



ADAMA SCIENCE AND TECHNOLOGY UNIVERSITY
SCHOOL OF CIVIL ENGINEERING AND ARCHITECTURE
DEPARTEMENT OF CIVIL ENGINEERING

SLOPE STABILITY RISK ASSESSMENT OF EMBANKMENT DAM,
CASE OF RIBB DAM

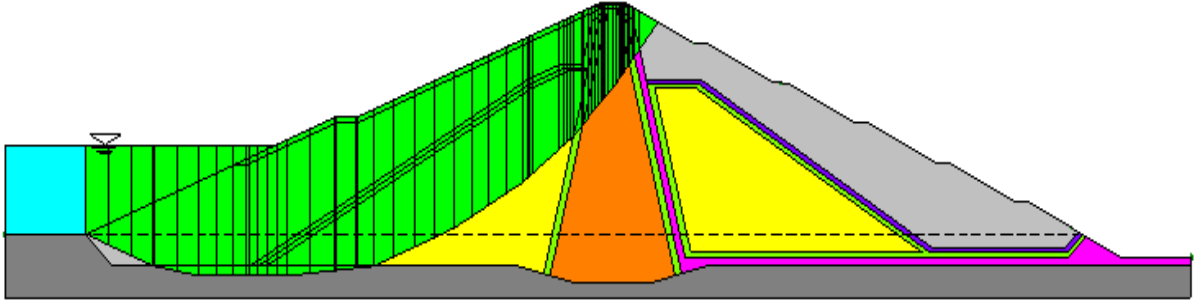
By

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Adama, Ethiopia

SLOPE STABILITY RISK ASSESSMENT OF EMBANKMENT DAM, CASE OF RIBB DAM



A Thesis submitted to Adama Science and Technology University in Partial Fulfillment of the Requirements for “*the Degree of M.Sc. in Civil Engineering (Geotechnical Engineering)*”

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DECLARATION

This thesis is my original work, has not been presented for a degree in any other university and that all sources of material used are duly acknowledged.

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DEDICATION

To Abaynew Takele that passed away suddenly (March 7, 2014).

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NOTATIONS

The following abbreviations and symbols are used to express the analogous phrases in a thesis:

b_i = slice base length

c = cohesion of the soil

COV = coefficient of variation

d = standard deviation corresponding to ε

DS = down stream

E_i = interslice force

$E[FS]$ = mean value of factor of safety

$E[x]$ = mean value of random variables

FE = finite element

FS = factor of safety

$FOSM$ = first order second moment

$f(x)$ = probability density function

$f(x_i)$ = half-sine function

Fx_i, Fy_i = slice forces acting x-y axes

g = gravitational acceleration

i = slice of slip surface

K_h = horizontal acceleration coefficient

l_i = slice base length

m = number of variables

M = mid point of slice base length

MCS = Monte Carlo simulation

$M-P$ = Morgenstern-Price

$M1_i$ = moment at the center of slice

N_i = slice base normal force

N_{mc} = number of Monte Carlo trials

O = center of Bishop/ordinary slip surface

PF = probability of failure

R = radius of slip surface from point O

RDD = rapid drawdown

RI = reliability index

SD = Standard deviation of materials

T_i = slice base shear force

US = upstream

$U.S.$ = United States

$USSD$ = United States society on dams

u_i = pore pressure within slice

W_i = slice weight

$MoWE$ = Ministry of Water and Energy

Wt = material weight

x = random variables like c, φ , etc.

x_i = boundary point of slices

y_{gi} = vertical coordinate of slice weight

y_M = the vertical coordinate of point M

Z_i = position of interslice force

Z_0, Z_1, Z_2, Z_3 = distances b/n intervals

$\Delta Z, \Delta Z'$ = partial sampling distances

$\rho(\Delta Z, \Delta Z')$ = soil coefficient correlation

σ_n = normal stress on shear plane

$\sigma[FS]$ = factor of safety standard deviation

σ_x, σ_y = normal stresses on x and y-axes

σ^2 = variance of random variables

τ_{xy} = shear stress on x-y plane

α_i = slice base inclination

β = reliability index

φ = angle of internal friction of the soil

δ_i = inclination of E_i from horizontal plane

ϑ = scale of fluctuation

γ = unit of weight the soil

ε = desired level of confidence

λ = dimensionless half-sine parameter

Γ = dimensionless variance function

ABSTRACT

The slopes stability of Ribb earth-rock fill dam, was carried out by SLOPE/W for searching the probability failure or reliability of the dam, using Spencer, M-P and Janbu method of slices. The analysis accounts the spatial variability of the input variables and the statistical uncertainty of the input parameters of unit weight, cohesion, angle of internal friction and seismic coefficient by performing 115000 Monte Carlo trials within 0.5 meter soil sampling distance. The pore water pressure analysis through the dam body has been carried out by SEEP/W finite element mesh norm.

The factor of safety was used to find the critical slip surface of slope by evaluating the slip surface that got the lowest factor of safety. From the selected critical surfaces, the sensitivity analysis, it was concluded that the variation in friction angle of the shell and foundation materials affect the factor of safety more than any other material parameter in both upstream and downstream slopes. For rapid drawdown, the stability is much more sensitive to changes in the rock fill material friction angle.

The materials in the zoned embankment dam were assigned some variation for the probability analysis which showed that the selected slip surfaces had probability of failure which is much greater than the acceptable limit. The reliability of index value is also unsatisfied compared to the acceptable limit. The most critical slip surface derived from deterministic analysis is not necessarily sufficient, but the probabilistic stability analysis is an appropriate estimation of the most critical slip surface search.

Key words: Slope stability, Deterministic method, Sensitivity analysis, Probabilistic analysis.

1. INTRODUCTION

1.1. General

Slope stability analysis is an important part of the design of embankments, cut slopes, excavations and dams. In practice, limit equilibrium methods are used in the analysis of slope stability. Failure is considered to occur along an assumed or a known failure surface and the shear strength required maintaining equilibrium is compared with the available shear strength of the soil. The requirements for static equilibrium of the soil mass are used to compute a factor of safety with respect to shear strength. The factor of safety is defined as the ratio of the available shear resistance to mobilized shear that required for equilibrium. Limit equilibrium analyses assumed the factor of safety is the same along the entire slip surface. The most common methods for limit equilibrium analyses are methods of slices. In these methods, the soil mass above the assumed slip surface is divided into vertical slices for purposes of convenience in analysis.

Deterministic approaches are employed in the analysis and design of embankment structures. These approaches are characterized by the use of specified minimum factors of safety or specified minimum material properties. Deterministic approaches did not rigorously account for uncertainties in slope stability analysis and design. In order to address uncertainty, probability theory has been widely accepted and used in embankment dam design in which some statistical knowledge of random variables such as their mean values and standard deviations are used to introduce them into applications (Calamak and Yanmaz, 2014).

During the last two decades, there has been an increasing recognition of uncertainties in geotechnical engineering (Bertoldi, 1988). Monte Carlo simulation is a very useful approach for modeling the problems with uncertainty in the inputs. This simulation method is based on input parameters those reflect the probability function of each parameter. Thus, the repetitive calculations take into account the randomly selected combinations of the statistical input parameters, generating the probability functions for the inputs. Based on those probability functions, a risk level representing the high end, central tendency (median or mean), or any other desired level of probability can be obtained (Manafi, 2012).

1.2. Ribb dam background

The main dam project is under construction separately from the Ribb irrigation and drainage projects. The type is an earth-rock fill zone embankment dam some 73 m high, 800 m length, and normal water level of the reservoir is 1940.0 m from mean seal level as shown below Table 1.1. The dam is being constructed mainly from natural materials of compacted clay core, compacted alluvium foundation, compacted alluvium shell, compacted gravel transition, compacted rock fill materials near by the site and filter material some 40 km distance from

the site. The location of the dam is between Ebinat and Farta weredas in South Gondar as the map showed in Figure 1.1 below, some 687 km from Addis Ababa (MoWE, 2010).

Table 1.1: General description and feature for Ribb embankment dam

<i>Description</i>	<i>Feature</i>	<i>Description</i>	<i>Feature</i>
Type of dam	Earth-rock fill	Dam height above river bed level	73.2 m
Top dam elevation	1946.2 m	Height above foundation bottom	83.2 m
Normal water level	1940 m	Upstream berm length	8 m
Maximum water level	1943 m	Downstream berms length	5 m
Rapid drawdown level	1901.4 m	Upstream slope	2.5 H:1 V
Sediment level	1901.2 m	Downstream slope	2 H:1 V
River bed level	1873 m	Clay core slopes	1 H:4 V
Bottom foundation level	1963 m	Shell material slopes	1.75 H:1 V
Upstream berm level	1910 m	Fine filter thickness	1-1.5 m
Crest length	800 m	Coarse filter thickness	2.5-2.8 m
Crest width	10 m	Core width top and bottom	3, 30.1 m

To obtain the factor of safety of both the upstream and downstream slopes and the rapid drawdown condition, the slope stability investigation of Ribb dam was carried out using the SLOPE/W product in Geo-Studio computer program based on limit equilibrium method particularly using Morgenstern-Price method (Ministry of Water and Energy, 2010).

The computed factor of safety against slope failures under different loading conditions including earthquake for the selected typical dam, were obtained as shown Table 1.2 below.

Table 1.2: Ribb dam factors of safety (Ministry of Water and Energy, 2010)

Loading condition	Upstream slope	Downstream slope
Steady state seepage condition	1.892	1.688
Steady state seepage + earthquake	1.178	1.225
Rapid drawdown condition	1.32	-



Figure 1.1: Location of Ribb dam project

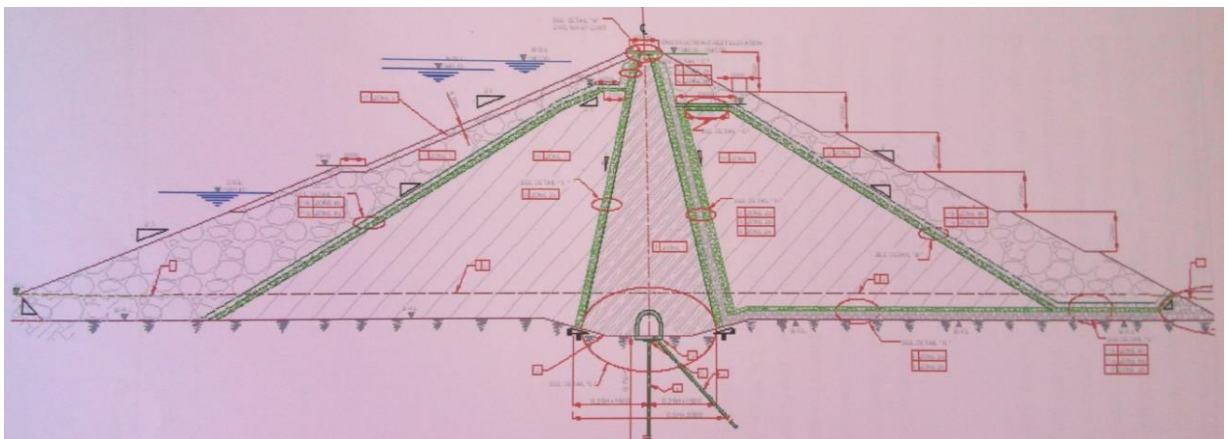


Figure 1.2: Ribb dam proposed cross section

1.3. Problem statement

The slope stability analysis of Ribb embankment dam was conducted using deterministic method or limit equilibrium method, particularly Morgenstren-Price method was used by assuming circular slip surface failure. But deterministic approaches or limit equilibrium method did not rigorously account uncertainties in slopes stability analysis.

A slope with a higher factor of safety value may be no more stable than a slope with a lower factor of safety value, depending on the nature and variability of the slope materials. There are many sources of uncertainty in deterministic slope stability analysis approaches. These uncertainties may come from

- Statistical uncertainties
- Natural soil variability
- Models uncertainty

These implied that deterministic approaches specially limit equilibrium method analyses are suffer from limitations. Now deterministic methods or limit equilibrium methods analysis role is being questioned, because neither factor of safety exceeding unity guarantees stability for such type of mass movement, nor the value less than unity always implies failure due to the mentioned uncertainties.

1.4. Objectives of the study

Examination of dam slopes stability after construction is an important issue for dam safety evaluation. Thus, the purpose of the present thesis is mainly aimed at assessing the uncertainties using probabilistic analysis perception to evaluate the slopes stability of Ribb embankment dam with risk based approaches. In this sense, the assessment of the uncertainties in the evaluation of soil properties affecting the slopes stability is vital and the possibility of reducing the uncertainty is attractive.

The results of this study will provide a better understanding of the effects of soil variability and statistical uncertainties on the stability of Ribb embankment dam. In this sense, the researcher will enable to quantify the effect of uncertainties on the stability of slopes of embankment dams.

In order to achieve the overall aim of this research, some of the specific objectives of the study are established, as summarized below:

- To compare and show the inherent limitations of various methods of slices of limit equilibrium analyses.
- To show graphically the data used in the analyses make it possible to see the influence of parameters along the slip surfaces.
- To evaluate the position of the critical slip surface with the lowest factor of safety of the given dam under long-term condition.
- To ensure that a consistent and systematic approach was adopted to investigate the slope stability of the given embankment dam.

1.5. Scope and limitations

Scope: particular interest is given during this thesis to modeling of Ribb embankment dam stability or instability caused by uncertainties using two useful indices: probability of failure and reliability index methods.

Limitation: this work is applied only for two-dimensional plain-strain problem using SLOPE/W software window with other parental analysis of SEEP/W for finite element pore-water pressure analysis. In this study, the analysis is performed only using secondary data availability and the risk assessment based on only limit equilibrium approaches definition of

failure surface mechanism. Finite element failure definition based probabilistic analysis couldn't use in this study due to lack of reliable data from different sources.

2. LITERATURES REVIEW

2.1. Introduction

In the assessment of the stability of slopes, Engineers primarily used factor of safety values to determine how close or far slopes are from failure. Conventional limit equilibrium techniques are the most commonly used analysis methods. Since in the past many years, slope stability of embankments have been carried out by limit equilibrium analyses of various methods such as Fellenius, Bishop, Janbu, Morgenstren-Price, Spencer, etc.

Recently, however; probabilistic analysis is a more realistic approach to the assessment of slope stability because the uncertainty and variability in soil properties can be explicitly taken into account. Unlike deterministic analysis, which is based on assumed characteristics values of soil properties, probabilistic analysis considers the variable nature of soil properties based on their statistical characteristics. The later approach leads to a more realistic measure of the stability of a slope, which is usually characterized by the probability of failure.

2.2. Limit equilibrium methods

Limit equilibrium methods consider force and/or moment equilibrium of a mass of soil above potential failure surface. The soil above the potential failure surface is assumed to be rigid. Because of the soil is assumed to be rigid, the factor of safety is constant over the entire failure surface. This topic reviews and compares the five commonly used limit equilibrium methods as listed below.

2.2.1. Ordinary/Fellenius method

This method is also sometimes referred to as the Swedish circle method. It is the simplest method of slices to use. The simplicity of the method made it possible to compute factors of safety using hand calculations. The method assumed that all interslice forces are ignored and the slice weight is resolved into forces parallel and perpendicular to the slice base. The force perpendicular to the slice base is the base normal force, which is used to compute the available shear strength. The weight component parallel to the slice base is the gravitational driving force (mobilized shear). Summation of moments about a point used to describe the trial slip surface which is also used to compute the factor of safety (Pettersen, 1955).

Only moment equilibrium was satisfied. In this respect, the factor of safety calculated by this method is typically conservative. The factor of safety calculated by ordinary or Fellenius method for slopes stability is very conservative by as much as 60 percent, when compared with values from more exact solutions (Whitman and Baily, 1967; cited in Fredlund and Krahn, 1977). For this reason, this method is not used much nowadays. Forces acting on individual slices are displayed in the following Figure 2.1.

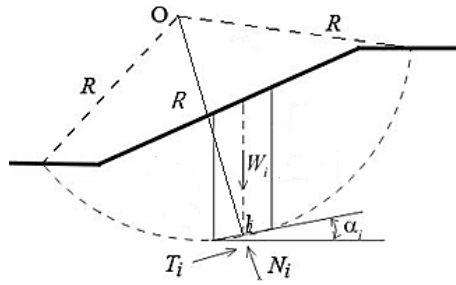


Figure 2.1: Static scheme of ordinary/Fellenius method

The factor of safety FS was derived from the summation of moments about a common point, O and expressed as:

$$FS = \frac{1}{\sum_i W_i \cdot \sin \alpha_i} \cdot \sum_i [c_i \cdot l_i + (N_i - u_i \cdot l_i) \cdot \tan \varphi_i]$$

Where:

O = center of the arc surface

W_i = slice weight

R = radius of slip surface

T_i = shear force on the slip surface

u_i = pore pressure within slice

N_i = base normal ($W \cos \alpha$)

c_i = effective cohesion

α_i = slice base inclination

φ_i = effective friction angle

l_i = slice base length

2.2.2. Bishop simplified method

The Bishop simplified method also used the method of slices to find the factor of safety for embankment structures. The method included interslice normal forces, but ignored the interslice shear forces. Bishop (1955) developed an equation for the normal at the slice base by summing slice forces in the vertical direction. The consequence of this is that the base normal becomes a function of the factor of safety. This in turn made the factor of safety equation nonlinear and an iterative procedure is consequently required.

Although the simplified Bishop method did not satisfy complete static equilibrium, the procedure gave relatively accurate values for the factor of safety. The Simplified Bishop method is more accurate than the ordinary method of slices, especially for effective stress analysis with high pore-water pressure. The primary limitation of the Simplified Bishop method is that it is limited to circular slip surface only.

To solve for the Bishop simplified factor of safety, it is necessary to start with the initial guess of ordinary factor of safety. The procedure is repeated until the last computed factor of safety is within a specified tolerance of the previous factor of safety. Fortunately, usually it only takes a few iterations to reach a converged solution. Forces acting on individual slices are displayed in the following Figure 2.2.

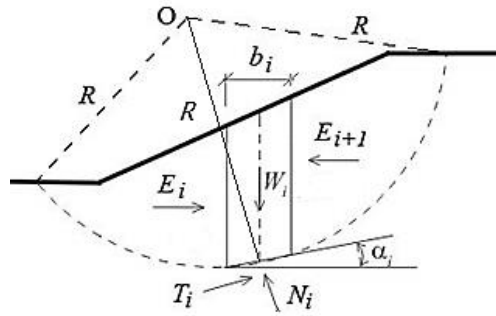


Figure 2.2: Static scheme of Bishop simplified method

The factor of safety FS was derived by taking moments about the center of the circle about a common point, O and expressed as:

$$FS = \frac{1}{\sum_i W_i \cdot \sin \alpha_i} \cdot \sum_i \frac{c_i \cdot b_i + (W_i - u_i \cdot b_i) \cdot \tan \varphi_i}{\cos \alpha_i + \frac{\tan \varphi_i \cdot \sin \alpha_i}{FS}}$$

Where:

R = radius of slip surface	T_i = shear force on the slip surface
u_i = pore pressure within slice	N_i = base normal on slice surface
c_i = effective cohesion	E_i, E_{i+1} = interslice normal forces
φ_i = effective friction angle	α_i = slice base inclination
W_i = slice weight	b_i = slice base length

In summary, the Bishop Simplified method: considered normal interslice forces but ignored interslice shear forces and satisfied over all moment and horizontal force equilibriums but not overall vertical force equilibrium.

2.2.3. Spencer method

The Spencer method (1967) is a general method of slices developed on the basis of limit equilibrium. It requires satisfying equilibrium of forces and moments acting on individual slices. Spencer developed two factors of safety equations: one with respect to moment equilibrium and another with respect to horizontal force equilibrium. It was adopted a constant relationship between the interslice shear and normal forces, and through an iterative procedure altered the interslice shear to normal ratio until the two factors of safety were the same. Forces acting on individual slices are displayed in the following Figure 2.3.

The following assumptions were introduced in the Spencer method to calculate the limit equilibrium of forces and moment on individual slices.

- Dividing planes between slices are always vertical.
- The line of action of weight of slices W_i passes through the center of the i^{th} segment of slip surface represented by point M.

- The normal force N_i is acting in the center of the i^{th} segment of slip surface, at M.
- Inclination of forces E_i acting between slices is constant for all slices and equals to δ , only at slip surface end points is $\delta = 0$.

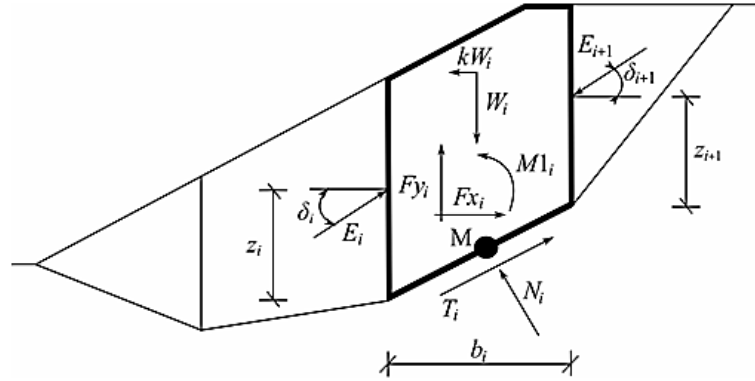


Figure 2.3: Static scheme of Spencer method

The following formula allowed calculating all forces E_i acting between slices for a given values of δ_i and FS . This solution assumed that at the slip surface origin the value of E is known and equal to $E_1 = 0$.

$$E_{i+1} = \frac{\left[(W_i - Fy_i) \cdot \cos \alpha_i - (K_h W_i - Fx_i) \cdot \sin \alpha_i - U_i + E_i \cdot \sin(\alpha_i - \delta_i) \right] \cdot \frac{\tan \varphi_i}{FS} + \frac{c_i}{FS} \cdot \frac{b_i}{\cos \alpha_i} - (W_i - Fy_i) \cdot \sin \alpha_i - (K_h W_i - Fx_i) \cdot \cos \alpha_i + E_i \cdot \cos(\alpha_i - \delta_i)}{\sin(\alpha_i - \delta_{i+1}) \cdot \frac{\tan \varphi_i}{FS} + \cos(\alpha_i - \delta_{i+1})}$$

The following formula allowed us calculating for a given value of δ all arm Z of forces acting between slices, knowing the value on the left at the slip surface origin, where $Z_1 = 0$.

$$Z_{i+1} = \frac{\frac{b_i}{2} \cdot \left[E_{i+1} (\sin \delta_{i+1} - \cos \delta_{i+1} \cdot \tan \alpha_i) + E_i (\sin \delta_i - \cos \delta_i \cdot \tan \alpha_i) \right] + E_i \cdot z_i \cdot \cos \delta_i - M1_i + K_h W_i \cdot (y_M - y_{gi})}{E_{i+1} \cdot \cos \delta_{i+1}}$$

Where:

W_i = block or slice weight

$K_h \cdot W_i$ = horizontal seismic force

K_h = horizontal seismic coefficient

N_i = normal force on slip surface

Fx_i, Fy_i = forces acting on slice

y_{gi} = vertical coordinate of slice

y_M = vertical coordinate at M

b_i = slice base length

T_i = shear force on the slip surface

c_i, φ_i = shear strength parameters

E_i, E_{i+1} = interslice forces inclined by δ

δ_i, δ_{i+1} = inclination of interslice forces

$M1_i$ = moment at the center of slice

U_i = pore pressure resultant on slice

α_i = slice base inclination

- Dividing planes between slices are always vertical.
- The line of action of weight of slices W_i passes through the center of the i^{th} segment of slip surface represented by point M.
- The normal force N_i is acting in the center of the i^{th} segment of slip surface, at M.
- Inclination of forces E_i acting between slices is different on each slice δ_i and at slip surface end points is $\delta = 0$.

