

**Modeling and Simulation of Intelligent Fuzzy-PID
Controller for a full car model to Ride Quality
Improvement of Active Suspension System**



Daniel Atomsa Bulti

A Thesis Submitted to the Department of Mechanical Engineering
School of Mechanical, Chemical and Materials Engineering
Presented in Partial Fulfillment of the Requirement for the Degree of
Master's in Automotive Engineering

**Office of Graduate Studies
Adama Science and Technology University**

November, 2022

Adama, Ethiopia

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DECLARATION

I hereby declare that this Master Thesis entitled “**Modeling and Simulation of Intelligent Fuzzy-PID Controller for a full car model to Ride Quality Improvement of Active Suspension System**” is my original work. That is, it has not been submitted for the award of any academic degree, diploma, or certificate in any other university. All sources of materials that are used for this thesis have been duly acknowledged through citation.

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Name of student

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I/we, the advisor(s) of this thesis, hereby certify that I/we have read the revised version of the thesis entitled “**Modeling and Simulation of Intelligent Fuzzy-PID Controller for a full car model to Ride Quality Improvement of Active Suspension System** prepared under my/our guidance by **Daniel Atomsa** submitted in partial fulfillment of the requirements for the degree of Mater’s of Science in Automotive Engineering. Therefore, I/we recommend the submission of a revised version of the thesis to the department following the applicable procedures.

Dr. Ramesh Babu Nallamothu _____

Major Advisor/Supervisor

Signature

Date

Co-advisor/Co-supervisor

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Date

APPROVAL

I/we, the advisors of the thesis entitled “**Modeling and Simulation of Intelligent Fuzzy-PID Controller for a full car model to Ride Quality Improvement of Active Suspension System**” and developed by **Daniel Atomsa**, hereby certify that the recommendation and suggestions made by the board of examiners are appropriately incorporated into the final version of the thesis.

<u>Dr. Ramesh Babu Nallamothu</u>		
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We, the undersigned, members of the Board of Examiners of the thesis by **Daniel Atomsa** have read and evaluated the thesis entitled “**Modeling and Simulation of Intelligent Fuzzy-PID Controller for a full car model to Ride Quality Improvement of Active Suspension System**” and examined the candidate during the open defense. This is, therefore, to certify that the thesis is accepted for partial fulfillment of the requirement of the degree of Master of Science in Automotive Engineering.

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TABLE OF CONTENTS

ACKNOWLEDGMENT	vi
LIST OF FIGURES	x
LIST OF TABLES	xii
ACRONYMS	xiii
ABSTRACT	xiv
CHAPTER ONE: INTRODUCTION	1
1.1. Background of the study.....	1
1.2. Statement of the problem.....	3
1.3. General and specific objectives	3
1.3.1. General objective.....	3
1.3.2. Specific objectives.....	3
1.4. Significance of the study	4
1.5. Scope of the study	4
1.6. Organization of the thesis	4
CHAPTER TWO: LITERATURE REVIEW	6
2.1. Introduction	6
2.2. Types of the suspension system	6
2.2.1. Passive suspension.....	6
2.2.2. Semi-active suspension	7
2.2.3. Active suspension.....	8
2.3. Mathematical model of the suspension system	8
2.3.1. Quarter car model	9
2.3.2. Half car model	11
2.3.3. Full car model.....	12
2.4. Control of active suspension system	14
2.4.1. Proportional integral derivative (PID) control.....	15
2.4.2 Fuzzy logic control	16
2.4.3 Intelligent controller	18
2.4.4. Fuzzy-PID controller	19

2.5. Hydraulic actuator	20
2.6. Research gap.....	21
CHAPTER THREE: MATERIALS AND METHODS	22
3.1. Introduction	22
3.2. Methods	22
3.3. Data Collection	23
3.4. Data Analysis.....	23
3.5. Materials	23
3.5.1. MATLAB	23
3.5.2. Simulink	24
3.6. Methodology.....	27
CHAPTER FOUR: MATHEMATICAL MODELING FOR ACTIVE SUSPENSION SYSTEM.....	29
4.1. Introduction	29
4.2. Quarter car model for the active suspension system	29
4.2.1. Actuator model for quarter car model	30
4.2.2. Quarter car model in SIMULINK	33
4.3. Half car model	33
4.3.1. Actuator model for half car Model	36
4.3.2. Half car Model in Simulink	36
4.4. Full car model.....	36
4.4.1. Actuator model for full car model	39
4.4.2. Full car model in Simulink.....	40
CHAPTER FIVE: DESIGN OF INTELLIGENT FUZZY-PID CONTROLLER FOR ACTIVE SUSPENSION	41
5.1. Introduction	41
5.2. Control objective	41
5.3. PID controller	42
5.4. Fuzzy logic	43
5.4.1. Fuzzy Algorithm.....	44

5.5. Fuzzy controller.....	45
5.5.1. Fuzzification.....	45
5.5.2. Rules Base.....	47
5.5.3. Defuzzification.....	48
5.6. Modeling of an intelligent fuzzy-PID controller in Simulink.....	48
CHAPTER SIX: RESULT AND DISCUSSION.....	50
6.1. Introduction.....	50
6.2. Simulation of quarter car model.....	50
6.2. Simulation of half car model.....	55
6.3. Simulation of full car model.....	59
6.4. Validation.....	61
CHAPTER SEVEN: CONCLUSION AND RECOMMENDATIONS.....	63
7.1. Summary.....	63
7.2. Conclusion.....	64
7.3. Recommendations.....	64
REFERENCES.....	65
APPENDIX - A.....	71
APPENDIX - B.....	75

LIST OF FIGURES

Figure 1. Passive suspension system	7
Figure 2. Semi-active suspension system	7
Figure 3. Active suspension system	8
Figure 4. Quarter car model	9
Figure 5. Half car model (Jazar, 2014).....	11
Figure 6. Full car model (Jazar, 2014).....	12
Figure 7. Flow chart of proposed research work	27
Figure 8. Quarter car model of the active suspension system.	29
Figure 9. The schematic diagram of the electro-hydraulic actuator	30
Figure 10. Hydraulic actuator Simulink model	32
Figure 11. Quarter car active suspension Simulink model.....	33
Figure 12. Half car model of the active suspension system.	34
Figure 13. Half car active suspension Simulink model	36
Figure 14. Full car model of active suspension system.....	37
Figure 15. Full car active suspension Simulink model.....	40
Figure 16. Block diagram of process control using PID	42
Figure 17. Fuzzy controller for active Suspension	45
Figure 18. Membership function for ‘error’	46
Figure 19. Membership function for ‘change in error’	46
Figure 20. Surface area of input and output parameter	47
Figure 21. Membership function for control force	48
Figure 22. Fuzzy PID controller structure.	49
Figure 23. IFPID controller structure.	49
Figure 24. Representation of an intelligent fuzzy-PID controller in SIMULINK.	49
Figure 25. Simulink model flow chart.....	50
Figure 26. Road profile 1	51
Figure 27. Body velocity (m/s ²) vs time (s) of active quarter car model.	51
Figure 28. Body displacement (m/s ²) vs time (s) of active quarter car model.	52
Figure 29. Body acceleration (m/s ²) vs time (s) of active quarter car model.	52
Figure 30. Suspension deflection (m) vs time (s) of active quarter car model.....	53

Figure 31. Wheel displacement (m) vs time (s) of active quarter car model.	53
Figure 32. Wheel deflection (m) vs time (s) of active quarter car model.....	54
Figure 33. Road profile 2.....	55
Figure 34. Front axle vertical displacement of active half car model with fuzzy-PID controller.	56
Figure 35. Rear axle vertical displacement of active half car model with fuzzy-PID controller.	56
Figure 36. Vertical body displacement of active half car model with fuzzy-PID controller. ...	57
Figure 37. Rotational body displacement of active half car model with fuzzy-PID controller.	57
Figure 38. Body acceleration of active half car model with fuzzy-PID controller.	58
Figure 39. Body displacement (m) vs time (s) of a full car model.....	59
Figure 40. Body acceleration (m/s^2) vs time (s) of a full car model.....	59
Figure 41. Pitch angle (radian) vs time (s) of a full car model.....	60
Figure 42. Roll angle (radian) vs time (s) of a full car model.	60
Figure 43. Road input signal for the simulation	61
Figure 44. Response of suspension system for road input signal.....	62
Figure 45. Simulation results of the half car model with Fuzzy-PID controller.	63

LIST OF TABLES

Table 1. Overview of Simulink libraries	26
Table 2. Quarter car suspension system parameters, (Agharkakli et al., 2012).	32
Table 3. Half car model for active suspension system parameters, (Gandhi et al., 2017).	35
Table 4. Full car model for active suspension system parameters.	38
Table 5. Fuzzy Rules for Controller	47

ACRONYMS

AVSS	Active Vehicle Suspension System
DOF	Degrees Of Freedom
EHA	Electro-Hydrostatics Actuator
FLC	Fuzzy Logic Control
FLST	Fuzzy Logic-Based Self-Tuning
FSMC	Fuzzy Sliding-Mode Control
IFPID	Intelligent Fuzzy Proportional Integral Derivative
MR	Magneto-rheological
LQR	Linear Quadratic Control
OASC	Optimal Active Suspension Control
PBW	Power-By-Wire
PID	Proportional Integral Derivative
PSO	Particle Swarm Optimization
SMC	Sliding-Mode Control
SIL	Software in the Loop
SWS	Suspension Working Space

ABSTRACT

The vehicle suspension is responsible for ride comfort and stability as the suspension system carries the vehicle mass and transmits all forces between body and road. The design of the suspension system involves an optimization process as it is not possible to provide both ride comfort and stability simultaneously. Thus, a better suspension can be designed through an optimization process to provide optimum ride comfort and optimum road safety for vehicles. Initially, the model-based controller is designed with a quarter car model for active suspension. But it cannot be used to measure the pitch and roll motion of the vehicle. Both quarter car and half car model does not model the actual system for practical applications. Hence, an accurate model for the actual system needs a full car model with seven degrees of freedom. The inherent complex nonlinear full car model for active suspension and the presence of parameter uncertainties in actuator dynamics have increased the difficulties in applying conventional linear control techniques to full car model-based hydraulic actuated active suspension systems. Recently, the combination of sliding mode, fuzzy logic, and neural network methodologies have emerged as a promising technique for dealing with complex uncertain systems. The objective of this thesis is to develop an intelligent fuzzy-PID controller for tuning the membership function of the fuzzy controller to optimize the performance of the active suspension. Intelligent control schemes can control the un-modeled part of the suspension dynamics which are simple to realize and can yield accurate control. The whole research work is classified into three different divisions namely system modeling, controller design and simulation. The performance of the developed Intelligent Fuzzy-PID controller measured by using Software in the Loop (SIL) simulation and also in the real-time control platform for the active suspension. The development of an Intelligent Fuzzy-PID control for realistic full model-based active suspension outperform the existing conventional controllers with regard to body acceleration, body displacement, roll angle and pitch angle. The real active suspension should be tested on real road with varying disturbance in order to assess the effectiveness of developed controller.

Keywords: Active Suspension, Intelligent fuzzy-PID, Full car, Ride comfort, Simulation.

CHAPTER ONE: INTRODUCTION

1.1. Background of the study

The suspension system consists of a spring and oil damper interconnected between the vehicle body and tire. The spring isolates the vehicle body from road disturbances by carrying the load of the vehicle. The damper contributes dissipation of energy to provide both ride comfort and stability, (Samsuria et al., 2018). The main purpose of suspension is to provide better ride comfort and stability to vehicles. Ride comfort is always linked with the amount of energy transmitted through the suspension system. Similarly, the stability of the vehicle is linked to the vertical motion of the tires. Hence, a soft suspension provides better ride comfort to passengers at the cost of low stability, while a hard suspension provides better stability to vehicles at the cost of low ride comfort.

The suspension system is made up of three major components: a structure that supports the weight of the vehicle and determines suspension geometry, a spring that converts kinematic energy to potential energy or vice versa, and a shock absorber, which is a mechanical device designed to dissipate kinetic energy.

A vehicle's suspension attaches the wheels to the body while supporting the vehicle's weight. It facilitates for relative motion between the wheel and the vehicle body; theoretically, a suspension system should reduce the degree of freedom (DOF) of a wheel from 6 to 2 on the rear axle and 3 on the front axle, despite the fact that the suspension system must support propulsion, steering, brakes, and their associated forces. Vertical movement, rotational movement about the lateral axes, and rotational movement about the vertical axes due to steer angle are the relative motions of the wheels.

As mentioned previously, it is a common misconception that a suspension system's primary purpose is to smooth out bumps in the road. In reality, a vehicle's suspension must meet a number of requirements, some of which are partially at odds with one another because of various operating circumstances. All forces and moments between the vehicle's body and the ground pass through the suspension since it connects the body to the ground. As a result, a vehicle's dynamic behavior is directly influenced by the suspension system. Three main principles are commonly used by automotive engineers when analyzing the suspension system's functioning.

- **Ride Comfort:** How a passenger feels inside a moving vehicle is used to describe ride comfort. The suspension system's primary function is to isolate the vehicle body from road disturbances. In general, the amount of vibration in the passenger compartment may be used to measure ride quality. A vehicle has several internal and external vibration sources. The engine and transmission of the car are examples of inner vibration sources, whereas road surface imperfections and aerodynamic forces are examples of outer vibration sources. According to frequency ranges, the vibration spectrum can be separated into pleasant (0–25 Hz) and noisy and severe categories (25–20,000 Hz).
- **Road Holding:** The vehicle body is subjected to forces at the point where a wheel makes contact with the road thanks to the suspension system. Road holding is one of the key responsibilities of the suspension system since the magnitude and direction of the forces impact the behavior and performance of the vehicle. The normal tire force, which promotes turning, traction, and braking capabilities, directly affects the lateral and longitudinal forces produced by a tire. If the variance in the typical tire load is kept to a minimum, these terms are enhanced. The suspension's support of the vehicle's static weight is its additional purpose. If the vehicle's rattling space needs are kept to a minimum, this operation may be completed successfully.
- **Handling:** An effective suspension system should make that the vehicle is stable during every maneuver. Perfect handling, though, goes beyond stability. The vehicle should follow the driver's steering, braking, and acceleration orders seamlessly and in accordance to the driver's inputs. The conduct of the vehicle must be predictable, and the driver should be informed of this behavior. Suspension systems have a variety of effects on how a vehicle handles, including minimizing roll and pitch motion, controlling wheel angles, and reducing lateral load transfer during turns.

The design of the suspension system involves an optimization process as it is not possible to provide both ride comfort and stability simultaneously. Therefore, through an optimization process, the improved suspension can be designed to provide the vehicle with optimum ride quality and optimum road safety.

1.2. Statement of the problem

Initially, a model-based controller is designed with a quarter car model for active suspension. But it cannot be used to measure the pitch and roll motion of the vehicle. Both quarter and half car model does not model the actual system for practical applications. Hence, an accurate model of the actual system requires a complete car model with 7 degrees of freedom.

The inherent complex full car model for active suspension and the parametric uncertainty of actuator dynamics have made it difficult to apply conventional linear control methods to hydraulically actuated active suspension based on the full car model.

Recently, a combination of fuzzy logic, sliding mode and neural network methodologies have emerged as a promising way to deal with complex and uncertain systems. Intelligent control schemes, such as fuzzy logic controllers, can control un-modeled parts of suspension dynamics which are easy to realize and provide precise control.

The development of intelligent fuzzy-PID control for realistic full car model-based active suspension will outperform the existing conventional controllers with regard to body acceleration, body displacement, roll angle and pitch angle.

1.3. General and specific objectives

1.3.1. General objective

The general objective of this thesis work is to develop an intelligent fuzzy-PID controller for vehicle active suspension based full car model of the vehicle.

1.3.2. Specific objectives

The specific objectives are as follows:

- To develop an intelligent fuzzy-PID controller for tuning the membership function of the fuzzy controller to optimize the performance of the active suspension.
- To measure the performance of the developed intelligent fuzzy-PID controller by using software in the loop (SIL) simulation.
- To compare the drive comfort and stability of the passive suspension system and active suspension system with PID controller and intelligent fuzzy-PID controller.

1.4. Significance of the study

The thesis is all about providing optimum ride comfort and optimum road safety for vehicles. The study provides an analysis of the 7 Degrees of Freedom (DOF) full car model for the active suspension system. It also indicates a good controller that outperforms the existing conventional controllers with regard to body acceleration, body displacement, roll angle and pitch angle.

1.5. Scope of the study

The scope of this research is modeling and simulation of an intelligent fuzzy-PID controller for a full car model to ride quality improvement of the active suspension system. And developing a mathematical model for 7 Degrees of Freedom (DOF) of full car model for an active suspension system based on the quarter car model. In this research quarter car model and half car model mathematical model also analyzed.

1.6. Organization of the thesis

This thesis is organized into 7 chapters. Chapter 1 discusses the introduction to the suspension system, a mathematical model for the suspension system and the control method for the active suspension system. The problem statement, research objective, significance of the study and research scope are also explained in this chapter.

In chapter 2 relevant literature on active suspension systems is reviewed. And also reviews previous works regarding the design of controllers for active suspension. The research work carried out on intelligent controllers is reviewed for quarter car, half car and full car model-based active suspension.

Chapter 3 explains about research methodology, methods used in this research to obtain results, data collection, data analysis and material used for this research.

Chapter 4 explains the mathematical model for the quarter car, half car and full car model of active suspension systems. The Simulink model of quarter car, half car and full car model is developed by using MATLAB software are also explained in this chapter.

Chapter 5 presents the controller design of an intelligent Fuzzy-PID controller and simulations. And also PID controller is explained in this chapter.

Chapter 6 includes comprehensive discussion for the result analysis. The comparison between the passive and active suspension system is performed by means of computer simulation using MATLAB/Simulink software.

Chapter 7 covers is conclusion and recommendation parts. The outcomes of this research and scope for future work has been suggested in detail.

CHAPTER TWO: LITERATURE REVIEW

2.1. Introduction

Literature on active suspension systems shows that many researchers proposed control systems for quarter car, half car and full car models. The model may be linear or nonlinear without considering the effect of actuator dynamics. Few works were carried out on active suspension by considering the effect of actuator dynamics. Generally, a model-based classical controller was proposed for nonlinear active suspension. Nowadays, model-free intelligent hybrid controllers were introduced for active suspension due to their ability to handle nonlinearity and parameter uncertainty in actuator dynamics.

The ideal vehicle suspension should meet the following basic characteristics in terms of ride comfort, reduction of dynamic road-tire forces, and reduction of relative motions between the vehicle bodies. Controllable suspension systems are required to improve the compromise between the conflicting demands, (Segla, 2007).

2.2. Types of the suspension system

The vehicle suspension is responsible for ride comfort and stability as the suspension system carries the vehicle mass and transmits all forces between body and road. The addition of adjustable dampers or springs greatly improves ride comfort and stability compared to fixed-speed suspension setups. To positively influence these characteristics, semi-active and active components are introduced, which allow the suspension system to adapt to different driving conditions. Automobile suspensions can therefore be divided into three categories namely passive, active and semi-active suspensions,(Avesh & Srivastava, 2012).

2.2.1. Passive suspension

The passive suspension can be considered as a spring (k) in parallel with a damper (c) placed at each corner of the vehicle as illustrated in Figure 1. The characteristics of the dampers used in a passive suspension are fixed. The choice of the damping coefficient is made considering the classic trade-off between ride comfort and vehicle stability.

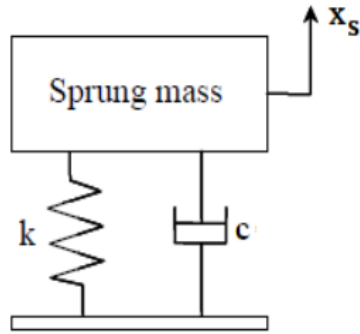


Figure 1. Passive suspension system

A low damping coefficient will result in a more comfortable ride but will reduce the stability of the vehicle. A vehicle with a lightly damped suspension will not be able to hold the road. Negotiating sharp turns becomes a safety issue for vehicles. A high damping coefficient yields a better road holding ability but reduces the ride comfort of the vehicle. Thus a compromise between ride comfort and stability leads to new types of vehicle suspensions.

2.2.2. Semi-active suspension

In semi-active suspensions, the passive dampers are replaced with dampers capable of changing their damping characteristics as illustrated in Figure 2. These dampers are called semi-active damper (Savaresi et al., 2010). External power is supplied to them to change the damping level. The control algorithm determines the damping level based on the information the controller received from the sensors. But the amount of power required for controlling the damping level of a semi-active damper is much less than that of the amount of power required for the operation of an active suspension.

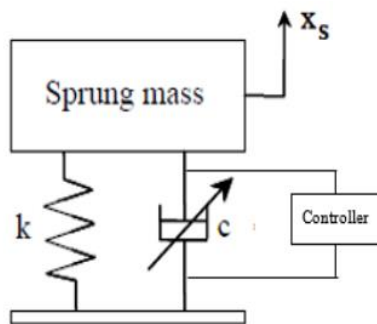


Figure 2. Semi-active suspension system

Semi-active suspensions are more expensive than passive suspensions but much cheaper than active suspensions, so they are becoming more and more popular in commercial vehicles.

2.2.3. Active suspension

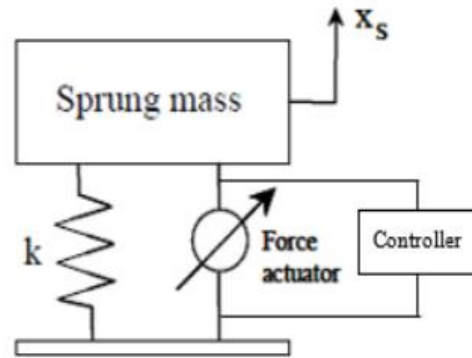


Figure 3. Active suspension system

In active suspension, an actuator is placed in between the wheel and vehicle body along with the suspension system, (Li et al., 2013) as depicted in Figure 3. The advantage of this is that the force actuator can generate a force in any direction, regardless of the relative velocity across it, while a passive damper can only dissipate energy. A well-designed controller for active suspension can provide a compromise between ride comfort and vehicle stability as compared to passive suspensions. Active suspensions can easily reduce the pitch and the roll of the vehicle. Nowadays, many researchers are focusing their research on active suspension systems due to their ability to operate in a wide range of frequencies. The development of computers and microprocessors further improved the practical implementation of active suspension in automotive industries.

2.3. Mathematical model of the suspension system

The first step in active suspension design is to develop appropriate mathematical models based on physical laws or experimental data. The mathematical model can be represented as a quarter car model, half-car model and full car model to find out the dynamic behavior of the active suspension.

2.3.1. Quarter car model

The quarter car model, which contains 1/4th of the vehicle mass, springs, and dampers, is the most commonly used and applicable model of the vehicle suspension system, (Jazar, 2014), as shown in Figure 4.

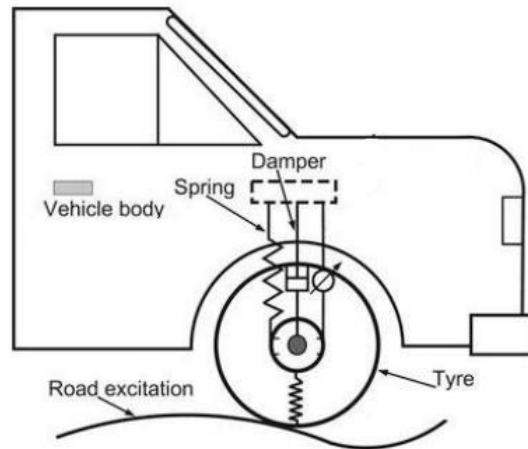


Figure 4. Quarter car model

Ahmed et al., (2015), to build a quarter car active suspension model controller for responsiveness to excitation from a road profile using a PID controller, researchers explored a mathematical model for passive and active suspension systems for the quarter car model. The road profiles are used to compare the passive and active suspension systems. The controller's performance is compared to that of a PID controller and a passive suspension system. The performance of this controller is determined by performing computer simulations using the Simulink toolbox.

Jain et al., (2020), studied an integrated model of a semi-active seat suspension with a human model for over a quarter. They presented an integrated model with eight degrees of freedom (8-DOF), which includes two degrees of freedom (2-DOF) for the quarter vehicle model, and two degrees of freedom (2-DOF) for the semi-active seat suspension, and four degrees of freedom (4-DOF) for the human model. The seat suspension is equipped with a magneto-rheological (MR) damper. The fuzzy logic-based self-tuning (FLST) proportional–integral–derivative (PID) controller allows regulating the controlled force based on sprung mass velocity error and its derivative as input. The Heaviside step function, which determines the

supply voltage for the MR damper, tracks the controlled force. The suggested integrated model's performance is evaluated in terms of human head accelerations for a variety of road layouts and speeds. The effectiveness of the suggested integrated model with a semi-active seat suspension is validated by comparing its performance to that of a traditional passive seat suspension. In comparison to the passive seat suspension, the simulation findings reveal that the semi-active seat suspension improves ride comfort by successfully lowering head acceleration. In comparison to the passive seat suspension, the semi-active seat suspension greatly enhances ride comfort by minimizing head acceleration.

