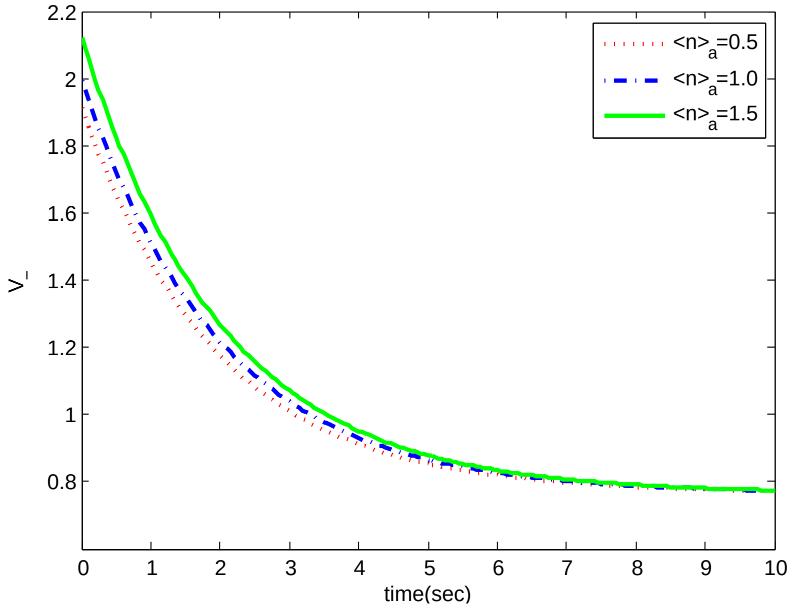


*Fig. 4.9:* Plots of minimum of the logarithmic negativity $(V_{-})$ versus time for $\kappa = 0.5$,$\overline{n}_b = 0$,$\epsilon = 0.05$ and different values of $\overline{n}_a$ ,for $\overline{n}_a = 1.5$(solid curve), $\overline{n}_a = 1.0$ (dashed curve) and $\overline{n}_a = 0.5$ (doted curve).

It is clearly shown in plots of Fig.4.9, the minimum logarithmic negativity decreases with time.

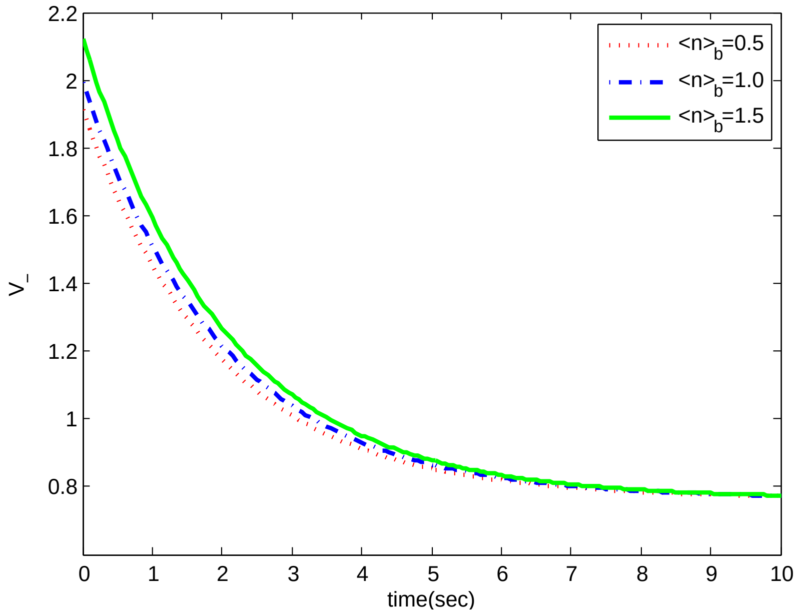


*Fig. 4.10:* Plots of minimum of the logarithmic negativity $(V_{-})$ versus time for $\kappa = 0.5$,$\overline{n}_a = 0$,$\epsilon = 0.05$ and different values of $\overline{n}_b$ ,for $\overline{n}_b = 1.5$(solid curve), $\overline{n}_b = 1.0$ (dashed curve) and $\overline{n}_b = 0.5$ (doted curve).