The only difference between Spencer and Morgenstern-Price method is shown in the above list of assumptions. Choice of inclination angles δ_i of forces E_i acting between the slices is realized with the help of half-sine function. One of the functions in the Figure 2.5 is automatically chosen. This choice of the shape of function has a minor influence on final results, but suitable choice can improve the convergence of method. Functional value of half-sine function $f(x_i)$ at boundary point x_i multiplied by parameter λ results the value of inclination angle δ_i $\{\delta_i = \lambda f(x_i)\}$.

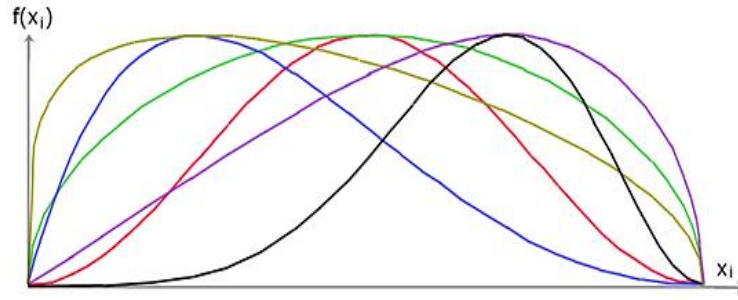


Figure 2.5: Graph of half-sine functions

The following formula allowed calculating all forces E_i acting between slices for a given values of δ_i and FS . This solution assumes that at the slip surface origin the value of E is known and equal to $E_1 = 0$.

$$E_{i+1} = \frac{\left[(W_i - Fy_i) \cdot \cos \alpha_i - (K_h W_i - Fx_i) \cdot \sin \alpha_i - U_i + E_i \cdot \sin(\alpha_i - \delta_i) \right] \cdot \frac{\tan \phi_i}{FS} + \frac{c_i}{FS} \cdot \frac{b_i}{\cos \alpha_i} - (W_i - Fy_i) \cdot \sin \alpha_i - (K_h W_i - Fx_i) \cdot \cos \alpha_i + E_i \cdot \cos(\alpha_i - \delta_i)}{\sin(\alpha_i - \delta_{i+1}) \cdot \frac{\tan \phi_i}{FS} + \cos(\alpha_i - \delta_{i+1})}$$

The following formula allowed us calculating all arm Z_i of forces acting between slices for a given value of δ_i , knowing the value on the left at the slip surface origin, where $Z_1 = 0$.

$$Z_{i+1} = \frac{\frac{b_i}{2} \cdot \left[E_{i+1} (\sin \delta_{i+1} - \cos \delta_{i+1} \cdot \tan \alpha_i) + E_i (\sin \delta_i - \cos \delta_i \cdot \tan \alpha_i) \right] + E_i \cdot z_i \cdot \cos \delta_i - M1_i + K_h W_i \cdot (y_M - y_{\bar{g}i})}{E_{i+1} \cdot \cos \delta_{i+1}}$$

Where:

W_i = block or slice weight	T_i = shear force on the slip surface
$K_h \cdot W_i$ = horizontal seismic force	c_i, φ_i = shear strength parameters
K_h = horizontal seismic coefficient	E_i, E_{i+1} = interslice forces inclined by δ
N_i = normal force on slip surface	δ_i, δ_{i+1} = inclination of interslice forces
Fx_i, Fy_i = forces acting on slice	$M1_i$ = moment at the center of slice
y_{gi} = vertical coordinate of slice	U_i = pore pressure resultant on slice
y_M = vertical coordinate at M	α_i = slice base inclination
b_i = slice base length	

In Morgenstern-Price method, the factor safety FS is determined by employing the following iteration process:

1. The initial value of angle δ_i is set according to half-sine function $\{\delta_i = \lambda f(x_i)\}$.
2. The factor of safety FS for a given value of δ_i follows from the 1st equation, while assuming the value of $E_{n+1} = 0$ at the end of the slip surface.
3. The value of δ_i is provided by the 2nd equation using the values of E_i determined in the previous step with the requirement of having the moment on the last slice equal to zero. The equation doesn't provide the value of Z_{n+1} as it is equal to zero. Functional values $f(x_i)$ are the same all the time during the iteration, only parameter λ is iterated.
4. Steps 2 and 3 are then repeated until the value of δ_i (respect parameter λ) doesn't change.

In summary, the Morgenstern-Price method: considered both shear and normal interslice forces, satisfied both moment and forces equilibrium and allowed for a variety of interslice forces relationship (shear-normal ratio is various).

2.2.5. Janbu method

Janbu method (1954, 1973) is a general method of slices developed on the basis of limit equilibrium. Janbu method of slices is an iterative procedure using vertical slices and any shape of slip surface. The Janbu method only satisfied overall force equilibrium. Moment equilibrium is only satisfied at the slices level. This method imposed a stress distribution on the potential sliding mass by defining a line of thrust. Forces acting on individual slices are displayed in the Figure 2.6. Assuming the position of the resultant made it possible to compute the interslice shear forces by taking moments about the slice base center.

The following assumptions were introduced in the Janbu method to calculate the limit equilibrium of forces and moment on individual slices.

- Dividing planes between slices are always vertical.

- The line of action of weight of slices W_i passes through the center of the i^{th} segment of slip surface represented by point M.
- The normal force N_i is acting in the center of the i^{th} segment of slip surface, at M.
- Position Z_i of forces E_i acting between slices is assumed, at slip surface end point is $Z = 0$.

Choice of position Z_i can have significant influence on convergence of method. If it is made a bad assumption of position Z_i for a given slope, it can become impossible to satisfy equilibrium conditions (algorithm doesn't converge). Heights Z_i above slip surface are set approximately to one third of height of interface between the slices. In the case of unsatisfying equilibrium conditions algorithm changes heights to a different position (e.g. slightly higher within passive zone, near the toe and lower within active zone, near the crest of slope).

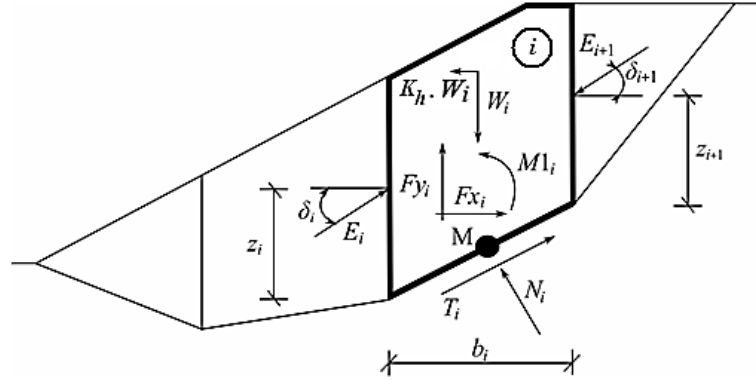


Figure 2.6: Static scheme of Janbu method

The following formula allowed calculating all forces E_i acting between slices for a given values of δ_i and FS . This solution assumes that at the slip surface origin the value of E is known and equal to $E_1 = 0$.

$$E_{i+1} = \frac{\left[(W_i - Fy_i) \cdot \cos \alpha_i - (K_h \cdot W_i - Fx_i) \cdot \sin \alpha_i - U_i + E_i \cdot \sin(\alpha_i - \delta_i) \right] \cdot \frac{\tan \varphi_i}{FS} + \frac{c_i}{FS} \cdot \frac{b_i}{\cos \alpha_i} - (W_i - Fy_i) \cdot \sin \alpha_i - (K_h \cdot W_i - Fx_i) \cdot \cos \alpha_i + E_i \cdot \cos(\alpha_i - \delta_i)}{\sin(\alpha_i - \delta_{i+1}) \cdot \frac{\tan \varphi_i}{FS} + \cos(\alpha_i - \delta_{i+1})}$$

The following formula allowed us calculating for a given value of δ all arm Z_i of forces acting between slices, knowing the value on the left at the slip surface origin, where $Z_1 = 0$.

$$\delta_{i+1} = \arctan \left(\frac{2 \cdot Z_{i+1}}{b_i} + \tan \alpha_i \right) - \arcsin \frac{E_i \left(\cos \delta_i \left(z_i - \frac{b_i \cdot \tan \alpha_i}{2} \right) + \sin \delta_i \cdot \frac{b_i}{2} \right) - M1_i}{E_{i+1} \sqrt{\left(z_{i+1} + \frac{b_i \cdot \tan \alpha_i}{2} \right)^2 + \left(\frac{b_i}{2} \right)^2}}$$

Where:

W_i = block or slice weight	T_i = shear force on the slip surface
$K_h \cdot W_i$ = horizontal seismic force	c_i, φ_i = shear strength parameters
K_h = horizontal seismic coefficient	E_i, E_{i+1} = interslice forces inclined by δ
N_i = normal force on slip surface	δ_i, δ_{i+1} = inclination of interslice forces
Fx_i, Fy_i = forces acting on slice	$M1_i$ = moment at the center of slice
y_M = vertical coordinate at M	α_i = slice base inclination
U_i = slice pore pressure resultant	b_i = slice base length

In Janbu method, the factor safety FS is determined by employing the following iteration process:

1. The initial value of angles set to zero, $\delta_i = 0$ and positions Z_i to approximately one third of interface height.
2. The factor of safety FS for a given value of δ_i follows from the 1st equation, while assuming the value of $E_{n+1} = 0$ at the end of the slip surface.
3. The value of δ_i is provided by the 2nd equation using the values of E_i determined in the previous step.
4. Steps 2 and 3 are then repeated until the value of FS doesn't change.

In summary, it is not necessarily better or worse than any of the other methods, as it has limitations just like all the other methods. The Janbu method factors of safety considered both shear and normal interslice forces, and satisfied overall both horizontal and vertical forces equilibrium, but not over all moment equilibrium.

2.3. Finite element based method

The limit equilibrium fundamentals discussed stress distributions obtained from limit equilibrium formulations, and showed that these stress distributions are not necessarily representative of the actual field stresses. To repeat, the limit equilibrium formulations gave stresses and forces that aimed to provide for equilibrium of each slice and made the factor of safety the same for each slice.

These inherent concepts and assumptions mean that it is not always possible to obtain realistic stress distributions along the slip surface or within the potential sliding mass. Some other concept of physics has to be added to the stability analysis to overcome these limitations. The missing concept of physics was a stress-strain relationship. Including such relationship means displacement compatibility should be satisfied, which in turn leads to much more realistic stress distributions.

The way of including a stress-strain relationship in a slope stability analysis was to establish the stress distribution in the ground using a finite element analysis and then used these

stresses in a stability analysis. SLOPE/W provided an alternative method of analysis using the stress state obtained from SIGMA/W. The basic information obtained from a finite element stress analysis is σ_x , σ_y and τ_{xy} within each element. The stresses σ_x , σ_y and τ_{xy} are known within each element (Krahn, 2004).

The dynamic programming method combined with a finite element stress analysis could be a viable and valuable tool for practical slope stability analyses. With the use of the finite element stress analysis, the presented method provided a solution of greater flexibility compared with those produced by conventional limit equilibrium methods of slices. More complex and rigorous stress-strain behavior of the soil such as elastic-plastic and nonlinear models could be used in the finite element stress analysis. Therefore, the effects of the stress history and volume-change behavior during shear could be taken into consideration as part of the analysis (Pham and Fredlund, 2003).

With the development of personal computer, finite element method has been increasingly used in slope stability analysis. The advantage of a finite element approach in the analysis of slope stability problems over limit equilibrium methods is that no assumption needs to be made in advance about the shape or location of the failure surface, slice forces and their directions. The method can be applied with complex slope configurations.

A potential sliding mass discretized into slices superimposed on the finite element mesh. Once the mobilized and resisting shear forces are available for each slice elements, the forces could be integrated over the length of the slip surface to determine a factor safety.

2.4. Probabilistic method of analysis

Probabilistic analyses combined with risk assessment are a better way to assess slope design in embankment dams compared to deterministic analyses. These analyses are suitable for use on evaluation of risk or when there is uncertainty in the input parameters. Probabilistic analyses required more computer power than deterministic analyses. In many cases the probabilistic analyses required ten to thousands more computer resources than an equivalent deterministic analyses. Methods like Monte Carlo simulation may require thousands of trials depending on the number of variables considered in the model (Gibson, 2011). Consequence should include the fatality risk. In this paper, the probability of failure and reliability index will be the concentrated design criteria.

Applications of probabilistic methods in geotechnical engineering have increased remarkably in recent years and major advances in this field of research have been made. Probabilistic slope stability analysis was one of the first applications of reliability-based design in geotechnical engineering. A lot of research efforts have been put into the probabilistic

description of the uncertainty in soil properties (Christian, 2004; cited in Muller, 2013). Probabilistic analyses allow uncertainty to be quantified and incorporated rationally into the design process. The application of a probabilistic approach to a geotechnical problem does not eliminate the uncertainties in an analysis or remove the need for judgments, but it can provide a way of assessing the uncertainties and of handling them consistently.

2.4.1. Probability density function

A probability density function is sometimes referred as probability distribution function. Probability density function is a type of function that links each outcome of a statistical experiment with its probability of occurrence. From the probability density function, the area under the function is established cumulative distributions function. Then the cumulative distribution function is inverted to form an inverse distribution function. The inverse distribution function is called a sampling function, since it is the function that is at the heart of sampling the various statistical parameters. According to different literatures reviewed, the normal distribution function and lognormal distribution function are the well-known types of probability distribution functions.

2.4.1.1. Normal distribution function

It is the most commonly used probability function, many natural data sets follow a bell-shaped distribution and measurements of many random variables appear to come from population frequency distributions that are closely approximated by a normal probability density function (Kramer, 1996). Figure 2.7, showed a typical normal distribution for the soil property ϕ when the mean is 28° and the standard deviation is 2° (Krahn, 2004).

Figure 2.8, presented the resulting sampling function of the normal probability density function. For the case of a normal probability function, the sampling function has a relatively straight segment in the middle. The implication of this is that parameters around the mean will be sampled more often than values at the extremities of the sampling function.

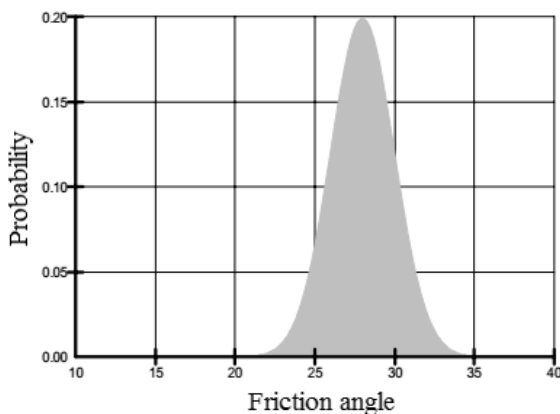


Figure 2.7: Normal distribution function

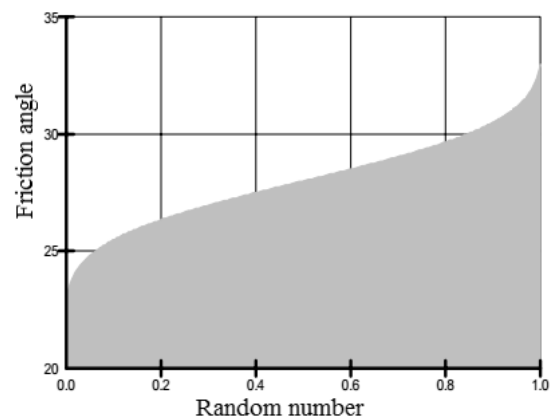


Figure 2.8: Normal sampling function

2.4.1.2. Lognormal distribution function

The Lognormal distribution has the property that it is always positive unlike the normal distribution. This is good for engineering problems which seldom deals with negative values. The Lognormal distribution have a random variable with a logarithm is normally distributed. Lognormal formula, where an offset value (θ) can be applied to shift the density function to better represent a probability density function. For example, if the actual mean value of the soil parameters is 18, it can be applied an offset parameter of 17 to the general density function. Figure 2.9 showed the density function with the applied offset.

The corresponding sampling function is shown in Figure 2.10. The sampling function indicated that when a random number is generated, there is an increased chance of obtaining a value between 17 and 19. This is consistent with the lognormal density function relatively compared with normal distribution function (Krahn, 2004).

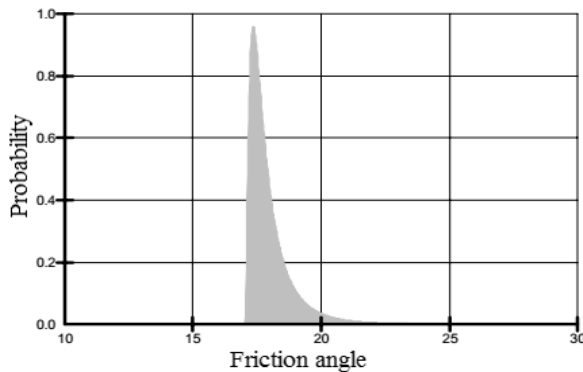


Figure 2.9: Lognormal distribution function

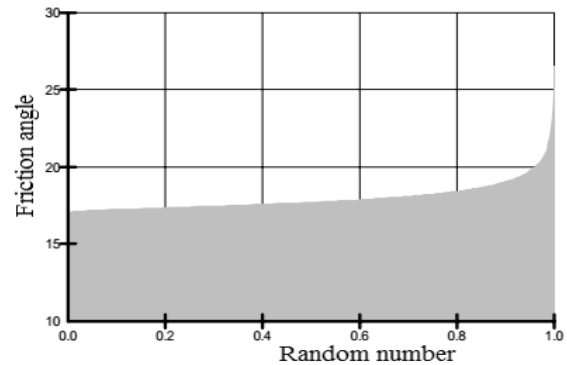


Figure 2.10: Lognormal sampling function

2.4.2. Spatial variability

Soil composition and properties vary from one location to another, even within homogeneous layers. The spatial variability of soil properties is a major source of uncertainty. Spatial variability is not a random process; rather it is controlled by location in space. Statistical parameters such as the mean and variance are one-point statistical parameters and cannot capture the features of the spatial structure of the soil. The performance of a structure is often controlled by the average soil properties within a zone of influence, rather than soil properties at discrete locations. Slope failure is more likely to occur when the average shear strength along the failure surface is insufficient rather than due to the presence of some local weak pockets. The uncertainty of the average shear strength along the slip surface, not the point strength, is therefore a more accurate measure of uncertainty.

The variance of the strength spatially averaged over some surface is less than the variance of point measurements. As the extent of the domain over which the soil property is being averaged increases, the variance decreases. At any location within a soil layer, a soil parameter is an uncertain quantity, or a random variable, unless it is measured at that

particular location. Each random variable is characterized by a probability distribution and is correlated with the random variables at adjacent locations. The set of random variables at all locations within the layer is referred to as a random field and is characterized by the joint probability distribution of all random variables. Figure 2.11, a realization of a random field of a variable x , with a mean $E[x]$, variance σ^2 and probabilistic density function $f(x)$, showing local averages over intervals Δz and $\Delta z'$ (Vanmarcke, 1983; cited El-Ramly et al., 2002).

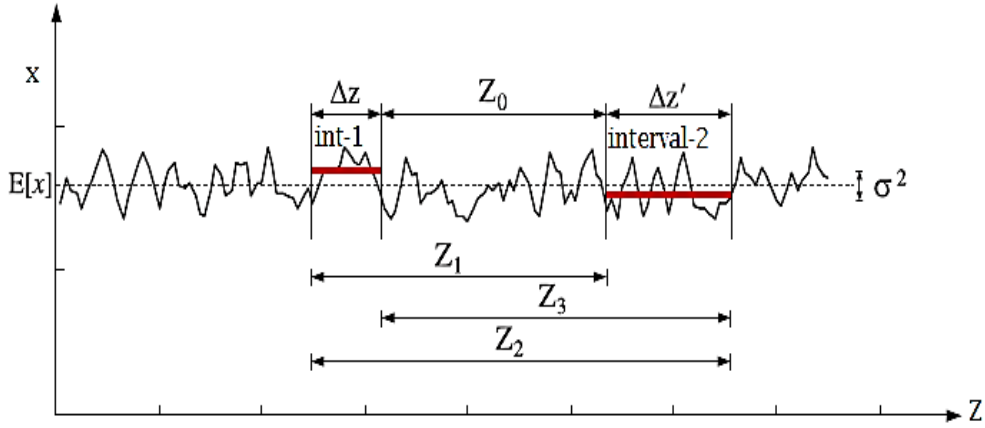


Figure 2.11: Realization of spatial variability

A partial sampling distance is considered to be correlated with the immediate preceding sampling distance. The coefficient of correlation between two soil sections can be computed with the equation as:

$$\rho(\Delta Z, \Delta Z') = \frac{Z_0^2 \Gamma(Z_0) - Z_1^2 \Gamma(Z_1) + Z_2^2 \Gamma(Z_2) - Z_3^2 \Gamma(Z_3)}{2 \Delta Z \Delta Z' [\Gamma(\Delta Z) \Gamma(\Delta Z')]^{0.5}}$$

Where:

$\Delta Z, \Delta Z'$ = partial sampling distances

Z_0 = specified sampling distance

$Z_1 = \Delta Z + Z_0$ = the distance from begin to begin of the intervals

$Z_2 = \Delta Z + Z_0 + \Delta Z'$ = the distance from begin to end of the intervals

$Z_3 = \Delta Z' + Z_0$ = the distance from the end to end of the intervals

$\Gamma(\Delta Z) = 1.0$ when $\Delta Z \leq \partial$ and Γ = dimensionless variance function

$\Gamma(\Delta Z) = \partial / \Delta Z$ when $\Delta Z > \partial$ ∂ = the scale of fluctuation

As evident from the above equations, the correlation is strong when the partial distance is short relative to the specified sampling distance, and the correlation is weak when the partial distance approaches the specified sampling distance. The probability of failure is the highest when the soil is sampled only once for each trial run and is the lowest when the soil is sampled for each slice.

2.4.3. Monte Carlo simulation

In probabilistic stability analysis: first order second moment (FOSM), point estimate and Monte Carlo simulation (MCS) are commonly used methods. In preliminary probabilistic slope stability studies, researchers applied FOSM for their probabilistic framework. Then, point estimated method was applied for probabilistic computations. Increasing computer facilities led researchers to adopt MCS method (Calamak and Yanmaz, 2014).

Monte Carlo simulation is the method that evaluates the problem many times using random input parameters. It provides a probabilistic approach to evaluate the physical behavior of infrastructures and the performance could be achieved in a more realistic manner. According to MCS method, a random value has been selected for each input parameter based on the assigned probability density function. For each analysis, probability of failure is computed by evaluating a target function. The method requires a definition of failure to be established prior to the analysis being undertaken (Zarghami and Rohaninejad, 2011).

Probabilistic slope stability analysis using the Monte Carlo method involves many trial runs. Theoretically, the more Monte Carlo trials the more accurate the solution will be, but the number of required Monte Carlo trials is dependent on the level of confidence in the solution and the amount of variables being considered. Statistically, the following equation has been recommended (Krahn, 2004; Chok, 2008):

$$N_{mc} = \left[\frac{(d^2)}{(4(1-\varepsilon)^2)} \right]^m$$

Where: N_{mc} = number of Monte Carlo trials

d = the standard deviation corresponding to the level of confidence

ε = desired level of confidence m = number of variables

The Monte Carlo method is a versatile computational procedure that is suitable for a high speed computer. In general, the implementation of the method involved the following steps:

- The selection of a deterministic solution, such as the Spencer, Morgenstern-Price, Janbu, finite element methods, etc.
- The decisions regarding which input parameters are to be modeled probabilistically and the representation of their variability in terms of a selected distribution model.
- The conversion of any density function into a sampling function, the random sampling of new input parameters and the determination of new factors of safety many times.
- The computation of the probability of failure based on the number of factors of safety less than one with respect to the total number of converged slip surfaces.

The number of Monte Carlo trials in an analysis is dependent on the number of variable input parameters and the expected probability of failure. In general, the number of required trials increases as the number of variable input increases or the expected probability of failure becomes smaller. It is usual to do thousands of trials in order to achieve an acceptable level of confidence in a Monte Carlo probabilistic slope stability analysis. Table 2.1 represents some recommended values of confidence levels (ϵ) with their relevant standard deviations (d).