Agharkakli et al., (2012), the goal of their research is to develop a mathematical model for passive and active suspension systems in a quarter car model. Current automotive suspension systems rely solely on passive components, such as fixed-rate spring and damping coefficients. The quality of a vehicle's suspension system to provide great road handling and improve passenger comfort is often graded. Only passive suspensions provide a middle ground between these two opposing criteria. By directly manipulating the suspension's force actuators, the active suspension has the potential to minimize the traditional design's compromise between handling and comfort. In their study, for a quarter car model, the Linear Quadratic Control (LQR) technique was applied to the active suspensions system. The use of various types of road profiles is used to compare passive and active suspension systems. The controller's performance is compared to that of the LQR controller and the passive suspension system.

Al-zughaibi & Davies, (2015), the goal of their work is to show how to model and manage a quarter-car active suspension system with unknown mass, time delay, and road disturbance. The goal of constructing the controller is to derive a control rule that may significantly increase ride comfort and road disturbance handling by achieving system stability and convergence. This is accomplished by using the Routh-Hurwitz criterion based on defined parameters. Mathematical proof is given to point out the flexibility of the designed controller to make sure the target of design, implementation with the active mechanical system and enhancement dispersion oscillation of the system despite these problems. Simulations were also performed to regulate quarter car suspension, where the results obtained from these simulations verify the validity of the proposed design.

Jamali et al., (2017), the response from a mathematical model that characterizes the transmissibility ratio of the vehicle's input and output is necessary when constructing suitable isolators to reduce undesired vibration in cars. The dynamic behavior performance of passive suspension for a lightweight electric vehicle is reviewed using a MATLAB Simulink model developed in this study. The Simulink model uses a two-degree-of-freedom quarter car model. The model is compared to theoretical graphs of the transmissibility ratios between the amplitudes of the sprung and unsprung masses displacements and accelerations to the amplitudes of the bottom, against frequencies at various damping values. It was found that the frequency responses obtained from the theoretical calculations and the Simulink simulation are similar to one another. Hence, the model could also be extended to a full vehicle model.

2.3.2. Half car model

The half-car model is used to represent the pitch and heave motion of the vehicle body as illustrated in Figure 5. The vehicle body is coupled between the front and the rear wheel through the center of gravity. Initially, a model-based controller is designed with a quarter car model for active suspension. But it cannot be used to measure the pitch motion of wheels.

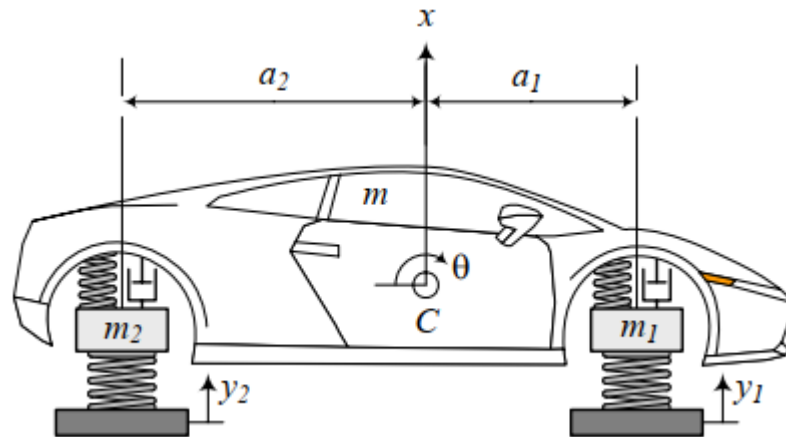


Figure 5. Half car model (Jazar, 2014)

Nassar & Al-Ghanim, (2018), studied half-car suspension of 4 degrees of freedom. Different controllers were developed and implemented within the study like PID, Fuzzy and Fuzzy-PID. Each controller aimed to reduce the deflection and also the acceleration of the mechanical system within the presence of road disturbances. The results of comparisons between passive

and active, linear simulation models was that the performance of the linear model was attuned better than the linear model, and therefore the Fuzzy-PID controller was suggested.

Susatio et al., (2018), designed the active suspension to scale back vibration on the passenger seat causing changes in road-surface shapes or disturbance. PID control is employed within the design of this active suspension, and also the Direct Synthesis method is proposed to tune PID parameters. The direct Synthesis optimizes PID tuning parameters to attain desired output response within the different road surfaces.

Ekoru et al., (2011), describe the design of a nonlinear, half-car active vehicle suspension system with four degrees of freedom (DOF) and a two-loop force/suspension travel PID control system (AVSS). An internal PID suspension travel control loop plus an external PID hydraulic actuator force control loop make up the two-loop system. The performance of the PID-based AVSS is compared to a passive nonlinear half-car suspension system with the same model parameters. The simulation results demonstrated the AVSS's outstanding performance in the presence of deterministic road disturbance.

2.3.3. Full car model

A full car model is shown in Figure 6. A full car model takes account of four vertical motions of wheel, pitch, roll and heave motion of vehicle body. Both quarter car and half car models do not model the actual system for practical application. Hence, an accurate model for the actual system needs a full car model with seven degrees of freedom.

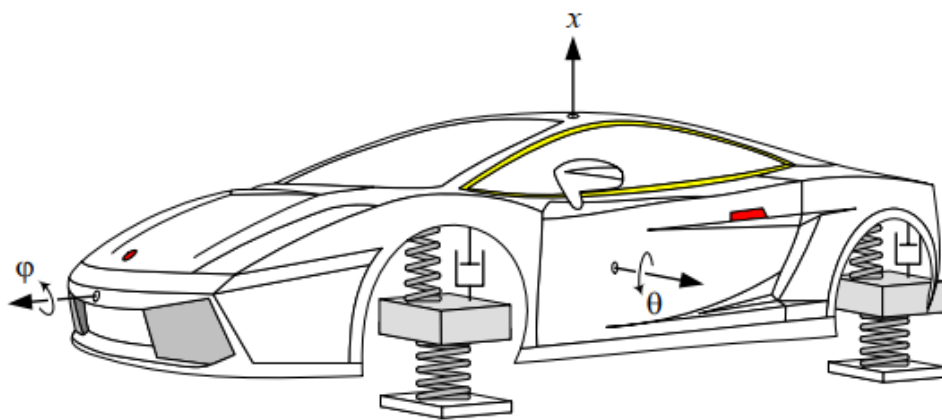


Figure 6. Full car model (Jazar, 2014)

Krishna et al., (2021), the ride comfort and road-holding capabilities of a car with a semi-active suspension system were investigated. To decrease the impact of road disturbance on vehicle performance, the researchers developed a full car semi-active suspension model with a 7 degrees of freedom (7 DOF) system and a fuzzy-logic control mechanism. The heave, roll, and pitch acceleration of the vehicle body around its center of gravity are used to evaluate the semi-active suspension system's performance. The performance of a vehicle equipped with a semi-active suspension system was compared to that of a vehicle equipped with a traditional passive suspension system. The study shows that a semi-active suspension system with a fuzzy-logic controller reduces vibration amplitude by roughly 43% at the vehicle's resonant frequency when compared to a passive suspension system.

Eski & Yildirim, (2009), to avoid vibration caused by road roughness, neural network control was proposed for the entire model-based active suspension. To model and design a neural network controller for active suspension, a seven-degree-of-freedom full-car model was developed. The neural network controller's response was compared to the active suspension's PID control. The simulation results suggest that the neural network controller improves active suspension vibration isolation. Passengers may not notice the transition from a severely rough to a smooth road profile.

Al-Rawashdeh et al., (2019), developed a robust active suspension system for a full car. Except for those related to the car's chassis, all characteristics are considered undetermined, and the center of gravity position is assumed to be fixed. The resulting uncertain system has a large number of unknown parameters, making the robust problem non-convex. To overcome this challenge, a particle swarm optimization technique is designed, culminating in an iterative PSO optimization process. SimMechanics is used to create the nonlinear full car model, as well as to enable future additions to consider more difficult instances, i.e., the passengers, goods, or the like might be uncertain or have their dynamics onboard.

Cheng et al., (2010), to improve the ride quality and reduce suspension deflection, have developed active suspension controls for vehicles with reduced vehicle observers. The proposed fuzzy sliding-mode control (FSMC) includes sliding-mode control (SMC) and fuzzy logic control (FLC), with the SMC reducing vehicle suspension deflection and the FLC

improving passenger ride comfort. In this study, the full-car model of a vehicle is investigated first. The Lyapunov stability study confirms the stability of the fuzzy sliding-mode controlled active suspension system. They also introduce the optimal active suspension control (OASC) scheme to make a comparison. To test the performance, three types of road profiles are used: a bumping road, random white noise, and a power spectral density road profile. Under all of these road profiles, all of the computer simulations show that the proposed FSMC can give the best ride comfort and the lowest suspension deflection of all the controllers tested.

Because of the various control parameters, complex objectives, and stochastic disturbances, designing a suspension system is a challenging task for car designers, (Mitra et al., 1997). Maintaining a high degree of ride comfort and vehicle handling under all driving conditions is always a challenge for vehicles. The objective of their work is to create a MATLAB/Simulink model of a full car to analyze ride comfort and handling. In addition, the detailed study of mathematical modeling with the step-by-step formation of a state-space matrix is to be developed and validation of the Simulink model with the analytical solution of a state-space matrix is to be done elaborately. They conclude that a speed range of 5 to 10 kmph must be an optimum speed to cross the bump without affecting the Human tolerance zone of 0.315 m/s^2 to 0.625 m/s^2 as per ISO standard. Presently, the effect of synthetic type bump is used in a city area, which is more dangerous for human health, as vehicle body acceleration is very high, even at the velocity of 10 kmph. The effect of a bump of the same amplitude nearly does not affect the roll angle and pitch angle of the vehicle. As per the results, spring stiffness and damping coefficient of 25000 N/m and 4000 N-s/m may provide better comfort.

2.4. Control of active suspension system

The objective of control of active suspension is the maximization of driving comfort and stability. Various control methods such as PID control, Linear Quadratic control, Sliding mode control, Fuzzy logic, and neural network have been proposed for active suspension, (Moaaz & Ghazaly, 2019).

While the passive suspension is made up of springs and dampers, the active suspension is made up of sensors, actuators, and controllers. The active suspension's actuator applies forces to the suspension, which improves ride performance and comfort. (Ahmed et al., 2016), presented the preliminary stages of modeling a railway vehicle's active suspension. The system's mathematical model is illustrated, along with the vehicle's early suspension control.

To begin, only the quarter car is considered and tested at a speed of 31 m/s on a random or irregular track input. The suspension becomes an active system after the skyhook control approach is implemented. After then, the active is compared to the passive. Active suspension enhances ride quality by lowering vehicle body accelerations and suspension deflections, according to preliminary studies.

The suspension system plays an important role in isolating the vehicle body from road shocks and vibrations, (Bharali & Buragohain, 2017). Three different active controllers are developed using a three-degree-of-freedom (DOF) quarter car model to evaluate their performance against a passive suspension system. PID controller, Linear Quadratic Controller (LQR), and Fuzzy logic controller are the three controllers designed. In their research, they employed the MATLAB/Simulink program for simulation, and the results reveal that the active suspension system outperforms the passive suspension system in terms of maximum amplitude, suspension deflection, body acceleration, and settling time. In addition, when compared to other control methods and passive as well, the result of the comparison shows that fuzzy logic control has higher working and stability.

Bhangal & Chaudhary, (2018), designed a linear active suspension system because it contains all of the fundamental performance parameters, including body deflection, suspension deflection, and body acceleration. Two control techniques PID and LQR are chosen to overturn the vibrations of the system. They have been made a comparison between passive and active suspension system using PID and LQR control technique with road disturbance as input. MATLAB/Simulink was used to conduct a simulation analysis of the active suspension system. According to the simulation, LQR control outperforms both PID control and a passive suspension system in terms of reducing vibrations.

2.4.1. Proportional integral derivative (PID) control

The most common controller used in car manufacturing is the PID due to its non-complex mechanism and the ability to tune the gains, controller, (Bharali & Buragohain, 2016). However, excess tuning of PID gains will make the system unstable.

Erenoglu et al., (2016), study, a design methodology is introduced that blends the classical PID and the fuzzy controllers in an intelligent way and thus a new intelligent hybrid controller has been achieved. Basically, in this design methodology, a combining mechanism based on a

specific function of actuating error has been used to merge the traditional PID and fuzzy controllers. Furthermore, the blending mechanism is induced by an intelligent switching method that determines the precedence of the two controller elements, namely the classical PID and the fuzzy constituents. When compared to pure classical PID or pure fuzzy controller applications, simulations on various processes employing the new hybrid fuzzy PID controller give 'superior' system responses in terms of transient and steady-state performances. The controller parameters are all tuned with the aid of a genetic search algorithm.

Kumar & Vijayarangan, (2007), conducted an experimental study on PID-controlled active suspension to improve the ride comfort of light passenger vehicles. The experiment was carried out on a quarter car test rig for bumpy road input. The result of the experiment shows that active suspension improves ride comfort by more than 50% compared to passive suspension. The road handling ability was also greatly improved in active suspension compare to passive suspension.

Shafiei, (2022), describes how to simulate a quarter vehicle model with an active suspension system using the MATLAB Simulink software. The initial goal is to develop the proper control system for a vehicle's active suspension system and to examine the closed-loop control system for hydraulic cylinders and servo valves in more detail. The second goal is to improve ride comfort and dependably maintain a vehicle's position on the road by correctly tuning the PID settings for an active suspension system to minimize the displacement and acceleration of the vehicle body. To achieve the desired passenger comfort while riding and the car's consistent ride-holding, the control system is tweaked via a PID controller utilizing the Ziegler-Nichols approach via the Control System Designer app. The simulation findings shown that the car's body displacement and acceleration have a smaller amplitude for an active suspension system than for a passive suspension system.

2.4.2 Fuzzy logic control

Zheng, (2010), developed a fuzzy logic controller for an active vehicle suspension. A suspension system has been modeled as a two-degree-of-freedom quarter-car model to represent passive and active suspension systems. MATLAB was used to simulate the suggested system and compare it to a passive suspension. According to the simulation results,

a logic-based suspension system improves ride comfort and road handling by reducing vehicle body acceleration and tire deflection.

Cheng & Li, (2007), designed a fuzzy logic control for an active suspension system to improve the riding comfort and minimize the suspension deflection. The optimum parameters of fuzzy logic were designed by minimizing fitness function using an evolutionary program to meet the desired performance of a full car model. The bumper road and white noise random road were modeled to test the performance of the designed controller for active suspension. The result of the simulation indicates that the designed controller improves ride comfort and road handling significantly.

Salem & Aly, (2014), proposed an active suspension system to improve ride comfort. The two-degrees-of-freedom (DOF) system is designed and engineered based on a four-wheel independent suspension concept to simulate the activities of a vehicle's active suspension. The goal of their research was to show how the fuzzy logic technique will be used to operate a continuously dampening vehicle suspension system. The ride comfort is improved by reducing the body acceleration caused by the car body when there are road disturbances from both smooth and rough roads. It also describes the models and controllers used in the study and discusses vehicle response results obtained from various road input simulations. In the end, a comparison of active suspension Proportional Integration derivative (PID) control and fuzzy control is shown using MATLAB simulations. Simulation results showed Fuzzy control is very effective and can be used in vehicles.

Palanisamy & Karuppan, (2016), the main objective is to investigate the performance of active suspension system, using suspension deflection of the vehicle body as the principal criterion of control and fuzzy-logic as the control scheme. And describes the application of fuzzy logic technique to the control of a continuously damping automotive suspension system. The proposed fuzzy control method for the vehicle's suspensions is simple to modify for control tasks due to the controller's inherent capacity to describe dynamics. The proposed fuzzy control strategy for the Active Suspension System's simulation results have been validated as being feasible.

2.4.3 Intelligent controller

Dhanaraju et al., (2015), proposed an intelligent approach (Fuzzy logic) for the design of a PID controller for better disturbance rejection. The proposed PID controller is designed using Ziegler-Nichols's tuning algorithm for the rejection of different disturbances. The proposed intelligent controller has got so many advantages/features over the conventional methods. Sudden ability to reject non-linear disturbances which occur in the system during operation, speed of operation and PID gains are altered online in accordance with the disturbances to reject. To show the efficacy of the proposed method a liquid control of the process tank is considered and an intelligent PID controller is designed. The designed intelligent controller is simulated under different disturbances using MATLAB/Simulink. The results are successfully verified.

Munawwarah & Yakub, (2021), to improve vehicle ride comfort and road handling performance, proposed an integrated chassis control of the quarter and half car active suspension systems. The PID-LQR and Fuzzy-PID integrated controls were designed to improve driving comfort by maintaining the vehicle wheels in contact with the road surface. The PID control, which is extensively used in automobile manufacturing, was chosen as a controller benchmark. Multiple road conditions such as speed bumps of different heights, $0.1 \text{ m} < h < 0.3 \text{ m}$ and widths, $0.3 \text{ m} < d < 0.5 \text{ m}$ were used in the system for analysis. In comparison to PID-LQR, Fuzzy-PID had the smallest peak amplitude and reached the stability state in the shortest amount of time when compared to the controller benchmark. They indicate that vehicle ride comfort and road handling performance can be optimized using the proposed Fuzzy-PID control.

Samsuria et al., (2021), propose an optimal control technique of an active suspension system using two degrees of freedom quarter car model. The primary goal of the study is to evaluate the performance of state feedback controllers based on the Linear Quadratic Regulator (LQR) and Sliding Mode Control (SMC), which are optimized using the Particle Swarm Optimization (PSO) algorithm for utilization of the active suspension system. The controllers' goal is to increase ride comfort while limiting the suspension travel and road disturbance-induced wheel deflection. Based on the road profile that the vehicle will travel through, the performance of the SMC-based-PSO controller is compared to the LQR-based-PSO controller and the current conventional suspension system. Simulations are conducted and tested under the input profile for a double bump road in order to assess the effectiveness of the suggested controller. The

results definitely indicate that the SMC technique outperforms the LQR and conventional suspension systems in achieving improved ride comfort. Simulation by (Simulink MATLAB) is carried out to illustrate system control and performances.

Tatsuo et al., (2015), presented a robust control approach for an active suspension that improves ride comfort and driving stability. The vertical and pitch vibrations of the car body affect the ride comfort. Loop shaping is applied for vertical and pitch motions based on ISO2631-1 to enhance ride comfort more effectively. On the other hand, the fluctuation of a vertical force of wheels, which is affected by pitch motion, is used to determine driving stability. To consider both of them more realistically, a half car model including vertical and pitch motions is analyzed. For practical application, considering uncertain parameters is important. Their paper focuses on the car body mass. With polytopic representation, the controller is designed to ensure robust stability for uncertain parameters. The LQ controller is utilized, which can take into account both performance and input energy.

2.4.4. Fuzzy-PID controller

The Fuzzy-PID controller is a combination of the conventional PID controller and the algorithm of fuzzy control designed to enhance ride comfort in vehicle suspension systems, (Munawwarah & Yakub, 2021).

(Lan & Ni, 2013), developed a fuzzy-PID controller for a half car with an active suspension system to improve ride comfort. The proposed controller's performance was verified in MATLAB/Simulink by comparing it to the passive control approach. According to the simulation results, the proposed fuzzy-PID controller improves the ride comfort performance of the vehicle's active suspension system by drastically reducing body acceleration and pitch angle.

Galab & Hurel, (2015), the fuzzy self-tuning PID controller is designed to control the active suspension system for the quarter car model. To limit the suspension working space of the sprung mass and its change rate and to maximize driver comfort, a fuzzy self-tuning is utilized to generate the ideal control gain for PID controller (proportional, integral, and derivative gains). The outcomes of the fuzzy self-tuning PID controller used in the active suspension system are visually displayed, along with comparisons to the PID and passive systems. It has

been discovered that employing fuzzy self-tuning effectively makes it possible to adjust the PID controller's gain parameters.

Shen et al., (2010), the mechanical, control, electronic, and hydraulic subsystems were taken into consideration when designing the active suspension. Electro-hydrostatics actuator (EHA) which is widely used in Power-By-Wire (PBW) system is applied to active suspension in this article instead of traditional hydraulic valve components. After the modeling, MATLAB is used to create the computer simulations and design the active suspension fuzzy PID controller. After that, better results were obtained by simulating the quarter-car active suspension system.

Divekar & Mahajan, (2017), focused on the development of modeling for 2 DOF and control algorithm in order to adjust damping rates of damper according to road disturbances. Proposed advanced Fuzzy Logic Controller (FLC) to minimize sprung mass displacement and Suspension Working Space (SWS) using MATLAB Simulink.

2.5. Hydraulic actuator

Wang et al., (2017), Any vehicle's suspension system is an important part since it conveys force and torque from the tire to the frame, ensuring that the ride is comfortable and the handling is stable. A seven-degree-of-freedom active suspension system model using electrohydraulic actuators is developed to address the issue of active suspension control. To regulate the hydraulic actuators for the active suspension, a fractional PID controller is employed. With the help of simulations of prototype vehicles and field tests on the road, the controller's precision and effectiveness are confirmed. Results indicate that the active suspension system performs better than conventional suspension systems. The reductions in vertical acceleration, pitch angle acceleration, and roll angle acceleration have a significant positive impact on ride comfort and handling stability.

Bello et al., (2015), using a four degree of freedom, nonlinear, half vehicle active suspension system model with hydraulic actuator, a double loop PID control of generated force and vehicle suspension parameters is designed. An inner hydraulic actuator PID force control loop and an exterior suspension parameter PID control loop make up the loops arrangement. A simulation study was carried out and comparisons between a nonlinear passive system and a nonlinear active PID base suspension system were made. The results obtained indicate that the

active system improves performance more than the passive system, but at the sacrifice of cost and power consumption.

Ghazali et al., (2017), developed mathematical modelling and designed an intelligent control strategy. The research started with the creation of a mathematical model based on the operational theory of the passive and active suspension systems. The suspension system was made active by the integration of an electro-hydraulic actuator. The model examined using the MATLAB and Simulink programs. Finally, the active suspension system is constructed with a proportional-integral-derivative (PID) controller and a fuzzy logic intelligent controller. As a result, the performance of the quarter-car model has been improved by creating a nonlinear active suspension system with an electro-hydraulic actuator. If not, the suspension control system may support the vehicle's body and provide a comfortable ride. The performance enhancements will improve both systems' road handling and ride comfort.

2.6. Research gap

From the literature review, it is clear that the active suspension based on the full car model is not focused on by the researchers. The pitch and roll motion of the vehicle can be measured only by using the full car model. The nonlinear hydraulic actuator dynamics due to control force are not much focused either. The main difficulties in fuzzy controller are proper selection of rules base and appropriate range of membership function which normally depends on knowledge of designer. In this research work, an intelligent Fuzzy-PID controller is proposed to overcome the issues in fuzzy logic. The active suspension systems are very good to bring significant performance of vibration isolation.

CHAPTER THREE: MATERIALS AND METHODS

3.1. Introduction

Each of the specific tasks to be performed in the research work is described here. Methods Methodology, data collection and data analysis methods are discussed in this chapter.

3.2. Methods

The whole research work is classified into three different divisions namely system modeling, controller design and simulation. Before and after title selection and problem identification, the previous research work was carried out on the quarter car, half car and full car model active suspension system. And also linear control, nonlinear control and intelligent control are reviewed for the active suspension. In the first division of this research, mathematical modeling for quarter car, half car and full car models was stated including actuator dynamics. After the mathematical model is analyzed based on Newton's law of motion the next task is designing the controllers. For the comparison, two types of controllers are selected and designed.

In the third division of this research work, the computer simulation is carried out and the result are discussed. There are various types of simulations depending upon the type of model. Physical modeling involves experimenting with physical prototypes of real-world systems. In this case, the actual input data (scaled) is applied to the model and the output data is compared to the real system. Sometimes electrical analog systems are also used for this type of simulation.

Another type of simulation is numerical simulation of mathematical models. This is very common in systems approach. Since the development of digital computers, this type of simulation has become popular because it is easy to develop and modify. Computers are used to imitate or simulate the operations of various kinds of real-world facilities or processes. The facility or process of interest commonly referred to as a system, and scientific study of it often requires several assumptions about how it works. These assumptions usually take the form of mathematical or logical relationships and constitute models used to understand how those systems work.

3.3. Data Collection

The model is developed in MATLAB/Simulink and estimated and validated using data collected from both primary and secondary sources.

3.4. Data Analysis

After reviewing the literature and the best concepts selected, the quarter car model, half-car model and full car model were numerically analyzed. Computer simulation carried out using MATLAB/Simulink software.