It is clearly shown in plots of Fig.4.10, the minimum logarithmic negativity decreases with time.

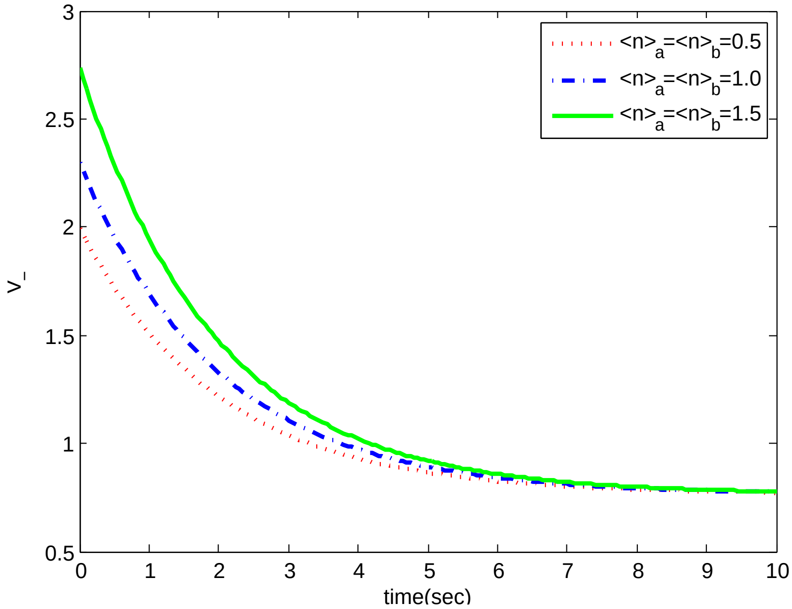


*Fig. 4.11:* Plots of minimum of the logarithmic negativity $(V_{-})$ versus time for $\kappa = 0.5$,$\epsilon = 0.05$ and different values of $\overline{n}_a = \overline{n}_b$ ,for $\overline{n}_a = \overline{n}_b = 1.5$(solid curve), $\overline{n}_a = \overline{n}_b = 1.0$ (dashed curve) and $\overline{n}_a = \overline{n}_b = 0.5$ (doted curve).

It is clearly shown in plots of Figs.4.9,4.10 and 4.11 the minimum logarithmic negativity decreases with time. This implies that the degree of entanglement increases with time. From Figs.4.9,4.10 and 4.11 one clearly observes that the mean photon number of chaotic light decreases the degree of entanglement.

# 5

# CONCLUSION

In this thesis, we have studied entanglement,quadrature squeezing and statistical properties of the light produced by a non degenerate parametric oscillator with the cavity modes being initially seeded by thermal light and coupled to vacuum reservoir. We determined the c-number Langevin equations associated with the normal ordering employing the pertinent master equation.Then, we obtained the solutions of the c-number Langevin equations and the correlation properties of the noise forces. Making use of the solutions of the resulting c-number Langevin equation, we calculate quadrature variances. We have seen that the quadrature squeezing at large time scale is independent of the variables of the chaotic light. This implies that the chaotic light in the cavity is converted to a correlated light or lost all the way which leads to diminishing effect of the seed on the properties of the cavity radiation. The degree of two mode squeezing is significantly impaired by chaotic light. The more intense the seeding chaotic light the more the squeezing is damaged. Hence at threshold and at steady state, the squeezing of the cavity radiation is equal to $50\%$.

On the other hand, employing the solution of the c-number Langevin equations, we have determined the mean and the variance of photon number. From the results obtained, we realize that the seeded thermal light increases the mean and the variance of the photon number at the beginning.

Finally with the aid of the solutions of c-number Langevin equation and Logarithmic Negativity criteria we calculate entanglement. It is clearly shown that the seeded thermal light decreases the degree of entanglement at the beginning. At longer time scale entanglement is independent of the strength of initially seeded thermal light.

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