Table 2.1: Standard deviations with confidence levels (Abramso et.al, 2001; Chok, 2008)

Confidence level (ϵ)	80%	90%	95%	99%
Standard deviation (d)	1.282	1.645	1.960	2.576

2.4.4. Risk evaluation methods

A factor of safety is really an index indicating the relative stability of the slopes. It does not imply the actual risk level of the slopes, due to the variability of input parameters. With a probabilistic analysis, two useful indices are available to quantify the stability or the risk level of the slopes. These two indices are known as the probability of failure and the reliability index (Krahn, 2004; Shen, 2012).

2.4.4.1. Probability of failure

It is the probability of obtaining a factor of safety less than unity. The probability of failure is determined by counting the number of safety factors below one and then taking this number as a percentage of the total number of converged of Monte Carlo trials. For example, if there are 1000 Monte Carlo trials with 980 converged safety factors and 98 of them are below one. Then the probabilistic of failure is 10%.

2.4.4.2. Reliability index

Another way of looking at the risk level of instability is known as reliability index. The reliability index described the stability by the number of standard deviations separating the mean factor of safety from its defined failure value of one. It can also be considered as a way of normalizing the factor of safety with respect to its uncertainty. The reliability index is defined in terms of the mean and the standard deviation of the trial factors of safety as shown in the following equation:

$$\beta = \frac{E[FS] - 1}{\sigma[FS]}$$

Where: β = reliability index

$E[FS]$ = average value of factor of safety and

$\sigma[FS]$ = standard deviation of factor of safety

2.5. Summary of stability methods

Table 2.2: Different slope stability methods limitations and advantages

Method	Limitations	Advantages
Ordinary/Fellenius Method	<ul style="list-style-type: none"> ▪ The failure surface assumed to be rigid ▪ Interslice forces are neglected ▪ Only moment equilibrium is satisfied ▪ Stress-strain compatibility not satisfied ▪ Only used for circular slip surfaces 	<ul style="list-style-type: none"> ▪ Easy to hand calculations
Bishop simplified Method	<ul style="list-style-type: none"> ▪ The failure surface assumed to be rigid ▪ Interslice shear forces are ignored ▪ Vertical force equilibrium not satisfied ▪ Only used for circular slip surfaces ▪ Stress-strain compatibility not satisfied 	<ul style="list-style-type: none"> ▪ Starts the solution of the nonlinear iteration factor safety
Spencer Method	<ul style="list-style-type: none"> ▪ The failure surface assumed to be rigid ▪ Stress-strain compatibility not satisfied ▪ Shear to normal ratio is constant 	<ul style="list-style-type: none"> ▪ Satisfied both equilibrium conditions ▪ Applicable to any slip surface shapes
M-P Method	<ul style="list-style-type: none"> ▪ The failure surface assumed to be rigid ▪ Stress-strain compatibility not satisfied ▪ Used random lambda solver technique 	<ul style="list-style-type: none"> ▪ Satisfied both equilibrium conditions ▪ Applicable to any slip surface shapes ▪ Shear/normal is various slice to slice
Janbu Method	<ul style="list-style-type: none"> ▪ The failure surface assumed to be rigid ▪ Moment equilibrium isn't satisfied ▪ Lambda is always equal to unity 	<ul style="list-style-type: none"> ▪ Applicable to any slip surface shapes ▪ assumed heights of side forces various slice to slice (1/3 height of interface)
FE based Method	<ul style="list-style-type: none"> ▪ In nonlinear models, the stresses-strain behaviors are quite complex ▪ Soil variability isn't considered (Note: for all deterministic approaches, the soil variability isn't satisfied) 	<ul style="list-style-type: none"> ▪ Applicable to any slip surface shapes ▪ Considered stress-strain compatibility ▪ No need of assumptions about failure, interslice forces and directions
Probabilistic Method	<ul style="list-style-type: none"> ▪ Required definition of slip failure from deterministic approaches 	<ul style="list-style-type: none"> ▪ The method runs many trials ▪ Assessing uncertainties and handling them consistently

3. ANALYSIS METHODOLOGY

3.1. General description

In the present study, it is tried to illustrate the application of Monte Carlo simulation method using the familiar and readily available Geo-Slope software to analyze the slopes stability of embankment dam. In this respect, upstream and downstream slopes of Ribb dam have been considered as a case study. Seismic, rapid/sudden drawdown and pore-water pressure effects should be considered in the analysis. The analysis has been done using Mohr-Coulomb constitutive model by SLOPE/W software, version 6.02. The strength parameters c and ϕ can be total strength parameters or effective strength parameters. From a slope stability analysis point of view, effective strength parameters are used together with realistic pore pressures, to obtain the most realistic solution position of the critical slip surface.

The analysis accounts for the spatial variability of the input variables and the statistical uncertainty. The input parameters are composed of from the values of unit weight (γ), cohesion (c), angle of internal friction (ϕ) and seismic coefficient. They are sampled from their specified probability density function. Then, using SLOPE/W search for the critical slip surface, factor of safety of each Monte Carlo simulation is calculated and corresponding probability distribution is obtained using the more reliable deterministic approaches like Spencer, M-P and Janbu methods. After obtaining the probability function of the factor of safety, the probability of failure of the slope and reliability index is determined.

3.2. Geo-Studio software

Geo-studio is a type powerful software which contains seven integrated products: SLOPE/W, SEEP/W, SIGMA/W, QUAKE/W, TEMP/W, CTRAN/W, and VADOSE/W. In this study, enough to use only the first two products, SLOPE/W and SEEP/W products. Geo-Studio environment, one can be thought of as having the following three components: defining the problem, solving the problem and viewing the results. By exercising this experience, it can be quickly obtained the required results and the overall understanding of the given problem. In this particular study, the required products are discussed below.

3.2.1. SLOPE/W product

SLOPE/W product has been introduced in the early 1980s made it economically viable. The initial code was developed by Professor D.G. Fredlund at the University of Saskatchewan. It is a powerful geotechnical software product available commercially for analyzing slope stability. Currently, SLOPE/W is being used by thousands of professionals both in education and in practice. One of the powerful features of this integrated approach is that it opens the

door to types of analyses of a much wider and more complex spectrum of problems, including the use of finite element computed pore pressures analysis.

SLOPE/W has many tools for inspecting the input data and evaluating the results. Tools like allowing to graph a list of different variables along the slip surface. These types of tools are vitally important to judging and being confident in the results. Using SLOPE/W sometimes required careful thought as to how to model a certain situation, but at the same time it greatly expands the range of possible situations it can be modeled. The general features allow for much greater creativity. Once the general features function is understood, the types of problems that can be analyzed are primarily limited by the researcher's creativity. SLOPE/W allowed easy integration with SEEP/W, SIGMA/W and QUAKE/W analyses to simulate the results of the available slope failure surface.

3.2.2. SEEP/W product

SEEP/W is a kind of Geo-Studio software which is used to analyze the seepage process in embankment structures or dams. In this study, the emphasis should not be on how much water is flowing through the dam, but on the state of pore-water pressure in the dam. Because of in slopes stability analysis, the more important issue is the pore-water pressure. The pore-water pressure whether positive or negative, it has a direct influence on the shear strength parameters and volume change characteristics of the soil. Negative pore-water pressure (suction) increases the soil strength where as positive pore-water pressure decreases the strength of the soil. So, it is important to accurately estimate the pore-water pressure in dams.

In Ribb embankment dam slopes stability analysis, the pore-water pressure has been done using R_u coefficients concept. But, R_u coefficients have difficulties. One of the difficulties with the R_u concept is the coefficient variation if the phreatic surface is not parallel to the embankment surface. When the phreatic surface is an irregular distance from the embankment surface, it is necessary to establish R_u at a number of points and then by some weighted averaging method to calculate one single overall average value for the slope. It is an impractical option. Because averaging is not recommended for any applications. In this study, the more powerful approach finite element method is used.

In SEEP/W, there are two fundamental types of finite element seepage analyses, steady-state and transient-state. The steady state seepage leaves out the time step concept and the volumetric water content function. Transient state analysis, however, it is essential to define initial condition pore-water pressure at all nodes and considers how long the soil takes to respond. However, in this current situation, there is no found relevant data regarding to transient state. So it better to use steady state analysis.

3.3. Ribb dam zoning

An embankment dam can be classified in three principal types: earth fill dam, rock fill and earth-rock fill dam, depending on the pre-dominant fill material used near the construction area. In the current situation, the Ribb Dam is the Zoned Embankment Dam as Figure 3.1, consists of clay core, fine filter, coarse filter, transition, shell, rock fill, riprap and foundation materials which is under the category of earth-rock fill material used.

Zoned dams are generally preferred since zoning permits the use of several different material types in the embankment that may be available from borrow areas or required excavations. One should keep in mind that embankment zoning is also established for economic reasons according to the availability of materials. The embankment zoning should provide an adequate impervious zone, transition zones between the core and the shells and between shells and rock fills, and seepage control zones. Table 3.1 showed the Ribb dam embankment zoning materials and their fundamental material properties. For zoned embankment dams, the zoning geometry and properties of the materials placed in the zones should be reviewed to determine the structural design and the types of internal features (Hasani et.al, 2013).

Table 3.1: Ribb embankment dam materials property (MoWE, 2010; Vahdati, 2014)

Item	Compacted materials	Unit weight, γ_{sat} (kN/m ³)	Cohesion, c' (kN/m ²)	Friction angle, ϕ' (°)	Hydraulic conductivity, (m/s)	Material size (mm)
[1]	Clay core	16	3	15	1.0×10^{-9}	0.0015-0.15
[2]	Fine filter	18	0	35	1.0×10^{-4} - 10^{-6}	0.14-2
[3]	Coarse filter	18	0	34	1.0×10^{-3} - 10^{-4}	1.33-4
[4]	Transition	18	0	34	5.0×10^{-3}	2.1-7.5
[5]	Shell	18	0	32	1.0×10^{-7}	0.075-2
[6]	Rock fill	22	0	40	1.0×10^{-2} **	31.25-900
[7]	Riprap	22	0	40	5.0×10^{-2} **	430-1560
[8]	Foundation	17	0	28	1.0×10^{-8}	...

** Sources from (Vahdati, 2014)

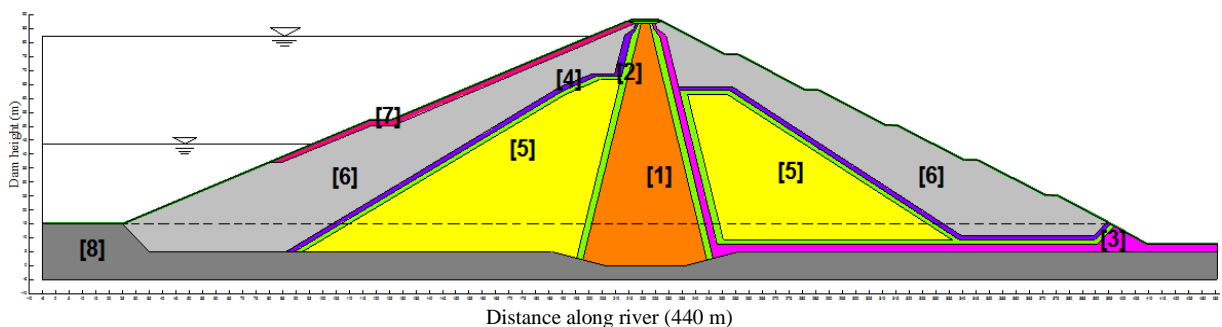


Figure 3.1: Ribb zoned dam typical cross section (scale 1:500)

3.4. Seismic consideration

Evaluations of seismic effects for embankments located in areas of low or negligible seismicity (0.05g or less) may be accomplished using the seismic coefficient in the pseudo-static method of analysis. The pseudo-static method assumed that the earthquake causes additional horizontal forces in the direction of potential failure (Kramer, 1996; Krahn, 2004). In this study, only horizontal seismic coefficient of 0.05 is considered, based on Seismic Hazard Map of Ethiopia prepared by the Intitute of Geophysical Observatory at Addis Ababa University, with a return period of 300 years, the nearest contour to Ribb dam site. The countours were developed based on a 33 years (1960 to 1993) data (Pierre Gouin, 1997).

The application of vertical earthquake coefficient often has little impact on the safety factor. The vertical earthquake forces alter the slip surface weight. This alters the base normal, which in turn alters the base shear resistance. For instance, if the vertical earthquake force has the effect of increasing the slice weight, the base normal increases and then the base shear resistance increase. The added mobilized shear arising from the added weight tends to be offset by the increase in shear strength. Of course, this is only true for frictional strength components, but not for cohesive strength components (Krahn, 2004).

3.5. Most critical slip surface search

Determining the position of the critical slip surface with the lowest factor of safety remains one of the key issues in a stability analysis. Finding the critical slip surface involves a trial procedure. Several methods were analyzed by using circular failure surfaces, including Ribb dam, for the generation of trial slip surface in slope stability analysis. However, in this study, the critical slip surface generation will be done in both circular and non-circular optimization technique to compare and construct the critical position of failure surface between them.

To obtain the most critical slip surface and the reliable factor of safety, the option of “Search for tension crack” should be considered at the crest, the normal at the base of the first slice will point away. It is always a good idea to start an analysis without the tension crack option and then add a tension crack option later to examine the effect of negative normal exist at the slices base in the crest area. The first slice with a negative normal is removed from the analysis, and then the whole procedure is repeated until there is no further negative base normal. But this is considered unrealistic, particularly for materials with little or no cohesion materials (Krahn, 2004).

3.5.1. Circular slip surface search

Circular trial slip surface was inherent in the earliest limit equilibrium formulations. The trial slip surface is an arc of circle. The arc is that portion of a circle that cuts through the slope. A circle can be defined by specifying the x-y coordinate of the center and the radius. A wide

variation of trial slip surfaces can be specified with a defined grid of circle centers and a range of defined radii. In SLOPE/W, this procedure is called the Grid and Radius method.

One of the difficulties with the historic Grid and Radius method is that it is difficult to visualize the extents and/or range of trial slip surfaces. This limitation can be overcome by specifying the location where the trial slip surfaces will likely enter the ground surface and where they will exit. This technique is called the Entry and Exit method in SLOPE/W.

SLOPE/W posted no restriction to the location of the Entry and Exit zones; it is recommended that the Entry and Exit zones should be carefully defined on locations where the critical slip surface is expected. Defining a large Entry and Exit zones on the ground surface blindly may lead to many impossible slip surfaces and may miss the real critical slip surface.

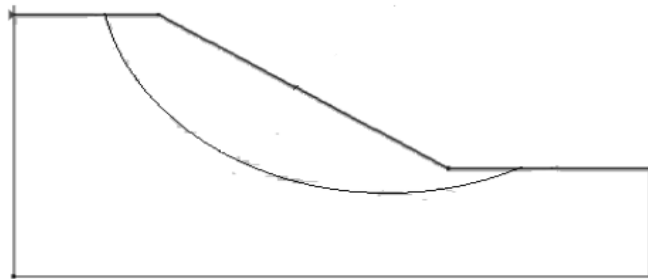


Figure 3.2: Circular slip surface shape

3.5.2. Non-circular slip surface search

In this thesis, non-circular slip surface search methods are referred to as specified slip surface search methods. In SLOPE/W, there are two types of specified slip surface search methods. These are, block specified method and fully specified methods. Block specified and fully specified methods are done by creating grids and series of data points on the suggested failure zone respectively.

Block specified method can be done by specifying two grids of points. The grids are referred to as the left block and the right block and the slip surface consists of three line segments. The middle segment goes from each grid point on the left to each grid point on the right. The other two segments are projections to the ground surface at a range of specified angles. The Block method needs a defined Axis about which to take moments. The difficulties with the Block method are, it is not always possible to find a converged solution when the corners along the slip surface become too sharp and it can result in a large number of trial slip surfaces with an undefined safety factor.

Fully Specified method is completely flexible with respect to trial slip surfaces shapes and position since it contains a series of data points. SLOPE/W can then compute the ground surface intersection.

A point needs to be created about which to take moments. This is called the Axis Point. The Axis Point should be specified. In general, the Axis Point should be in a location close to the approximate center of rotation of all the specified slip surfaces. It is usually somewhere above the slope crest and between the extents of the potential sliding mass. The Fully Specified method has a unique feature that any points on the slip surface can be specified as fixed. When a point is fixed, the point will not be allowed to move during the slip surface optimization process.

While the Fully Specified method is completely flexible with respect to trial slip surfaces shapes and position, it is limited in that each and every trial slip surface needs to be specified. The method is consequently not suitable for doing a large number of trials to find the critical slip surface.

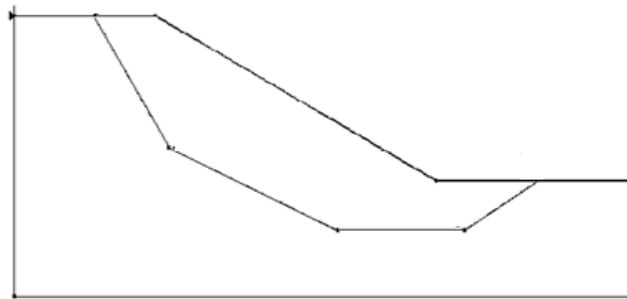


Figure 3.3: Non-circular slip surface shape

3.5.3. Critical slip surface optimization

Recent researches have explored the possibility of incrementally altering only portions of the slip surface. After finding the critical slip surface by one of the above methods, the new segmental technique is applied to optimize the solution (Krahn, 2004).

The first step in the optimization process is to divide the slip surface into a number of straight line segments. Next, the end points of the line segments are moved to probe the possibility of a lower safety factor. The process starts with the point where the slip surface enters ground surface. Then move the point backward and forward randomly along the ground surface until the lowest safety factor is found. Then, adjustments are made to the next point along the slip surface until again the lowest safety factor is found. This process is repeated for all the points along the slip surface. Next, the longest slip surface line segment is subdivided into two parts and new point is inserted into the middle. This process is repeated over and over until changes in the computed safety factor are within a specified tolerance.

3.5.4. Auto-Search critical slip surface

The Auto-Locate method is in essence a combination of the Entry and Exit method with the Optimization procedure, except that SLOPE/W automatically attempts to locate the possible Entry and Exit position in order to find a preliminary approximate solution. The success of

the method consequently depends on whether the approximation is realistic. If the behind the scenes approximation is realistic, then the Auto-Locate final solution should be very similar to the more deliberate Optimization method.

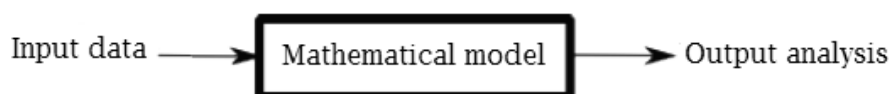
When Auto-Locate method is used, SLOPE/W generates 1000 trial slip surfaces to find the most probable minimum slip surface before optimization is applied. As a result, Auto-Locate will usually lead to a reasonable result. It is a new method and has as yet not been used extensively. At this stage, it is recommended that try the optimization search from the above methods. If the optimized solution appears intuitively more realistic, then perhaps it is acceptable to trust on the new procedure.

3.6. Simulation methodology

In the context of Monte Carlo analysis, simulation is the process of approximating the output of a model through repetitive random application of a model algorithm. In Monte Carlo simulation, the component random variables are generated in a fashion consistent with their probability distribution. The process is repeated many times to obtain an approximate probability density function for appropriate sampling technique.

In Monte Carlo simulation, variance reduction technique has been developed in order to improve the computational efficiency of the method by reducing the statistical error inherent in Monte Carlo method and keeping the sample size to the minimum possibility (Sung, 2007). The following steps are typically performed in Monte Carlo simulation modeling technique of a physical process.

Static model generation: Every Monte Carlo simulation starts off with developing a deterministic model which closely resembles the real scenario. In this deterministic model, it can be used the most likely value of the input parameters. Mathematical relationships are applied to which use the values of the input variables and transform them into the desired output. The value of each output parameter is one particular outcome scenario in the simulation run. Such output values are collected from a number of simulation runs. Finally, the values of the output parameters are performed on statistical analysis, to make decisions about the course of action. This step of generating the static model closely resembles the schematic diagram as shown below.



Input distribution identification: When researchers are satisfied with the deterministic model, it should be added the risk components to the model. As mentioned before, since the risks originate from the stochastic nature of the input variables, it is tried to identify the

underlying distributions, if any, which govern the input variables. This step needs historical data for the input variables. When there are existing historical data for a particular input parameter, numerical methods are used to identify the most suitable probability distribution function for a given set of data.

Random variable generation: After the identification of the underlying distributions for the input variables, it should be generated a set of random numbers (also called random variants or random samples) from these distributions. One set of random numbers, consisting of one value for each of the input variables, will be used in the deterministic model, to provide one set of output values. Then repeat this process by generating more sets of random numbers, one for each input distribution, and collect different sets of possible output values. This part is the core of Monte Carlo simulation. In this section, the author will use the most common method for generating random variants which is called inverse transformation method (inverse distribution function).

Analysis and decision making: The result of the Monte Carlo simulation of a model is typically subjected to statistical analysis. As mentioned before, for each set of random numbers (trials) generated for each of the random variable, the model formula is used to arrive at a trial value for the output variables. After collecting a sample of output values in from the simulation, the statistical analyses are performed based on those values. This step provides with statistical confidence for the decisions which it might be made after running the simulation.

4. DETERMINISTIC APPROACHES

4.1. Introduction

Deterministic approach slope stability analyses compute the factor of safety of a slope based on a fixed set of conditions and material parameters. If the factor of safety is greater than unity, the slope is considered to be stable, if the factor of safety is less than unity, the slope is considered to be unstable or susceptible to failure using the commonly used deterministic approaches such as limit equilibrium methods.

Limit equilibrium methods require a continuous surface passes of the soil mass. This surface is essential in calculating the minimum factor of safety against sliding or shear failure. These methods, in general, require the soil mass to be divided into slices. The directions of the forces acting on each slice in the slope are assumed. This assumption is a key role in distinguishing one limit equilibrium method from another.

4.2. Pore-pressure analysis

One of the basic requirements for design of embankment dams is to ensure safety against development of excessive pore-water pressure through the dam body. In this thesis the main emphasis is given to the development of pore-water pressure against slopes stability. For the case of Ribb Dam drains are composed of from fine and coarse filters, to safely discharge the seepage water and to protect erosion of fines and slopes failure. In this study, as mentioned previously, in chapter three, steady state pore-water pressure analysis has been conducted.

Steady seepage develops after a reservoir pool has been maintained at a particular elevation for a sufficient length of time to establish a steady line of saturation through the embankment. The seepage forces which develop in the steady state condition act in a downstream direction. The condition of steady seepage throughout an embankment may be critical for downstream slope stability. In the case of Ribb dam, it isn't a serious problem because seepage control materials have been existed in the embankment.

The pore water pressures which exist within an embankment at any given time are generated as the result of gravity seepage flow actions which can be considered independent for practical purposes. The full reservoir stability condition is nearly always analyzed using the effective stress method of analysis and the pore water pressures acting are assumed to be those governed by gravity flow through the embankment.

4.2.1. Rapid draw dawn analysis

Stability analysis during rapid drawdown is an important consideration in the design of embankment dams. During rapid drawdown, the stabilizing effect of the water on the upstream face is lost, but the pore water pressures within the embankment may remain high.

As a result, the stability of the upstream face of the dam can be much reduced. The dissipation of pore water pressure in the embankment is largely influenced by the permeability and the storage characteristic of the embankment materials. Highly permeable materials drain quickly during rapid drawdown, but low permeability materials take a long time to drain. Stability during rapid drawdown can be modeled in two approaches, namely the simple effective strength approach and rigorous effective strength approach methods.

4.2.1.1. Simple effective method

One simple and conservative approach is to assume that the rapid drawdown process occurs instantaneously. With this assumption, the stabilizing force due to the upstream water is immediately reduced but the piezometric surface follows the upstream slope surface and remains unchanged within the embankment dam. In this method, the piezometric surface is used to compute the pore-water pressures along the slip surface.