3.5. Materials

3.5.1. MATLAB

MATLAB is a high-performance technical computing language, (Davis & Sigmon, 2010). Its visualization, integrates, computation and programming in a user-friendly interface where problems and solutions are represented in a familiar mathematical notation. Typical uses include;

- Math and computation.
- Algorithm development.
- Modeling, simulation, and prototyping.
- Data analysis, exploration, and visualization.
- Scientific and engineering graphics.
- Application development, including graphical user interface building.

MATLAB is a cooperative system with an array as its basic data element that doesn't need dimensioning. This permits you to solve many technical computing problems in a fraction of the time it takes to build a program in a scalar non-interactive language like C or FORTRAN, especially ones using matrix and vector formulations.

The name MATLAB stands for matrix laboratory. In industry, MATLAB is the tool of choice for high-productivity research, development, and analysis. Toolboxes are a type of application-specific solution available in MATLAB. The toolbox, which is very important to most MATLAB users, allows you to learn and apply special technologies. The toolboxes are a comprehensive collection of MATLAB functions (M-fi files) that extend the MATLAB environment to solve specific classes of problems. Areas, where toolboxes are available,

contain control systems, signal processing, fuzzy logic, neural networks, wavelets, and simulation.

3.5.2. Simulink

Simulink® is a software package used for modeling, analyzing, and simulating a wide variety of dynamic systems. Simulink provides a graphical interface for constructing the models, (Chaturvedi, 2010). It features a library of standard components that enables creating block diagrams easier and faster. Simulink is an advanced learning tool for simulating real-world operational problems, as simulation algorithms and parameters can be changed between simulation and visual results. It is particularly useful for studying the effect of nonlinearities on the behavior of the system.

Features of Simulink

- A complete library for crating linear, nonlinear, discrete, or multiple input/output systems.
- Mask facility to create custom blocks.
- Unlimited hierarchical model structure.
- Connections of scalar and vector.
- Simulations of interaction with the live display.
- One can easily perform a what-if analysis can be easily performed by changing the parameters of the model.
- Simulink block library can be extended with special block sets.
- Custom blocks and block libraries can be created by using your icons and user interfaces from MATLAB®, FORTRAN, or C code.

In general, Simulink is a visual programming interface designed to intuitively create simulation systems. Provides a way to numerically solve equations using a graphical user interface rather than code.

Models include **signals**, **blocks** and **annotation** on a background:

- **Blocks** are mathematical functions and can have different numbers of inputs and outputs.

- **Signals** are lines connecting blocks that pass values between blocks. Signals are different types of data, for example, numbers, matrices, or vectors. Signals can be **flagged**.
- **Annotations** of images or text can be contributed to the model, and while they aren't used in the computations, they can help others comprehend the model's design decisions.

Simulink toolbar

The toolbar is located above the main canvas of a Simulink model.



Model settings

All of the setting related to how to numerically solve the equations of the model are found in “model configuration parameters” cog.

Run model

To run the simulation, press the green arrow. Be aware there are different modes e.g. ‘Normal’.

If you are working with hardware the mode will be ‘External’.

The textbox is indicates time you want to run the simulation.

Build model

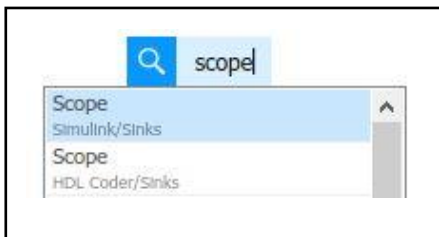
If your model is interacting with hardware, you will need to build the model before it can run. The current

Working with blocks

The library browser or the quick search are the two ways to add blocks to a model.



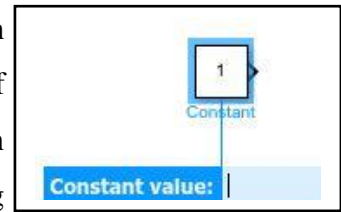
Library Browser: Shows all Simulink blocks, organized by folders like 'Math Operations' and 'Signal Routing.' On the upper left, there is a search bar. Drag and drop library blocks into your model canvas.



Quick Search: directly search for blocks by single-clicking on the background of your model and typing in a search term. Select a block from the search results to quickly add it to your model.

The automatic block input box

When adding a block to a model for the first time, the most common parameter will often pop up automatically for a value to be specified. If you add a gain block, for example, it will prompt you to enter the gain value. Interacting with this can help you save time by avoiding having to enter the block parameters menu.



Positioning blocks

- Clicking and dragging can be used to move blocks.
- Connect blocks by clicking and dragging the output of one block into the input of another block.
- Once a signal connects two blocks, it can be clicked and dragged to be repositioned.
- To create a branch from an existing signal, hold ctrl while clicking and dragging.
- Clocks can also be rotated/flipped for better positioning by selecting "rotate and flip" from the right-click menu.

Table 1 show Simulink libraries which is used mainly for building Simulink block

Table 1. Overview of Simulink libraries

Library	Types of blocks	Example of block
Source	Provide inputs to your model	Constant, Sine Wave, Step
Sink	Provide ways to view or export data	Scope, XY Graph, To Workspace
Math operations	Common mathematical functions to apply data	Add, Divide, Abs
Ports and subsystems	Create different subsystems (resettable, triggered, etc.)	Subsystem, Enable Ports, Inputs And Outputs: In1 And Out1
User-defined functions	Implement custom functions	Fcn, MATLAB Fcn
Lookup table	Use functions defined as discrete data	1-D Lookup Table
Signal routing	Organize signal from blocks	Mux, Bus Creator, Goto, Switch

Continuous	Systems with continuous states	Integrator, Derivative
Discrete	Systems with discrete states	Unit Delay, Discrete Derivative
Logical and bit operations	Boolean operators for comparisons	Compare to Zero, Logical Operator

3.6. Methodology

The overall research methodology for intelligent fuzzy-PID control of hydraulic actuated active suspension is depicted in a flowchart in Figure 4. Initially, a literature survey is carried out on active suspension based on the quarter car model, half-car model and full car model.

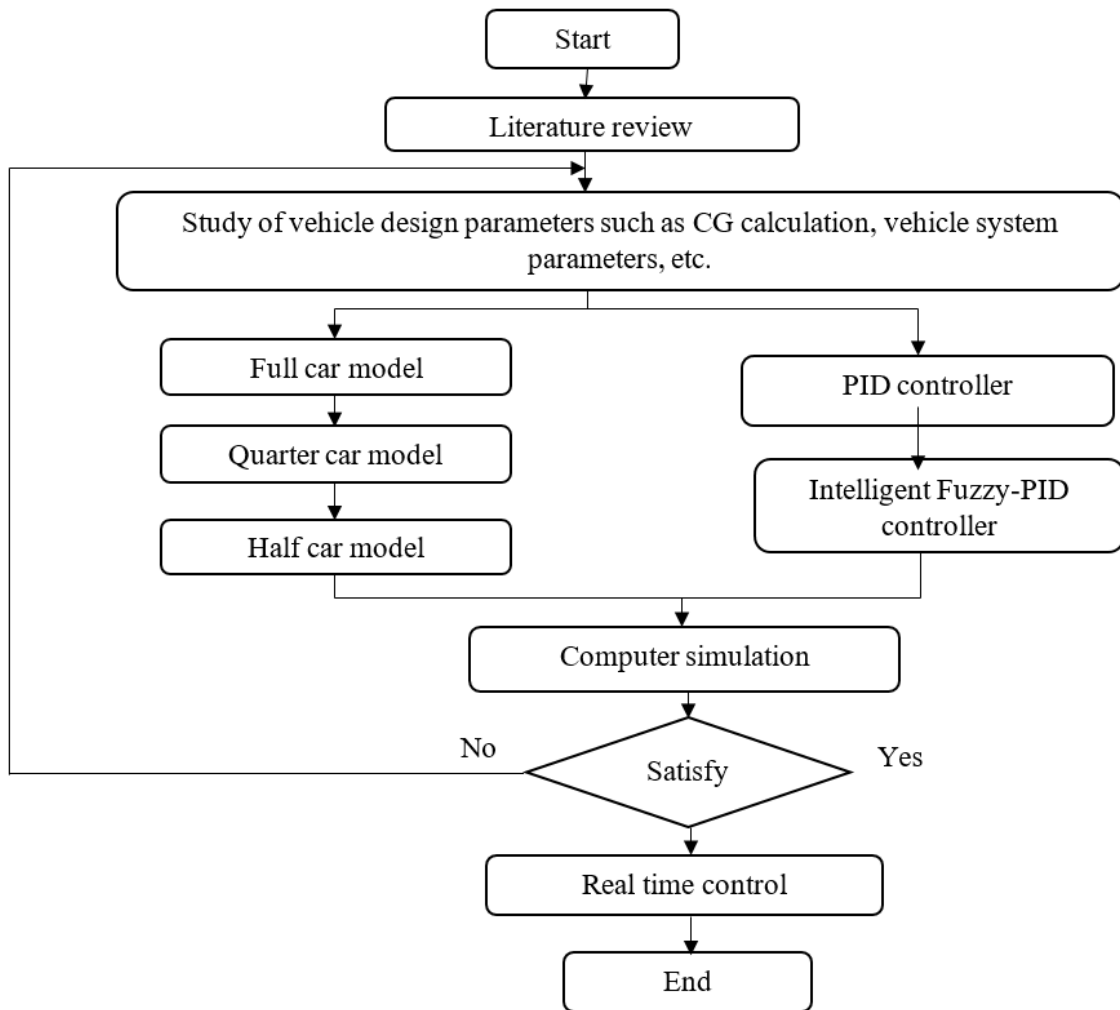


Figure 7. Flowchart of proposed research work

Initially, the governing equation for the quarter car model is established without considering the effect of actuator dynamics. Then the hydraulic actuator model is developed for the quarter car model by considering the effect of actuator dynamics. The half-car model for hydraulic actuated active suspension is developed based on a quarter car model with actuator dynamics. Finally, 7 degrees of freedom full car model is developed based on the quarter car and half car model of vehicle.

The PID controller is developed for the quarter car, half car and full car model-based hydraulic actuated suspension system. The performance of the developed PID controller is compared with that of the passive.

The computer simulation of full model-based hydraulic active suspension is carried out separately for the PID controller and intelligent Fuzzy-PID controller. The parameters of the intelligent Fuzzy-PID controller are tuned until it gives satisfying performance for the hydraulic actuated active suspension system. Finally, the performance of an intelligent Fuzzy-PID controller is verified experimentally by using SIL simulation.

The results of the simulation should indicate that an intelligent Fuzzy-PID controller outperforms the existing conventional controllers with regard to body acceleration, body displacement, roll angle and pitch angle.

CHAPTER FOUR: MATHEMATICAL MODELING FOR ACTIVE SUSPENSION SYSTEM

4.1. Introduction

The main objective of this chapter is to develop a mathematical model of active suspension based on the full car model. Initially, this chapter discusses the development of a quarter car model of active suspension with 2 DOF systems. Then, the development of half-car model active suspension with 4 DOF systems. Finally, the mathematical model of the full car model is developed based on a quarter car and half car model of active suspension with 7 DOF systems.

4.2. Quarter car model for the active suspension system

A quarter car model for an active suspension system consists of 1/4th of vehicle mass, spring, damper and actuator as shown in figure 8. The quarter car model is a simplified model of a car with one wheel using which, the vertical motion of the car body is measured. Quarter car simulation assumptions are:

- There is no rotational motion in the wheel or body, therefore the tire is modeled as a linear spring with no damping.
- The spring and damper have a linear behavior.
- The tire is always in touch with the road surface, and the effects of friction are neglected, so residual structural damping is not taken into account while modeling the vehicle.

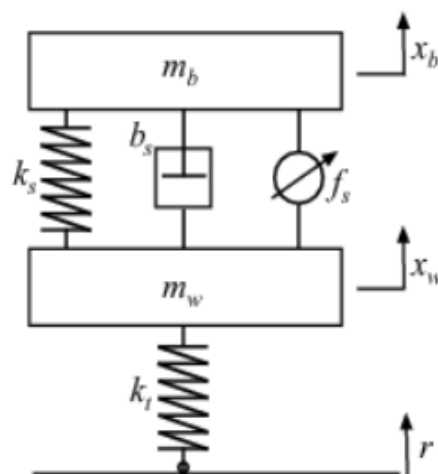


Figure 8. Quarter car model of the active suspension system

We may represent the vertical vibration of a vehicle using a quarter car model made of two solid masses m_b (kg) and m_w (kg) denoted as sprung and unsprung masses, respectively. The sprung mass m_b represents 1/4 of the body of the vehicle, and the unsprung mass m_w represents one wheel of the vehicle. A spring of stiffness k_s (N/m), and a shock absorber with viscous damping coefficient b_s , (Ns/m) support the sprung mass and are called the main suspension. The unsprung mass m_w is in direct contact with the ground through a spring k_t (N/m), representing the tire stiffness. The actuation force f_s (N) is acting between the sprung and unsprung masses. The state variables x_b (m) and x_w (m) are the vertical displacements of the sprung and unsprung masses, respectively and r is the vertical road profile.

The equation of motion based on Newton's second law of motion that is related to the body mass is given by;

$$m_b \ddot{x}_b = -k_s(x_b - x_w) - b_s(\dot{x}_b - \dot{x}_w) + f_s \quad \text{Eq. (1)}$$

$$\ddot{x}_b = \frac{1}{m_b} [-k_s(x_b - x_w) - b_s(\dot{x}_b - \dot{x}_w) + f_s] \quad \text{Eq. (2)}$$

The equation of motion based on Newton's second law of motion that is related to the wheel mass is given by;

$$m_w \ddot{x}_w = k_s(x_b - x_w) + b_s(\dot{x}_b - \dot{x}_w) - k_t(x_w - r) - f_s \quad \text{Eq. (3)}$$

$$\ddot{x}_w = \frac{1}{m_w} [k_s(x_b - x_w) + b_s(\dot{x}_b - \dot{x}_w) - k_t(x_w - r) - f_s] \quad \text{Eq. (4)}$$

4.2.1. Actuator model for quarter car model

Electro-hydraulic systems are chosen as actuators for active control, and their dynamics are taken into consideration.

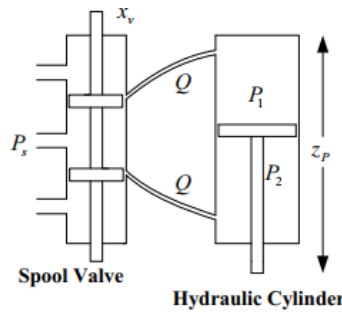


Figure 9. The schematic diagram of the electro-hydraulic actuator

The modeling of the hydraulic actuator with a four-way piston valve arrangement, (Mittal & Bhandari, 2015), is shown in Figure 9. The force exerted by the actuator is $f_s = A_P P_L$. Where A_P and P_L represent the area of piston and pressure drop inside cylinder respectively.

The rate of change of pressure drop is expressed as in equation (5)

$$\frac{V_t}{4\beta_e} P_L = C_{tp} P_L - A_P (x_b - x_w) + Q \quad \text{Eq. (5)}$$

In the above equation, P_L is a pressure drop, V_t indicates the volume of an actuator, β_e is the effective bulk modulus, C_{tp} is piston coefficient and Q is the hydraulic flow which is derived from Equation (6).

$$Q = C_d w x_v \sqrt{\frac{1}{\rho} [P_s - \text{sgn}(x_v) P_L]} \quad \text{Eq. (6)}$$

Where C_d is the coefficient of discharge, w is the slope of a spool valve, x_v is the displacement of a spool valve, ρ is the density of the fluid and P_s is the pressure source. Hence, the actuator force developed by the hydraulic actuator is described by the nonlinear equation (7).

$$f_s = -\beta P_L - \alpha A_P^2 (x_b - x_w) + \gamma A_P x_v \sqrt{P_s - \frac{\text{sgn}(x_v) P_L}{A}} \quad \text{Eq. (7)}$$

Where $\alpha = 4\beta_e/V_t$, $\beta = \alpha C_{tp}$, $\gamma = \alpha C_d w \frac{1}{\rho}$ Kc is the gain of conversation and u is the voltage of the servo valve, (Chantranuwathana & Peng, 2014). The parameters and values assigned for the quarter car active suspension system with the hydraulic actuator, (Shafie et al., 2015) are represented in Table 2.

Also, spool valve velocity, \dot{x}_v is given in equation (8), as a *linear* filter. Where τ is the time constant and k_c denotes the servo valve gain.

$$\dot{x}_v = \frac{1}{\tau} [-x_v + k_c u] \quad \text{Eq. (8)}$$

The hydraulic actuator as shown in figure 10 is modeled in Simulink using equation (7).

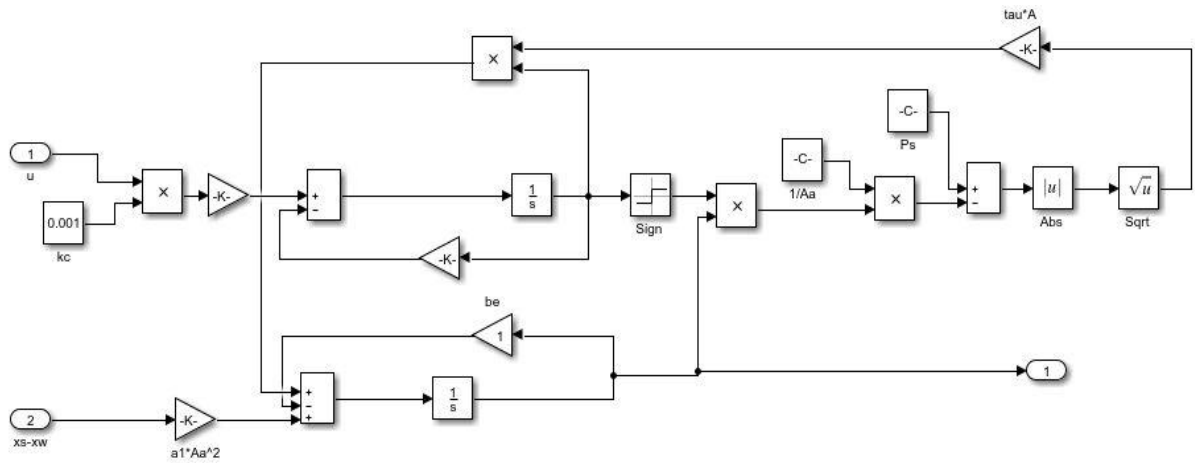


Figure 10. Hydraulic actuator Simulink model

The data in Table 2 indicates the parameters of the quarter car model and values used in the simulation.

Table 2. Quarter car suspension system parameters, (Agharkakli et al., 2012)

<i>Parameters</i>	<i>Symbols</i>	<i>Units</i>	<i>Value</i>
<i>Sprung masses</i>	m_b	<i>Kg</i>	290
<i>Unsprung masses</i>	m_w	<i>Kg</i>	59
<i>Spring of stiffness</i>	k_s	<i>N/m</i>	16,812
<i>Tire stiffness</i>	k_t	<i>N/m</i>	190,000
<i>Damping constant</i>	b_s	<i>Ns/m</i>	1000
<i>Hydraulic pressure</i>	P_s	<i>Pa</i>	10,342,500
<i>Actuator ram area</i>	A_p	m^2	3.35×10^{-4}
<i>Conversion gain</i>	K_c	<i>m/v</i>	0.001
α	A	<i>N/ms</i>	4.515×10^{13}
γ	γ	$N/m^{5/2}kg^{1/2}$	1.545×10^9
β	β	s^{-1}	1
<i>Time constant</i>	τ	s^{-1}	0.003

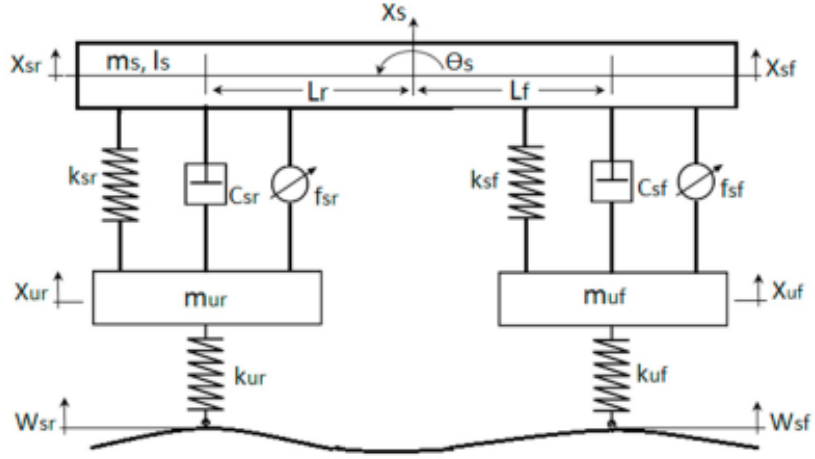


Figure 12. Half car model of the active suspension system

The dynamics equation of the vertical moment at an unsprung mass of vehicle front and rear is described by the equation (9) and (10) respectively.

$$m_{ur}\ddot{x}_{ur} - k_{sr}(x_{sr} - x_{ur}) - C_{sr}(\dot{x}_{sr} - \dot{x}_{ur}) + k_{ur}(x_{ur} - w_{sr}) + f_{sr} = 0 \quad \text{Eq. (9)}$$

$$m_{uf}\ddot{x}_{uf} - k_{sf}(x_{sf} - x_{uf}) - C_{sf}(\dot{x}_{sf} - \dot{x}_{uf}) + k_{uf}(x_{uf} - w_{sf}) + f_{sf} = 0 \quad \text{Eq. (10)}$$

The governing equation for the vertical motion at the vehicle body can be obtained by applying Newton's second law of motion.

$$m_s\ddot{x}_s + k_{sr}(x_{sr} - x_{ur}) + C_{sr}(\dot{x}_{sr} - \dot{x}_{ur}) + k_{sf}(x_{sf} - x_{uf}) - C_{sf}(\dot{x}_{sf} - \dot{x}_{uf}) - f_{sf} - f_{sr} = 0 \quad \text{Eq. (11)}$$

The pitch motion of a vehicle obtained by taking moment about the pitch axis can be represented as,

$$I_s\ddot{\theta}_s + L_s[k_{sr}(x_{sr} - x_{ur}) + C_{sr}(\dot{x}_{sr} - \dot{x}_{ur}) - f_{sr}] + L_f[k_{sf}(x_{sf} - x_{uf}) - C_{sf}(\dot{x}_{sf} - \dot{x}_{uf}) - f_{sf}] = 0 \quad \text{Eq. (12)}$$

The equations (11) and (12) are further substituted by the following constraints,

$$x_s = \left(\frac{L_f x_{sr} + L_r x_{sf}}{L} \right) \quad \text{Eq. (13)}$$

$$\theta_s = \left(\frac{x_{sf} - x_{sr}}{L} \right) \quad \text{Eq. (14)}$$

$$\begin{aligned}
\ddot{x}_{sf} = & \left[\frac{-1}{m_s} + \frac{L_f L_r}{I_s} \right] [C_{sr}(\dot{x}_{sr} - \dot{x}_{ur}) + k_{sr}(x_{sr} - x_{ur}) - f_{sr}] \\
& - \left[\frac{1}{m_s} - \frac{L_f^2}{I_s} \right] [C_{sf}(\dot{x}_{sf} - \dot{x}_{uf}) + k_{sf}(x_{sf} - x_{uf}) - f_{sf}]
\end{aligned} \tag{Eq. (15)}$$

$$\begin{aligned}
\ddot{x}_{sr} = & - \left[\frac{1}{m_s} + \frac{L_r^2}{I_s} \right] [k_{sr}(x_{sr} - x_{ur}) + C_{sr}(\dot{x}_{sr} - \dot{x}_{ur}) - f_{sr}] \\
& + \left[\frac{-1}{m_s} + \frac{L_f L_r}{I_s} \right] [k_{sf}(x_{sf} - x_{uf}) + C_{sf}(\dot{x}_{sf} - \dot{x}_{uf})] - f_{sf}
\end{aligned} \tag{Eq. (16)}$$

Where:- f_{sf} = Force at front actuator, f_{sr} = Force at rear actuator, \ddot{x}_s = Sprung mass acceleration at Vehicle body, \dot{x}_{sf} = Sprung mass velocity at front body, \dot{x}_{uf} = Unsprung mass velocity at front body, x_{sf} = Sprung mass displacement at front body, x_{uf} = Unsprung mass displacement at front body, \dot{x}_{sr} = Sprung mass velocity at Rear body, \dot{x}_{ur} = Unsprung mass velocity at Rear body, x_{sr} = Sprung mass displacement at Rear body, x_{ur} = Unsprung mass displacement at Rear body, θ_s = Rotary angle at vehicle's CG point, \ddot{x}_{uf} = Unsprung mass acceleration at front body, \ddot{x}_{ur} = Unsprung mass acceleration at Rear body, \ddot{x}_{sr} = Sprung mass acceleration at Rear body, w_{sf} = Road Input to Front wheel, w_{sr} = Road Input to Rear wheel.