4.2.1.2. Rigorous effective method

A more sophisticated approach is to model the rapid drawdown and evaluate the pore water pressure conditions by doing a finite element seepage analysis. The advantage of this approach is that the hydraulic properties of the materials are appropriately considered in the analysis. With this approach, rapid drawdown is not just instantaneous but can be modeled as a process. The factor of safety of the embankment dam at different times during the entire drawdown process can be evaluated. So it is better to use rigorous effective method in this study for pore-water pressure analysis during rapid down condition.

4.2.2. Results visualization

When the researchers get to the visualization of results stage of a finite element pore-water pressure analysis, they can congratulate themselves for having completed the hardest parts: setting up the embankment geometry, defining meaningful soil property functions, and applying appropriate boundary conditions to the mesh as shown Figure 4.1. At this point, the researcher should have the understandings of how to interpret the massive amount of data that may have been generated by the SEEP/W solver.

The power using advanced graphic interfaces with finite element analysis is that the software can quickly convert thousands of pieces of data into meaningful pictures. In this section shown below Figure 4.2 and Figure 4.3, SEEP/W gave an indication of pressure head gradients for Ribb embankment dam in the condition of reservoir at full level and at rapid drawdown condition respectively. It is quite simple and much more meaningful to interpret the pore-water pressure distribution throughout the dam.

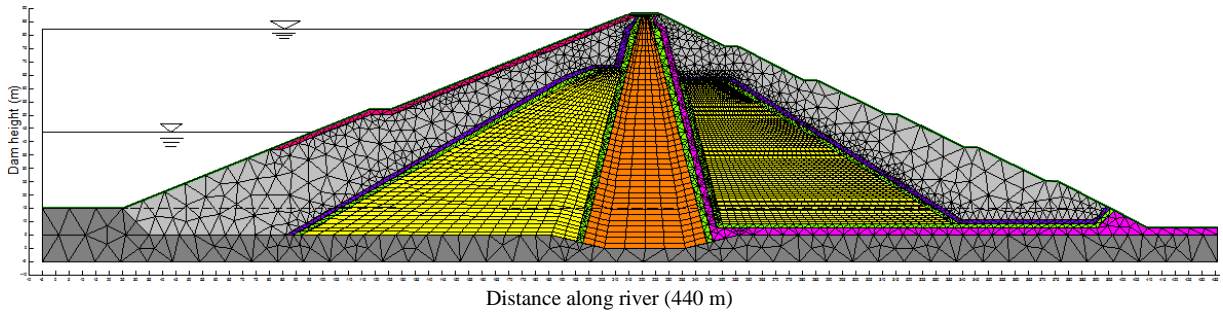


Figure 4.1: Finite element mesh during pore-pressure analysis

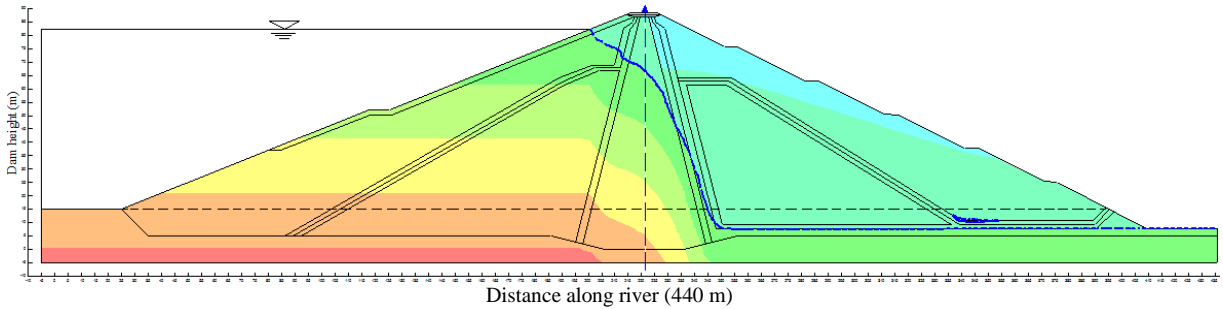


Figure 4.2: Pore-pressure counter before drawdown

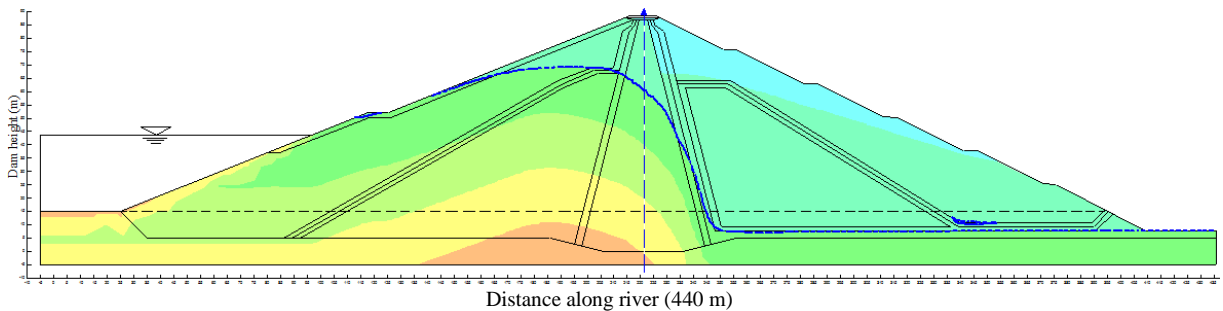


Figure 4.3: Pore-pressure counter during drawdown

4.2.3. Results convergence

Convergence means repeated solving of the nodal seepage equations until the computed solution doesn't change by more than a specified amount on successive iterations. There are various ways to determine convergence from mathematical perspective but not all of the ways are suited to seepage analysis (Krahn, 2004). In this study, vector norms are suited for steady seepage analysis.

The residual of the vector norm is simply the change in the value between two successive iterations. If the change in were exactly zero, then there would no change in solution on successive iterations. In general, there are always small differences between iterations so it is not reasonable to expect a residual vector norm of zero (Krahn, 2004). The following figures, Figure 4.4 and figure 4.5 represented Ribb dam residual vector norm converging values of pore pressure analysis using SEEP/W product in Geo-Studio software, before drawdown and during rapid drawdown respectively.

In summary, the graph of the residual vector norm values versus the number of iterations is a straight line in the case of reservoir at full level whereas in the case of rapid drawdown, the line is somewhat zigzag. This shows that the pore pressure determination in rapid drawdown condition is somewhat inaccurate relative to the pore pressure before drawdown or the full reservoir level condition. The values approach zero or are very small or acceptable.

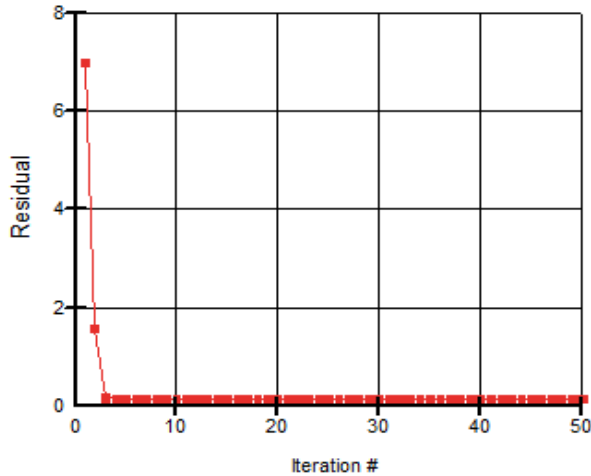


Figure 4.4: Converging before drawdown

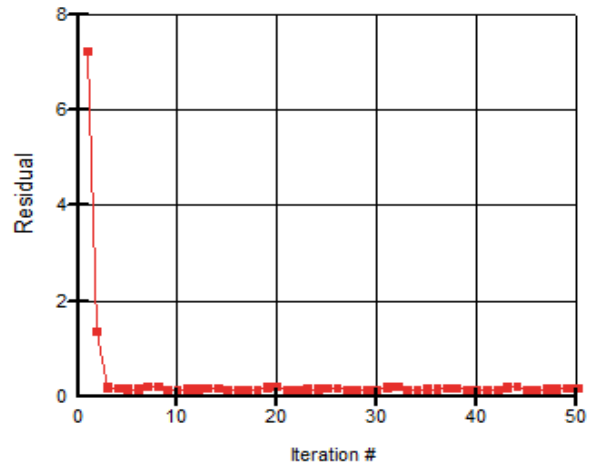


Figure 4.5: Converging during drawdown

4.3. Limit equilibrium analysis

Limit equilibrium analyses are commonly used in slope stability calculations and are based on discretizing the probable sliding mass into vertical slices. There are a number of methods having various assumptions for analyses. These assumptions are about the satisfaction of force and moment equilibrium, and consideration of interslice forces as mentioned in the chapter two, literature review part. The limit equilibrium analysis has been performed on Ribb typical zoned embankment dam using SLOPE/W integrated with SEEP/W pore pressure results.

Critical failure surface of the slope can be determined from various techniques. In this study, the Entry and Exit technique is applied. The entry and exit ranges are defined as left or right ranges. Whether the slip surface enters or exits the left range, for example, will depend on the direction of movement failure surface. In general, to define the entry and exit ranges:

- Define the left range by dragging the mouse along the left side of the ground surface line as shown a thick red line. Specify the left range values in the dialog box.
- Define the right range by dragging the mouse along the right side of ground surface line, and then specify the right range values in the dialog box.
- Enter the number of increments over range to specify the number of points along which the slip surface can intersect. The total number of slip surface intersection points along the range is one more than the specified number of increments over range value.

4.3.1. Spencer method

The Spencer method satisfied both force and moment equilibrium condition and is restricted to a constant interslice force function (i.e. the ratio interslice shear force to interslice normal force is constant for the vertical slices). SLOPE/W uses the “Rapid Solver” technique to compute the lambda value that results in the same factor of safety for both moment and force equilibrium condition.

Table 4.1 showed the computed factors of safety for different group of number of slices in Spencer method. From this, it could be developed the graph of factors of safety against number of slices as shown Figure 4.6 below. In all three cases (upstream, downstream and rapid drawdown), the factors of safety regarding to circular surfaces are greater than the optimized non-circular failure surfaces. At rapid drawdown condition, the factors of safety in the optimized non-circular surfaces are slightly approached to circular surface along the numbers of slices. This implies that the factors of safety using circular slip surface searching might be acceptable during drawdown, in this particular dam.

Table 4.1: Spencer method: Finding minimum factors of safety for Ribb typical dam

Number slices		80	90	100	110	120	130	140	150
US	Circular	1.549	1.550	1.534	1.534	1.527	1.538	1.525	1.542
	Optimized	1.430	1.436	1.439	1.446	1.457	1.464	1.444	1.438
DS	Circular	1.393	1.393	1.393	1.393	1.393	1.393	1.393	1.393
	Optimized	1.309	1.332	1.323	1.328	1.350	1.320	1.308	1.345
RDD	Circular	1.252	1.252	1.252	1.252	1.252	1.252	1.257	1.257
	Optimized	1.234	1.235	1.234	1.236	1.227	1.239	1.247	1.233

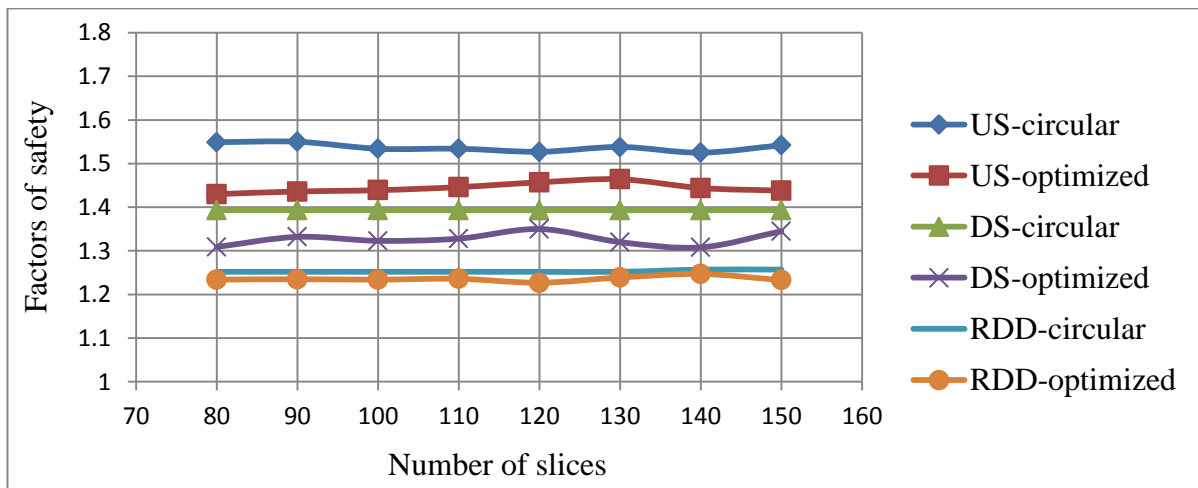


Figure 4.6: Spencer: Graph of factors of safety against number of slices

4.3.2. M-P method

The Morgenstern-Price method satisfied both force and moment equilibrium and used a selected interslice force function (i.e. the ratio of interslice shear forces to interslice normal forces is various for the vertical slices). SLOPE/W used the "Rapid Solver" technique to compute the lambda value that results in the same factor of safety for both moment and force equilibrium.

Table 4.2 showed the computed factors of safety for different group of number of slices in Morgenstern-Price (M-P) method. From this, it could be developed the graph of factors of safety against number of slices as shown Figure 4.7 below. In all three cases (upstream, downstream and rapid drawdown), the factors of safety regarding to circular surfaces are greater than the optimized non-circular failure surfaces. At rapid drawdown condition, the factors of safety in the optimized non-circular surfaces are slightly approached to circular surface along the numbers of slices. This implies that the factors of safety using circular slip surface searching might be acceptable during drawdown, similar to Spencer method.

Table 4.2: M-P: Finding minimum factors of safety for Ribb typical dam

Number slices		80	90	100	110	120	130	140	150
US	Circular	1.537	1.543	1.528	1.527	1.520	1.532	1.519	1.536
	Optimized	1.493	1.453	1.459	1.455	1.451	1.499	1.469	1.473
DS	Circular	1.389	1.389	1.389	1.389	1.389	1.389	1.389	1.389
	Optimized	1.311	1.325	1.344	1.306	1.313	1.341	1.338	---
RDD	Circular	1.252	1.252	1.252	1.252	1.252	1.252	1.256	1.256
	Optimized	1.243	1.235	1.244	1.240	1.235	1.231	1.247	1.240

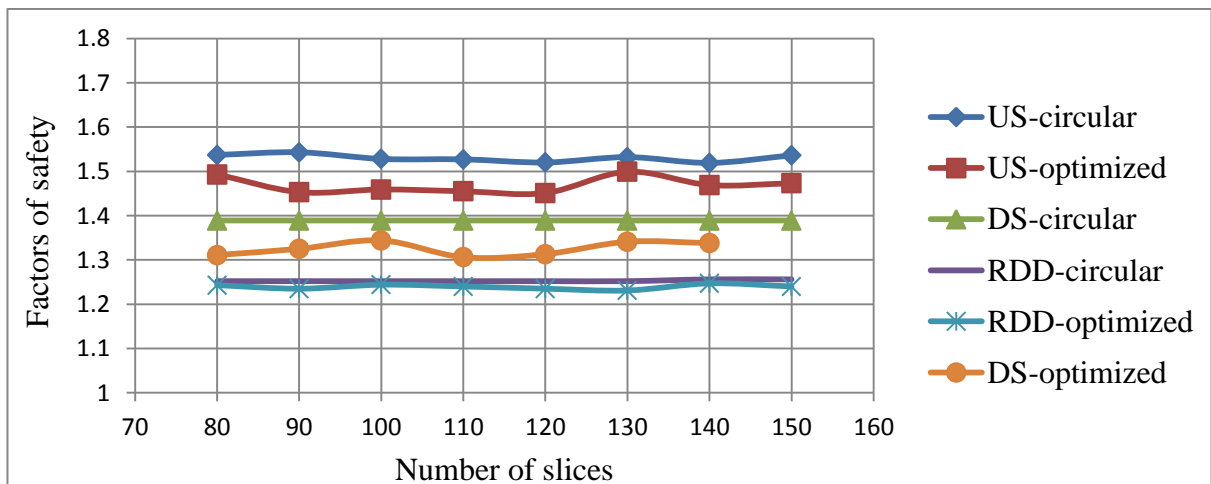


Figure 4.7: M-P: Graph of factors of safety against number of slices

4.3.3. Janbu method

The Janbu method satisfied only force equilibrium. The method imposed a stress distribution on the potential sliding mass by defining a line of thrust. The direction of the interslice force is assumed to act along the line of thrust which is set at the lower one-third point along the sides of the slices. Lambda is always equal to unity.

Table 4.3 showed the computed factors of safety for different group of number of slices in Janbu method. From this, it could be also developed the graph of factors of safety against number of slices as shown Figure 4.8 below. In all three cases (upstream, downstream and rapid drawdown), the factors of safety regarding to circular surfaces are greater than the optimized non-circular failure surfaces. At rapid drawdown condition, the factors of safety in the optimized non-circular surfaces are slightly approached to circular surface along the numbers of slices. This implies that the factors of safety using circular slip surface searching might be acceptable during rapid drawdown, in this particular dam. In general, the factors of safety conducted in the mentioned three methods, more or less similar.

Table 4.3: Janbu method: Finding minimum factors of safety for Ribb typical dam

Number slices		80	90	100	110	120	130	140	150
US	Circular	1.538	1.410	1.397	1.396	1.391	1.40	1.389	1.404
	Optimized	1.495	1.327	1.334	1.329	1.326	1.368	1.345	1.346
DS	Circular	1.278	1.278	1.278	1.278	1.278	1.278	1.278	1.278
	Optimized	1.222	1.230	1.245	1.214	1.222	1.242	1.237	---
RDD	Circular	1.218	1.218	1.218	1.218	1.218	1.218	1.223	1.223
	Optimized	1.208	1.202	1.210	1.205	1.202	1.197	1.213	1.207

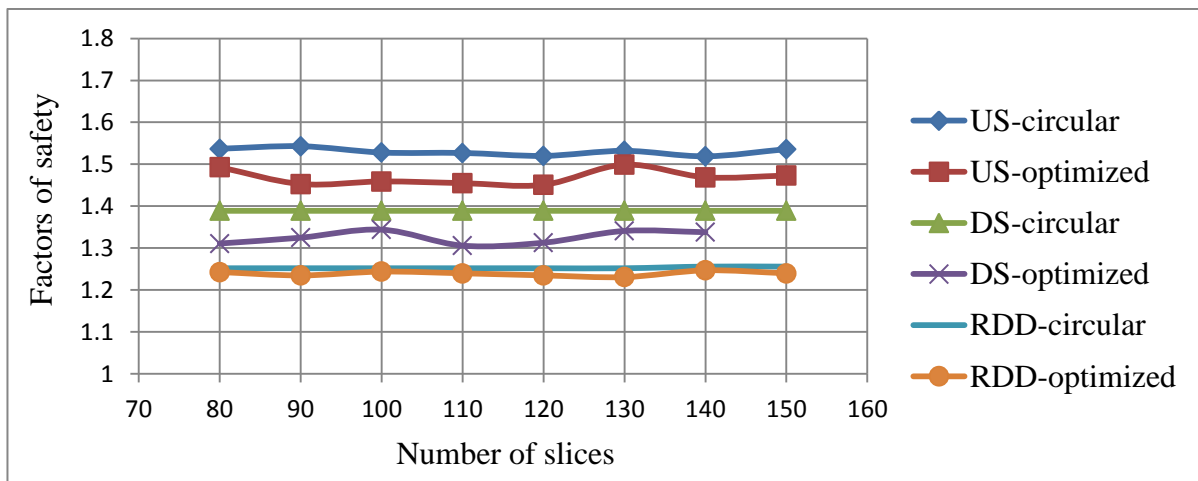


Figure 4.8: Janbu: Graph of factors of safety against number of slices

4.4. Selection of critical surface

The factor of safety is often used to find the critical slip surface of slope by evaluating a slope in order to find the slip surface that got the lowest factor of safety.

The factor of safety is often used as a design criterion. The standards in different parts of the world have somewhat different values on the factors of safety that have to be obtained for ensuring a safe design. Often are these values based on experience. Table 4.4 showed the recommendations that the safety factor have to meet in designs according to the U.S. Army Corps of Engineers' slope manual (Cederstrom, 2014; USSD, 2007) and U.S. Reclamation managing water in west standard no. 13, 2011.

Table 4.4: Minimum required factors of safety for embankment dam upstream slopes.

Types of slopes	Minimum factors of safety (FS)	
	For long term steady state	For rapid drawdown
Slopes of dams, levees, dikes, and other embankments and excavation slopes	1.50	1.0-1.2

Special care, and possibly higher factor of safety, should be used in long term steady state condition. FS = 1.0 applies to drawdown from maximum storage pool, for conditions where these water levels are unlikely to persist for long enough to establish steady state. FS = 1.2 applies to maximum storage pool level, likely to persist for long periods prior to drawdown.

For slopes where either sliding or large deformation have occurred, and back analyses have been performed to establish design shear strengths, lower factor of safety may be used. In such cases, probabilistic analysis may be useful in supporting the use of lower factors of safety for design. Lower factors of safety may also be justified when the consequences of failure are small. The deterministic approaches are required chosen and used to calculate the safety factor as being dependent on parameters which should be modelled probabilistically, such as Spencer M-P, Janbu, in this particular study.

In this study, the minimum factors of safety have been obtained through Entry and Exit slip surface search method, in SLOPE/W software to compare the recommended values listed in Table 4.4 above. As shown below Figure 4.9, 4.10, 4.11 or above table 4.1, 4.2, 4.3, all the factors of safety before drawdown, are less than the required value 1.5, whereas during rapid drawdown, the factors of safety are greater or equal to the required value 1.2 mentioned above Table 4.4. Nevertheless, it would be gone to the probabilistic analysis in order to assure the stability or instability of the typical dam. Before doing the probability analysis, there should be necessary to know the influence of variables which used to analyze the slopes

stability along the slip surface using sensitivity analysis method. Table 4.5 below showed the judged values leads to sensitivity analysis and probabilistic analysis.