Table 3. Half car model for active suspension system parameters, (Gandhi et al., 2017)

Symbol	Unit	Value	Definition
m_s	kg	1794.4	Vehicle body sprung mass
C_{sf}	Ns/m	1190	Front suspension damper co-efficient
C_{sr}	Ns/m	1000	Rear suspension damper coefficient
k_{sf}	N/m	66824	Stiffness coefficient at the front suspension
k_{sr}	N/m	18615	Stiffness co-efficient at Rear suspension
I_s	kgm ²	3443.05	Moment of Inertia
m_{uf}	kg	87.15	Unsprung mass at front vehicle body
m_{ur}	kg	140.14	Unsprung mass at rear vehicle body
l_f	m	1.27	Inter-space between Front axle and Vehicle CG point
l_r	m	1.72	Inter-space between Rear axle and Vehicle CG point
L	m	2.99	Inter-space between Front and rear axle
$k_{uf} = k_{ur}$	N/m	101115	Stiffness co-efficient at front and rear wheel

4.3.1. Actuator model for half car Model

The nonlinear hydraulic actuator force equation used in the quarter car is also used for the half car model. The actuator force developed by the hydraulic actuator for the half-car model is described by the equation (17) and (18).

$$f_{sf} = -\beta f_{sf} - \alpha A^2 (x_{sf} - x_{uf}) + \gamma A x_v \sqrt{P_s - \frac{\text{sgn}(x_v) f_{sf}}{A}} \quad \text{Eq. (17)}$$

$$f_{sr} = -\beta f_{sr} - \alpha A^2 (x_{sr} - x_{ur}) + \gamma A x_v \sqrt{P_s - \frac{\text{sgn}(x_v) f_{sr}}{A}} \quad \text{Eq. (18)}$$

4.3.2. Half car Model in Simulink

Half car Simulink model illustrated in figure 13.

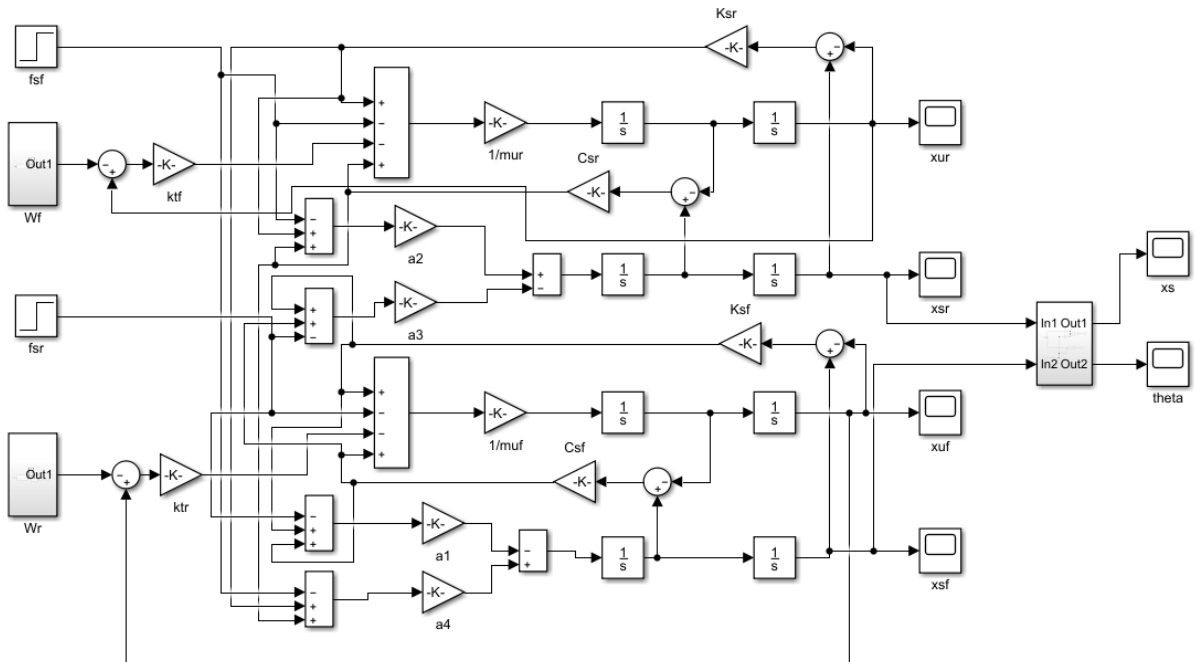


Figure 13. Half car active suspension Simulink model

4.4. Full car model

A seven-degrees of freedom (7-DOF) full vehicle active suspension system model is constructed as shown in Figure 14. For the complexity of the various components of the vehicle, a suspension system is an object with strong nonlinearity and uncertainty. Thus, the accurate mathematical model of the full car system is difficult to describe. However, the

vertical response can be modeled with a relatively simple set of dynamic equations by simplifying the system into mass blocks and spring-damper systems.

A vehicle system can be decomposed into four parts: sprung mass, suspension system, unsprung mass and road inputs. For sprung mass, it contains the vehicle body, chassis and other elements like seats and passengers. In the simplified model, it is treated as a mass block with geometric parameters. For the suspension system, we neglect its connecting structure and simplified it into spring-damper blocks. The four suspension subsystems can be established separately for modern independent suspension. For unsprung mass, it mainly represents the four wheels, which are simplified into mass blocks and considered its elasticity as a spring with large stiffness

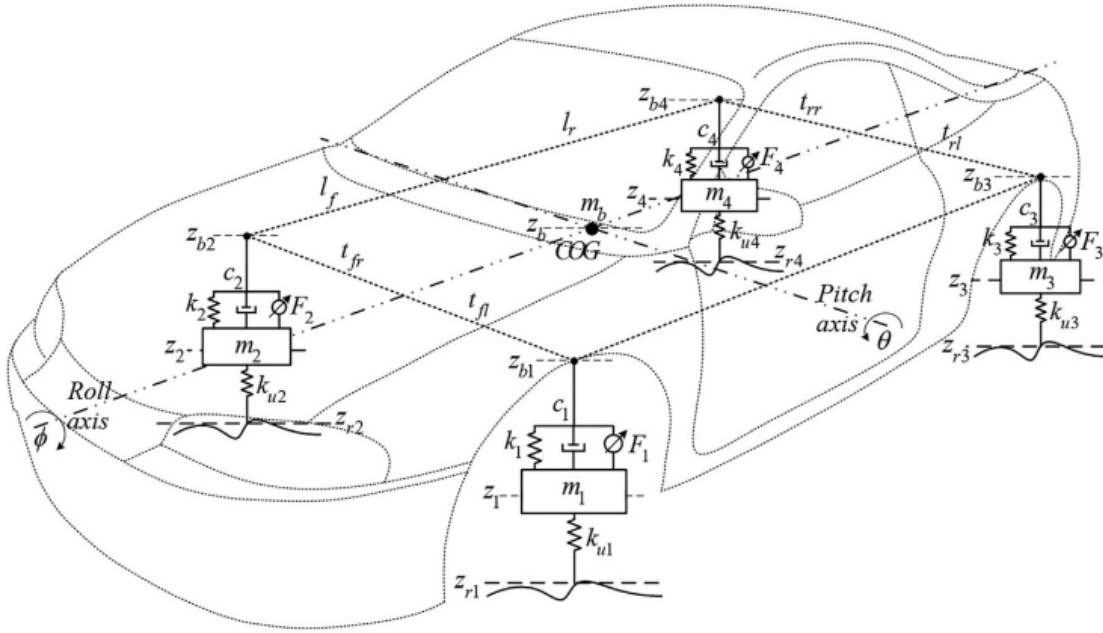


Figure 14. Full car model of active suspension system

Full vehicle suspension system parameters are given in table 4. Using Newton's second law the dynamic equations of each wheel are established as follows;

$$m_1 \ddot{z}_1 = k_{u1}(z_{r1} - z_1) - k_1(z_1 - z_{b1}) - c_1(\dot{z}_1 - \dot{z}_{b1}) - F_1 \quad \text{Eq. (19)}$$

$$m_2 \ddot{z}_2 = k_{u2}(z_{r2} - z_2) - k_2(z_2 - z_{b2}) - c_2(\dot{z}_2 - \dot{z}_{b2}) - F_2 \quad \text{Eq. (20)}$$

$$m_3 \ddot{z}_3 = k_{u3}(z_{r3} - z_3) - k_3(z_3 - z_{b3}) - c_3(\dot{z}_3 - \dot{z}_{b3}) - F_3 \quad \text{Eq. (21)}$$

$$m_4 \ddot{z}_4 = k_{u4}(z_{r4} - z_4) - k_4(z_4 - z_{b4}) - c_4(\dot{z}_4 - \dot{z}_{b4}) - F_4 \quad \text{Eq. (22)}$$

A full car model can simulate the vertical vibration, rolling and pitching movements of sprung mass, as are the most important characteristics of system dynamics. By applying Newton's second law of motion the following equations.

The vertical motion of the vehicle body;

$$\begin{aligned}
 m_b \ddot{z}_b = & k_1(z_1 - z_{b1}) + c_1(\dot{z}_1 - \dot{z}_{b1}) + k_2(z_2 - z_{b2}) + c_2(\dot{z}_2 - \dot{z}_{b2}) \\
 & + k_3(z_3 - z_{b3}) + c_3(\dot{z}_3 - \dot{z}_{b3}) + k_4(z_4 - z_{b4}) + c_4(\dot{z}_4 - \dot{z}_{b4}) \\
 & + F_1 + F_2 + F_3 + F_4
 \end{aligned} \tag{23}$$

The pitch motion of the vehicle body is obtained by taking moment along the pitch axis;

$$\begin{aligned}
 J\ddot{\theta} = & [k_1(z_1 - z_{b1}) + c_1(\dot{z}_1 - \dot{z}_{b1}) + k_2(z_2 - z_{b2}) + c_2(\dot{z}_2 - \dot{z}_{b2}) + F_1 + F_2] \\
 & - [k_3(z_3 - z_{b3}) + c_3(\dot{z}_3 - \dot{z}_{b3}) + k_4(z_4 - z_{b4}) + c_4(\dot{z}_4 - \dot{z}_{b4}) \\
 & + F_3 + F_4] l_r
 \end{aligned} \tag{24}$$

The roll motion of the vehicle body is obtained by taking moment along the roll axis;

$$\begin{aligned}
 I\ddot{\phi} = & [k_1(z_1 - z_{b1}) + c_1(\dot{z}_1 - \dot{z}_{b1}) + F_1] t_{fl} \\
 & - [k_2(z_2 - z_{b2}) + c_2(\dot{z}_2 - \dot{z}_{b2}) + F_2] t_{fr} \\
 & + [k_3(z_3 - z_{b3}) + c_3(\dot{z}_3 - \dot{z}_{b3}) + F_3] t_{rl} \\
 & - [k_4(z_4 - z_{b4}) + c_4(\dot{z}_4 - \dot{z}_{b4}) + F_4] t_{rr}
 \end{aligned} \tag{25}$$

If we assume that the roll angle ϕ and pitch angle θ change in a small range, we can gain the equations of z_i as follows;

$$z_{b1} = z_b + l_f \sin \theta + t_{fl} \cos \phi \approx z_b + l_f \theta + t_{fl} \phi \tag{26}$$

$$z_{b2} = z_b + l_f \sin \theta - t_{fr} \cos \phi \approx z_b + l_f \theta - t_{fr} \phi \tag{27}$$

$$z_{b3} = z_b - l_r \sin \theta + t_{rl} \cos \phi \approx z_b + l_r \theta + t_{rl} \phi \tag{28}$$

$$z_{b4} = z_b - l_r \sin \theta - t_{rr} \cos \phi \approx z_b - l_r \theta - t_{rr} \phi \tag{29}$$

Table 4. Full car model for active suspension system parameters

Symbol	Unit	Value	Definition
l_f	m	1.011	Distance between front axle and COG
l_r	m	1.803	Distance between rear axle and COG
t_{fl}	m	0.761	Distance between the front left tire and roll axis
t_{fr}	m	0.761	Distance between front right tire and roll axis

t_{rl}	m	0.755	Distance between the rear left tire and roll axis
t_{rr}	m	0.755	Distance between rear right tire and roll axis
m_b	kg	1460	Body mass
$m_{1,2}$	kg	40	Front tire masses
$m_{3,4}$	kg	35.5	Rear tire masses
$k_{u1,2,3,4}$	N/m	175500	Tire spring constants
$k_{1,2}$	N/m	19960	Front suspension spring constants
$k_{3,4}$	N/m	17500	Rear suspension spring constants
$c_{1,2}$	Ns/m	1290	Front suspension damping coefficients
$c_{3,4}$	Ns/m	1690	Rear suspension damping coefficients
I	kgm^2	460	Roll inertia
J	kgm^2	1460	Pitch inertia
$F_{1,2,3,4}$	N		Actuator forces
z_b	m		Tire vertical positions
$z_{r1,2,3,4}$	m		Body vertical position
$z_{b1,2,3,4}$	m		Suspension and body link point positions
ϕ	radian		Roll angle
θ	radian		Pitch angle
z_r	m		Road excitations

4.4.1. Actuator model for full car model

The full car model consists of four hydraulic actuators placed between the car body and each wheel. The nonlinear hydraulic actuator force equation used in the quarter car will be used for the full car model. Hence, the actuator force developed by each hydraulic actuator is described by the nonlinear equation (30- 33)

$$F_1 = -\beta F_1 - \alpha A^2(z_{b1} - z_1) + \gamma A x_v \sqrt{P_s - \frac{sgn(x_v)F_1}{A}} \quad \text{Eq. (30)}$$

$$F_2 = -\beta F_2 - \alpha A^2(z_{b2} - z_2) + \gamma A x_v \sqrt{P_s - \frac{sgn(x_v)F_2}{A}} \quad \text{Eq. (31)}$$

$$F_3 = -\beta F_3 - \alpha A^2(z_{b3} - z_3) + \gamma A x_v \sqrt{P_s - \frac{sgn(x_v)F_3}{A}} \quad \text{Eq. (32)}$$

$$F_4 = -\beta F_4 - \alpha A^2(z_{b4} - z_4) + \gamma A x_v \sqrt{P_s - \frac{sgn(x_v)F_4}{A}} \quad \text{Eq. (33)}$$

F_1, F_2, F_3, F_4 indicates hydraulic force generated by actuators at wheel 1,2,3,4 respectively

4.4.2. Full car model in Simulink

Full car active suspension Simulink model illustrated in figure 15.

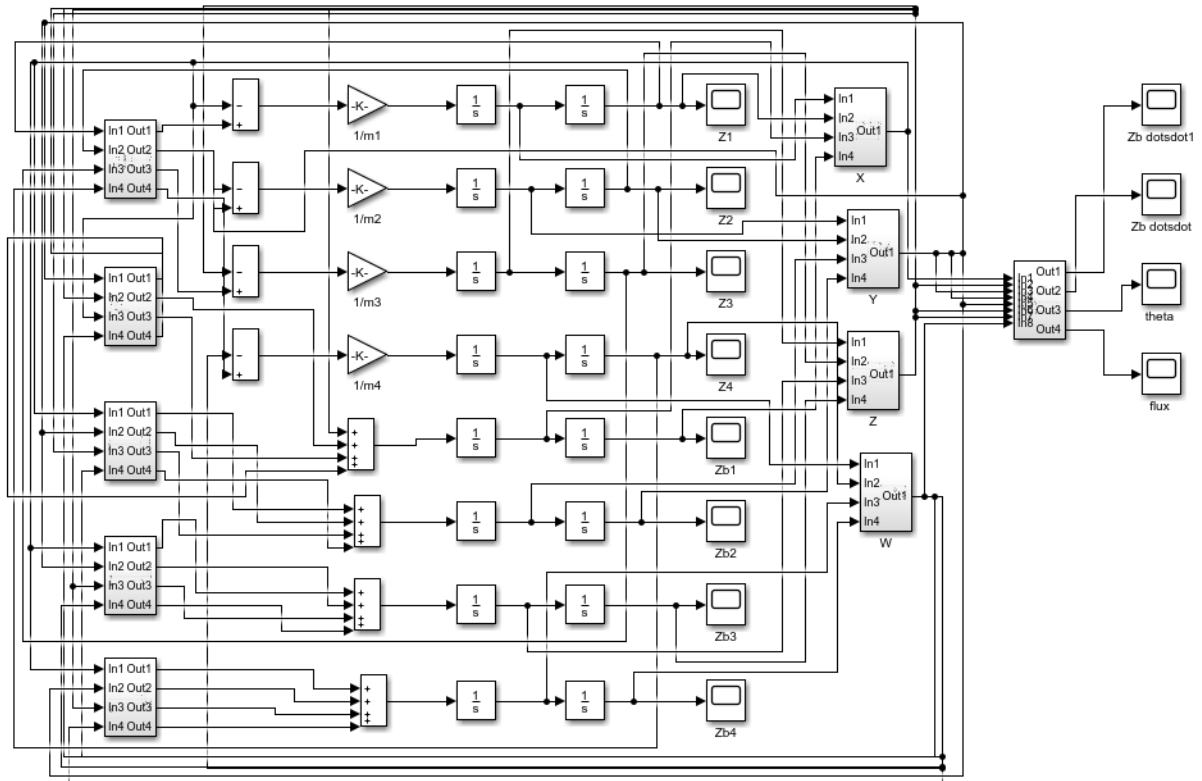


Figure 15. Full car active suspension Simulink model

CHAPTER FIVE: DESIGN OF INTELLIGENT FUZZY-PID CONTROLLER FOR ACTIVE SUSPENSION

5.1. Introduction

This chapter emphasizes the development of an intelligent fuzzy controller for active suspension. A model-free fuzzy controller is designed for a full car model-based nonlinear hydraulic actuated active suspension. The designed fuzzy controller is implemented using Fuzzy Toolbox in MATLAB. The performance of the designed fuzzy controller is measured in terms of body displacement, body acceleration, pitch angle and roll angle. The designed fuzzy controller is combined with sliding mode control in order to remove the chattering effect in sliding mode control.

5.2. Control objective

The essential function of the vehicle suspension is to connect the vehicle body with the wheels. Thereby the body can be carried along the drive route and forces can be transmitted in the horizontal plane. The suspension allows the wheel to move in a vertically aligned manner. As a result, the wheel follows a route with uneven road surfaces to a certain extent. By using spring and damping elements, the resulting body movements are reduced and driving safety and comfort are ensured.

Furthermore, the geometry of the vehicle suspension, as well as the spring and damping rate, influence the position of the wheel relative to the road. This enables for a systematic influence on the vehicle's dynamic driving characteristics. Because the criteria of appropriate driving behavior and great comfort are frequently incompatible, the modification of these features necessitates a compromise. Therefore, in designing the control law for a suspension system, usually, we need to consider the following aspects, (Du et al., 2014):

- Ride comfort: it is well-known that ride comfort is an important performance for vehicle design, which is usually evaluated by the body acceleration in the vertical, longitudinal and lateral directions.
- Road holding ability: The dynamic tire load should not exceed the static tire load in order to provide a firm, uninterrupted contact between the wheels and the road, (Xu et al., 2017).

- Maximum suspension deflection: Due to mechanical structure constraints, the maximum permitted suspension strokes must be considered to avoid excessive suspension bottoming, which can result in ride comfort deterioration and even structural damage.
- Saturation effect of the actuator: because of the limited power of the actuator, the control force for the suspension system should be confined to a certain range.
- Reliability of closed-loop systems: the closed-loop systems should be reliable when meeting with non-ideal situations caused by actuators, such as the problems of actuator input delay, sampled data, and fault accommodation for unknown actuator failures.

5.3. PID controller

A PID controller is the most common controller in industry applications. It consists of three terms: the proportional (P) term, the integral (I) term, and the derivative (D) term. In an ideal form, the output $u(t)$ of a PID controller is the sum of the three terms, (L. Wang, 2020).

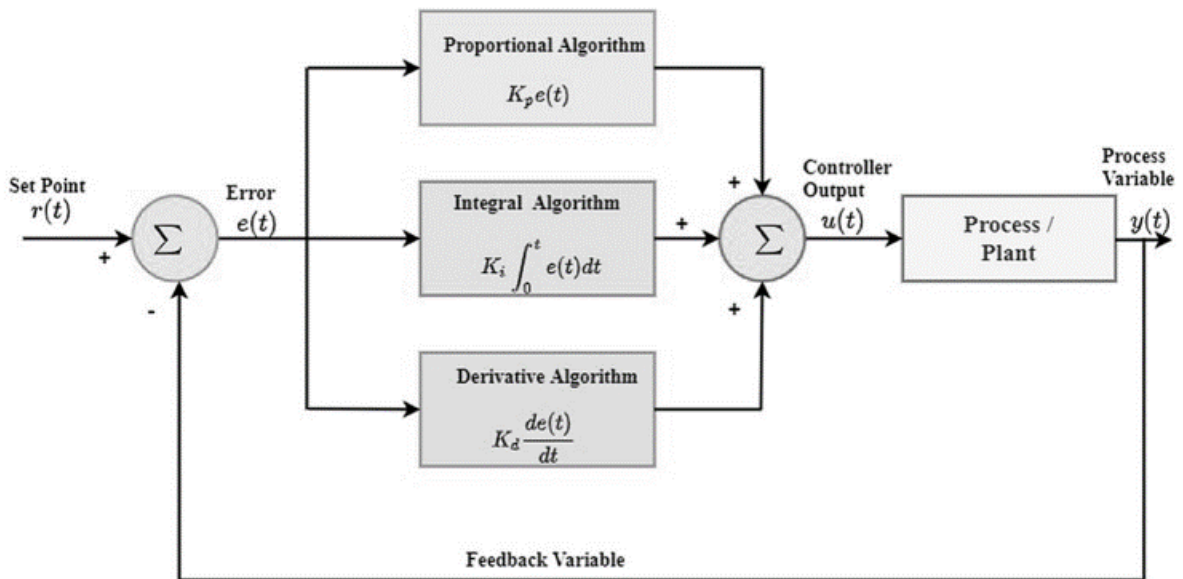


Figure 16. Block diagram of process control using PID

The mathematic form of PID controller can be expressed as follows:

$$u(t) = k_p e(t) + k_i \int_0^t e(t) dt + k_d \frac{de(t)}{dt} \quad \text{Eq. (34)}$$

Where, $e(t) = \text{set-point} - \text{plant output} = \text{proportional gain} = \text{integral gain} = \text{derivative gain}$

5.4. Fuzzy logic

The concept of fuzzy logic was proposed by Dr. Lofti Zadeh, at the University of California in 1960. Fuzzy logic is an extension of a multivalued logic system where the true value is of real numbers between 0 to 1. Fuzzy logic has the ability to handle partial truth which lies somewhere in between true and false. Fuzzy logic works based on the principle of fuzzy set theory, (Bustince Sola et al., 2015). The fuzzy set theory consists of a class of objects with unsharp boundaries. The values of membership function in fuzzy set desired the performance of fuzzy logic. Some of the salient features of fuzzy logic are:

- Fuzzy logic has tolerant of imprecise data.
- Fuzzy logic has the ability to model nonlinear systems with less complexity.
- Fuzzy logic can be combined with classical control techniques more easily.
- The concept of fuzzy logic is flexible and easy to understand.

The main aspects of fuzzy logic are fuzzy set, linguistic variable, membership function and fuzzy rules. The fuzzy set contains elements with a degree of membership which lies between 0 to 1. In contrast, the classical set contains elements with a degree of freedom either 1 or 0.

A classical set A with a collection of object x is defined as:

$$A = \{x, \mu(x) | x \in X\} \quad \mu(x) = \begin{cases} 1 & x \in X \\ 0 & x \notin X \end{cases} \quad \text{Eq. (35)}$$

Where $\mu_A(x)$ is called degree of membership of fuzzy set A.

$$A = \{x, \mu(x) | x \in X\} \quad \mu(x) = [0 \quad 1] \quad \text{Eq. (36)}$$

A fuzzy set A with a collection of object x is defined as follows:

Where $\mu_A(x)$ is called degree of membership of fuzzy set A.

Linguistic variables are words of natural language to represent the numerical value of fuzzy input-output variables. The linguistic variables consist of a set of the linguistic term.

The membership function converts real data into linguistic terms during the fuzzification and defuzzification process. The linguistic terms are quantified using the membership function. There are different types of membership functions such as triangular, trapezoidal, Gaussian and bell shape, etc.

Fuzzy rules are created to control output parameters, (Bai & Wang, 2016). A fuzzy rule consists of a simple IF-THEN rule with the condition and conclusion parts. Fuzzy sets and fuzzy operators are the subjects and verbs of fuzzy logic. The conditional statements that make up fuzzy logic are formed using these if-then rule statements.

If x is A, then y is B is the form of a single fuzzy if-then rule. Where A and B are linguistic values on the ranges (universes of discourse) X and Y, respectively, defined by fuzzy sets. The antecedent or premise of the rule "x is A" is known as the if-part, while the consequent or conclusion of the rule "y is B" is known as the then-part. In general, the input to an if-then rule is the current value of the input variable (in this case the service) and the output is the entire fuzzy set (in this case the average). Later this set is purged and the output is assigned a single value.

5.4.1. Fuzzy Algorithm

The working of fuzzy logic explained using a fuzzy algorithm, (Akgun et al., 2012), is as follows.

1. Identify linguistic variables and its term.
2. Construct membership function for input and output.
3. Construct a rule base for the fuzzy system.
4. Fuzzification of real data into fuzzy values using membership function.
5. Evaluate rules on rules-based using an inference mechanism.
6. Defuzzification of fuzzy data into real values using membership function.

Initially, the linguistic variable, type of membership function and rules base are initiated for fuzzy logic. The real data of the system is converted into a fuzzy set using linguistic terms and membership functions. This process is known as fuzzification. A set of rules are created using a rules base. Finally, the fuzzy output is mapped with real values. This process is known as defuzzification.

5.5. Fuzzy controller

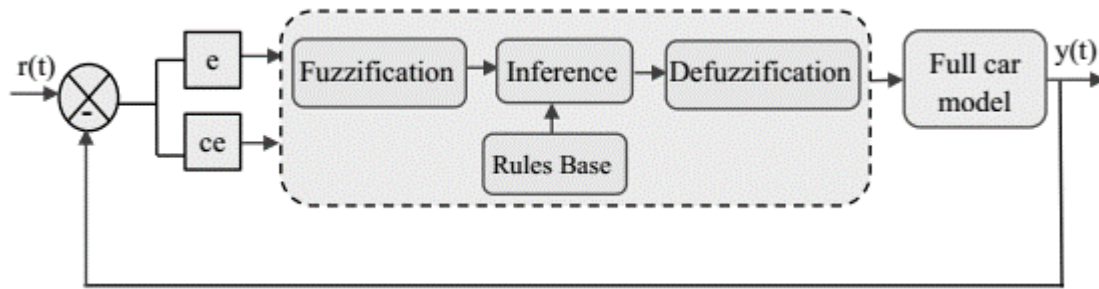


Figure 17. Fuzzy controller for active Suspension

A fuzzy logic controller has the ability to handle complexity, nonlinearity and unpredictable behavior of actuator dynamics in an active suspension system. The overall layout of an intelligent fuzzy-PID controller used in a full car modeled active suspension system is shown in Figure 17. The actual suspension travel of each wheel act as a control parameter for the fuzzy controller.

The linguistic variables used for fuzzy control of active suspension are error (e) and change in error (ce) of suspension travel of each wheel. The linguistic terms are Negative Big (NB), Negative Medium (NM), Negative Small (NS), Zero (ZE), Positive Small (PS), Positive Medium (PM) and Positive Big (PB). The fuzzification stage converts the error (e) and error change (ce) of suspension deflection into fuzzy values with the help of the membership function. The triangular membership function with five linguistic variables is used for the fuzzification process. The fuzzy rules which are designed based on expert knowledge are applied using an inference mechanism.

5.5.1. Fuzzification

Fuzzification is an important concept in fuzzy logic theory. Fuzzification is the process of converting crisp values to fuzzy values. The fuzzy values are created by identifying some of the uncertainty existing in the crisp values. The conversion of fuzzy values is represented by the membership functions. In many practical applications, in industries, etc., measurement of voltage, current, temperature, etc., there might be a negligible error. This causes imprecision in the data. This imprecision can be represented by the membership functions. As a result, fuzzification is performed. Assigning membership values for the given crisp quantities may be part of the fuzzification process. There are various methods to assign membership values or

membership functions to fuzzy variables. The assignment can be just done by intuition, inference, angular fuzzy set and using some algorithms like neural networks, and genetic algorithms.

The membership function of error and change in error is represented in Figures 18 and 19 using MATLAB Fuzzy Logic Toolbox. The first stage is to use membership functions to determine the degree to which the inputs belong to each of the suitable fuzzy sets. The output is a fuzzy degree of membership in the qualifying linguistic set, and the input is always a crisp numerical value constrained to the universe of discourse of the input variable. The range of input variables for error and change in error is [-2, 2].

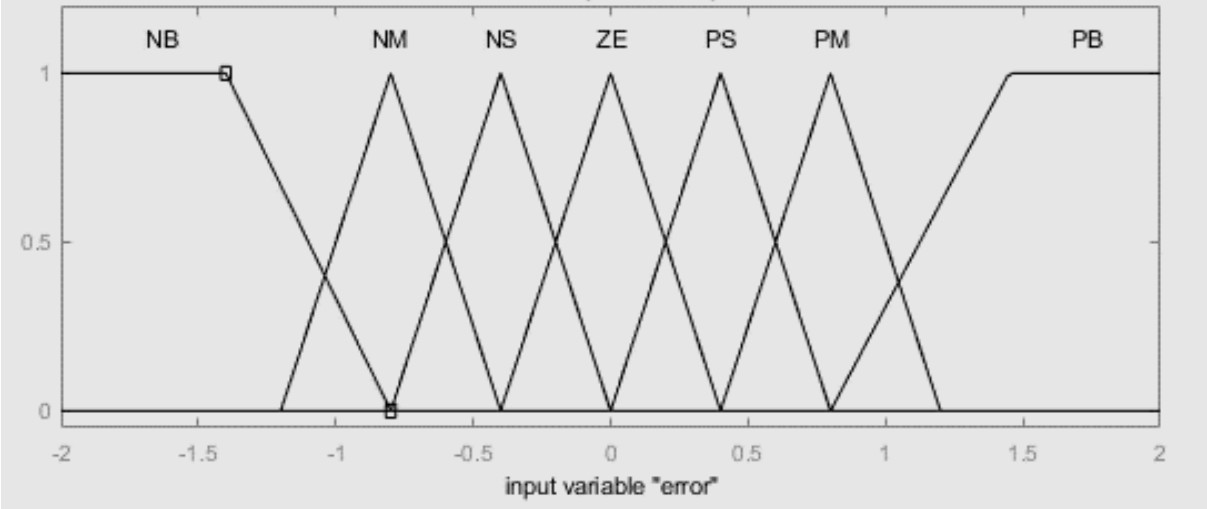


Figure 18. Membership function for 'error'

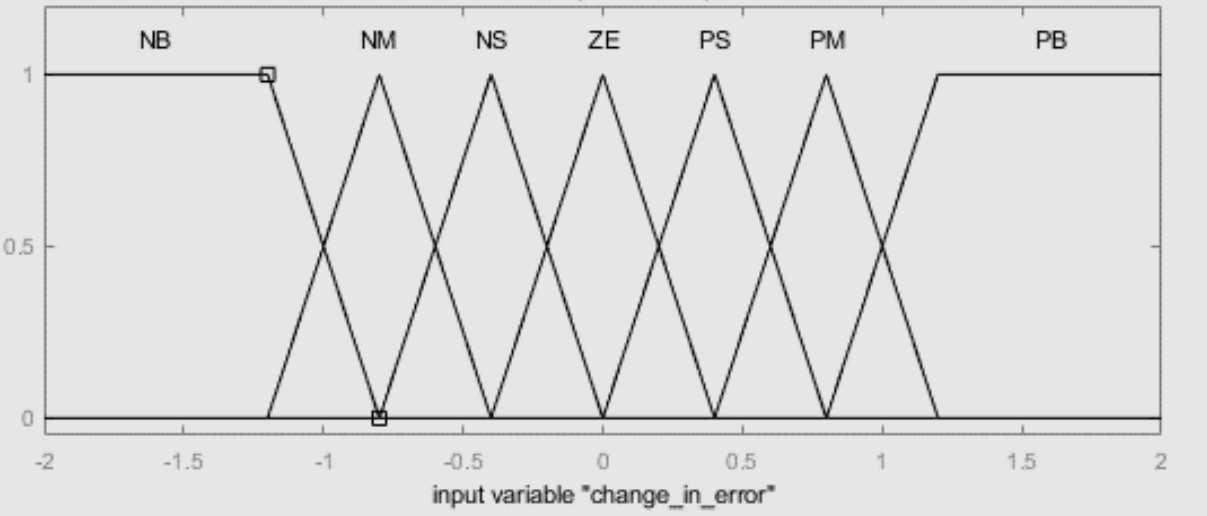


Figure 19. Membership function for 'change in error'

5.5.2. Rules Base

Fuzzy rules are a collection of linguistic statements that describe how the FIS should decide to classify an input or control an output. Each linguistic variable has five linguistic terms. So totally, 49 rules can be formed from the linguistic variables.

Table 5. Fuzzy Rules for Controller

e/ce	NB	NM	NS	ZE	PS	PM	PB
NB	NB	NB	NB	NB	NM	NS	ZE
NM	NB	NB	NB	NM	NS	NS	PS
NS	NB	NB	NM	NS	ZE	ZE	PM
ZE	NB	NM	NS	ZE	PS	PS	PB
PS	NM	NS	ZE	PS	PM	PB	PB
PM	NS	ZE	PS	PM	PB	PB	PB
PB	ZE	PS	PM	PB	PB	PB	PB

The list of rules for the rules base is represented in Table 5. The abbreviations used correspond to **NEB**: Negative Big; **NES**: Negative Small; **ZER**: Zero; **PES**: Positive Small; **PEB**: Positive Big; **e (k)**: Error; **ec(k)**: change in error.

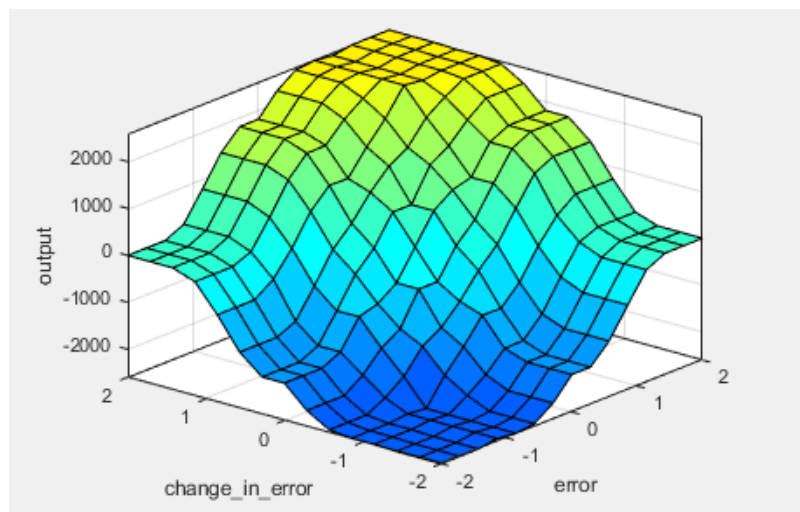


Figure 20. Surface area of input and output parameter

5.5.3. Defuzzification

Because the fuzzy results created cannot be used directly in applications, they must be converted into crisp values for further processing. This can be accomplished through the use of the defuzzification procedure. Defuzzification has the capability to reduce a fuzzy to a crisp single-valued quantity or as a set or convert to the form in which a fuzzy quantity is present, (Runkler et al., 2015). The membership function for output control voltage is represented in Figure 21.

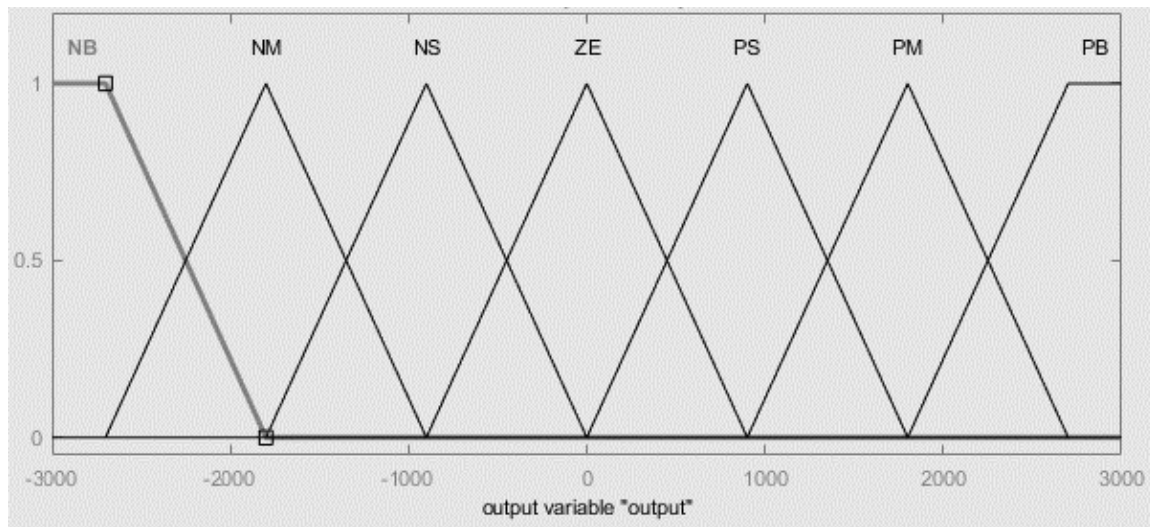


Figure 21. Membership function for control force

The fuzzy inference diagram is the composite of all the smaller diagrams. It simultaneously displays all parts of the fuzzy inference process. The flow starts in the lower left with the inputs, then moves across each row or rule, and finally down the rule outputs to the lower right. This compact flow shows everything at once, from linguistic variable fuzzification to defuzzification of the aggregate output.

5.6. Modeling of an intelligent fuzzy-PID controller in Simulink

The Fuzzy-PID controller is a combination of the conventional PID controller and the algorithm of fuzzy control designed to enhance the ride comfort in vehicle suspension system.

The developed controller comprises PID and Fuzzy-PID controllers to blend the benefits of both controllers. The parallel combination of PID combination of both controllers enhances the performance of individual controller. FPID, and proposed Intelligent FPID controller structures are illustrated in Figure. 22 and 23 respectively.

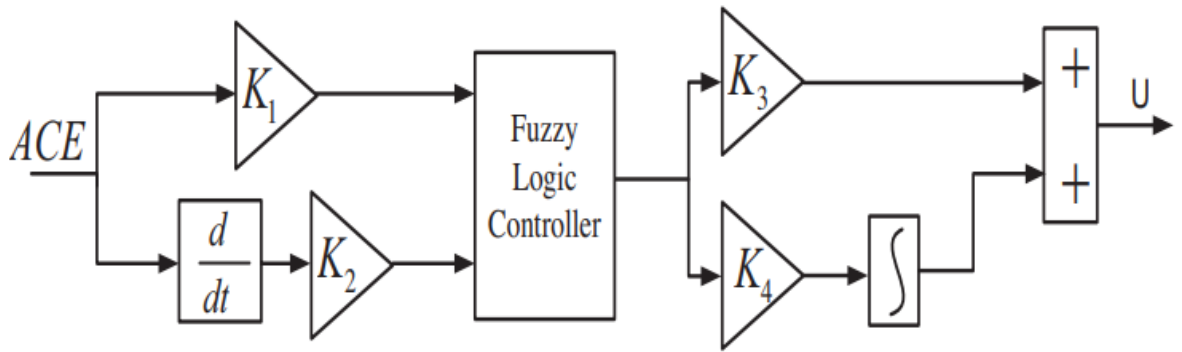


Figure 22. Fuzzy PID controller structure

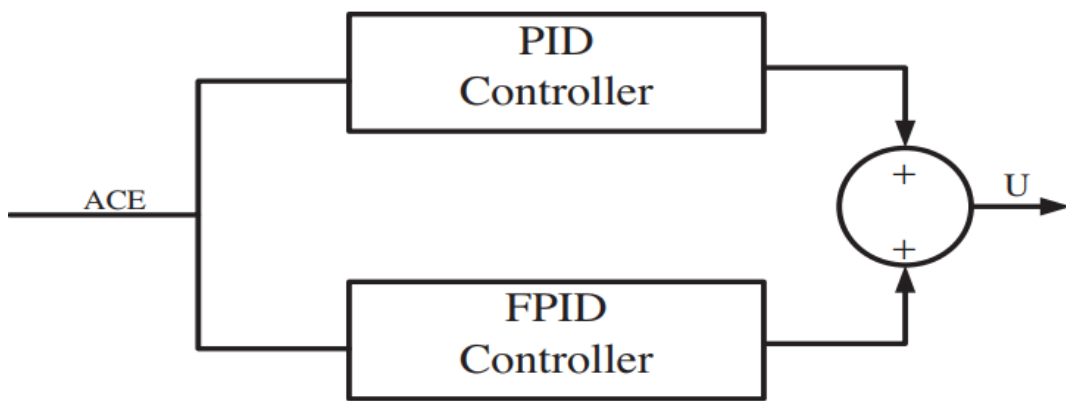


Figure 23. IFPID controller structure

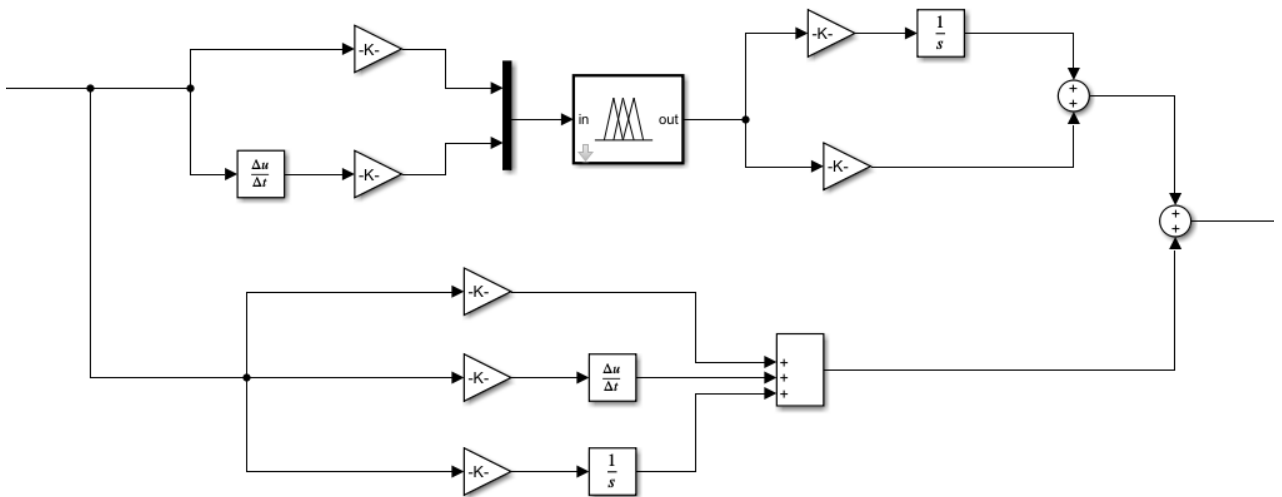


Figure 24. Representation of an intelligent fuzzy-PID controller in SIMULINK

CHAPTER SIX: RESULT AND DISCUSSION

6.1. Introduction

This chapter discusses about active suspension system's performance. Simulation based on the mathematical model for quarter car, half car and full car model performed by using MATLAB/SIMULINK software. It is possible to observe how the suspension system performs in terms of stability and ride quality when road disturbance is supposed to represent the system's input. The suspension travel, body displacement, and car body acceleration for quarter- and full-car models are among the parameters that are measured. For suspension travel, body displacement, and car body acceleration, a minimal amplitude value is desired. Each component's steady state should also be quick.

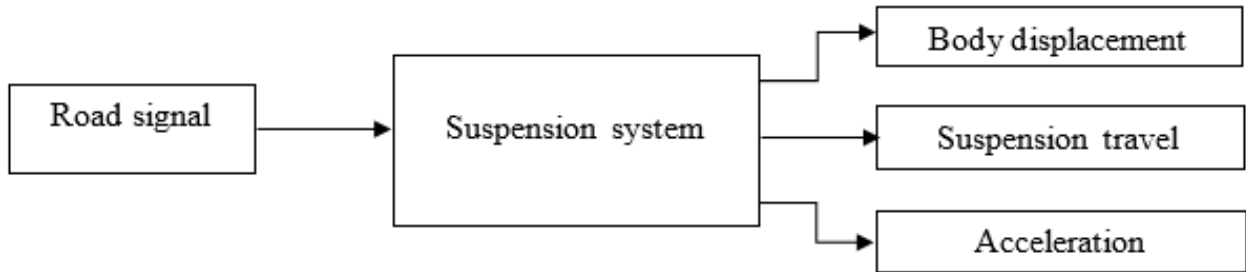


Figure 25. Simulink model flowchart

6.2. Simulation of quarter car model.

The most relevant for ride studies are ground input disturbances caused by road roughness. There are many possible ways to analytically describe the road inputs, which can be classified as shock or vibration. A simple model of the vertical road displacement $z_r(t)$ resulting from a singular disturbance event is given by;

$$z_r(t) = \begin{cases} \frac{a}{2}(1 - \cos 8\pi t), & 0.5 \leq t \leq 0.75 \\ 0, & \text{otherwise} \end{cases} \quad \text{Eq. (36)}$$

Where a represents the bump amplitude, ($a = 0.05$ (road bump height 10 cm))

A single bump road input (z_r) as shown in the figure 26 is used to express the road status, and conditions, and to verify the developed control system, this road bump is called road profile or road disturbance.

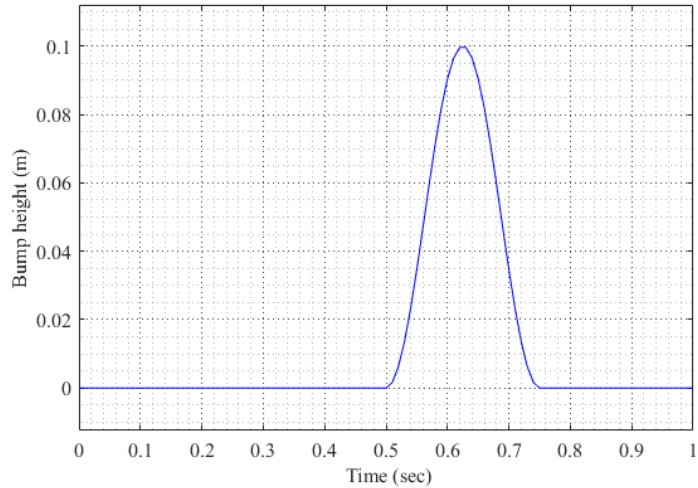


Figure 26. Road profile 1

The performance of the quarter car active suspension system is evaluated between the passive and active suspension controlled by PID and IFPID controller. Computer simulations have resulted and performed based on the equations of motion of a quarter-car model using MATLAB / SIMULINK. The performance of the car suspension system can be illustrated in terms of driving comfort, quality, and road handling, as the road bump is assumed to be the input signal of the suspension.