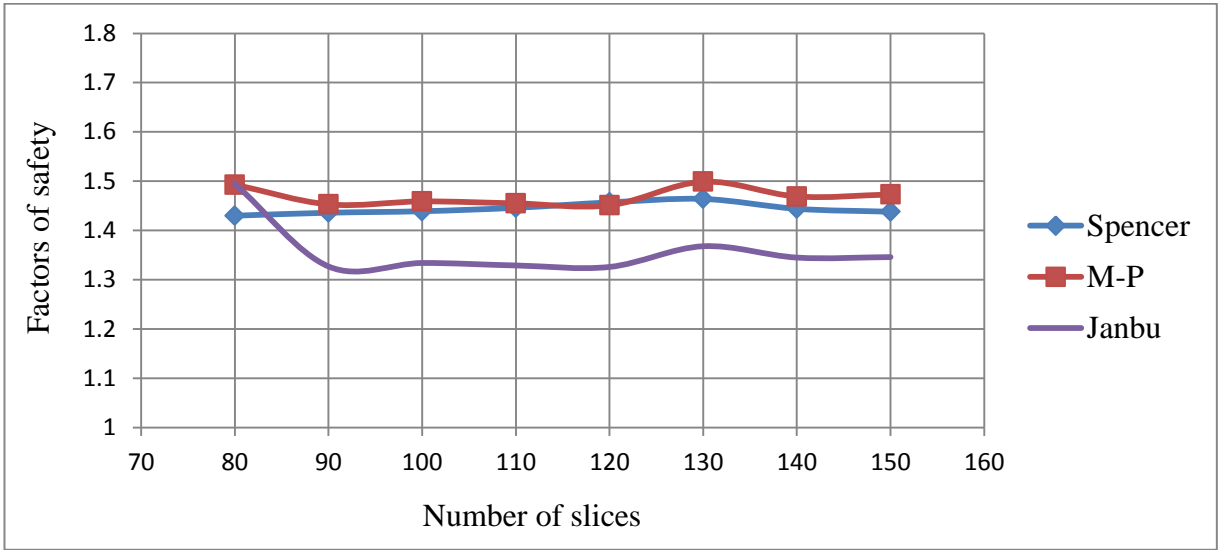


Figure 4.9: Upstream slope most critical slip surfaces

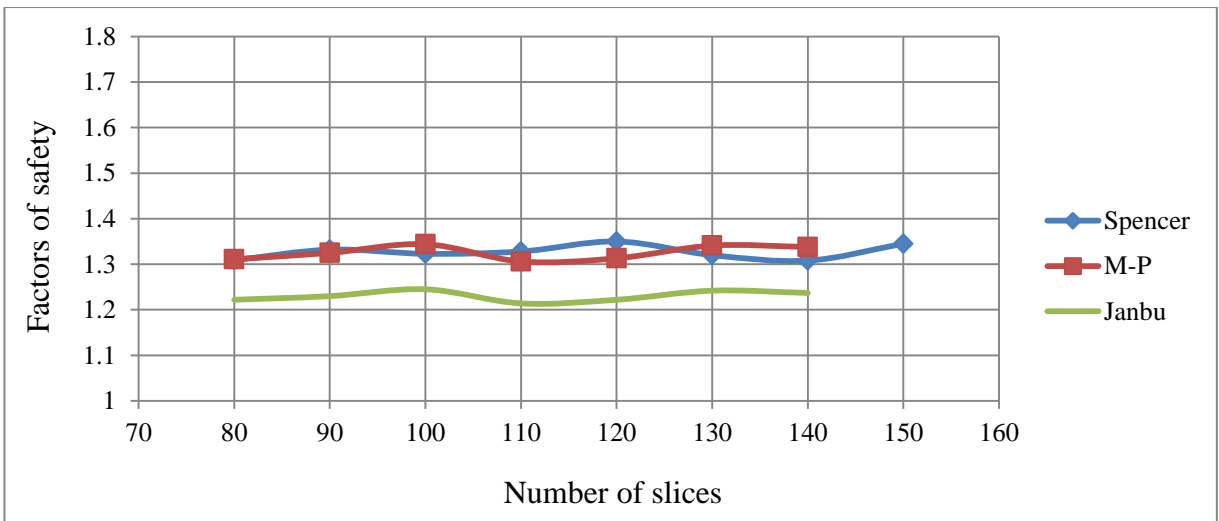


Figure 4.10: Downstream slope most critical slip surfaces

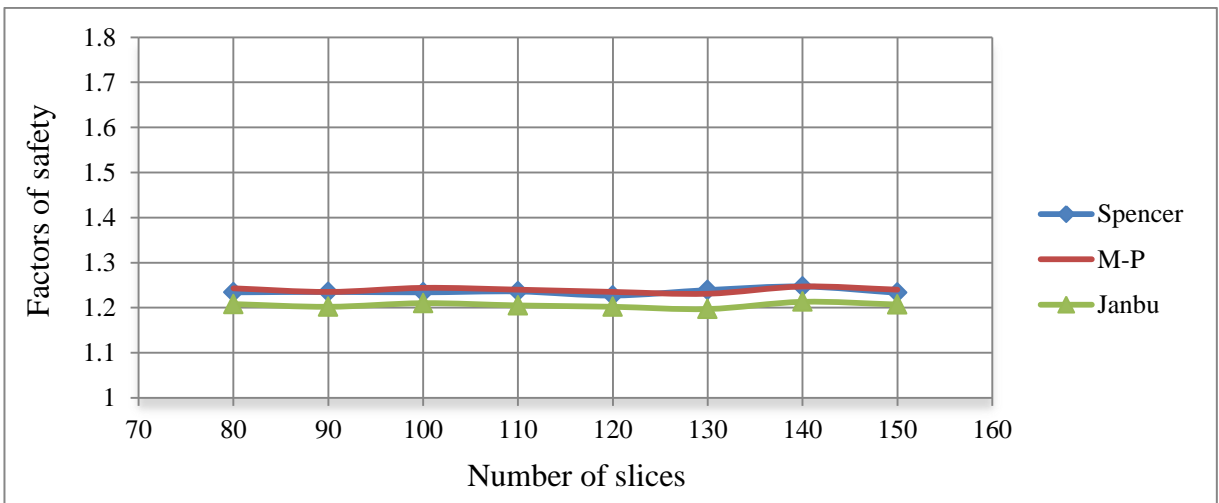


Figure 4.11: Most critical slip surfaces during rapid drawdown

In summary, as it could be seen in the above figures, the factors of safety are more or less similar for Spencer and M-P methods in all positions (i.e. upstream, downstream and rapid drawdown) corresponding to different numbers of slices. But the slight difference may come from the interslice forces assumption. Whereas Junbu method factors of safety graphs are completely different from the Spencer and M-P methods. This happens basically due to the following reasons:

1. In Junbu method, moment equilibrium isn't satisfied
2. Lambda is always assumed to be one or unity
3. Interslice forces assumed at one-third of the slices interface.

Table 4.5: Selected factors of safety for further analysis from deterministic methods

<i>Methods</i>	<i>Spencer method</i>		<i>M-P method</i>		<i>Junbu method</i>		<i>Minimum required FS</i>
	FS	# slice	FS	# slice	FS	# slice	
Upstream slope	1.457	120	1.451	120	1.326	120	1.50
Downstream slope	1.35	120	1.313	120	1.222	120
Rapid drawdown	1.227	120	1.235	120	1.202	120	1.20

4.5. Sensitivity analysis

The deterministic analysis is unable to account for variation in slope properties and parameters and other variable conditions. In reality, each parameter has a range of values and that value will affect the stability of slopes. Therefore, geotechnical properties of slope parameters have always pose some uncertainties in the simulation. This uncertainty imposes a limit on our confidence in the response or output of the model.

Sensitivity analysis involves a series of calculations in which each significant parameter is varied systematically over its maximum credible range in order to determine its influence upon the factor of safety. Sensitivity analysis is an interactive process adopted to simulate slope instability more realistically and determine the influence of the different parameters on the factor of safety. It indicates which input parameters may be critical to the assessment of slope stability, and which input parameters are less important.

The results will depend on how the initial value was chosen because a small initial value leads to a small variation and a greater initial value leads to a larger variation. Therefore, the parameter variation is considered in which the parameters are varied by a fixed percentage of the valid parameter range. The results of the sensitivity analysis are then displayed on a single graph where the computed factors of safety are plotted versus a normalized sensitivity range. A normalized value of 0.0 means the lowest specified parameter value was used and 1.0

means the highest specified value of each parameter was used. By normalizing the sensitivity range, all parameters selected for the analysis can be plotted onto a single graph for comparison purposes (Krahn, 2004).

In general, a sensitivity analysis simply means the following:

- For one or more selected input parameters, the user specifies a Minimum and a Maximum value.
- Each parameter is then varied in uniform increments, between the Minimum and Maximum values, and the safety factor of the Global Minimum slip surface is calculated at each value (Note: while a parameter is being varied, all other input parameters are held constant, at their mean values).
- This results in a plot of safety factor versus the input parameters, and allowed to determine the sensitivity of the safety factor, to changes in the input parameters.
- A steeply changing curve on a sensitivity plot, indicates that the safety factor is sensitive to the value of the parameter.
- A relatively flat curve indicates that the safety factor is not sensitive to the value of the parameter.

Table 4.6: The judged change in values of Ribb dam materials for sensitivity analysis

Variables	Unit weight		Frictional angle		Cohesion,		Steps from given values
	γ_{sat} , kN/m ³	Delta	φ' (°)	Delta	c' kN/m ²	Delta	
Clay core	16	1.0	15	2.5	3	0.75	4
Filters	18		34		0	-	
Transition			35				
Shell			32				
Foundation			28				
Rock + riprap			22				

The sensitivity results of the upstream face of Ribb dam were presented as shown in Figure 4.12 in three methods: Spencer, Janbu and M-P. For this study, the stability is much more sensitive to changes in the shell and foundation material friction angles in all three methods. This is intuitively correct, since a major portion of the slip surface is in shell and foundation as shown Figures (A1.1, A2.1, A3.1) in the appendix A. The point where the all sensitivity curves cross is the deterministic factor of safety or the factor of safety at the mid-point of the ranges for each of the parameters.

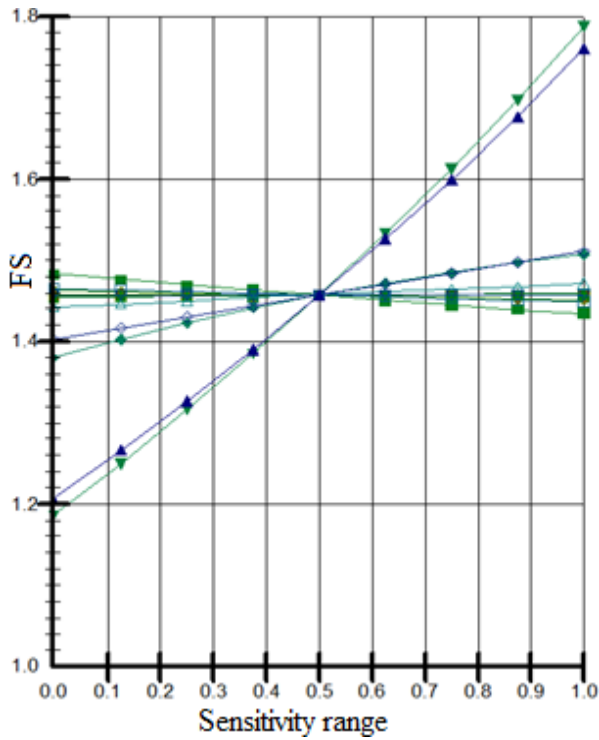


Figure 4.12a: Spencer method

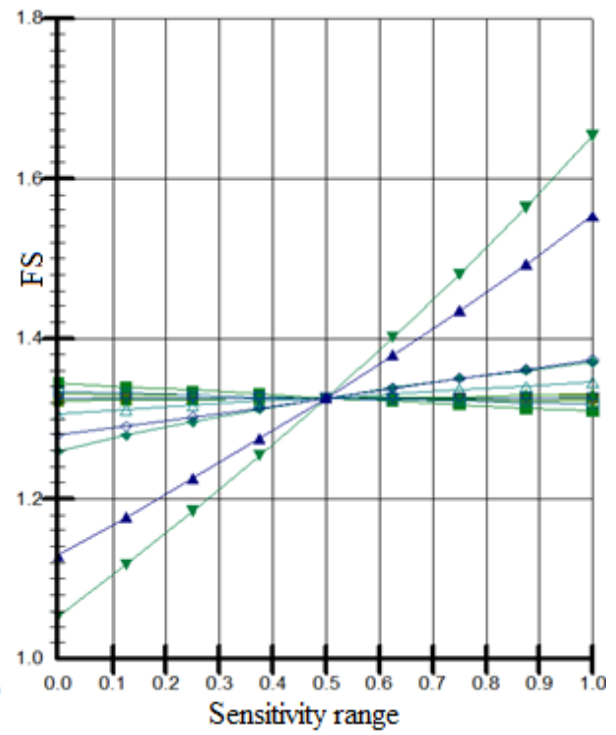


Figure 4.12b: Janbu method

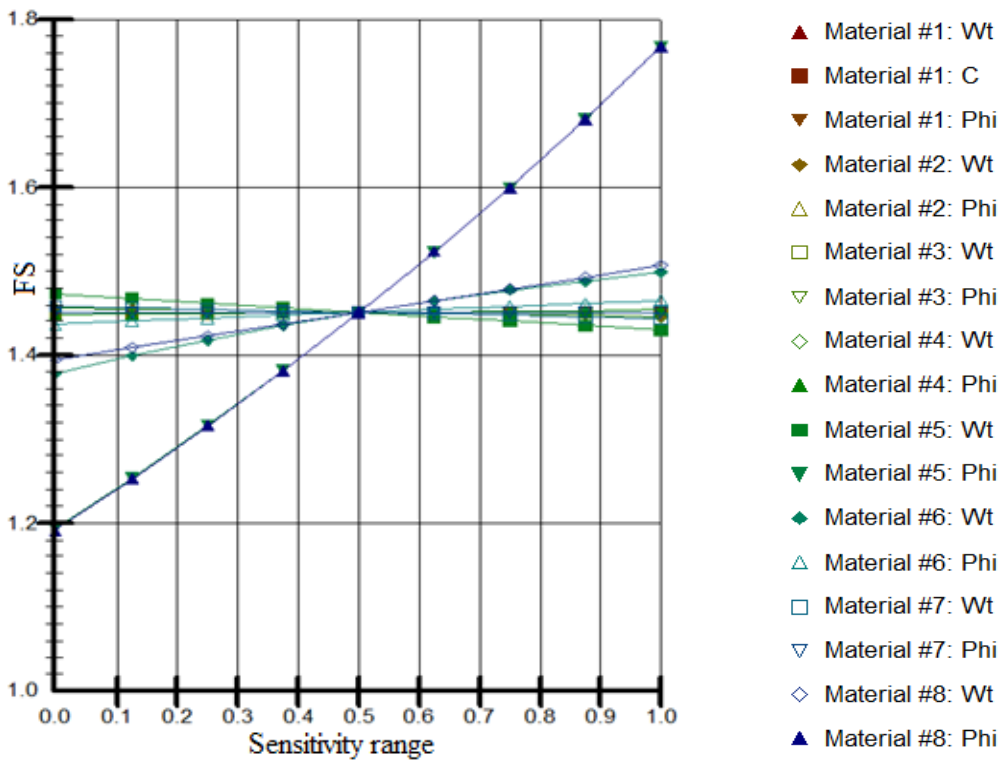


Figure 4.12c: M-P method

Figure 4.12: Upstream slope of Ribb dam FS sensitivity functions

The sensitivity results of the downstream face of Ribb dam were presented as shown in Figure 4.13 in three methods: Spencer, M-P and Janbu. For this analysis, the stability is much more sensitive to changes in the shell and foundation material friction angles like upstream slope. But it could be seen slight difference in the graphs. This is intuitively also correct, since a major portion of the slip surface is in shell and foundation as shown Figures (A1.2, A2.2,

A3.2) in the appendix A. The point where the all sensitivity curves cross is the deterministic factor of safety or the factor of safety at the mid-point of the ranges for each of the parameters.

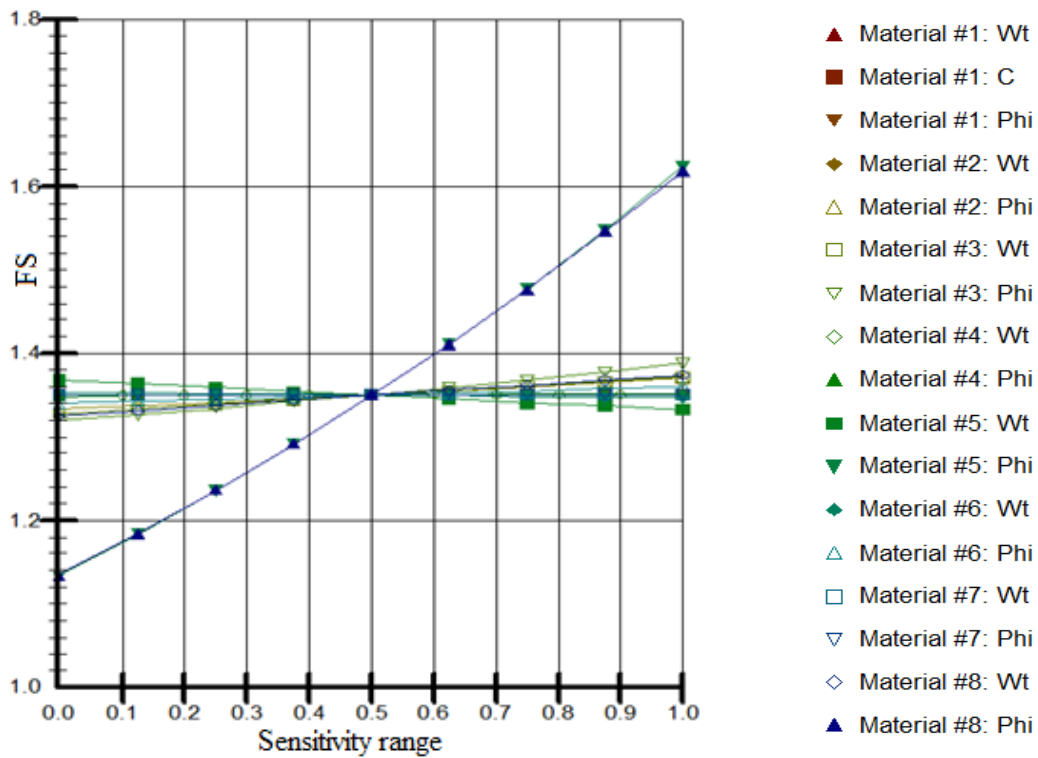


Figure 4.13a: Spencer method

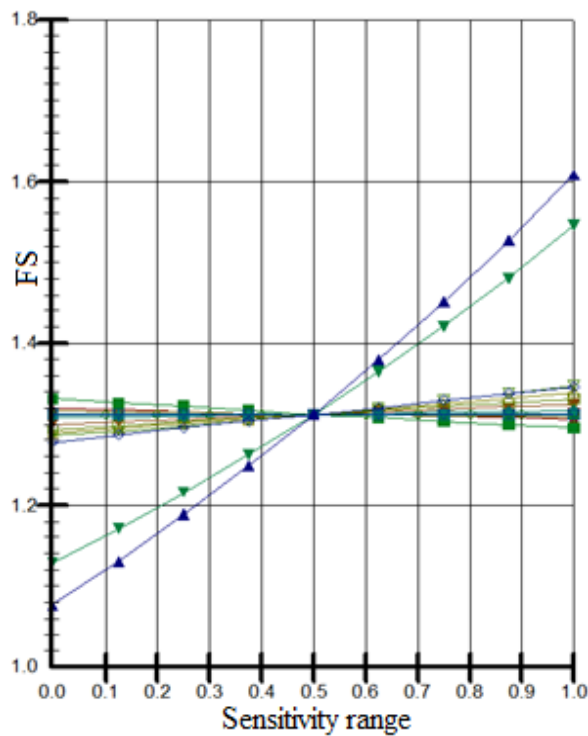


Figure 4.13b: M-P method

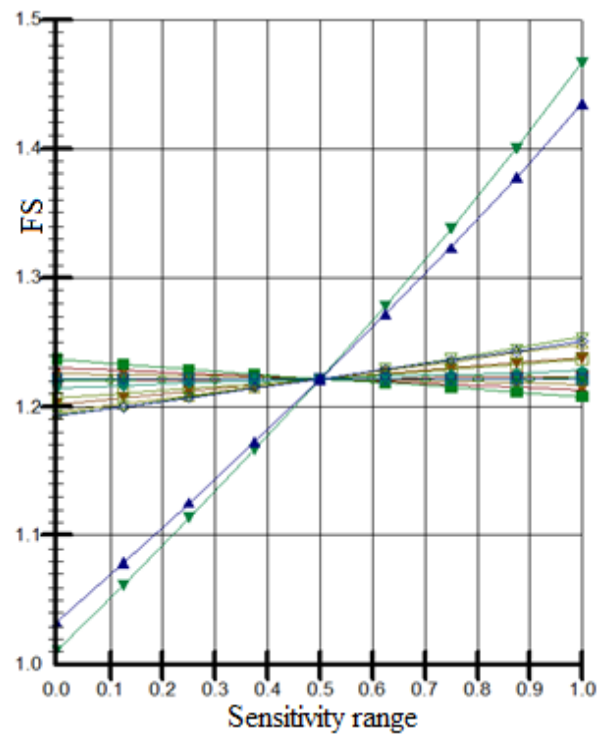


Figure 4.13c: Janbu method

Figure 4.13: Downstream slope of Ribb dam FS sensitivity functions

The sensitivity results of the rapid drawdown of Ribb dam were presented as shown in Figure 4.14 in three methods: Spencer, Janbu and M-P. For this study, the stability is much more sensitive to changes in the rock fill material friction angles in all three methods. This is intuitively correct, since all portion of the slip surface is in in the rock fill zone as Figures (A1.3, A2.3, A3.3) in the appendix A. The point where the all sensitivity curves cross is the deterministic factor of safety or the factor of safety at the mid-point for each of parameters.

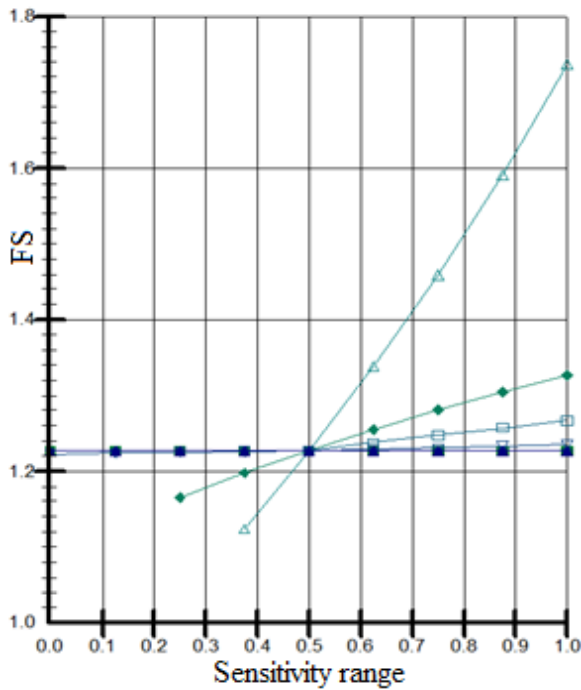


Figure 4.14a Spencer method

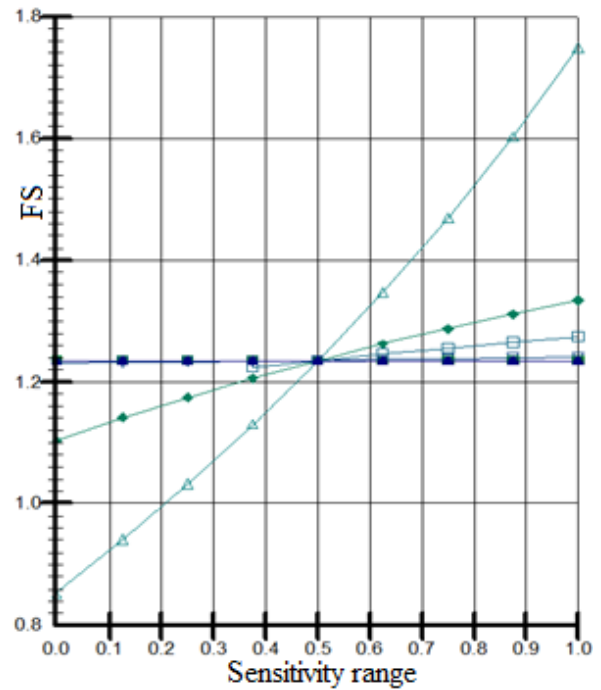


Figure 4.14b M-P method

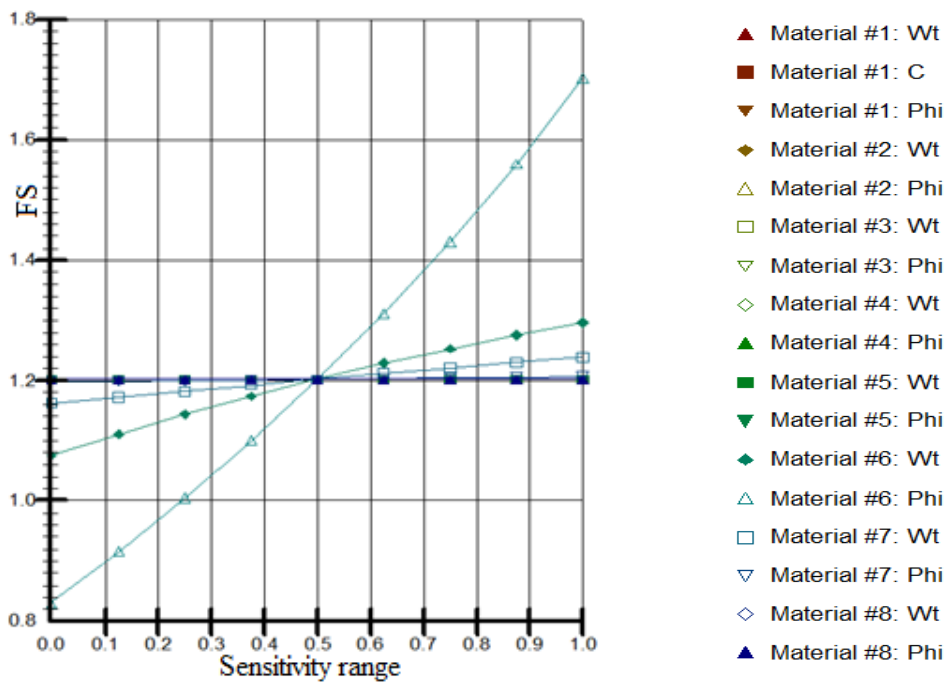


Figure 4.14c: Janbu method

Figure 4.13: Ribb dam FS sensitivity functions during rapid drawdown

5. PROBABILISTIC ANALYSIS

5.1. Introduction

Geotechnical engineering is to a large extent the art of making decisions in the presence of uncertainty, i.e. of managing risks. In comparison to other nearby fields of engineering, such as structural or mechanical engineering, materials provided by nature rather than man-made materials are treated. The soil deposits found on Earth are characterized by irregular layers of various soil materials with widely varying properties that must be inferred from a limited number of relatively costly observations (such as soundings, samplings, etc.) made during geotechnical site investigations. In addition to the uncertainty arising from the randomness in the underlying natural phenomena (e.g. the soil property), the total uncertainty in a geotechnical analysis is also dependent on how well the real world (e.g. the failure mechanism) is modeled.