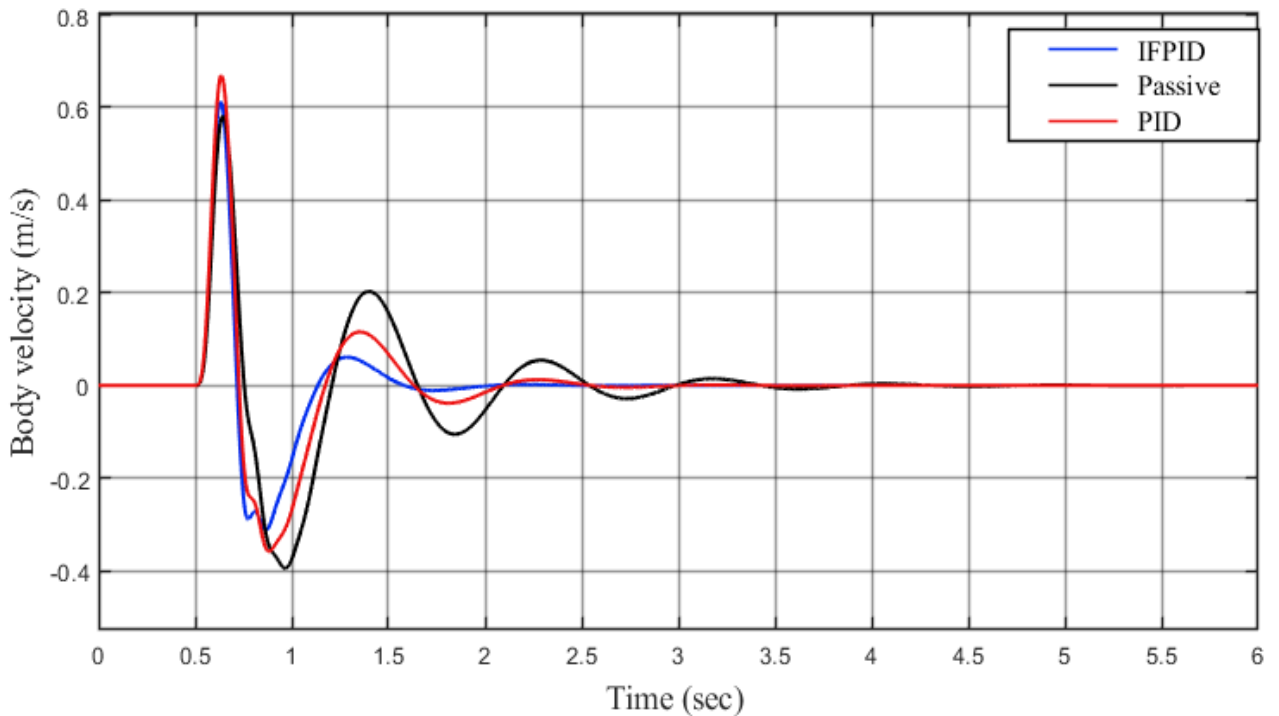


Figure 27. Body velocity (m/s²) vs time (s) of active quarter car model

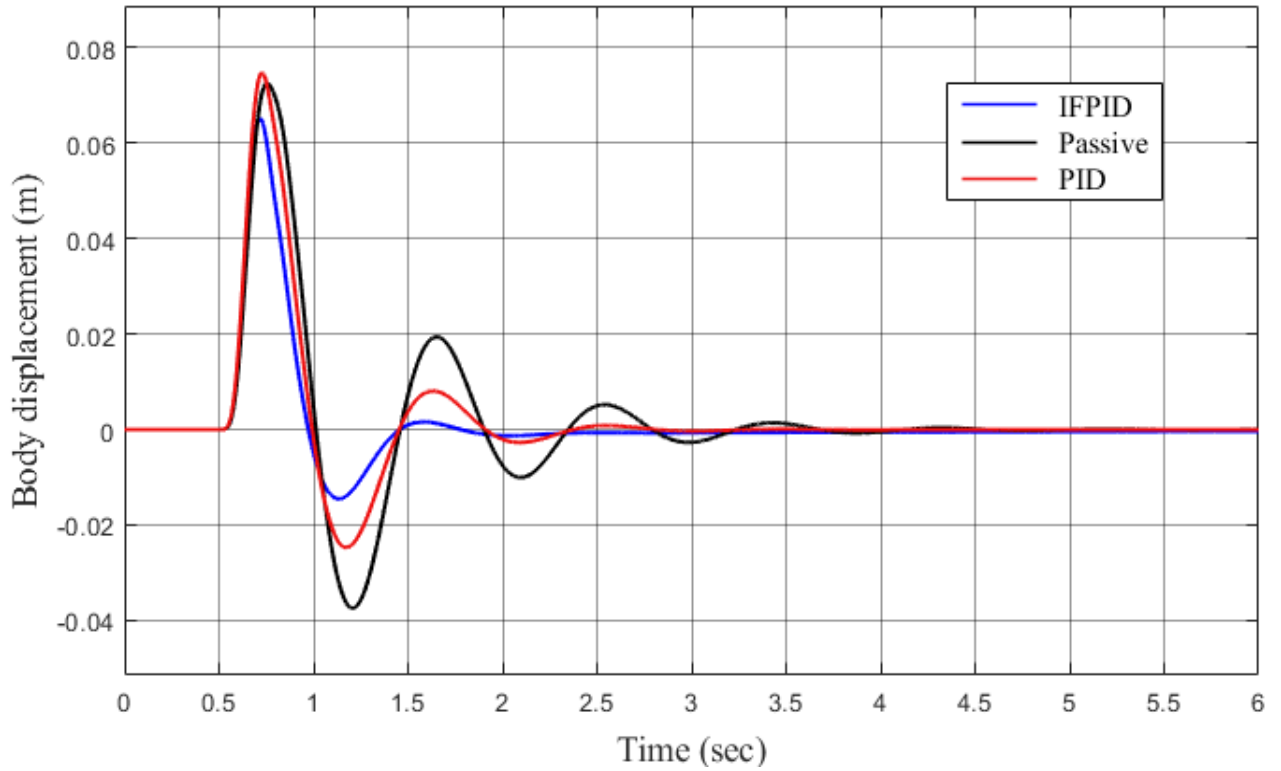


Figure 28. Body displacement (m/s^2) vs time (s) of active quarter car model

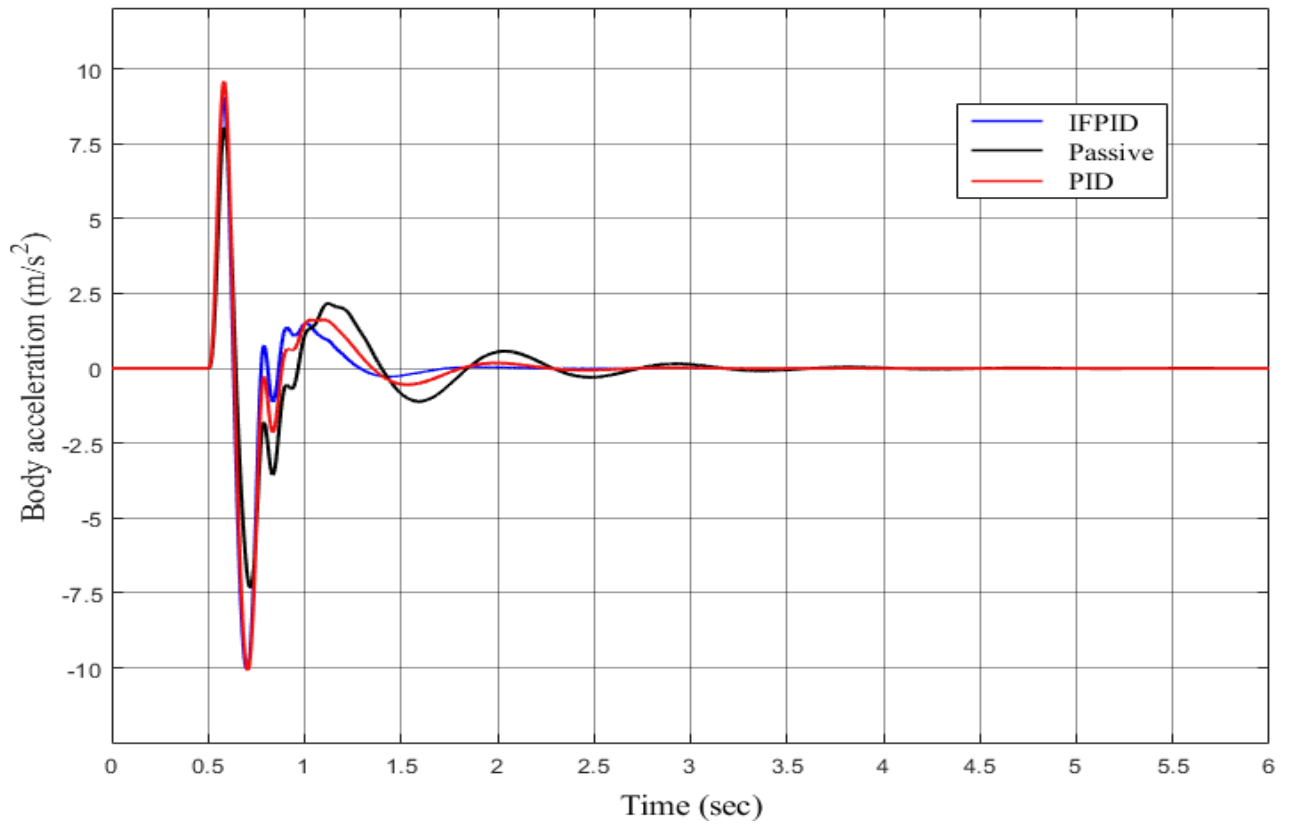


Figure 29. Body acceleration (m/s^2) vs time (s) of active quarter car model

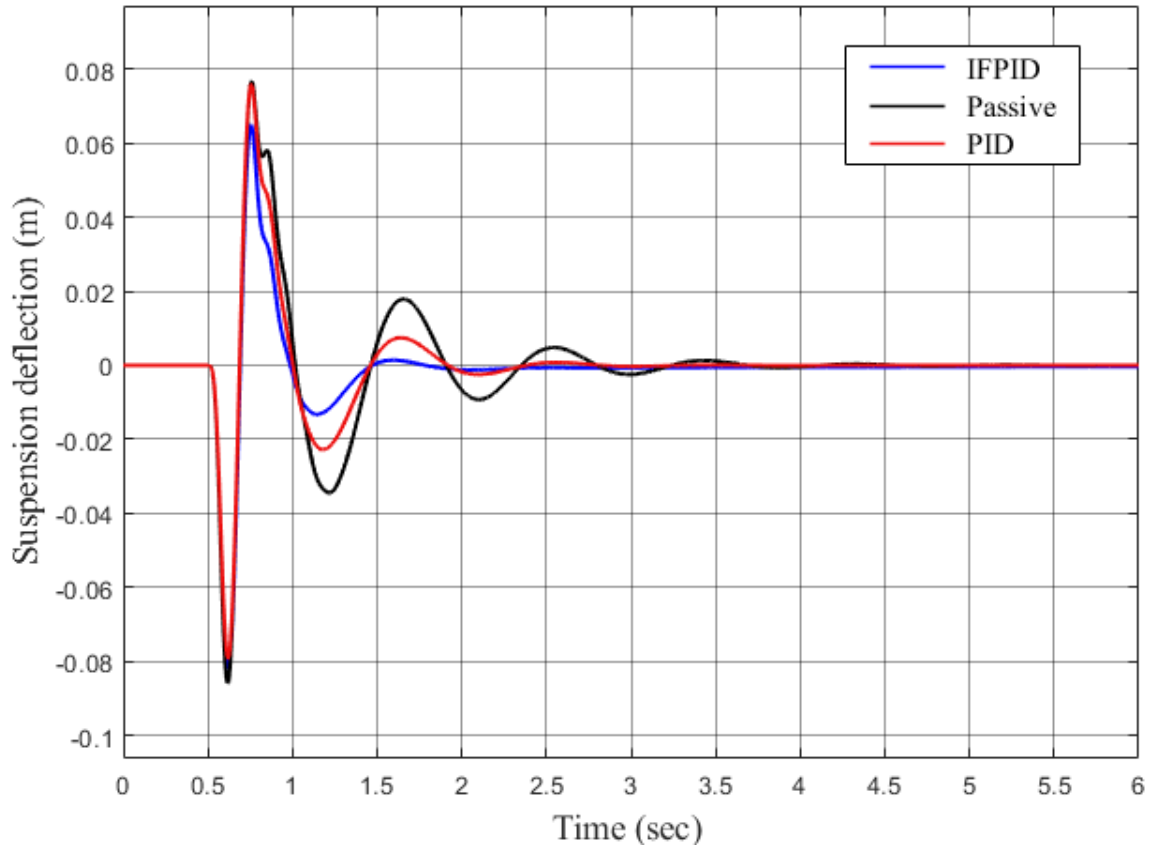


Figure 30. Suspension deflection (m) vs time (s) of active quarter car model

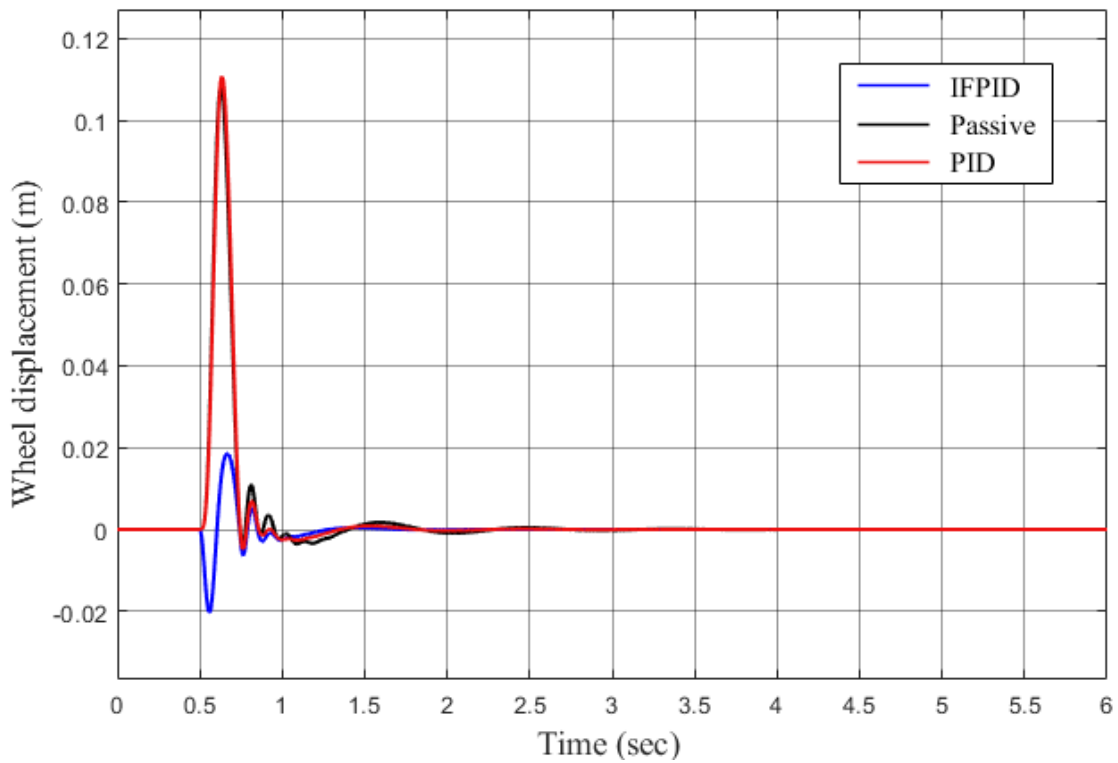


Figure 31. Wheel displacement (m) vs time (s) of active quarter car model

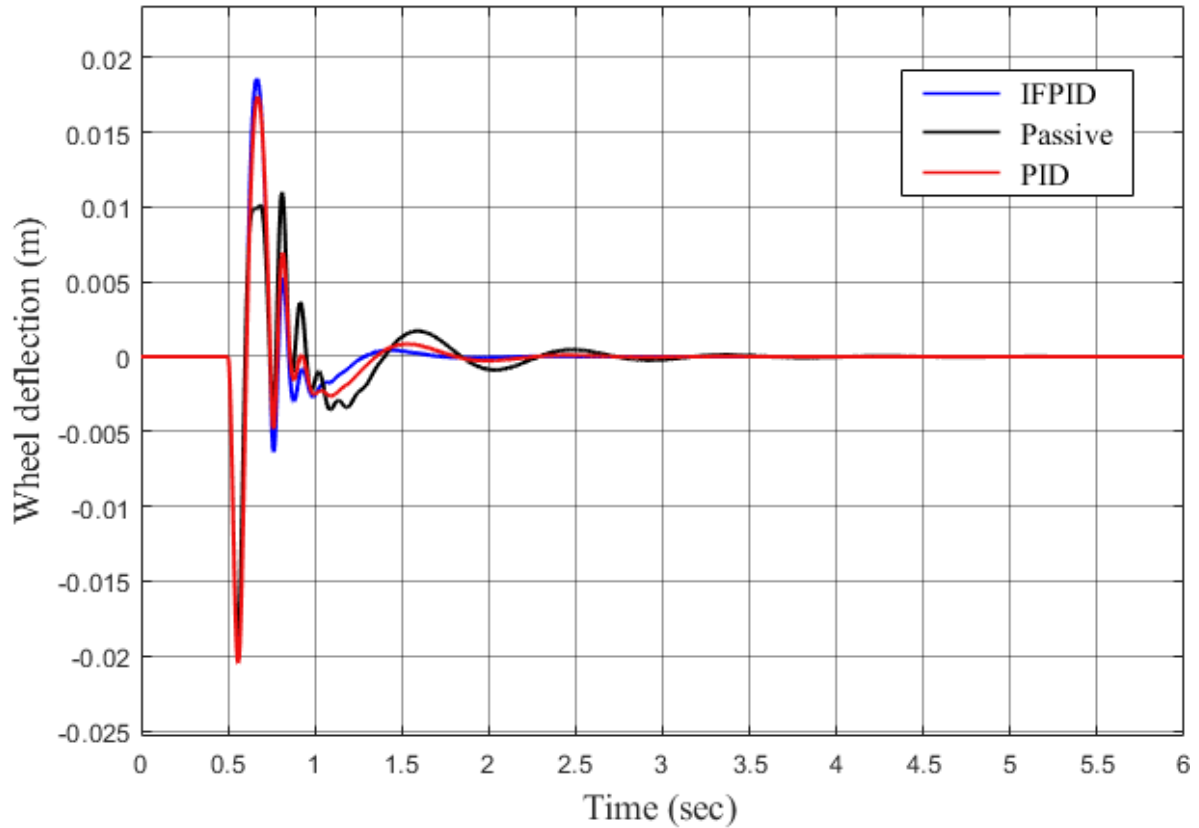


Figure 32. Wheel deflection (m) vs time (s) of active quarter car model

The simulation results are shown in figure 26 -32 for road profile 1. It shows the comparison between Passive, PID and intelligent fuzzy-PID controlled systems for body deflection, suspension deflection and body acceleration with road disturbance.

By comparing the performance of the passive, and active suspension system using PID and intelligent fuzzy-PID control technique it is clearly shows that active suspension controlled with intelligent fuzzy-PID can give lower amplitude and faster settling time. Suspension travel/deflection for can reduce the amplitude and settling time compare to passive suspension system. Even though the amplitude is slightly higher compared to a passive suspension system, body displacement has improved, and the settling period is incredibly short. The term "riding quality" is used to describe body displacement. Wheel Displacement also gives slightly higher amplitude but assure fast settling time. The main purpose to observe Wheel Displacement it represents car handling performance.

Simulation results shows that there is improvement in the ride comfort performance and suppression of vibrations with intelligent fuzzy-PID control as compared to passive and PID.

6.2. Simulation of half car model.

The vehicle is excited by a sinusoidal bump on an otherwise smooth road. This is illustrated in Figure 33 and expressed in equation. (37), and (38) where w_f and w_r are the fronts and rear-wheel input disturbances. The road profile 2 illustrated in figure 33 is simulation of road disturbance used in half and full car model.

$$w_f(t) = \begin{cases} \frac{a}{2}(1 - \cos 8\pi t), & 0.5 \leq t \leq 0.75 \\ 0, & \text{otherwise} \end{cases} \quad \text{Eq. (37)}$$

$$w_r(t) = \begin{cases} \frac{a}{2}(1 - \cos 8\pi t), & 3 \leq t \leq 3.25 \\ 0, & \text{otherwise} \end{cases} \quad \text{Eq. (38)}$$

Where a represents the bump amplitude, ($a = 0.05$ (road bump height 10 cm))

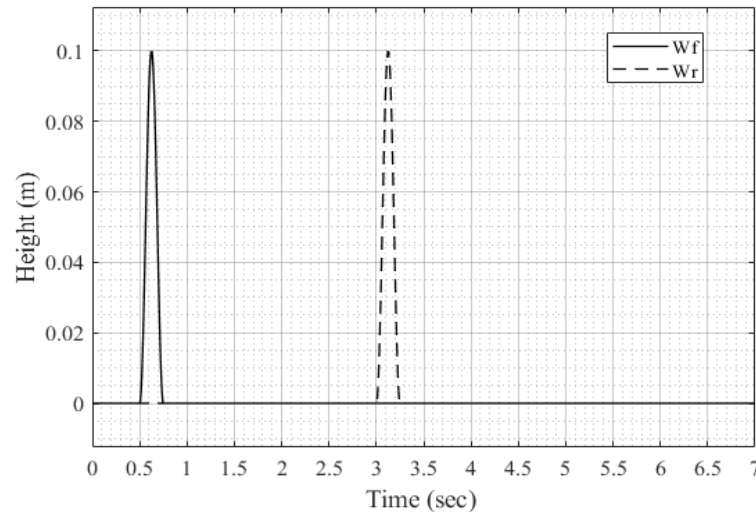


Figure 33. Road profile 2

The performance of the half car active suspension system is evaluated between the passive and active suspension controlled by PID and IFPID controller. Computer simulations have resulted and performed based on the equations of motion of a half car model using MATLAB / SIMULINK.

To make the comparison easier between the active and passive response plot in varying work environment. The developed Simulink basically, consists of two main subsystems with the road disturbance being injected to simulate the actual vehicle performance. The controllers need to be carefully tuned to get the best response. The time response plot is obtained during simulation while considering passive and active system.

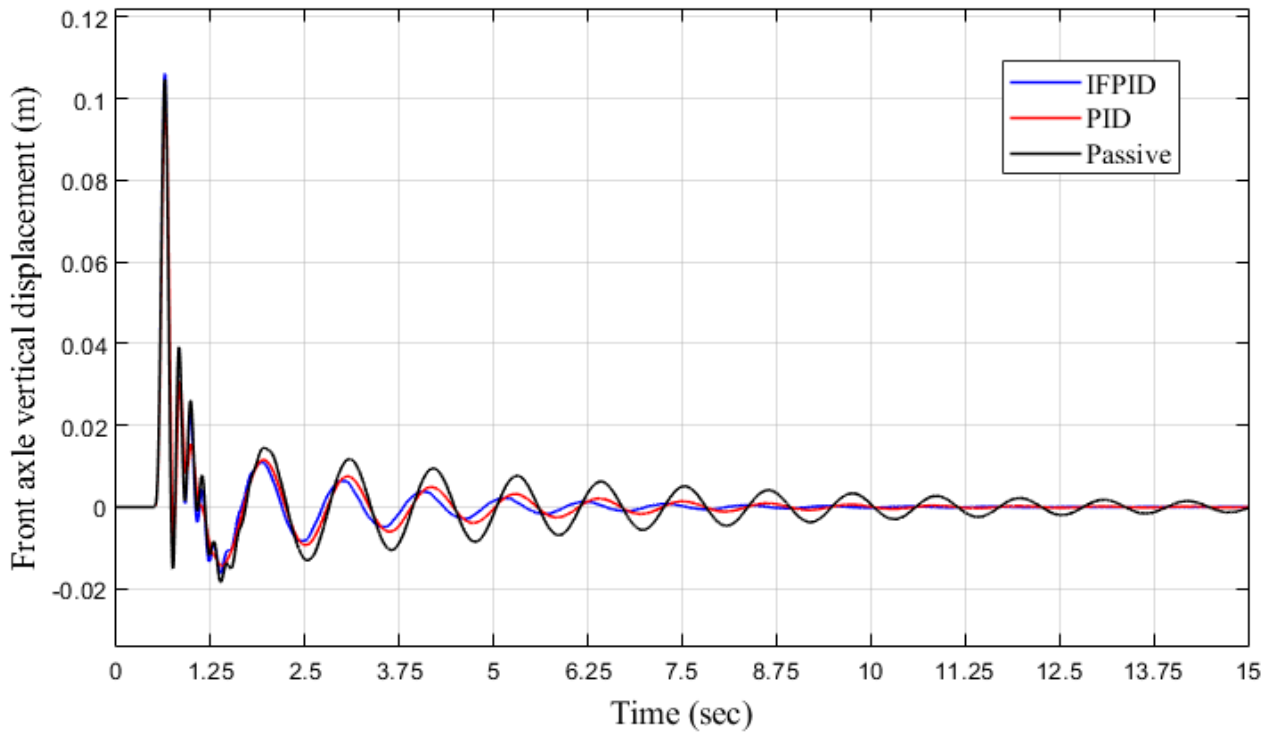


Figure 34. Front axle vertical displacement of active half car model with fuzzy-PID controller

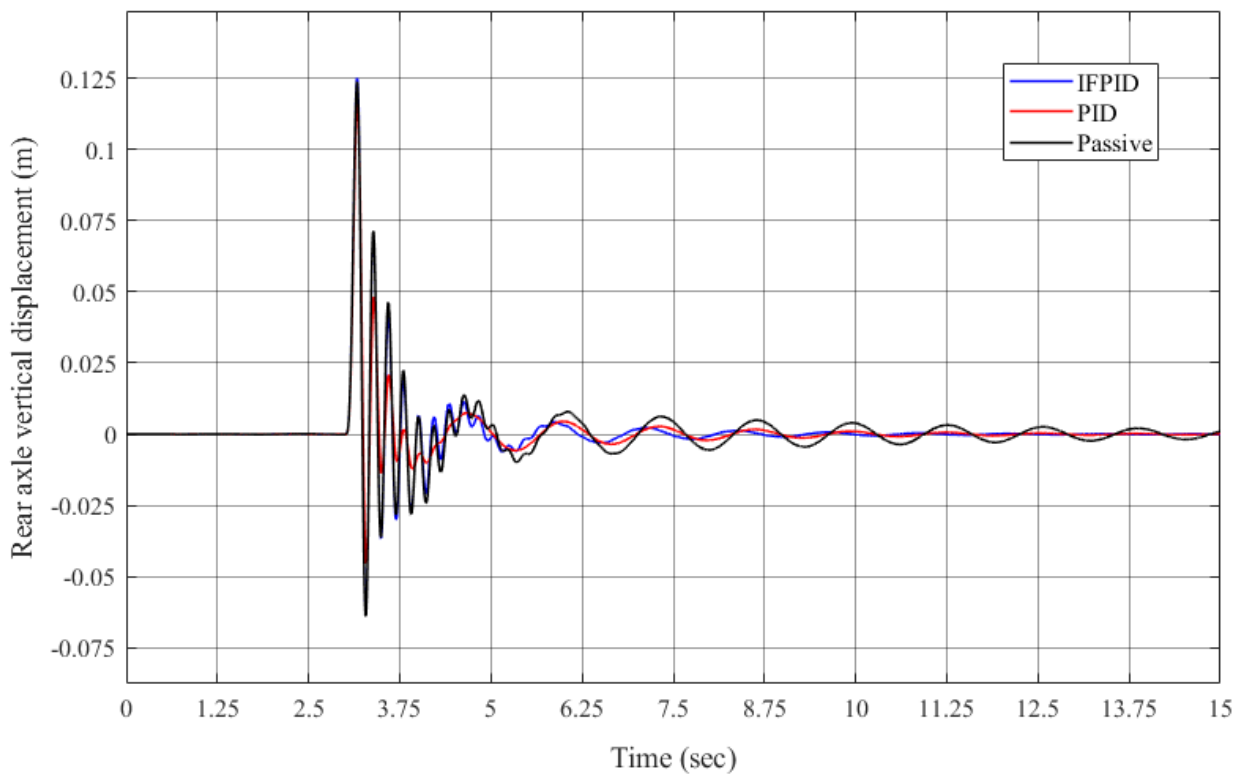


Figure 35. Rear axle vertical displacement of active half car model with fuzzy-PID controller

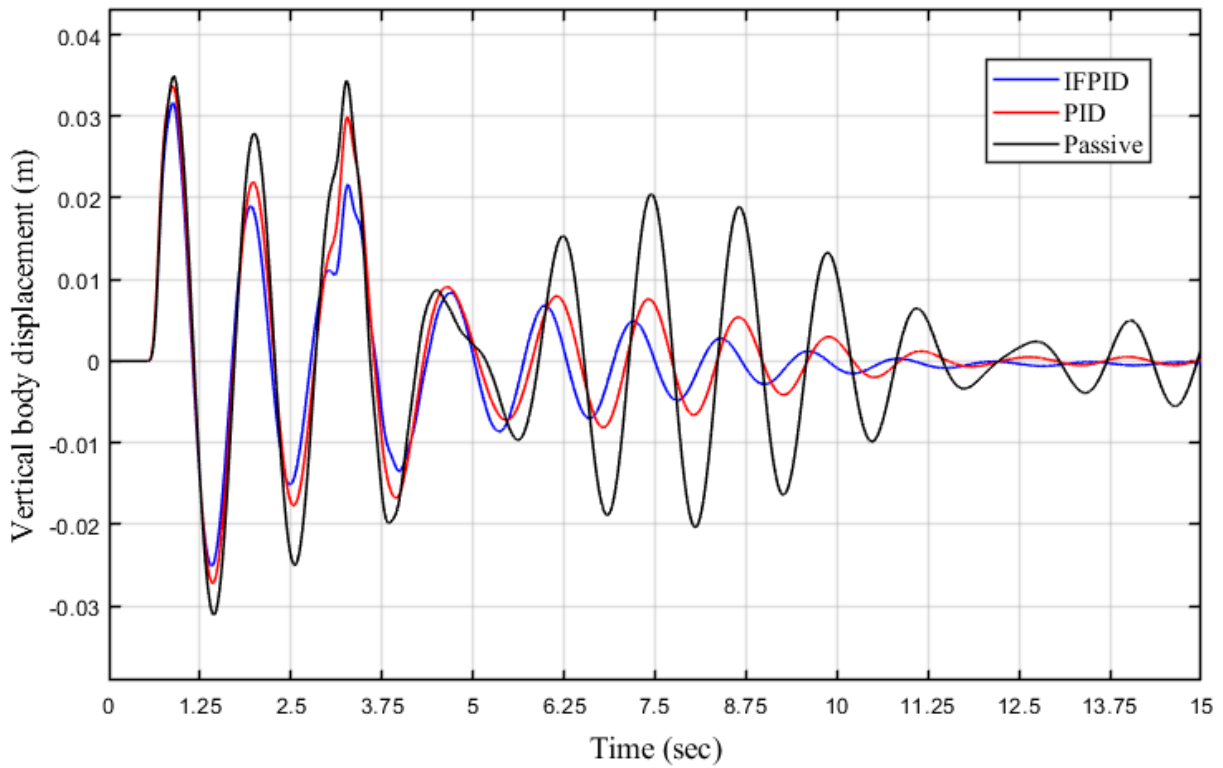


Figure 36. Vertical body displacement of active half car model with fuzzy-PID controller

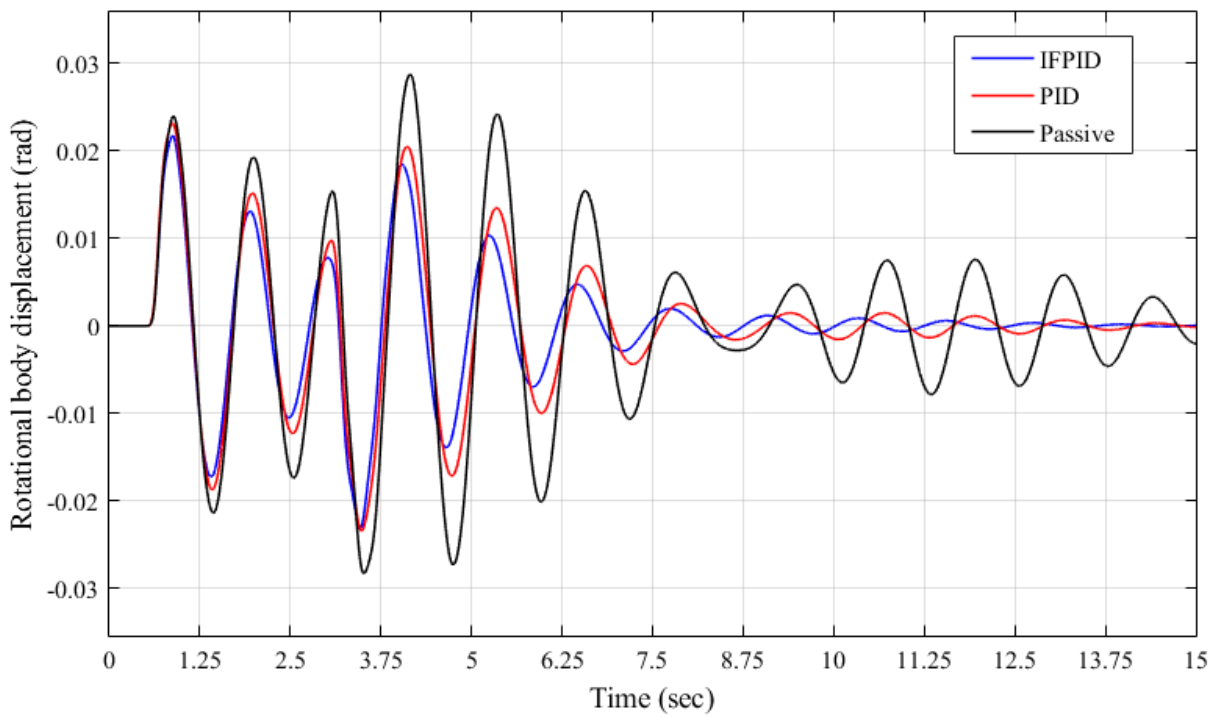


Figure 37. Rotational body displacement of active half car model with fuzzy-PID controller

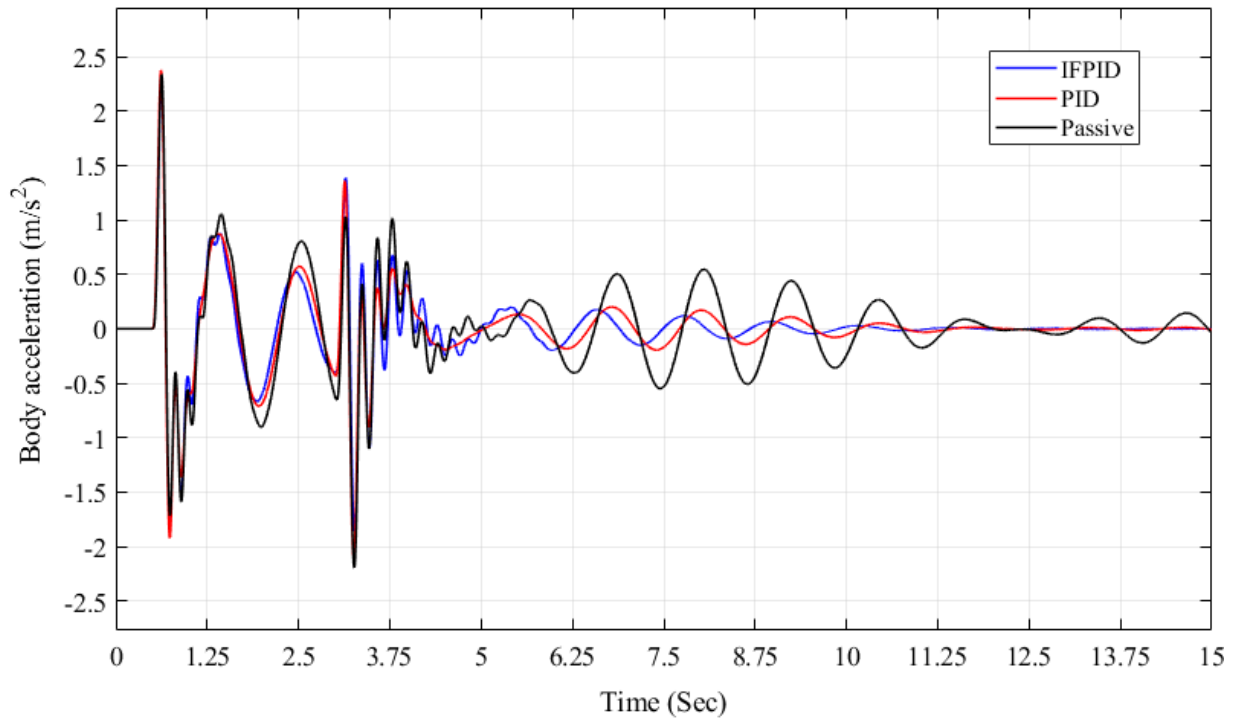


Figure 38. Body acceleration of active half car model with fuzzy-PID controller

Simulation results show that there is improvement in the ride comfort performance and suppression of vibrations with intelligent fuzzy-PID control as compared to passive and PID based systems in terms of settling time and body acceleration, suspension deflection, and percentage overshoot. Intelligent fuzzy-PID based control technique delivers faster response and lower amplitude in case of body deflection, suspension deflection, and body acceleration compared to passive and active suspension using PID control scheme.

To look into the robustness of the controllers' parameters in depth, the model is changed to the half car active suspension system. This is to ensure that the controller can perform well as in the quarter car model. The same criteria are measured by the same parameters which are: car body displacement, car body acceleration, and wheel deflection. The front and rear axle vertical displacement presented in Figures 34 and 35 reveal that intelligent Fuzzy-PID is able to reduce the vibration effectively compared to passive and PID. On the other hand, the output responses of the front and rear suspensions demonstrate ride comfort, as shown in Figures 36. The response for body acceleration shows that intelligent Fuzzy-PID managed to reduce vibration effectively.

6.3. Simulation of full car model.

A full car model simulation results and comparison between passive, PID, and intelligent fuzzy-PID controller are shown in figure 39 - 42.

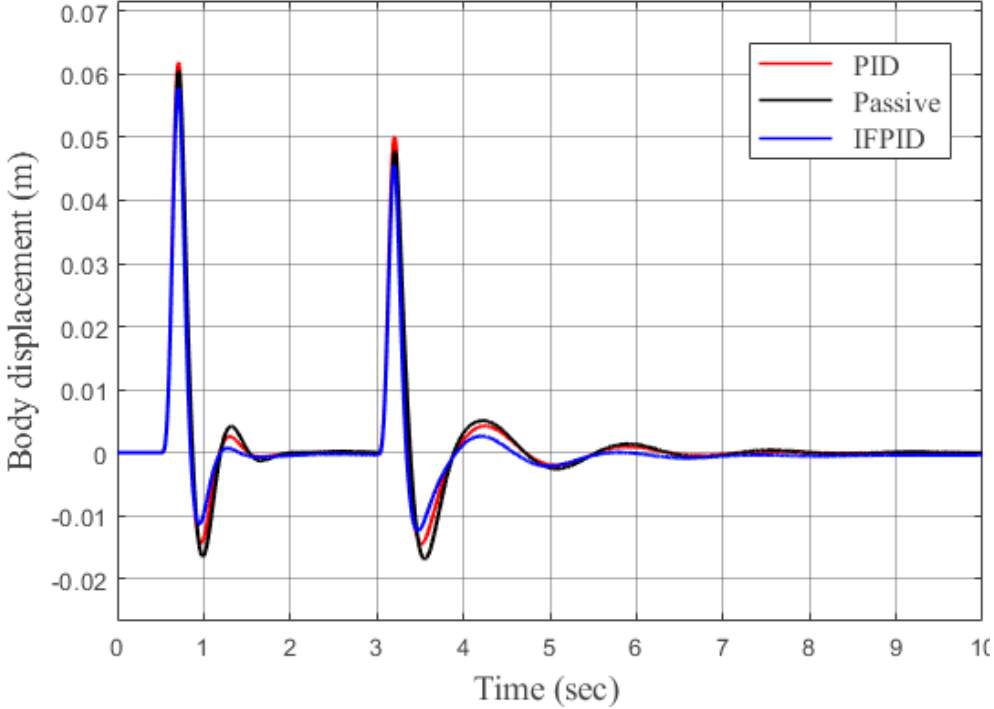


Figure 39. Body displacement (m) vs time (s) of a full car model

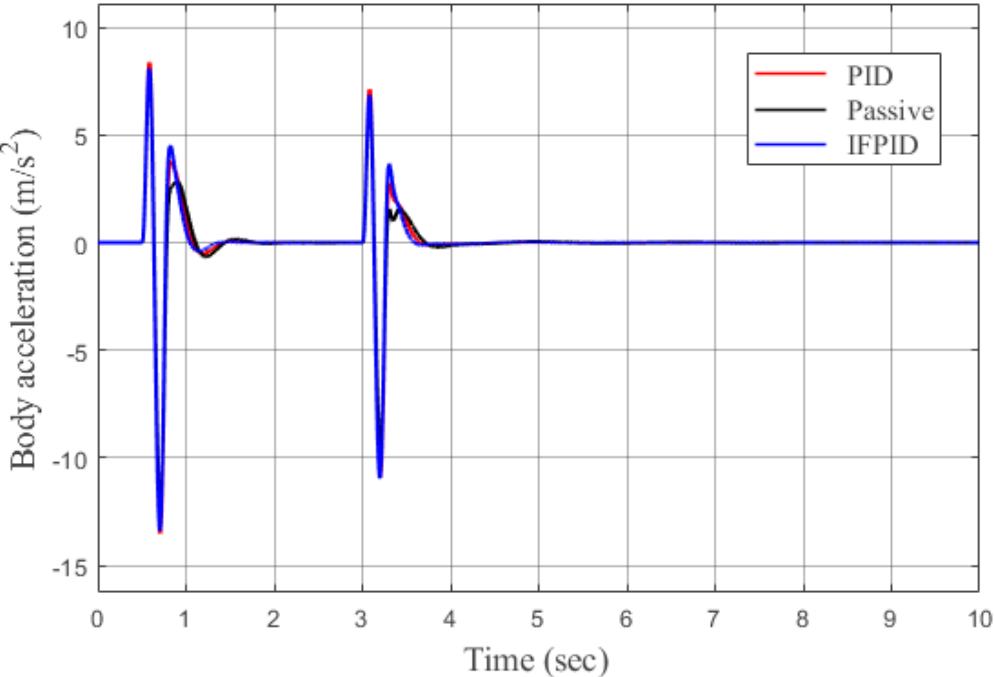


Figure 40. Body acceleration (m/s²) vs time (s) of a full car model

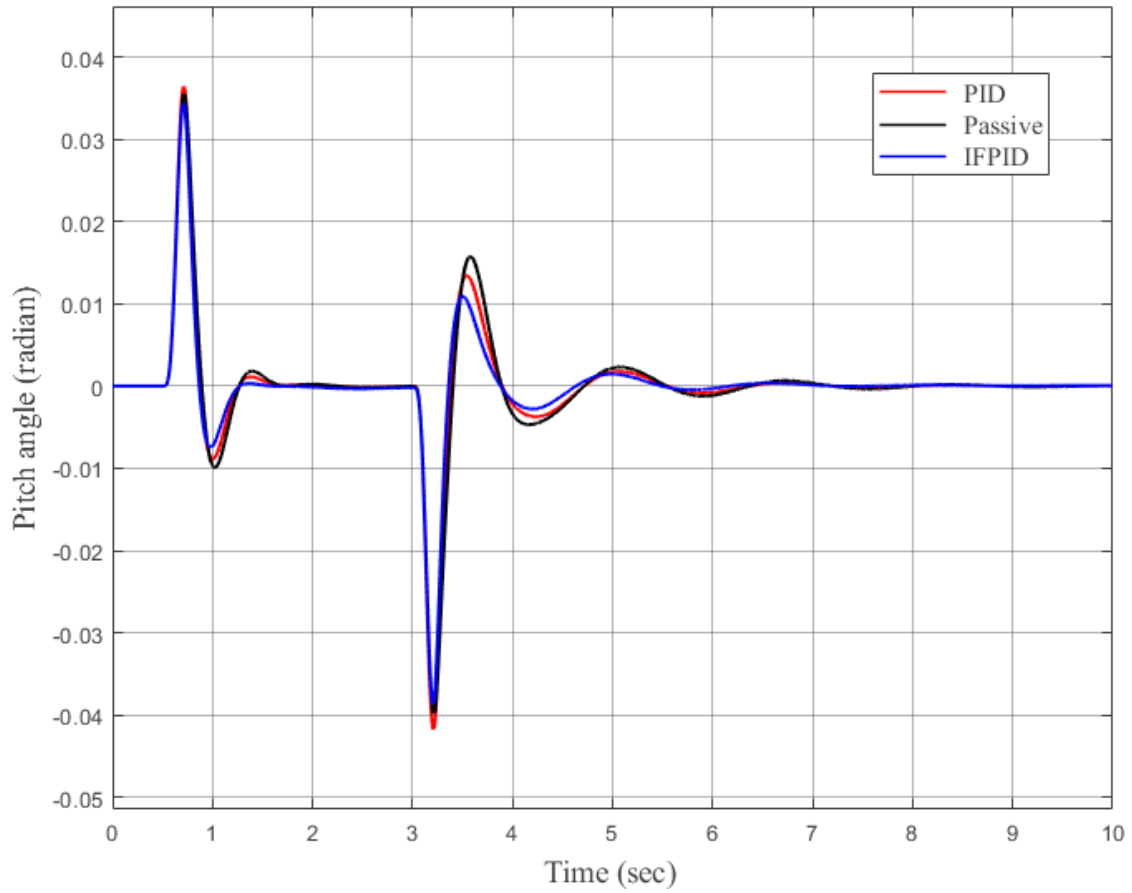


Figure 41. Pitch angle (radian) vs time (s) of a full car model

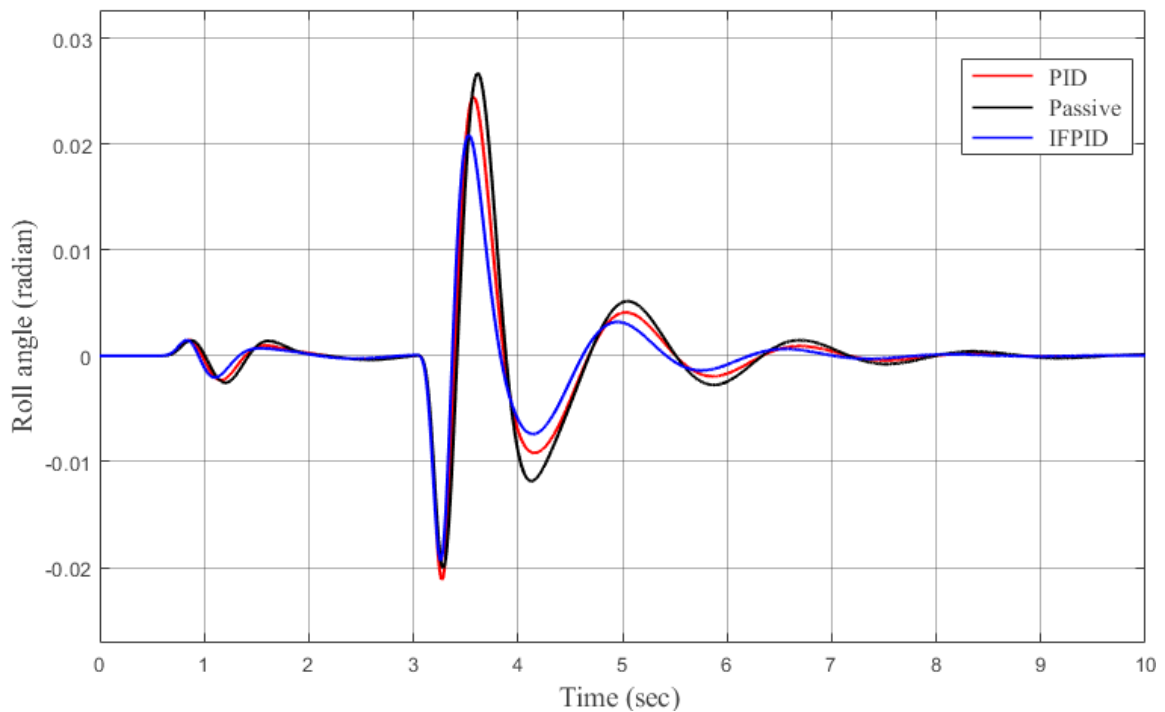


Figure 42. Roll angle (radian) vs time (s) of a full car model

Full-car model with hydraulic actuator and disturbance is modeled by the dynamics equation. One of the desired point of this study is to decrease the amplitude of car body vibration, when the suspension system exited from the road profile. Therefore, the effect of PID and intelligent fuzzy-PID controllers is simulated for impulse excitation.

By comparing the performance of the passive and active suspension system using intelligent fuzzy-PID control technique for full car model, it is clearly shows that there is a problem with robustness. Body Displacement improved even the amplitude is slightly lower compare with passive suspension system and the settling time is fast. The term "riding quality" is used to describe body displacement. Each pairs of wheel set have same output performance due to mathematical modeling that has been use shows that there are relationship exist among these wheel. The conclusion for the relationship between these wheels is when front wheel right and left receive same types of disturbance therefore output performance will be exactly same. This also refers to the rear wheel right and left. According to the results intelligent fuzzy-PID controller can be clearly seen that considerable improvement the ride comfort.

6.4. Validation

The performance of the proposed fuzzy control scheme is illustrated in this section through a series of simulations. Simulation results are carried out using a quarter car model with active suspension, with road profile as shown in Figure 43, to demonstrate the efficiency of the proposed Fuzzy controller. For comparison purposes, the numerical results of the vehicle model with PID controller are also presented. PID control is perhaps the most widely used control method. It can provide fast response, well system stability and small steady state errors in a linear system with known parameters, (Palanisamy & Karuppan, 2016).