In every deterministic slope stability analyses, the uncertainties in slope engineering are not considered. This common shortcoming sometimes causes collapse of slope even though the safety factor is greater than unity. Historical record showed that around 15% of cause of embankment dams failure are slope slides (Singh and Varshney, 1995). For this reason, the evaluation of the role of uncertainty necessarily required the implementation of probability concepts. Probabilistic slope stability analysis allows for the consideration of variability in the input parameters and it quantifies the probability of failure of the slopes. Basically, the method consists of re-running the analysis many times by inputting new parameters estimated from the mean and standard deviation values of the chosen parameters.

5.2. Random variables

In a probabilistic analysis, the parameters which affect the results of analysis significantly and have some uncertainties are considered as random variables. The uncertainty in the stability of a slope is the result of many factors such as ignorance of geological details in exploration, statistical uncertainty, measurement or calibration error, natural inherent variability, etc. In slope probabilistic analysis, the soil parameters, which represent the major sources of uncertainties, are treated as random variables. Instead of a single value, random variables are considered as a range of values in accordance with a probability density function or probability distribution. The probability distribution quantifies the frequency of values in any given interval.

In general, there is no way of predicting exactly what the value of one of these parameters will be at any given location. Hence these parameters are described as random variables as shown below (Abramson et.al, 2001).

- ✚ The arithmetic mean $E[x] = \frac{1}{n}(\sum x_i)$, often referred to as simply the mean or average.
- ✚ The variance $Var[x] = \delta^2$ or $(SD)^2$, described the extent of the range of the random variable about the mean.
- ✚ The coefficient of variation (COV) is a dimensionless parameter and it is a particularly useful measure of uncertainty. A small uncertainty would typically be represented by a $COV = 0.05$ while considerable uncertainty would be indicated by a $COV = 0.25$.
- ✚ The standard deviation is given by $SD = COV * E[x]$. A small standard deviation will indicate a tightly clustered data set while a large standard deviation will be found for a data set in which there is a large scatter about the mean.

As mention in chapter two, in literatures review, there are two commonly used distribution methods, the normal distribution method and lognormal distribution method. But it should be considered the proper distribution method that best represents the variables included in the model. For example, normal distribution method might produce negative numbers that have no physical meaning like negative friction or cohesion. In these cases, it is better to use a log normal distribution. In SLOPE/W, this is done using a random number generation function that is the fundamental input into a deterministic model.

5.3. Probability density function

Probability density function of a continuous random variable is a function that describes the relative likelihood for this random variable to occur at a given point. The probability density function integral over the entire space is equal to one. The probability density function defines the distribution of the random variables and can take many shapes, but the most common ones used in geotechnical applications are the normal and lognormal (Yang, 2005).

Since soils are naturally formed materials, and their physical properties vary from point to point. The variability of soil properties is a major source of the uncertainties in embankments. Laboratory results on natural soils indicated that most soil properties can be considered as random variables conforming to the normal or lognormal probability distribution functions. As mentioned in the methodology part, lognormal probability distribution function has been selected for this study.

The component input parameters, in a probabilistic slope stability analysis, are modeled as random variables, and used to estimate the probability density and sampling functions of the geotechnical parameters. This probability density function is characterized by its mean value and standard deviation. It is usually assumed that COV of the materials are used to determine standard deviations (Timpong et.al, 2007). Most recommended values of COV for zoned

embankment dam variables: unit weight (γ_{sat}), friction angle (φ) and cohesion (c), are 3-5%, friction angle 10-20% and 20-50% respectively (Timpong et.al, 2007; Manafi, 2012; Ray and Baidya, 2011). It is better assigned coefficient of variation of seismic coefficient greater than any of the above values such as 50% because seismicity is the worst condition in slopes stability. Based the recommended and judged values above, the standard deviations of Ribb dam parameters could be summarized as shown below Table 5.1. The developed lognormal distribution Figures regarding to this study, particularly for Ribb embankment dam, were listed in appendix B.

Table 5.1: Calculated standard deviation (SD) values for Ribb zoned embankment dam

Variables	COV = 3%		COV = 10%		COV = 4%	
	γ_{sat} , kN/m ³	SD	φ (°)	SD	c, kN/m ²	SD
Clay core	16	0.48	15	1.50	3	0.12
Filters (both)	18	0.54	34	3.4	0	-
Transition			35	3.5		
Shell			32	3.2		
Rock fill and riprap	22	0.66	40	4.0		
Foundation	17	0.51	28	2.8		
Seismic coefficient	0.05		COV = 50%		SD = 0.025	

5.4. Monte Carlo trials

Monte Carlo simulation is a computerized mathematical technique that allows to account for variability in their process to enhance quantitative analysis and decision making. The term Monte Carlo was coined in the 1940s by physicists working on nuclear weapon projects in the Los Alamos National Laboratory. Monte Carlo simulation performs variation analysis by building models of the possible results, by substituting a range of values, a probability distribution, for any factor that has inherent uncertainty. It then calculates results over and over, each time using a different set of random values from the probability functions. Depending on the number of uncertainties and the ranges specified for them, a Monte Carlo simulation could involve thousands or tens of thousands of recalculations before it is complete. Monte Carlo simulation produces distributions of possible outcome values.

In general, the Monte Carlo method simulation can be carried out following steps:

- A deterministic method, such as the limit equilibrium method, is chosen and used to calculate the safety factor as being dependent on parameters of the problem which should be modelled probabilistically.

- The probability density distribution obtained from experimental data measurements can be constructed in a histogram form.
- The cumulative distribution is constructed for each random variable probability density distribution. This cumulative curve can be drawn, by dividing, for instance, the variation field of each probabilistic parameter into ten intervals, in such a way that an increasing value between 0 and 1 corresponds to each central value of these intervals.
- Random values are generated and the correspondent values of the random variables are determined. Random values varying in the 0-1 field, are generated and for each generation, the correspondent parameter is determined.
- The random variable values obtained by the random generation are used as input data for the determination of the correspondent safety factor with the chosen deterministic method.

In this study, the given values assigned for unit weights, cohesion, internal friction angle of all materials and seismic coefficient have been considered along slip surface. The following trials have been calculated with respect to the recommended confidence levels and their standard deviations as shown in Table 5.2. In this study, 115000 trials have been done at a sampling of 0.5 m, with respect to confidence level of 80% and standard deviation of 1.282 for this probabilistic analysis. The remaining three number of trials are too large difficult to analyze or beyond the capacity the processor.

Table 5.2: Number of trials with respect to confidence levels and their standard deviations

Confidence level ε	80%	90%	95%	99%	Number of variable, $m=5$
Standard deviation d	1.282	1.645	1.960	2.576	$N_{mc} = \left[\frac{d^2}{4(1-\varepsilon)^2} \right]^m$
Number of trials N_{mc}	$1.15 \cdot 10^5$	$1.42 \cdot 10^9$	$8.37 \cdot 10^{12}$	$1.27 \cdot 10^{21}$	

5.5. Probabilistic stability analysis

Traditional methods are used principles of static equilibrium to evaluate the balance of driving and resisting forces. The factor of safety is defined as the resisting forces divided by the driving forces, or alternatively as the shear strength divided by the calculated shear stresses. A factor of safety greater than one indicates a stable slope; a value less than one indicates impending failure.

A probabilistic analysis is based on a randomness of the parameters affected by uncertainties. According to Monte Carlo simulation method, a random value has been selected for each input parameter based on the assigned probability density function and its amplitude. The

probabilistic analysis model requires the knowledge or the reliable estimation of the independence of the random variables. Many variables are involved in slope stability evaluation and the calculation of the factor of safety. It requires shear strength parameters (i.e. cohesion and angle of internal friction), pore-water pressures, the unit weights, seismic acceleration, etc. Traditional slope stability analysis uses single value for each variable to calculate the factor of safety. The output of a traditional stability analysis is a single-value of factor of safety in deterministic estimate. Single value of the factor of safety approach cannot quantify the probability of failure, associated with a particular design. A probabilistic approach to studying geotechnical issues offers a systematic way to treat uncertainties, especially slope stability. Probability analysis is generally understood to deal with the probability of failure of a slope of embankment, given a particular state of knowledge about the loads and the properties of materials. A far useful approach is to deal with other measures of reliability. After performing 115000 Monte Carlo trials in SLOPE/W within 0.5 meter soil variability or sampling distance, the following results obtained from probabilistic analysis, for Ribb typical dam: upstream and downstream slopes.

5.5.1. Probability of failure and reliability index

In this study, random field is composed of random values of unit weight, γ , cohesion, c , and angle of internal friction, ϕ . They are sampled from their specified probability distribution function. The distributions are defined with a type, a mean, and a coefficient of variation. Then, using Geo-Studio software for the critical slip searching and limit equilibrium methods, factor of safety of each Monte Carlo simulation is calculated and corresponding probability distribution is obtained. After obtaining probability distribution of the factor of safety, probability of the failure of the slope and reliability index is determined.

Factor of safety, ratio of resisting to driving forces, gives an idea about the relative stability of the system. However, risk of failure of the stability of a slope is assessed by calculation of its probability of failure or P (failure) (%). Failure probability is assessed by determining the mean, the standard deviation and related probability density function of factor of safety obtained from Monte Carlo simulation method. In general, probability of failure is the probability of the factor of safety getting a value less than unity.

Another way of quantifying the risk is calculating a reliability index. The reliability index described the stability by the number of standard deviations separating the mean factor of safety from its defined failure value of one. It can also be considered as a way of normalizing the factor of safety with respect to its uncertainty. The reliability index is defined in terms of the mean and the standard deviation of the trial factors of safety (see page 20). The following Figures and Tables justified/showed the performance of the Ribb typical dam.

Table 5.3: Upstream slope of Ribb dam computed probabilistic values before drawdown

Method	Mean factor of safety	Reliability index	P (failure) (%)	Standard deviation
Spencer	1.4786	2.847	0.89442	0.168
M-P	1.4721	2.819	0.92683	0.550
Janbu	1.3443	2.172	2.86522	0.158

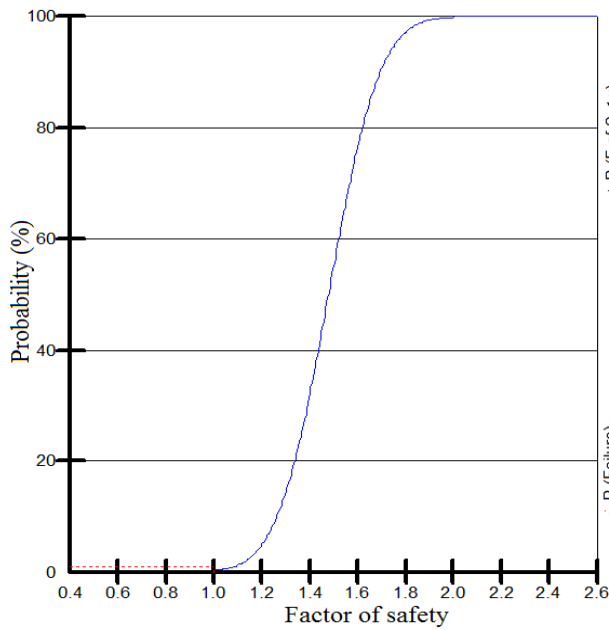


Figure 5.1a: Spencer method

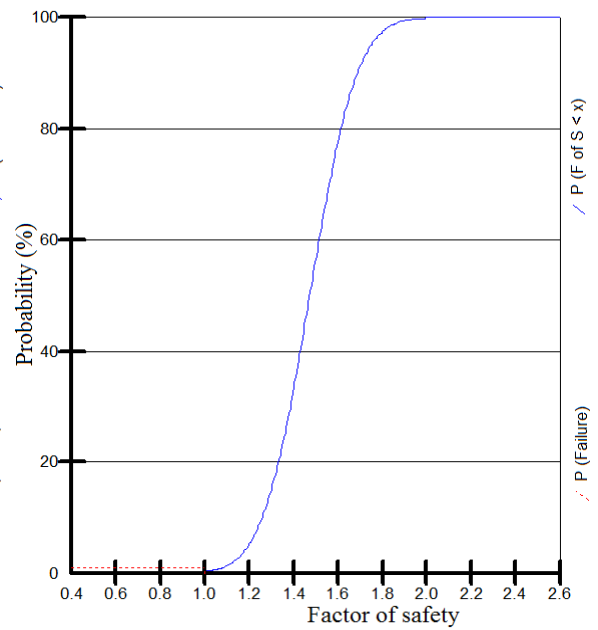


Figure 5.1b: M-P method

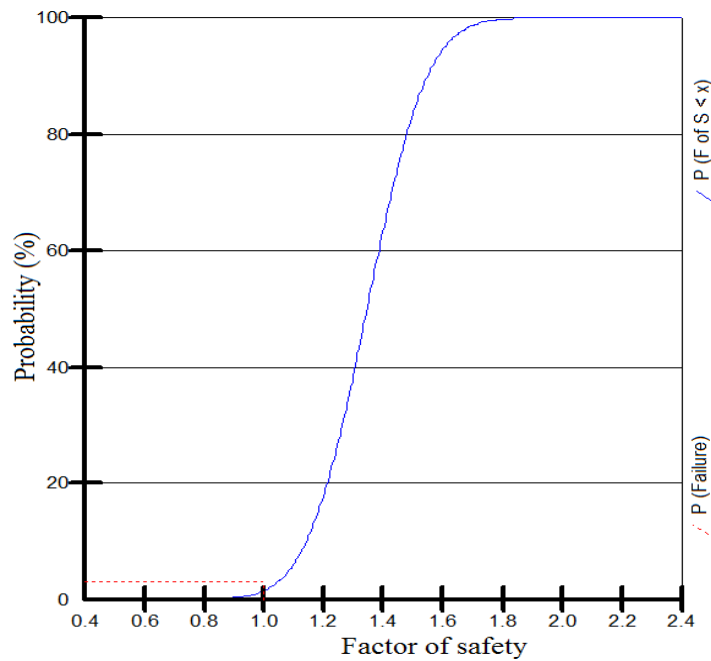


Figure 5.1c: Janbu method

Figure 5.1: Upstream slope of Ribb dam probability of failure models

Table 5.4: Downstream slope of Ribb dam computed probabilistic values before drawdown

Method	Mean factor of safety	Reliability index	P (failure) (%)	Standard deviation
Spencer	1.3604	4.423	0.17803	0.081
M-P	1.3225	4.029	0.24744	0.08
Janbu	1.2264	2.900	1.16617	0.078

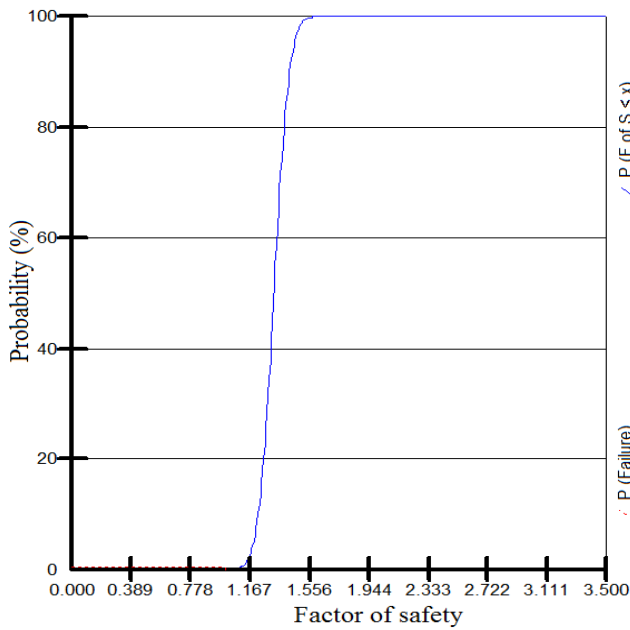


Figure 5.2a: M-P method

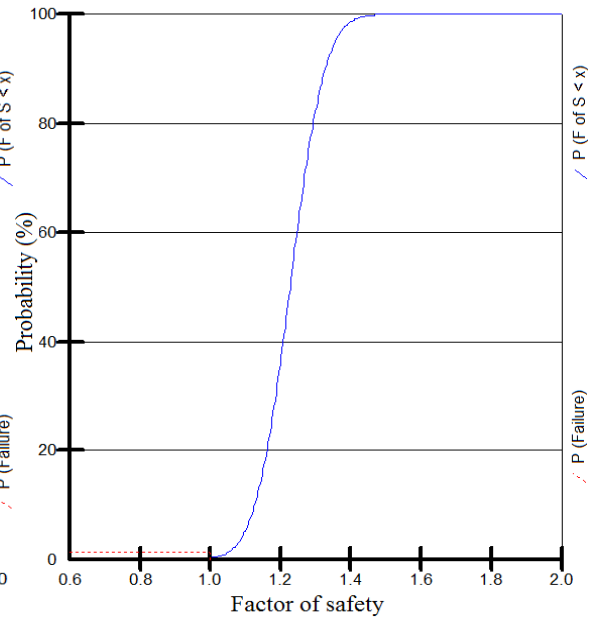


Figure 5.2b: Janbu method

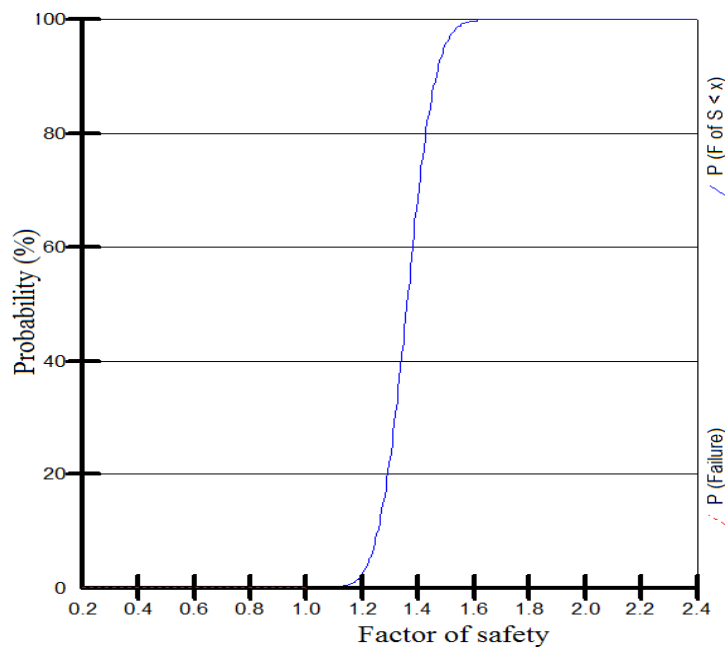


Figure 5.2c: Spencer method

Figure 5.2: Downstream slope of Ribb dam probability of failure models

Table 5.5: Computed probabilistic values of Ribb dam during rapid drawdown (upstream)

Method	Mean factor of safety	Reliability index	P (failure) (%)	Standard deviation
Spencer	1.2517	3.232	0.49202	0.079
M-P	1.2511	2.879	1.05678	0.087
Janbu	1.2112	2.425	2.19213	0.078

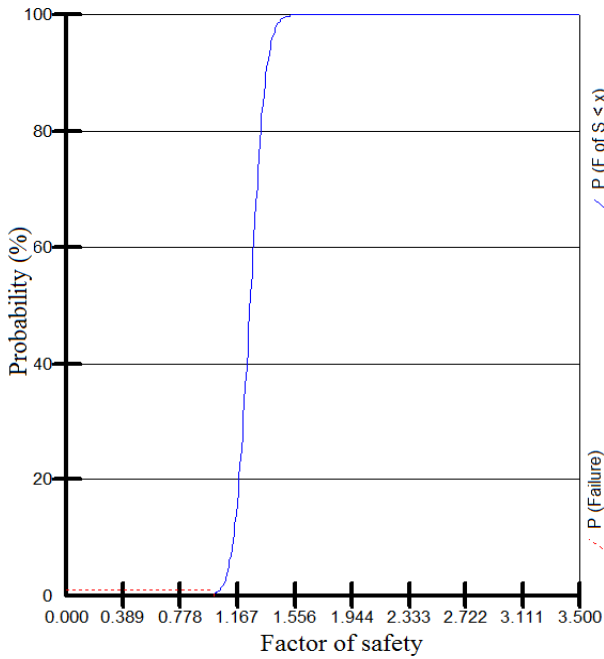


Figure 5.3a: M-P method

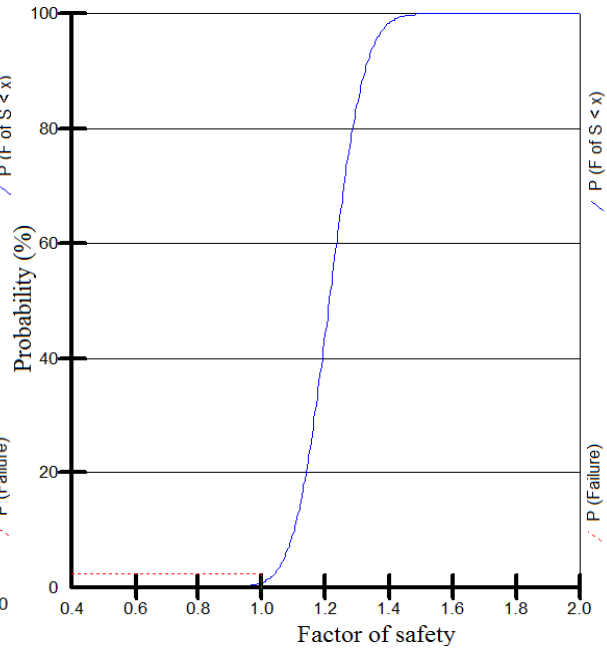


Figure 5.3b: Janbu method

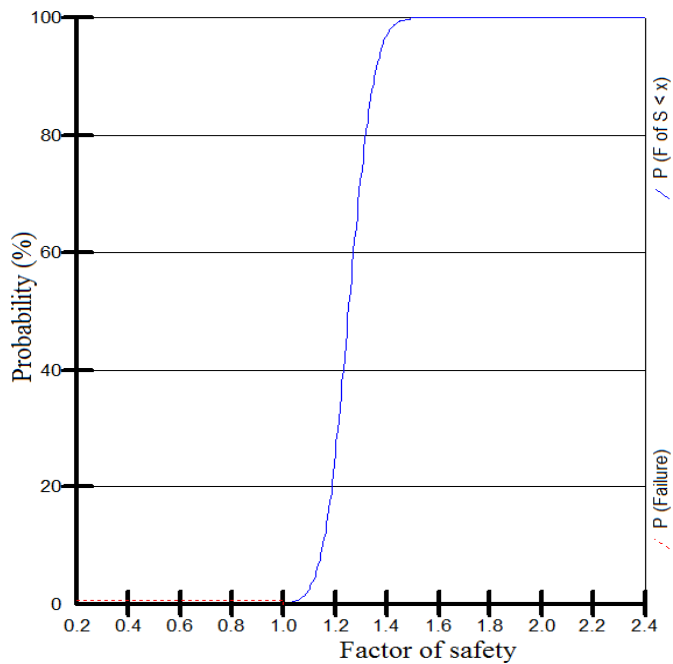


Figure 5.3c: Spencer method

Figure 5.3: Ribb dam probability of failure models during rapid drawdown

5.6. Comparison of results

A number of acceptability criteria based on probability of failure have been found in the literatures by using the mean parameters. There is some consensus that a maximum probability of failure of 0.01%, is considered generally acceptable criterion for all slopes (Telford, 2004; Khan and Malik, 2013). For a general, which could be considered in parallel with consequence of failure considerations, it is proposed that additional criteria be used according to U.S. Army Corps of Engineers, for embankment dams, slopes with reliability index of more than 3.0 are stable (Manafi, 2012). These are currently suggested values only and are being evaluated along side the conventional deterministic factors of safety already in use in the existing methodology.

Table 5.6: Upstream slope comparison of computed values with recommended values

Methods	Deterministic FS		P (failure) (%)		Reliability index		Remark
	Current	Required	Current	Required	Current	Required	
Spencer	1.457	1.50	0.89442	0.01	2.847	3.0	Unaccepted
M-P	1.451		0.92683		2.819		Unaccepted
Janbu	1.326		2.86522		2.172		Unaccepted

Table 5.7: Downstream slope comparison of computed values with minimum required values

Methods	Deterministic FS		P (failure) (%)		Reliability index		Remark
	Current	Required	Current	Required	Current	Required	
Spencer	1.350	0.17803	0.01	4.423	3.0	Unaccepted
M-P	1.313		0.24744		4.029		Unaccepted
Janbu	1.222		1.16617		2.900		Unaccepted

Table 5.8: Rapid drawdown comparison of computed values with minimum required values

Methods	Deterministic FS		P (failure) (%)		Reliability index		Remark
	Current	Required	Current	Required	Current	Required	
Spencer	1.227	1.20	0.49202	0.01	3.232	3.0	Unaccepted
M-P	1.235		1.05678		2.879		Unaccepted
Janbu	1.202		2.19213		2.425		Unaccepted

As shown above Tables, it is evident that for a factor of safety of 1, there is a 0.17803-2.86522 percent probability of failure for all the material parameter variability tried in the selected methods analysis. This probability of failure is of serious concern to the designer and calls for some improvement in slopes in order to reduce to the probability of failure.

From the following the Figure 5.4, the results indicated that Janbu method has the lowest deterministic factor of safety, the lowest reliability index and the largest probability of failure among selected methods, in all the three conditions. As it appears from Figure 5.4c or Table 5.8 during rapid drawdown, although M-P method has the largest deterministic safety factor among selected methods, but didn't indicate the highest reliability index or the lowest probability of failure. The highest standard deviation of safety factors of M-P method is the reason of this fact. The results showed the inability of deterministic analysis to determine accurate risk level. This also showed that the deterministic factor of safety didn't have direct relationship with the reliability index or probability of failure of slopes.

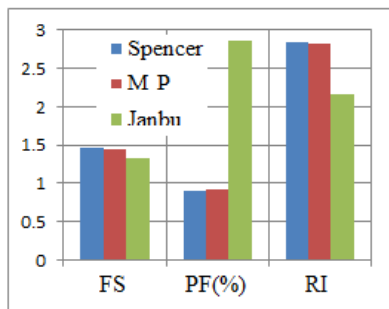


Figure 5.4a: Upstream

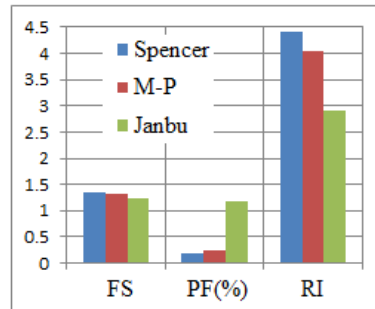


Figure 5.4b: Downstream

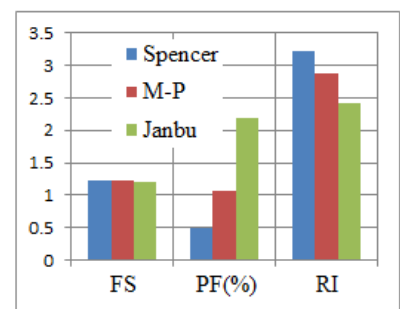


Figure 5.4c: Rapid Drawdown

Figure 5.4: Deterministic FS, probability of failure and reliability index of selected methods

Where, FS = factor of safety

PF = probability of failure

RI = reliability index

Figure 5.4 above gave as the summarized results about this risk assessment for Ribb dam upstream and downstream slopes and the upstream slope during rapid drawdown condition. The performance evaluation methods showed that the results of conventional deterministic approaches factors of safety are not much significant.

In general, it is important to keep in mind that the most critical slip surface derived from deterministic analysis is not necessarily sufficient, but the most critical slip surface of probabilistic stability analysis is an appropriate estimation of the most critical slip surface of embankment dams. Such an approach allowed dam owners to evaluate how safe their dams are in terms of probability of failure or reliability index.

6. CONCLUSION AND RECOMMENDATION

6.1. Conclusion

The probabilistic stability analysis was performed for both upstream and downstream slopes of Ribb earth-rock fill dam. The probabilistic analysis allowed for quantifying the influence of uncertainties and presented a risk limit design criteria based on probability of failure and reliability index methods. Based on the results obtained in this study, the following significant conclusions could be drawn:

- Deterministic methods, such as limit equilibrium methods, are not sufficient for searching the most critical slip surface in embankment dams slopes since the methods are dependent of the single input parameters analysis.
- Both the upstream and downstream slopes stability of Ribb dam are sensitivity to angle of internal friction of shell and foundation materials. Where as the upstream slope stability during rapid drawdown is sensitive to rock fill material friction angle.
- During rapid drawdown, the position of most critical failure surface was found above the level of water which is different from the critical failure surface found by water work design and supervision enterprise.
- In Ribb dam slopes stability analysis, the selected slip surfaces had probability of failure with much greater than the acceptable limit and the reliability of index value is more or less unsatisfied compared the limit.
- The probability of failures of the upstream slope with the large factors of safety is always larger than the downstream slope of the small factor of safety or the reliability index of the upstream slope is always smaller than the downstream slope. Because the downstream part has been protected by seepage control appliances.
- There is no direct relationship between factors of safety and probability of failure or reliability index. In other words, a slope with a higher factors of safety values mayn't be more stable than a slope with a lower factors of safety.
- The probability of failure is always decreased when reliability index is increased. This showed that the probability of failure and the reliability index have always inverse relationship.
- Spencer method is the least effectual one giving the highest reliability indices or the lowest probability of failures, whereas Janbu method is the most effectual yielding lowest reliability indices or the highest probability failures.
- For this typical dam, the Morgenstren-Price method is not appropriate method since limit equilibrium methods haven't considered uncertainty or they are dependent of single values. Resuts showed that the slopes are under risk.

6.2. Recommendation

This research was focused on risk assessment of Ribb dam slopes stability analysis with the aim of understanding the probabilistics analysis basics with the definition of limit equilibrium failure surface mechanism of an earth-rockfill dam application. In this study, the following points are recommended:

- It is necessary to investigate the probabilistic analysis using the definition of failure from finite element-stress based method for finding the more realistic slip surface position in order to obtain accurate risk level.
- In slopes stability analysis, the more important issue is the pore-water pressure. It is recommended to investigate the pore-water situations such as modeling of infiltration of precipitation and transient state conditions through the dam.
- It is recommended to take slope stabilization measures in order to safe the dam. Such as reinforcements controlled the failure surface either by the strength of the bar itself or by the shear resistance between the bonding grout and the soil.
- In method of slices stability analysis, there are many limit equilibrium methods which are developed based on static equilibrium assumptions. If necessary, researchers can do this using the remaining limit equilibrium methods in order to compare the results from the existed method such as Spencer, M-P, and Janbu.

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Appendix A: Critical slip surfaces

Appendix A1: Spencer method

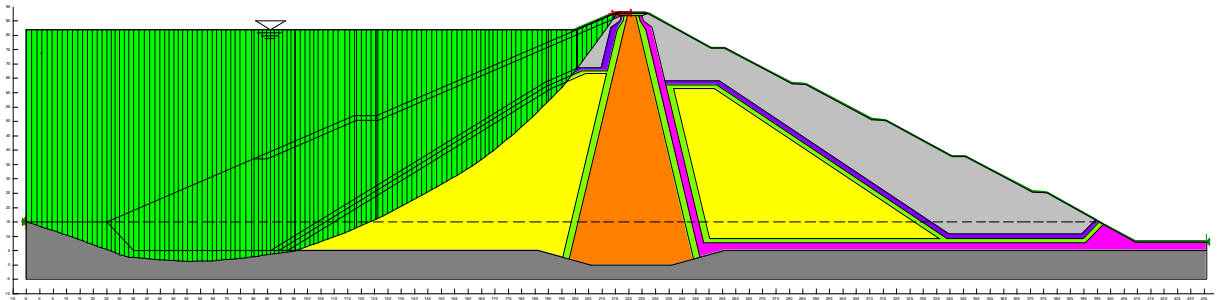


Figure A1.1: Spence method-upstream critical surface (FS=1.457)

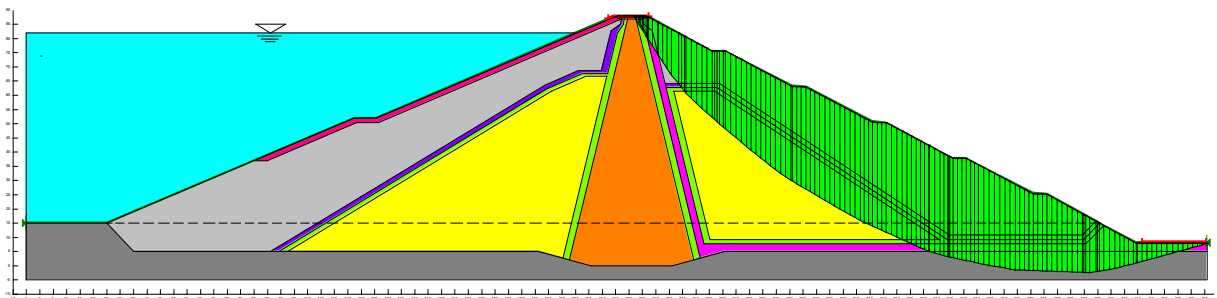


Figure A1.2: Spence method-downstream critical surface (FS=1.35)

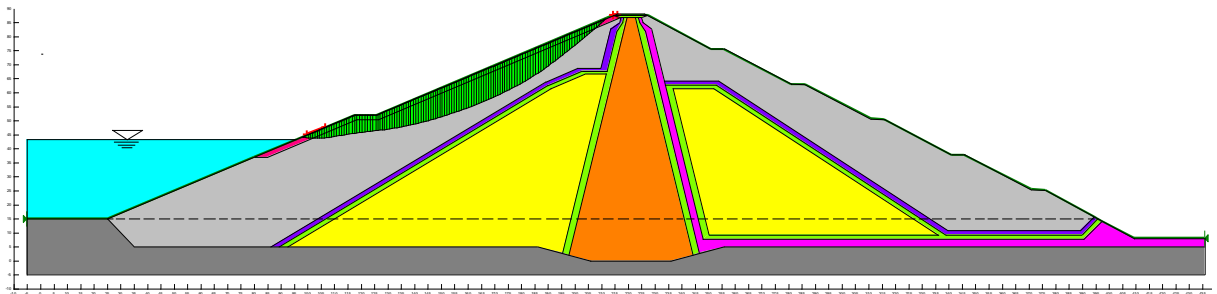


Figure A1.3: Spence method-rapid drawdown critical surface (FS=1.227)

Appendix A2: M-P method

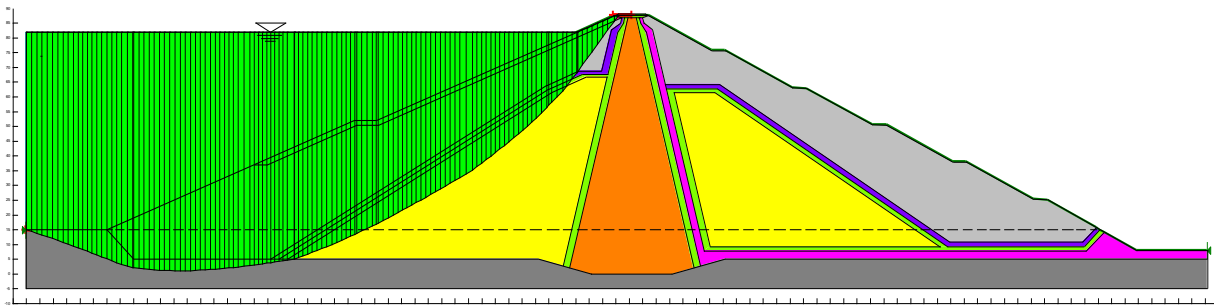


Figure A2.1: M-P method-upstream critical surface (FS=1.451)

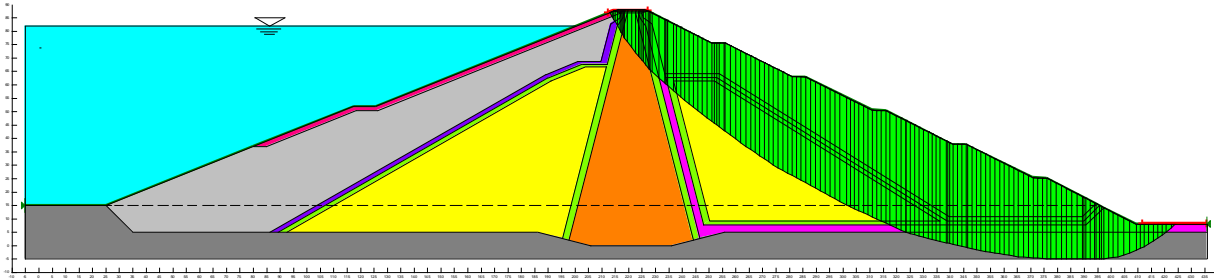


Figure A2.2: M-P method-downstream critical surface (FS=1.313)

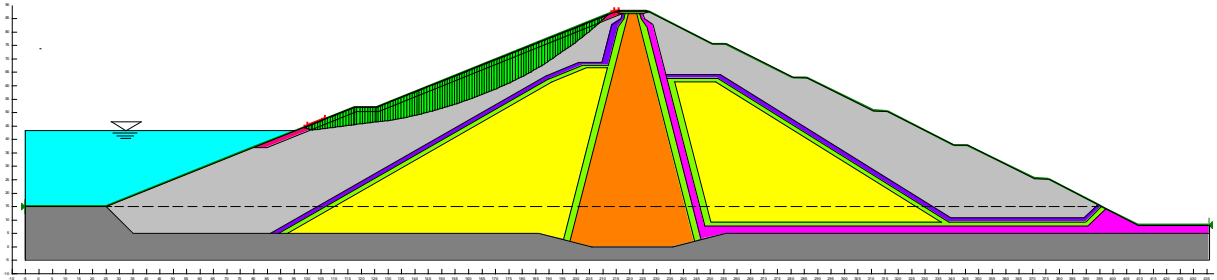


Figure A2.3: M-P method-rapid drawdown critical surface (FS=1.235)

Appendix A3: Janbu method

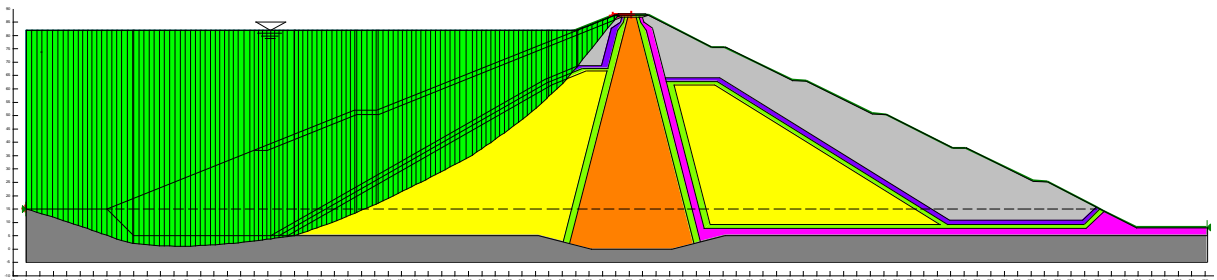


Figure A3.1: Janbu method-upstream critical surface (FS=1.326)

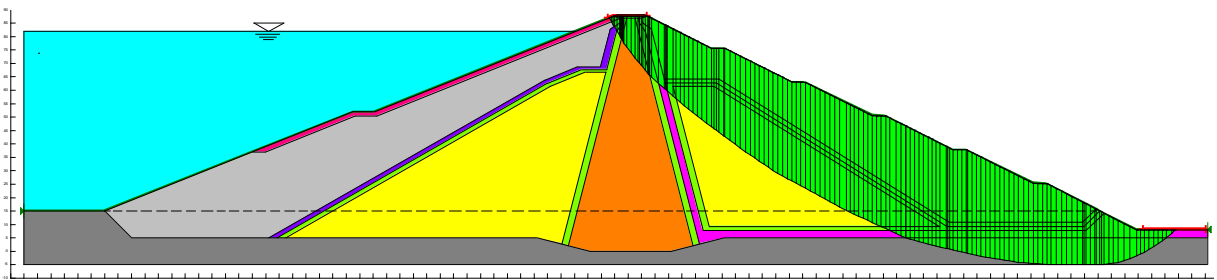


Figure A3.2: Janbu method-downstream critical surface (FS=1.222)

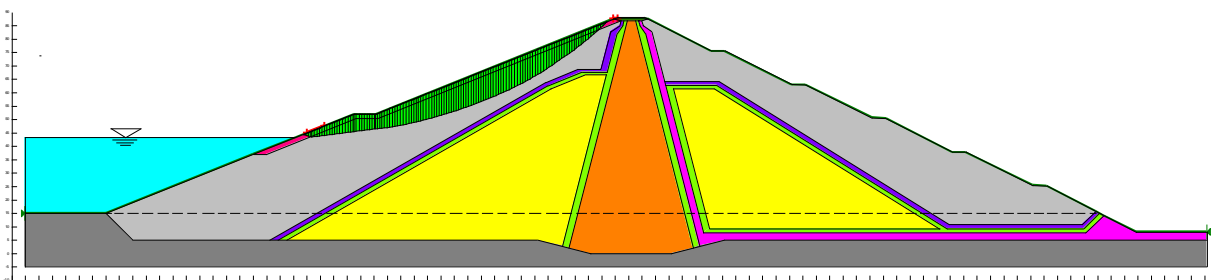


Figure A3.3: Janbu method-rapid drawdown critical surface (FS=1.202)