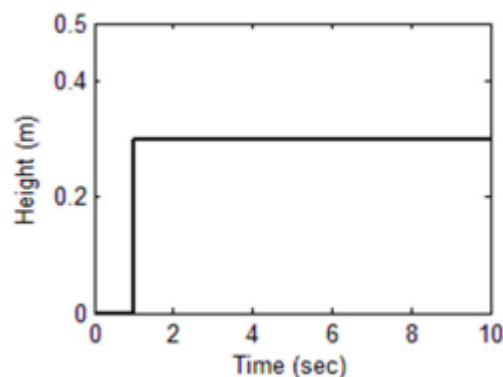


Figure 43. Road input signal for the simulation

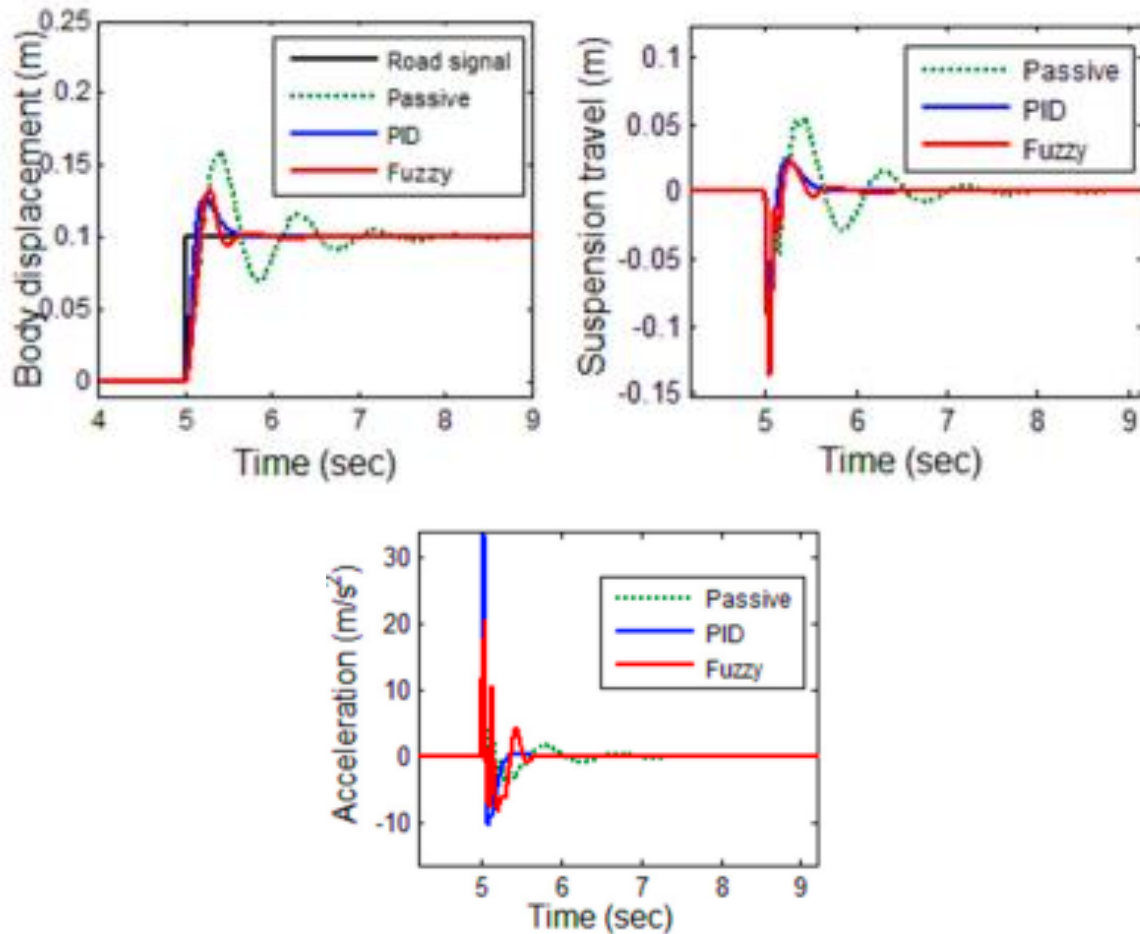
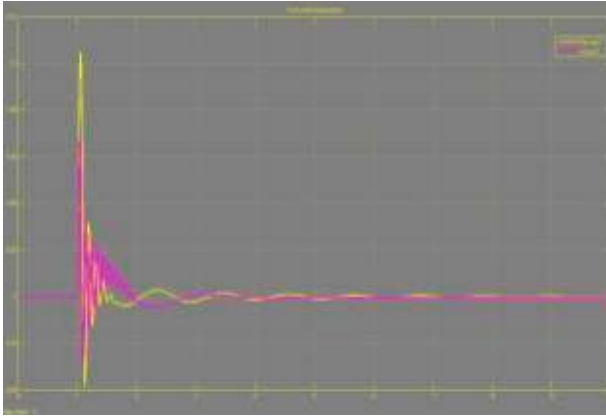


Figure 44. Response of suspension system for road input signal

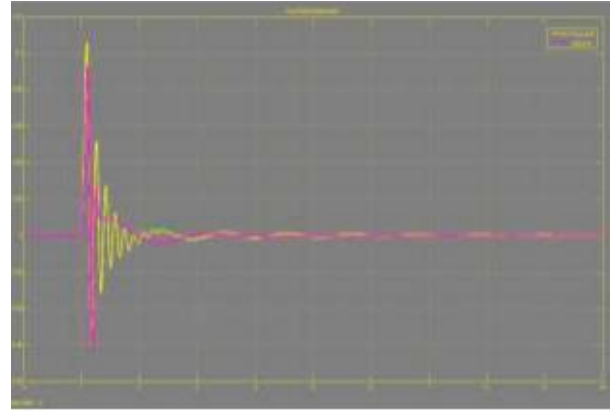
Figure 44 show the response of suspension system for the road input signal. The comparison is between passive, PID and fuzzy controller. In this comparison fuzzy controller outperform than passive and PID regarding to body displacement, suspension travel and acceleration. The graph is have similarity with this thesis work. So then this similarity indicate this thesis work result is validate.

In addition Figures 45 (A) to (E), respectively shown the simulation results of the half car model with Fuzzy-PID controller, (Nassar & Al-Ghanim, 2018).

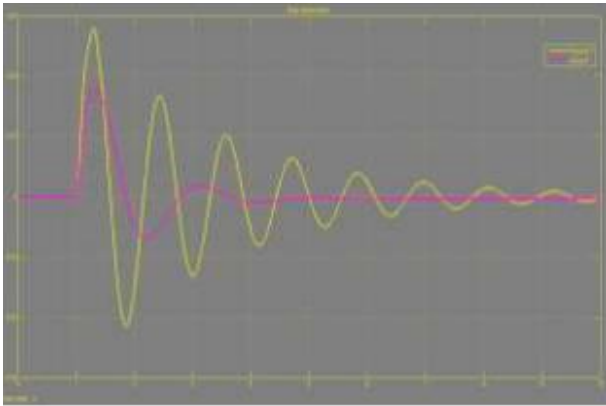
The comparison is done between passive, PID and fuzzy PID controllers. As we see from the graph the fuzzy PID controller outperform than passive and PID controller. In this thesis work the result of half car model have similarity with the graphs shown in figure 45. Therefore this similarity confirms the results obtained in this thesis work is validate.



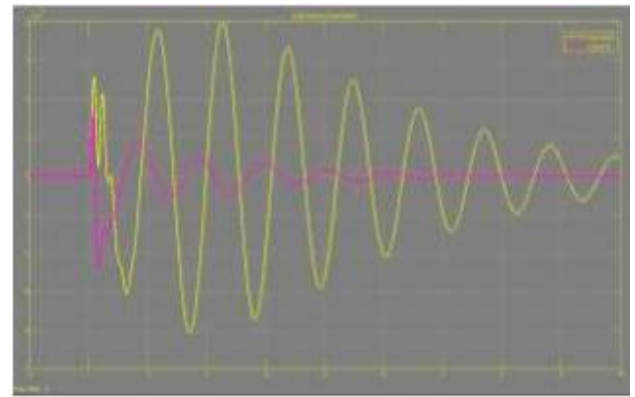
(A) Front axle vertical displacement of active half car model with fuzzy-PID controller.



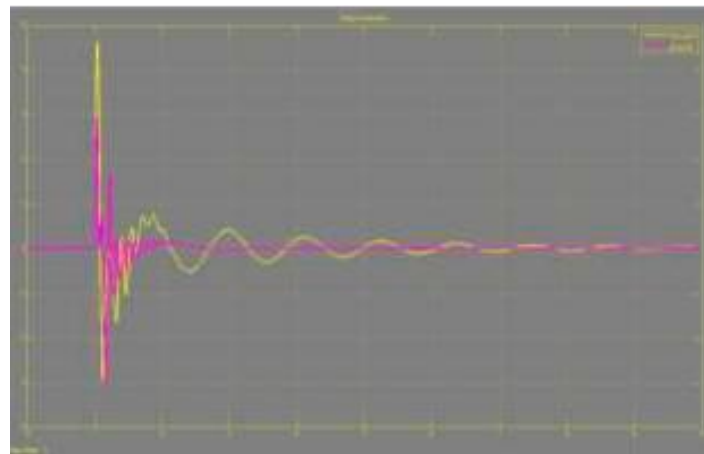
(B) Rear axle vertical displacement of active half car model with fuzzy-PID controller



(C) Vertical body displacement of active half car model with fuzzy-PID controller.



(D) Rotational body displacement of active half car model with fuzzy-PID controller



(E) Body acceleration of active half car model with fuzzy-PID controller.

Figure 45. Simulation results of the half car model with Fuzzy-PID controller

CHAPTER SEVEN: CONCLUSION AND RECOMMENDATIONS

7.1. Summary

The suspension system consists of a spring and oil damper interconnected between the vehicle body and tire. The spring isolates the vehicle body from road disturbances by carrying the load of the vehicle.

The design of the suspension system involves an optimization process as it is not possible to provide both ride comfort and stability simultaneously. Therefore, through an optimization process, the improved suspension can be designed to provide the vehicle with optimum ride quality and optimum road safety.

Initially, a model-based controller is designed with a quarter car model for active suspension. But it cannot be used to measure the pitch and roll motion of the vehicle. Both quarter and half car model does not model the actual system for practical applications. Hence, an accurate model of the actual system requires a complete car model with 7 degrees of freedom.

The objective of this thesis work is to develop an intelligent fuzzy-PID controller for vehicle active suspension based full car model of the vehicle.

The whole research work is classified into three different divisions namely system modeling, controller design and simulation. Before and after title selection and problem identification, the previous research work was carried out on the quarter car, half car and full car model active suspension system.

The mathematical model of the full car model is developed based on a quarter car and half car model of active suspension with 7 DOF systems. Electro-hydraulic systems are chosen as actuators for active control, and their dynamics are taken into consideration.

The designed fuzzy controller is implemented using Fuzzy Toolbox in MATLAB. The performance of the designed fuzzy controller is measured in terms of body displacement, body acceleration, pitch angle and roll angle.

Simulation based on the mathematical model for quarter car, half car and full car model performed by using MATLAB/SIMULINK software. It is possible to observe how the

suspension system performs in terms of stability and ride quality when road disturbance is supposed to represent the system's input.

By comparing the performance of the passive, and active suspension system using PID and intelligent fuzzy-PID control technique it is clearly shows that active suspension controlled with intelligent fuzzy-PID can give lower amplitude and faster settling time.

7.2. Conclusion

An intelligent fuzzy-PID controller was developed for full car model based hydraulic actuated active suspension to measure pitch and heave motion of vehicle which cannot be measured using quarter car model of vehicle.

The major difficulties in an intelligent fuzzy-PID controller were selection of fuzzy rules base and appropriate value of membership function.

It is clear from simulation that acceleration, pitch and roll angle of vehicle are considerably reduced by using an intelligent fuzzy-PID controller which further improve ride comfort and stability of vehicle.

The results of simulation confirms the feasibility of intelligent fuzzy-PID controller for hydraulic actuated active suspension in terms of body displacement, body acceleration, roll angle and pitch angle.

7.3. Recommendations

The performance of the proposed an intelligent fuzzy-PID controller is demonstrated by simulation under bump road. Further, its performance can be validated for various types of ISO road profile and random road profile to assess the robustness of proposed an intelligent fuzzy-PID controller.

An intelligent fuzzy-PID controller is developed based on modeling and simulation of full car model of active suspension. Hence, the real active suspension should be tested on real road with varying disturbance in order to assess the effectiveness of proposed controller.

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APPENDIX - A

State-space equation

$$\dot{x}(t) = Ax(t) + Bu(t) + Dw(t)$$

$$y(t) = x(t) + Dw(t)$$

Where, x = State input variable matrix, w = Rod input matrix, A = State Matrix, D = Input Matrix, B = Feedback Matrix, u = Controller input variable matrix

Quarter car model state-space equation

The states space of the quarter car model is defined as,

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_b} & -\frac{b_s}{m_b} & \frac{k_s}{m_b} & \frac{b_s}{m_b} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_w} & \frac{b_s}{m_w} & -\frac{k_s + k_t}{m_w} & -\frac{b_s}{m_w} \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ -1 \\ m_w \end{bmatrix} f_s + \begin{bmatrix} 0 \\ 0 \\ 0 \\ k_t \end{bmatrix} d$$

$$y = [1 \quad 0 \quad 0 \quad 0] \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}$$

Half car model state-space equation

The states space of the half-car model is defined as,

$$x = [\dot{x}_{sf} \quad \dot{x}_{uf} \quad \dot{x}_{sr} \quad \dot{x}_{ur} \quad x_{sf} \quad x_{uf} \quad x_{sr} \quad x_{ur}]^T$$

$$w = \begin{bmatrix} W_{sf} \\ W_{sr} \end{bmatrix}$$

$$u = \begin{bmatrix} f_{sf} \\ f_{sr} \end{bmatrix}$$

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

$$A_1 = \begin{bmatrix} -\frac{c_{sf}}{m_s} - \frac{l_f^2 c_{sf}}{l_s} & \frac{c_{sf}}{m_s} + \frac{l_f^2 c_{sf}}{l_s} & -\frac{c_{sr}}{m_s} - \frac{l_f l_r c_{sr}}{l_s} & \frac{c_{sr}}{m_s} - \frac{l_f l_r c_{sr}}{l_s} \\ \frac{c_{sf}}{m_{uf}} & -\frac{c_{sf}}{m_{uf}} & 0 & 0 \\ -\frac{c_{sf}}{m_s} + \frac{l_f l_r c_{sf}}{l_s} & \frac{c_{sf}}{m_s} - \frac{l_f l_r c_{sf}}{l_s} & -\frac{c_{sr}}{m_s} - \frac{l_r^2 c_{sf}}{l_s} & \frac{c_{sr}}{m_s} + \frac{l_r^2 c_{sf}}{l_s} \\ 0 & 0 & \frac{c_{sr}}{m_{ur}} & -\frac{c_{sr}}{m_{ur}} \end{bmatrix}$$

$$A_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} -\frac{k_{sf}}{m_s} - \frac{l_f^2 k_{sf}}{l_s} & \frac{k_{sf}}{m_s} + \frac{l_f^2 k_{sf}}{l_s} & -\frac{k_{sr}}{m_s} - \frac{l_f l_r k_{sr}}{l_s} & \frac{k_{sr}}{m_s} - \frac{l_f l_r k_{sr}}{l_s} \\ \frac{k_{sf}}{m_{uf}} & -\frac{(k_{sf} + k_{uf})}{m_{uf}} & 0 & 0 \\ -\frac{k_{sf}}{m_s} + \frac{l_f l_r k_{sf}}{l_s} & \frac{k_{sf}}{m_s} - \frac{l_f l_r k_{sf}}{l_s} & -\frac{k_{sr}}{m_s} - \frac{l_r^2 k_{sf}}{l_s} & \frac{k_{sr}}{m_s} + \frac{l_r^2 k_{sf}}{l_s} \\ 0 & 0 & \frac{k_{sr}}{m_{ur}} & -\frac{(k_{sf} + k_{uf})}{m_{ur}} \end{bmatrix}$$

$$A_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$D = \begin{bmatrix} 0 & 0 \\ \frac{k_{uf}}{m_{uf}} & 0 \\ 0 & 0 \\ 0 & \frac{k_{ur}}{m_{ur}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{1}{m_s} + \frac{l_f^2}{l_s} & \frac{1}{m_s} - \frac{l_f l_r}{l_s} \\ -\frac{1}{m_{sf}} & 0 \\ \frac{1}{m_s} - \frac{l_f l_r}{l_s} & \frac{1}{m_s} + \frac{l_r^2}{l_s} \\ 0 & -\frac{1}{m_{ur}} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Full car model state-space equation

The states space of the full car model is defined as,

$$A = \begin{bmatrix} 0 & \frac{-k_{u1}-k_1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{k_1}{m_b} & 0 & \frac{l_f k_1}{j} & 0 & \frac{t_{fl} k_1}{I} \\ 1 & -\frac{c_1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{c_1}{m_b} & 0 & \frac{l_f c_1}{j} & 0 & \frac{t_{fl} c_1}{I} \\ 0 & 0 & 0 & \frac{-k_{u2}-k_2}{m_2} & 0 & 0 & 0 & 0 & 0 & \frac{k_2}{m_b} & 0 & \frac{l_f k_2}{j} & 0 & -\frac{t_{fr} k_2}{I} \\ 0 & 0 & 1 & -\frac{c_2}{m_2} & 0 & 0 & 0 & 0 & 0 & \frac{c_2}{m_b} & 0 & \frac{l_f c_2}{j} & 0 & -\frac{t_{fr} c_2}{I} \\ 0 & 0 & 0 & 0 & 0 & \frac{-k_{u3}-k_3}{m_3} & 0 & 0 & 0 & \frac{k_3}{m_b} & 0 & -\frac{l_r k_3}{j} & 0 & \frac{t_{rl} k_3}{I} \\ 0 & 0 & 0 & 0 & 1 & -\frac{c_3}{m_3} & 0 & 0 & 0 & \frac{c_3}{m_b} & 0 & -\frac{l_r c_3}{j} & 0 & \frac{t_{rl} c_3}{I} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{-k_{u4}-k_4}{m_4} & 0 & \frac{k_4}{m_b} & 0 & -\frac{l_r k_4}{j} & 0 & -\frac{t_{rr} k_4}{I} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -\frac{c_4}{m_4} & 0 & \frac{c_4}{m_b} & 0 & -\frac{l_r c_4}{j} & 0 & -\frac{t_{rr} c_4}{I} \\ 0 & \frac{k_1}{m_1} & 0 & \frac{k_2}{m_2} & 0 & \frac{k_3}{m_3} & 0 & \frac{k_4}{m_4} & 0 & -\frac{k_1+k_2+k_3+k_4}{m_b} & 0 & \frac{(l_r k_3 + l_r k_4)}{j} & 0 & \frac{(t_{fr} k_2 - t_{fl} k_1)}{I} \\ 0 & \frac{c_1}{m_1} & 0 & \frac{c_2}{m_2} & 0 & \frac{c_3}{m_3} & 0 & \frac{c_4}{m_4} & 1 & -\frac{c_1+c_2+c_3+c_4}{m_b} & 0 & \frac{(l_r c_3 + l_r c_4)}{j} & 0 & \frac{(t_{fr} c_2 - t_{fl} c_1)}{I} \\ 0 & \frac{k_1 l_f}{m_1} & 0 & \frac{k_2 l_f}{m_2} & 0 & -\frac{k_3 l_f}{m_3} & 0 & -\frac{k_4 l_f}{m_4} & 0 & \frac{(l_r k_3 + l_r k_4)}{m_b} & 0 & \frac{(-l_f^2(k_1 + k_2))}{j} & 0 & \frac{(l_f t_{fr} k_2 - l_f t_{fl} k_1)}{I} \\ 0 & \frac{c_1 l_f}{m_1} & 0 & \frac{c_2 l_f}{m_2} & 0 & -\frac{c_3 l_f}{m_3} & 0 & -\frac{c_4 l_f}{m_4} & 0 & \frac{(l_r c_3 + l_r c_4)}{m_b} & 1 & \frac{(-l_f^2(c_1 + c_2))}{j} & 0 & \frac{(l_f t_{fr} c_2 - l_f t_{fl} c_1)}{I} \\ 0 & \frac{k_1 t_{fl}}{m_1} & 0 & -\frac{k_2 t_{fr}}{m_2} & 0 & \frac{k_3 t_{rl}}{m_3} & 0 & -\frac{k_4 t_{rr}}{m_4} & 0 & \frac{(k_2 t_{fr} + k_4 t_{rr})}{m_b} & 0 & \frac{(l_r t_{rl} k_3 - l_r t_{rr} k_4)}{j} & 0 & \frac{(-t_{fl}^2 k_1 - t_{fr}^2 k_2)}{I} \\ 0 & \frac{c_1 t_{fl}}{m_1} & 0 & -\frac{c_2 t_{fr}}{m_2} & 0 & \frac{c_3 t_{rl}}{m_3} & 0 & -\frac{c_4 t_{rr}}{m_4} & 0 & \frac{(-k_1 t_{fl} - k_3 t_{rl})}{m_b} & 0 & \frac{(l_f t_{fr} k_2 - l_f t_{fl} k_1)}{j} & 1 & \frac{(-t_{fl}^2 k_3 - t_{rr}^2 k_4)}{I} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & -\frac{1}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{1}{m_b} & 0 & \frac{l_f}{J} & 0 & \frac{t_f}{I} \\ 0 & 0 & 0 & -\frac{1}{m_2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{m_b} & 0 & \frac{l_f}{J} & 0 & -\frac{t_f}{I} \\ 0 & 0 & 0 & 0 & 0 & -\frac{1}{m_3} & 0 & 0 & 0 & \frac{1}{m_b} & 0 & -\frac{l_f}{J} & 0 & \frac{t_f}{I} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -\frac{1}{m_4} & 0 & \frac{1}{m_b} & 0 & -\frac{l_f}{J} & 0 & -\frac{t_f}{I} \end{bmatrix}^T$$

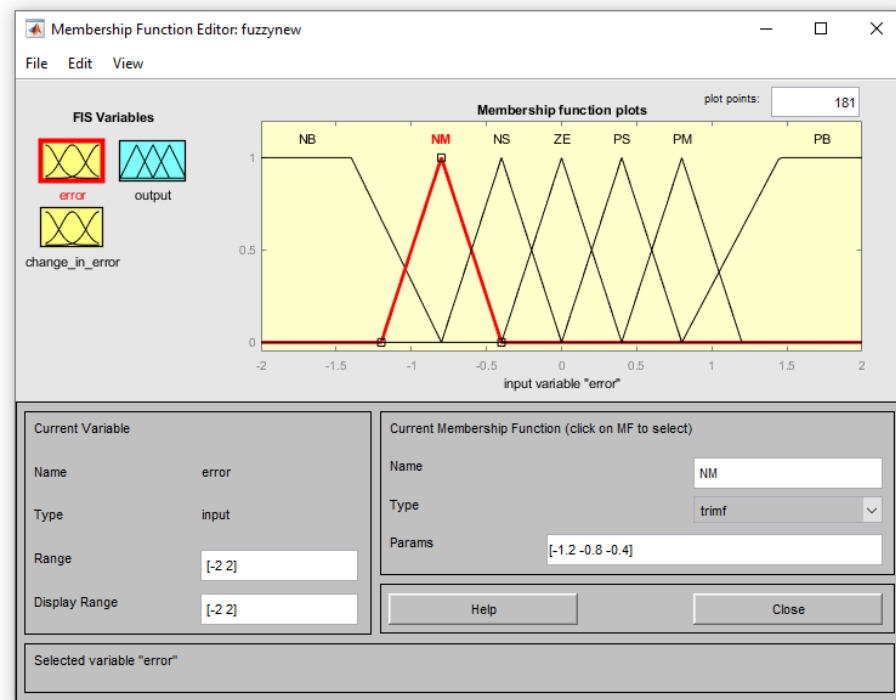
$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$D = 0$$

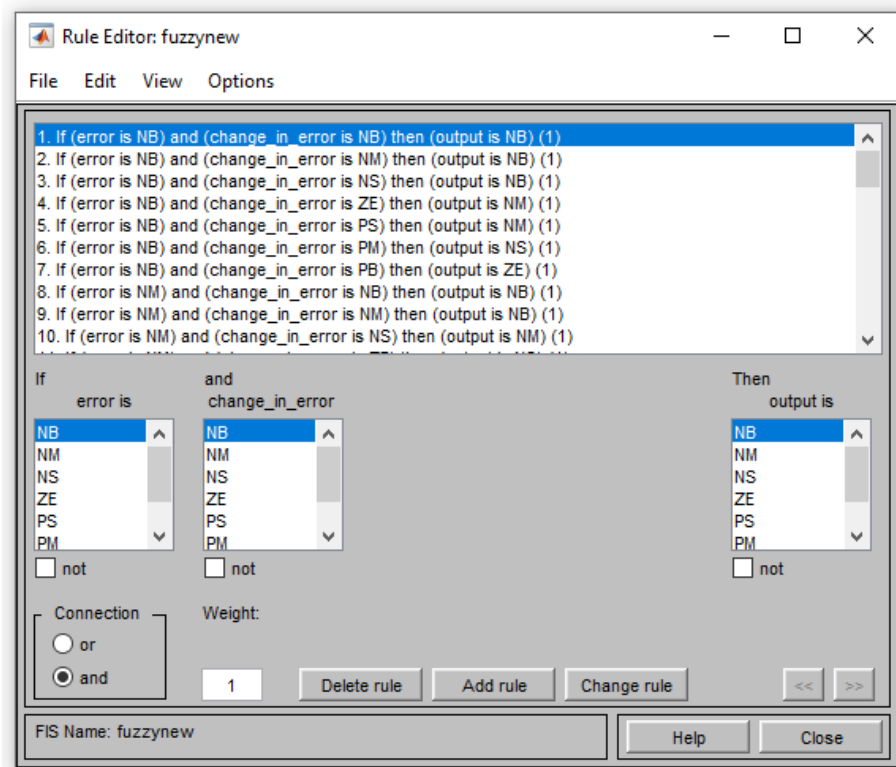
$$E = \begin{bmatrix} 0 & \frac{k_{u1}}{m_1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{k_{u2}}{m_2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{k_{u3}}{m_3} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \frac{k_{u4}}{m_4} & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T$$

APPENDIX - B

The membership function editor in Fuzzy Logic Toolbox



The list of rules used in the rules base



Rules of fuzzy