Appendix B: Lognormal functions

Appendix B1: Clay core

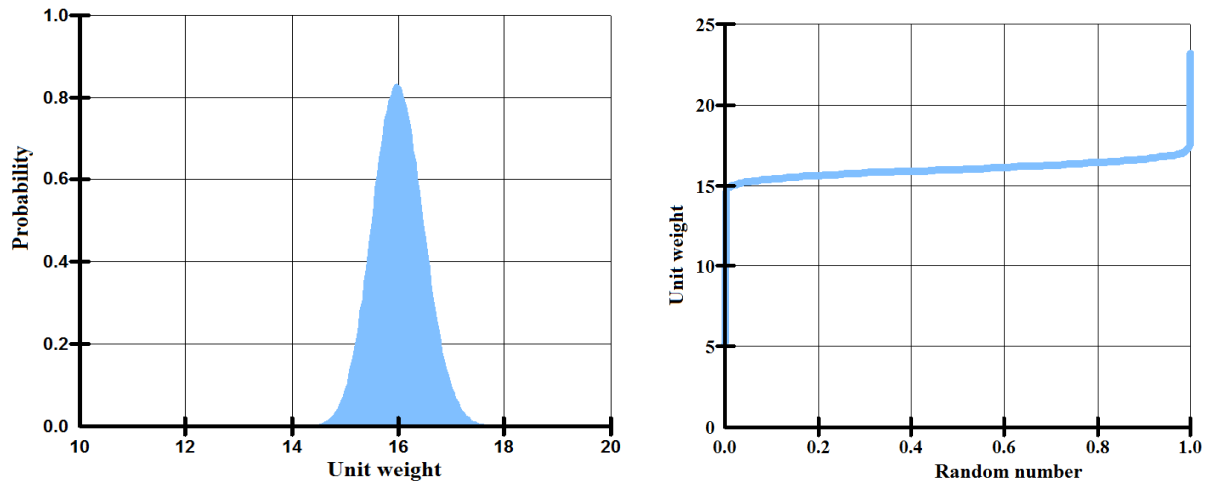


Figure B1.1: Probability distribution and sampling function of unit weight

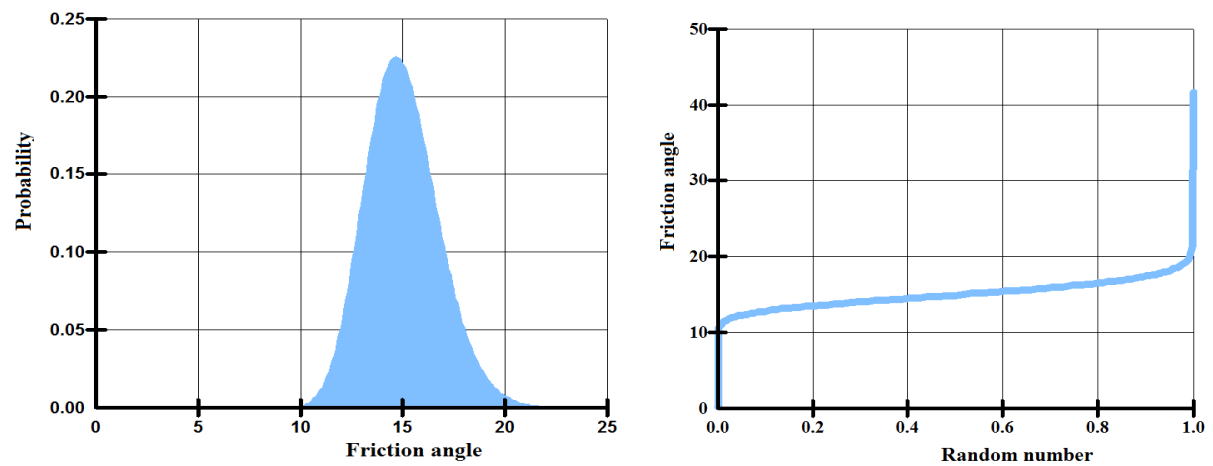


Figure B1.2: Probability distribution and sampling function of friction angle

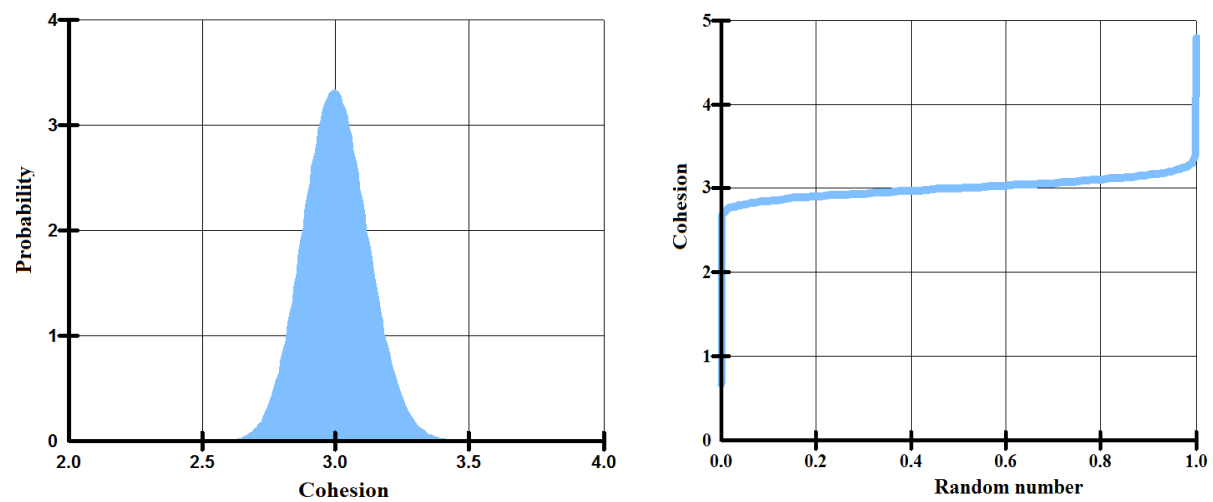


Figure B1.3: Probability distribution and sampling function of cohesion

Appendix B2: Filters-both

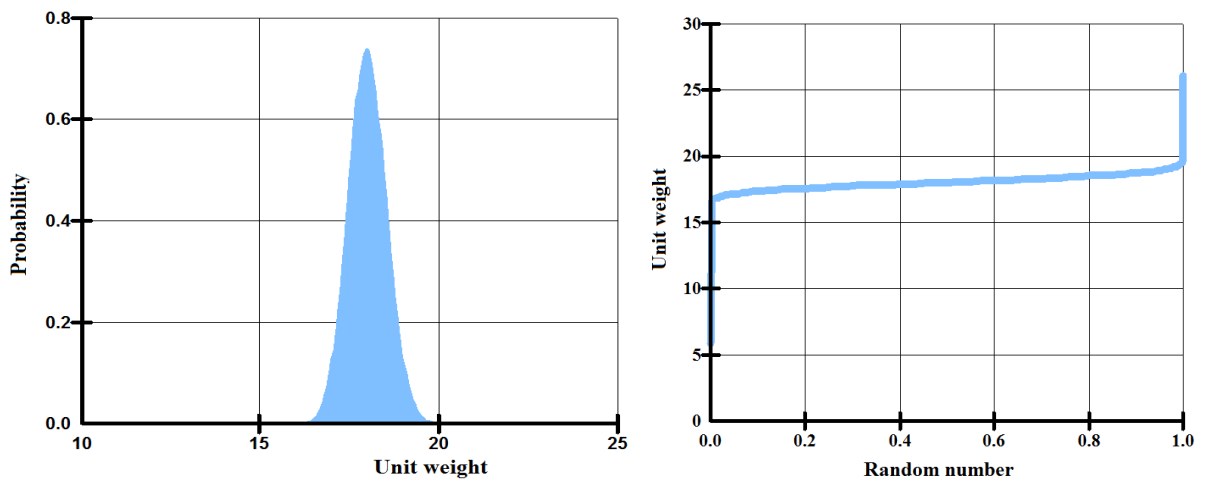


Figure B2.1: Probability distribution and sampling function of unit weight

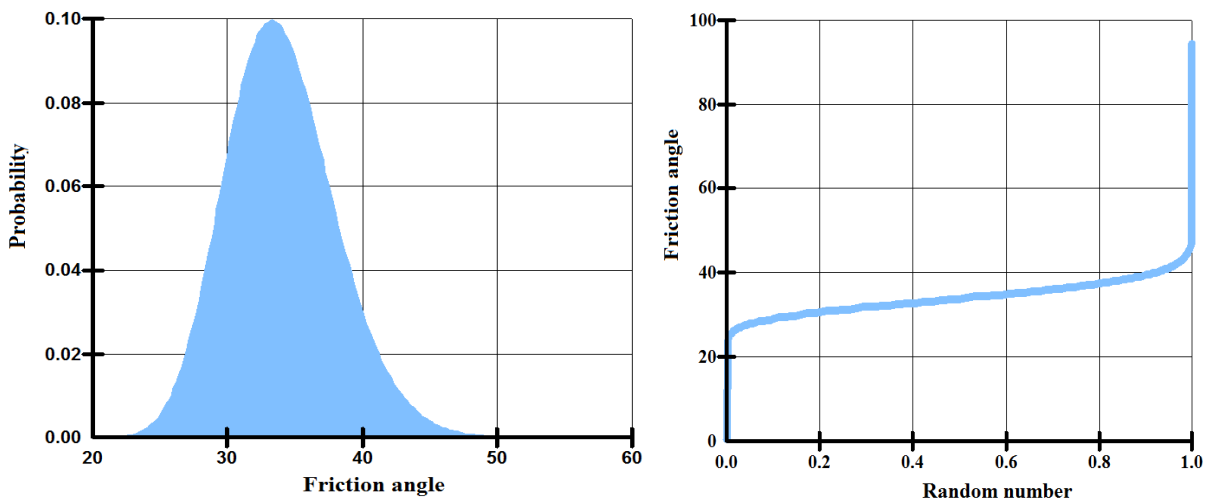


Figure B2.2: Probability distribution and sampling function of friction angle

Appendix B3: Transition

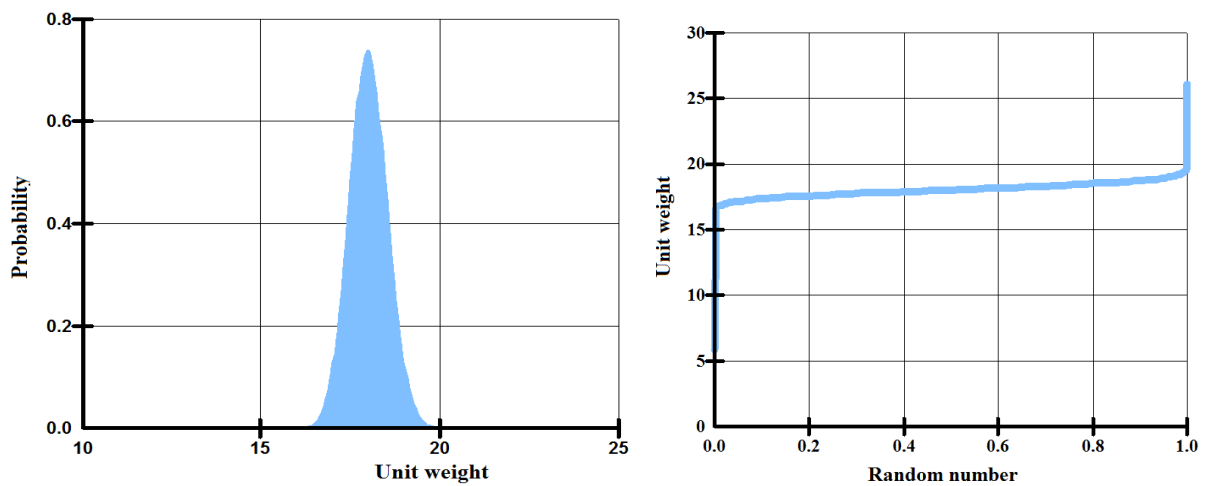


Figure B3.1: Probability distribution and sampling function of unit weight

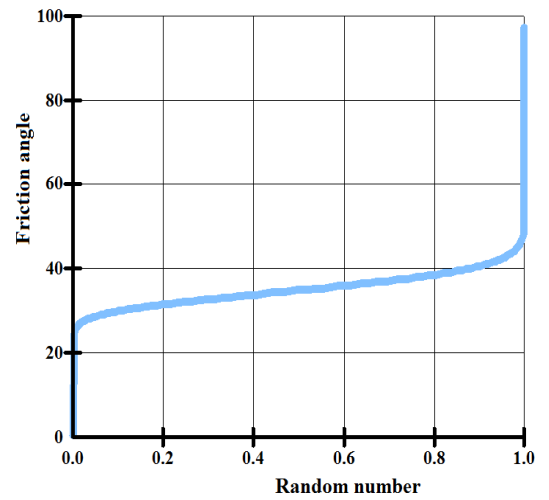
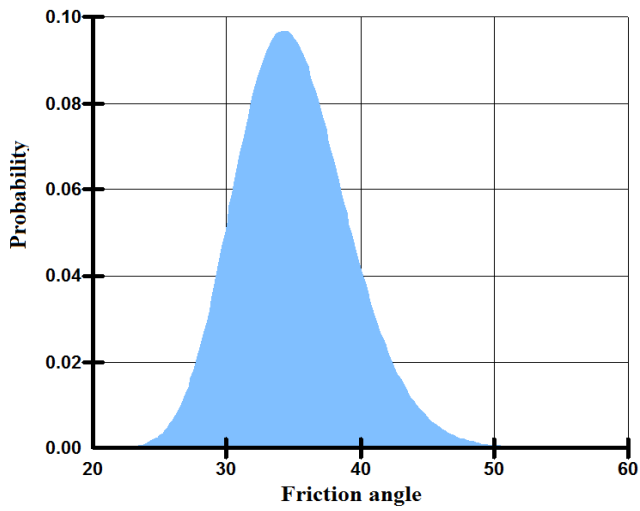


Figure B3.2: Probability distribution and sampling function of friction angle

Appendix B4: Shell

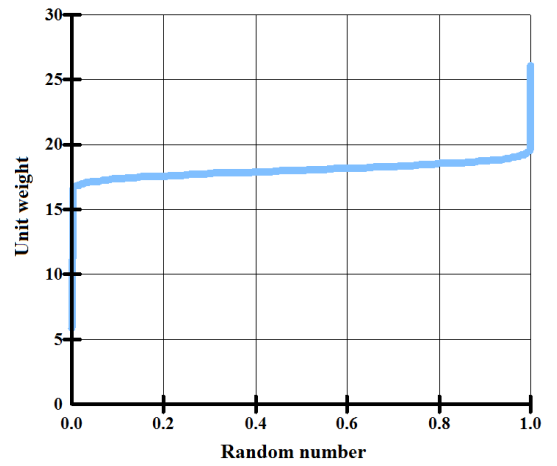
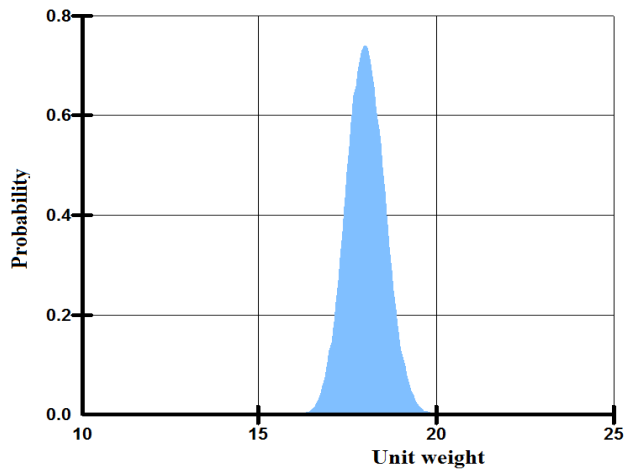


Figure B4.1: Probability distribution and sampling function of unit weight

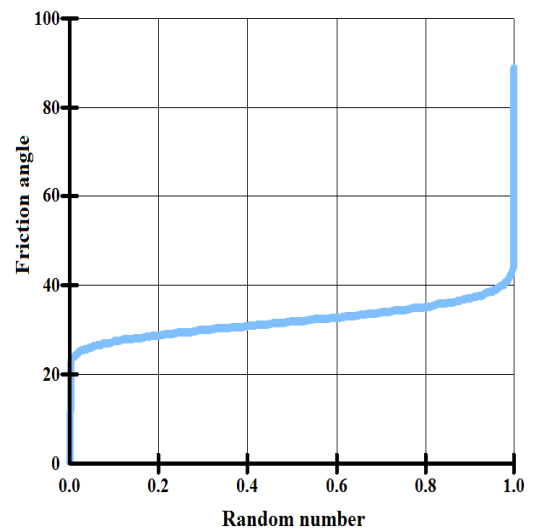
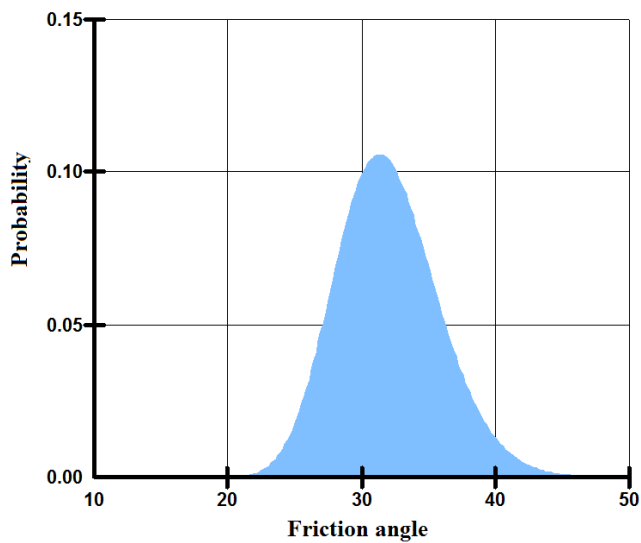


Figure B4.2: Probability distribution and sampling function of friction angle

Appendix B5: Rock + riprap

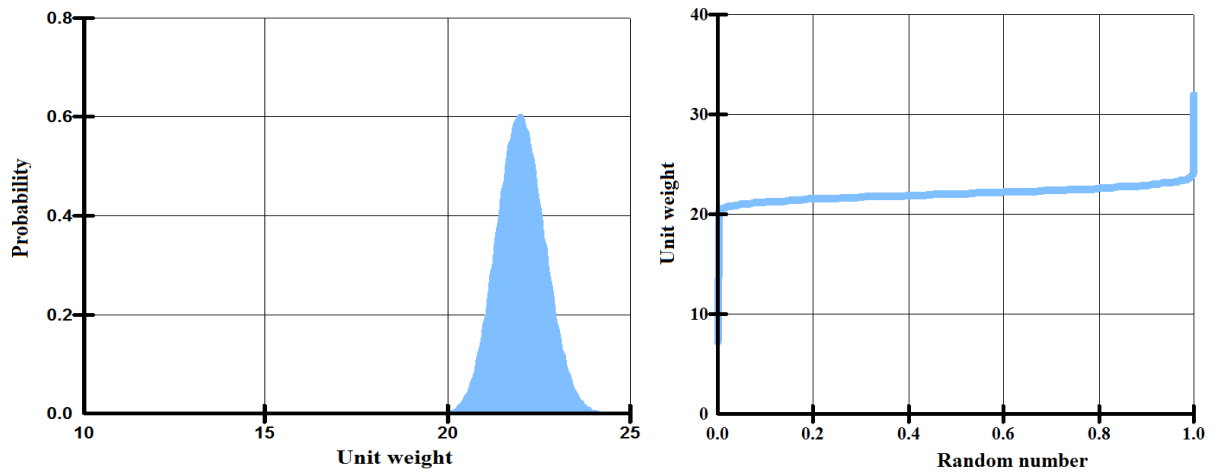


Figure B5.1: Probability distribution and sampling function of unit weight

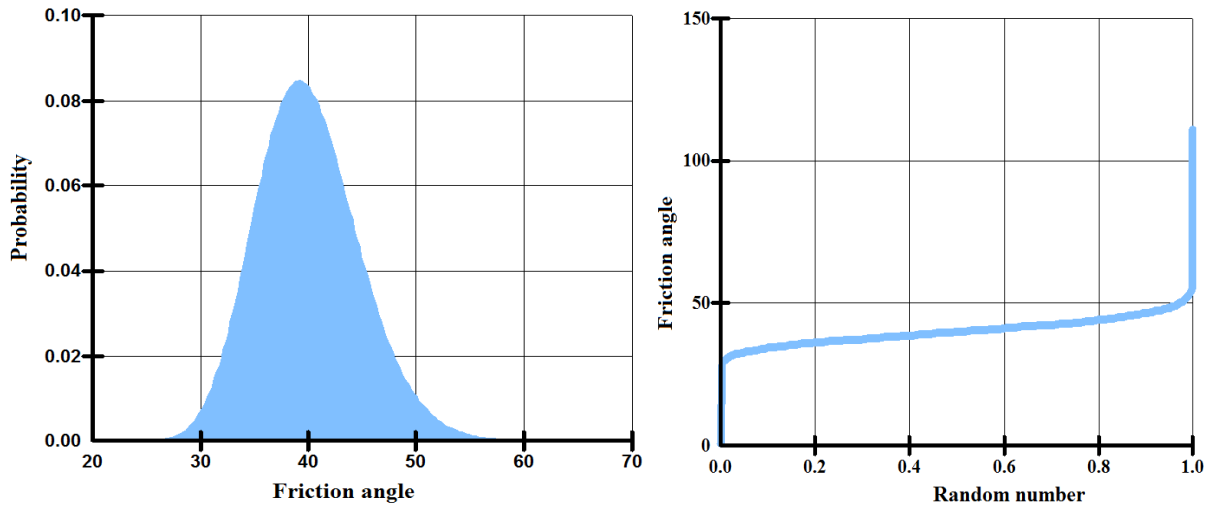


Figure B5.2: Probability distribution and sampling function of friction angle

Appendix B6: Foundation

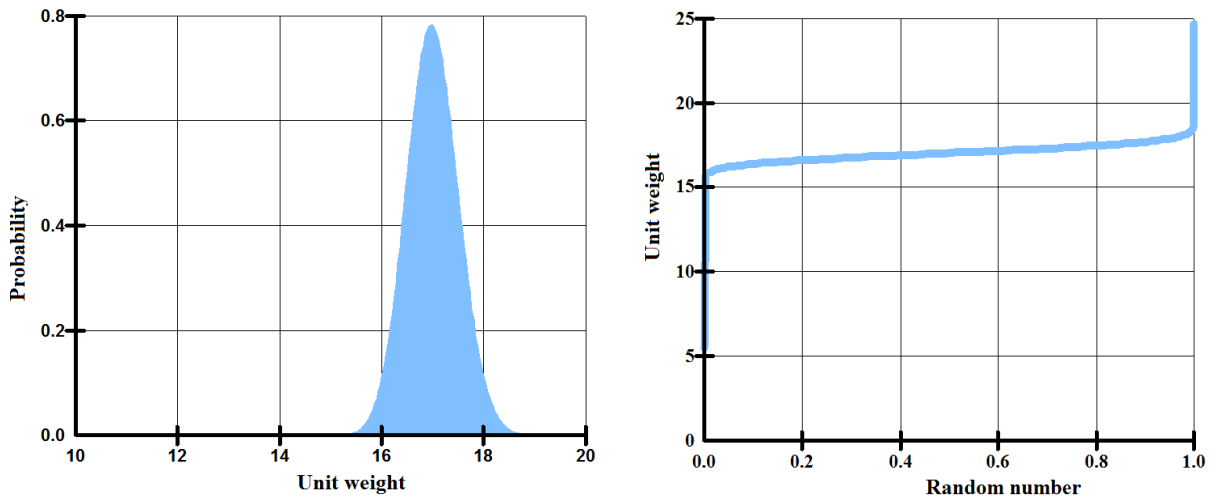


Figure B6.1: Probability distribution and sampling function of unit weight

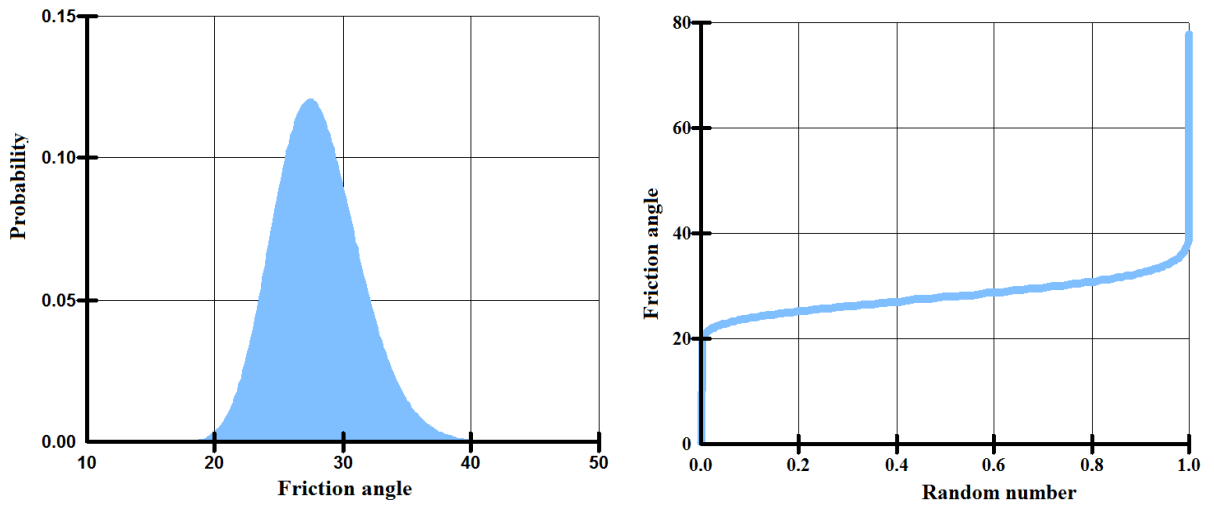


Figure B6.2: Probability distribution and sampling function of friction angle

Appendix B7: Seismicity

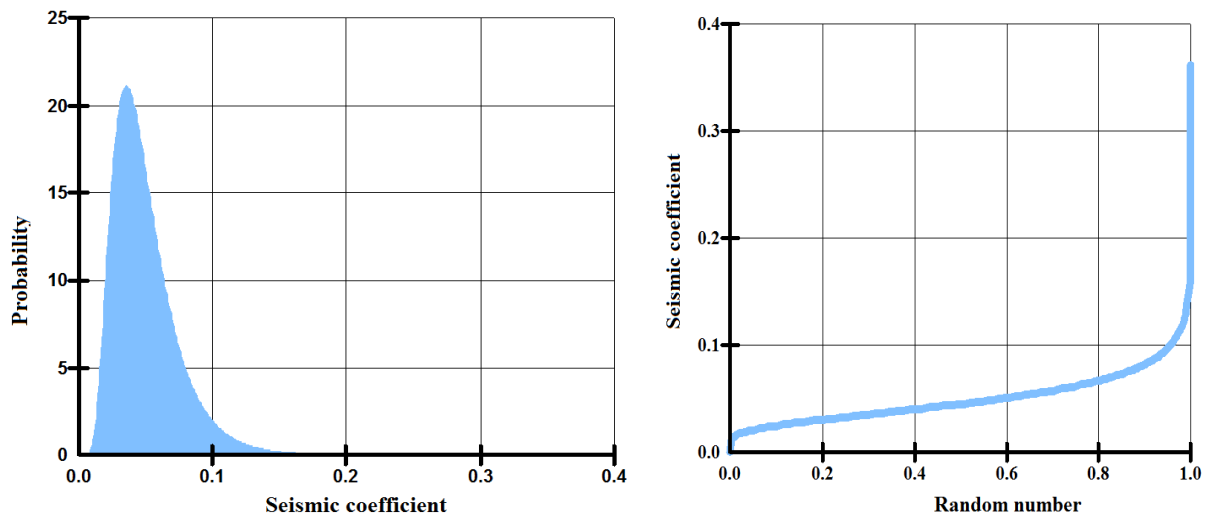


Figure B7: Probability distribution and sampling function of seismic coefficient